Deriving Special Relativity from A Moving Mirror Problem?

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Traditionally (1) reflection of light from a mirror moving at a constant velocity is solved using Lorentz transformations (i.e. special relativity). One transforms the incident ray in the lab frame into a moving frame in which the mirror is stationary. One then uses angle of incidence equals angle of reflection and finally transforms back to the lab. Alternatively (2) this problem may be solved using Fermat's principle to find a relationship between the angle of incidence and reflection in the lab and (3) wave principles to find a relationship between the incident and reflected momenta. (Alternatively one could bypass (3) and use conservation of momentum along the mirror surface.)

In (4) we suggested that this problem may be solved by introducing a hypothetical velocity H. In (5) we showed how a hypothetical velocity approach which yields sin(A) 1/v(relative a) = sin(B) / v(relative b) (where A and B are incident and reflected angles in the lab) may be linked to special relativity through: v(relative a) ta (lab) = c ta (moving frame) . {We note that sin(A) = x/(c ta) where c is the speed of light in a vacuum and sin(B) = (L-x)/ (ctb) where L is the length of the mirror whose face is taken to lie along the x axis. }

In this note we try to show that one may derive special relativity directly from the constantly moving mirror problem using the fundamental idea that c, the speed of light in a vacuum, is the same for the lab frame and moving frame. This is completely different from the idea of a Galilean transformation and suggests that perhaps time and distance values (y) may change in the moving frame. We suggest that one compare c t'a where t'a is the interval in the moving frame to the same distance computed without the notion of special relativity i.e. time intervals are the same in the lab and moving frame and one uses a relative speed in the moving frame not c. This equivalence defines t'a (time interval in the moving frame) in terms of v, ta and y in the lab frame thus partly leading to the form of the Lorentz matrix. There are still functions of v present and we calculate these by noting that -Et+px=0 for light because E=px. The Lorentz transformation is such that -E't'+p'x'=0 as well so it is an invariant. From this one may calculate a lingering function of v. Furthermore we argue that this transformation also applies to particles with rest mass.

Traditional Treatments of the Moving Mirror Problem

Define a moving mirror problem in the following way. A mirror of length L along the x axis moves in the negative y direction with speed v. Incident light making an angle A with the normal strikes the mirror at a point x and then reflects with an angle B (to the normal) and travels such that its x-projection is: L-x. L is chosen so that the y projection is the same for the incident and reflected rays.

The moving mirror problem was already solved by Einstein (1) in the first decade of the 1900s by using two sets of Lorentz transformations. First one takes the incident and reflected rays in the lab frame and finds their py (momentum y) value and energies (they are different) and transforms these into the moving frame using a Lorentz transformation. In the moving frame the ray reflects such that the angle of reflection equals the angle of reflection. In the lab frame the incident ray hits the mirror at x and then travels such that its x-projection is L-x. The y projection is the same for both the incident and reflected rays because one may make the mirror L as long as one wants. sin(A) = x/ cta and sin(B) = (L-x)/tb. One may then transform back to the lab. This approach allows one to find a relationship for sin(A) in terms of sin(B), but also of p(incident) versus p(reflected) which also involves sin(A). Full details are given in (1).

Two alternative approaches may be used to solve this same problem with no knowledge or use of special relativity whatsoever. In (2) Fermat's principle of least time is used to find a relationship between A and B. Then using this relationship together with the two sets of wavefronts (3) one may find the relationship between incident (p,E) and reflected. Alternatively one may use p(incident) sin(A) = p(reflected) sin(B) as we have pointed out earlier so Fermat's principle suffices.

In (4) we suggest that one may solve the moving problem using the idea of a hypothetical velocity H which does not require taking derivatives to minimize time. This hypothetical velocity lies along the x axis and is conserved with H sin(A) = 1/ (v relative a) and H sin(B) = 1/ (v relative b). Thus the relative ray velocities are projections of H which is a hypotenuse even though it lies along the x axis.This approach, however, is developed by observing the result from Fermat's principle as opposed to developing it a priori.

Suggested Derivation of Special Relativity from the Moving Mirror Problem

Consider the situation in which one does not know that special relativity exists i.e. one assumes that time intervals are the same in a lab and moving frame and the relative speeds apply to light. From the point of view of the lab c is the speed of light while c-vcos(A) and c+vcos(B) are the speeds that the person in the lab thinks that the person in the moving frame would see for the incident and reflected rays. The time interval would be the same so the ratio distances (assuming no special relativity) would be:

v(relative a) ta / (c ta)   and (vrelative b) ta / (c ta)   thus ta divides out ((2))

The assumption of special relativity, however, is that people in both the lab and moving frame would see the speed of light as c. A person in the moving frame does not know he or she is moving and light propagates with a speed linked to the magnetic and electric permeabilities which are not frame dependent. As a result adjustments must be made to allow for the same speed of light in both frames. In particular time intervals must be different in each frame which is the beginning of special relativity. Thus:

C ta' / cta and c tb' / c tb are the ratios corresponding to ((2)) with ta' and tb' being the new time intervals as seen in the moving frame. Thus:

C ta' (interval) = v(relative-a) ta   and c tb' (interval) = v(relative-b) tb   ((3))

((3)) links a time interval in the moving frame to quantities (y,t) in the lab frame because:

v(relative a) = c- vcos(A) = c- v Y/ cta   and v(relative b) = c+ v cos(B) = c+v Y/ctb ((4))

Consider the incident ray in the lab. Its coordinates for y and t are:

Initial (y=Y, t=0) Final (y=0, t=ta)   We assume that there may be changes in time and in the y direction, but not the x direction which is perpendicular to motion, in the moving frame.

Thus ((4)) shows Y being multiplied by v to create a new time. Thinking in terms of matrices, this yields:

| AA BB | where AA, BB and g are unknown and v is speed ((5))
| Vg   g |

Assuming a symmetric matrix yields | g vg| ((6))
|vg g|

Interestingly this matrix holds not only for light, but also for a particle with rest mass. Consider a particle with rest mass mo at x=0 at time to. (We switch from y to x as notational convenience.) Then x' = v g to' and t'=gto so the particle is seen as moving with speed x'/t' = v which is as it should be. The question becomes: How does one fix g? One way to do this is the following. Consider for light the quantity:

A=-Et+px   Then: dA=0 = -E dt + p dx → speed of light = E/p   ((7))

In the moving frame: -E't' + p'x' → same speed of light = E'/p'

-Et+px has the form of a special dot product. Given that -Et+px = p(-ct+x) = 0 this should be an invariant because x/t=c. Thus:

-E't'+p'x' = gg (1-vv) (-Et+px) so g=1/sqrt(1-vv/cc)   ((8))

One may also take the dot product -EE + pp and apply it to a particle with rest mass. Then:

-E'E' + p'p' = gg { pp(1-vv) + EE (1-vv) } so g= 1/sqrt(1-vv/cc) (c=1) for -EE+pp to be an invariant. In this case it would equal momo for c=1. Thus with the (1,-1) metric one has a unitary type of transformation i.e. the Lorentz transformation.

Finding the Angle of Incidence/ Angle of Reflection Relationship

Although this is unrelated to deriving special relativity ((2)) allows one to solve the angle relationship in the moving mirror problem .

Ta' sin(A') = x and tb' sin(A') = L-x because angle of reflection = angle of incidence in the moving frame and x and L-x distances do not change (they are perpendicular to motion).

((2)) indicates that 1/c = ta'/ (ta v(relative a)) = tb' / (tb v(relative b)) so:

(x/ta) (1/v(relative a)) = (L-x)/tb 1/(vrelative b) or sin(A)(1/v(relative a)) = sin(B) 1/v(relative b)

One could introduce the hypothetical H velocity from this result without using the result of Fermat's principle. One assumes that the incident and reflected rays move in straight lines so one is actually using parts of Fermat's principle.

Conclusion

In conclusion we argue that one may derive the Lorentz transformation (x,t) and (p,E) from a problem involving the reflection of light from a mirror moving at a constant speed v. The key idea seems to be that one may consider the problem in two ways. The first involves the idea that time is the same in a lab and moving frame, but that the speed of light differs (i.e. relative speeds apply) i.e. is c in the lab and c-vcos(A) for the incident ray and c+vcos(B) for the reflected. The distances are then v(relative) times ta (incident) or tb (reflected). If one assumes that the speed of light in a vacuum is the same in the two frames (the assumption of special relativity) then one is forced to assume that time intervals differ, but the distances computed both ways must be equal i.e.

v(relative a) ta = c ta' and v(relative b tb = c tb' where ta' and tb' are time intervals in the moving frame

Given that one knows v(relative a) and v(relative b) explicitly in terms of v and cos(A) = y/cta and cos(B) = y/ ctb one may formulate the form of the Lorentz matrix as shown above. In other words, one may find the full Lorentz matrix and also apply it to particles with rest mass as argued above

References

1. Einstein, A. https://einsteinpapers.press.princeton.edu/vol2-trans/178 page 164

2.Gjurchinovski, A. Einstein's Mirror and Fermat's Principle of least time (Am. J. of Phys

3. Gjurchinovski,A. Einstein's Mirror and Fermat's Principle of Least Time (2004) http://muj.optol.cz/~richterek/data/media/ref_str/gjurchinovski2004.pdf

4. Ruggeri, Francesco R. Fermat's Principle, Hypothetical Velocity, Lorentz Transformation and a Moving Mirror (preprint, zenodo, 2022)
5. Ruggeri, Francesco R. Special Relativity Linked to Generalized Index of Refraction i.e. 1/Relative Velocity For a Moving Mirror? (preprint, zenodo, 2022)