

Analysis of Optimal Altitude for UAV Cellular Communication in Presence of Blockage

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Abstract—In this paper, a novel framework for outage probability analysis consisting of unmanned aerial vehicles (UAV) base stations and ground users is proposed, which includes the blockage model for line-of-sight (LoS) and none-LoS (NLoS) probability and a tractable approach based on stochastic geometry. Specifically, a three dimensional (3D) LoS ball model is introduced to obtain the probabilistic propagation in UAV communication systems. By utilising this model, a tractable expression is derived for signal-to-noise ratio (SNR) outage probability. This approach leads to a closed-form expression for the optimal altitude of UAV which in turn helps to investigate the impacts of blockage height, density and length on the outage probability. Simulations are preformed to investigate the performance and accuracy of the proposed approach.

Index Terms—unmanned aerial vehicles, cellular networks, signal to noise ratio, analytical models, outage probability

I. INTRODUCTION

Unmanned aerial vehicle (UAV) mounted base stations have captivated significant interest from wireless system architects because of their cost effectiveness, flexibility, mobility and the ability of on-demand for future wireless channels [1], [2]. UAV-BSs allow terrestrial BS offloading in extremely crowded areas as well as wireless connectivity in battlefields or disaster areas. Before full use can be made of the UAV-BSs gains and many potential applications, some remaining technical challenges, such as optimal UAV-BS placement still need to be studied [3]. There exist several previous studies on performance analysis of wireless networks incorporating UAVs. The horizontal and/or vertical positions of the UAVs could be optimized for their deployment, leading to various two-dimensional (2D) or 3D UAV placement designs [4]–[5]. In [6], the joint optimization of UAV-BS altitude and beamwidth was proposed to maximize the sum rate of multiuser communications. The authors studied three different models based on proposed fly-hover-and-communicate protocol. In [7], a novel analytical framework for the coverage probability was developed. The authors demonstrated that the LOS ball model is an excellent candidate for tractable analysis of UAV networks, while maintaining satisfactory accuracy. In [8], a study was given on energy-efficient 3D placement of a UAV-BS by adopting UAV-BS antenna tilting to minimize the total UAV-BS energy consumption. The authors converted

the 3D placement problem into a 2D placement problem by obtaining the minimum altitude based on the elliptical characteristics produced by the tilted antenna. Motivated by the above works, to realize the full potential of UAV-enabled communication, it is essential to exploit the fully controllable UAV mobility in 3D space. One of the key factors of UAV mobility is the altitude that UAV is operating in cellular networks. So far, the studies on the altitude optimization are mostly based on optimization and finding the optimal altitude numerically. However, it is crucial to conduct a mathematical approach to find a closed form expression for the optimal altitude. Thus, in this work, we propose a new analytical framework for cellular connected UAV networks, which leads to results that are more tractable than those provided by previous studies. First, we introduce a new probabilistic model for LoS and NLoS propagations in these networks, which is inspired from the 3D LoS ball model, that not only achieves high accuracy but also remains tractable. Then, with the aid of the proposed analytical framework, we evaluate the coverage probability of cellular-connected UAV which serves multiple ground users in the covered region. Accordingly, we derive a novel and tractable formula for optimal altitude of cellular-connected UAV networks, which is separated to three region corresponding to three integral operations in the coverage probability expression. Based on our numerical results, the impact of the cell radius, height of the blockages, the density and the length of the blockages are investigated. In one of the scenarios, an optimal altitude is derived to maximizes the coverage probability. And in other scenarios, a lower bound on the optimal altitude is derived. To the best of the authors' knowledge, this is the first analytical result on the optimal altitude of cellular networks which also studies the impacts of blockages on the system performance.

The rest of this paper is organized as follows. Section II, describes the system model. In Section III, the outage probability expression is introduced. An analysis of UAV optimal altitude and closed-form formula for optimal altitude are given in Section IV. In Section V, the impact of system parameters on altitude are examined. Finally, Secion VI concludes this paper.

II. SYSTEM MODEL

In this section, we present the channel and blockage model for cellular-connected UAV networks. Consider a UAV-enabled wireless network, where a typical UAV is flying in the sky to

execute certain objectives. We assume that the UAV is located at the origin and it is equipped with a directional antenna of adjustable beamwidth. We focus on the performance analysis of a single UAV flying at altitude H , serves ground users which are uniformly distributed.

In order to model the wireless link between a ground user and the UAV, the LoS and NLoS components are considered separately. The path loss is defined as follows

$$l = K_s r^{\beta_s} \quad (1)$$

where r is distance from user to the UAV, K_s and β_s are the pathloss constant and exponent in the s state (LoS/NLoS), respectively. In this paper, we assume the small-scale fading H_F has exponential distribution. Thus, the SNR is given by

$$SNR = \frac{P_{tx} H_F l^{-1}}{N} = \frac{P_d H_F}{N (K r^\beta)} \quad (2)$$

where P_d is the transmitted power, H_F is the fading component and N is the noise power. Furthermore, we distribute N number of users and the 3D distance distribution between the ground users and the UAV can be written as follows,

$$f_r(r) = \frac{2r}{\bar{r}^2} \quad (3)$$

where \bar{r} is the radius of the covered region.

Inspired by random shape theory and stochastic geometry [9], we present the LoS probability and blockage model in the following. Firstly, we define a LoS radius D_h , which represents the radius of a 3D ball. In such manner, a certain link is in LoS path, when the user is located within radius D_h , and it is zero for NLoS path. It is worth mentioning that the 3D LoS ball model is not only 2D distance-dependent, but also altitude-dependent in the 3D space. In general, the LoS radius D_h should be a monotonically increasing function of the UAV altitude H . Therefore, the higher the UAV flies, the more ground users can be in the LoS path. It should be noted that the radius of the ball depends on the environment parameters (rural or urban).

Thus, according to 3D LoS ball model, we define the LoS and NLoS probabilities as follows,

$$P_{LoS}(r, H, h_{BLK}) = 1(r \leq D_{H, h_{BLK}}) \quad (4)$$

$$P_{NLoS}(r, H, h_{BLK}) = 1(r > D_{H, h_{BLK}}) \quad (5)$$

where $1()$ denotes the indicator function, $D_{H, h_{BLK}} = D \max\{H/h_{BLK}, 1\}$, $D = \frac{2}{\mu_{BLK}}$ and $\mu_{BLK} = \frac{2\lambda_{BLK} l_{BLK}}{\pi}$. Here h_{BLK} is height of the blockage, μ_{BLK} is the density and l_{BLK} is the length of the surrounding buildings (blockages).

III. OUTAGE PROBABILITY DERIVATION

In this section, we introduce a closed-form expression for outage probability by taking in to account the LoS and NLoS links from the ground users. With respect to the 3D distribution of distances, we can write the outage probability as follows,

$$P = E_r [P_{SNR}(LoS) + P_{SNR}(NLoS)] \\ = \int_{r^{3D}} (P_{SNR}(LoS) + P_{SNR}(NLoS)) f_{r^{3D}} dr^{3D} \quad (6)$$

where

$$P_{SNR}(LoS) = P(SNR < \tau | LoS) \times P_{LoS} \quad (7)$$

$$P_{SNR}(NLoS) = P(SNR < \tau | NLoS) \times P_{NLoS} \quad (8)$$

The outage probability can be defined as follows,

$$P(SNR < \tau) = P\left(\frac{P_{tx} H_F}{N (K r^\beta)} < \tau\right) \quad (9)$$

By considering the exponential distribution of channel fading and setting $A = N\tau P_{tx}^{-1} K$, we can write

$$P(H_F < A r^\beta) = 1 - \exp(- (A r^\beta)) \quad (10)$$

Based on the indicator functions in probability of LoS and NLoS formulation, we can separate the expression of outage probability in three regions,

$$P_{outage} = I_1 + I_2 + I_3 \quad (11)$$

where I_1, I_2 and I_3 correspond to

$$P(SNR < \tau) = \begin{cases} I_1 & d_{min} > D_{H, h_{BLK}} \\ I_2 & d_{max} > D_{H, h_{BLK}} > d_{min} \\ I_3 & D_{H, h_{BLK}} \geq d_{max} \end{cases} \quad (12)$$

where d_{min} and d_{max} are the minimum and maximum possible distance from user to UAV, respectively. Each of the terms in (11) refers to a different region with respect to D , d_{max} and d_{min} . It should be noted that I_1 is the case where all the ground users in the cell are in NLoS state, I_2 is the case where both LoS and NLoS users are present in the cell and I_3 is the case where all the ground users in the cell are in LoS state. In Table I, the expression for I_1 , I_2 and I_3 are presented.

Proof. A step by step derivation of (11) is given in Appendix A. \square

IV. OPTIMAL ALTITUDE FOR MINIMIZING OUTAGE PROBABILITY

In this section, we first present an analysis of UAV's optimal altitude for minimizing outage probability for the cases of I_1 , I_2 and I_3 , and then derive a novel closed-form formula for optimal attitude.

As discussed in previous section, we can separate the integrals based on different parameters of D , d_{max} and d_{min} . Thus, we have optimal H for three sets of parameters as described below,

$$H^* = \begin{cases} H_1 & d_{min} > D_{H, h_{BLK}} \\ H_2 & d_{max} > D_{H, h_{BLK}} > d_{min} \\ H_3 & D_{H, h_{BLK}} \geq d_{max} \end{cases} \quad (13)$$

Table I: Closed form expressions for the outage probability corresponding to different regions from (6)

$I_1 = \frac{(d_{\max})^2}{R^2} \left(1 + \frac{2(A(d_{\max})^{\beta_{NLoS}})^{-2/\beta_{NLoS}} \Gamma\left(\frac{2}{\beta_{NLoS}}, A(d_{\max})^{\beta_{NLoS}}\right)}{\beta_{NLoS}} \right) - \frac{(d_{\min})^2}{R^2} \left(1 + \frac{2(A(d_{\min})^{\beta_{NLoS}})^{-2/\beta_{NLoS}} \Gamma\left(\frac{2}{\beta_{NLoS}}, A(d_{\min})^{\beta_{NLoS}}\right)}{\beta_{NLoS}} \right)$
$I_2 = \frac{(D_{H,h_{BLK}})^2}{R^2} \left(1 + \frac{2(A(D_{H,h_{BLK}})^{\beta_{LoS}})^{-2/\beta_{LoS}} \Gamma\left(\frac{2}{\beta}, A(D_{H,h_{BLK}})^{\beta_{LoS}}\right)}{\beta_{LoS}} \right) - \frac{(d_{\min})^2}{R^2} \left(1 + \frac{2(A(d_{\min})^{\beta_{LoS}})^{-2/\beta_{LoS}} \Gamma\left(\frac{2}{\beta}, A(d_{\min})^{\beta_{LoS}}\right)}{\beta_{LoS}} \right) + \frac{(d_{\max})^2}{R^2} \left(1 + \frac{2(A(d_{\max})^{\beta_{NLoS}})^{-2/\beta_{NLoS}} \Gamma\left(\frac{2}{\beta}, A(d_{\max})^{\beta_{NLoS}}\right)}{\beta_{NLoS}} \right) - \frac{(D_{H,h_{BLK}})^2}{R^2} \left(1 + \frac{2(A(D_{H,h_{BLK}})^{\beta_{NLoS}})^{-2/\beta_{NLoS}} \Gamma\left(\frac{2}{\beta}, A(D_{H,h_{BLK}})^{\beta_{NLoS}}\right)}{\beta_{NLoS}} \right)$
$I_3 = \frac{(d_{\max})^2}{R^2} \left(1 + \frac{2(A(d_{\max})^{\beta_{LoS}})^{-2/\beta_{LoS}} \Gamma\left(\frac{2}{\beta}, A(d_{\max})^{\beta_{LoS}}\right)}{\beta_{LoS}} \right) - \frac{(d_{\min})^2}{R^2} \left(1 + \frac{2(A(d_{\min})^{\beta_{LoS}})^{-2/\beta_{LoS}} \Gamma\left(\frac{2}{\beta}, A(d_{\min})^{\beta_{LoS}}\right)}{\beta_{LoS}} \right)$

where H_1 is the optimal altitude for I_1 , H_2 is the optimal altitude for I_2 and H_3 is the optimal altitude for I_3 .

A. Optimal altitude for H_1 and H_3

Referring to I_1 from Table I, for H_1 , after taking the derivative and simplifying it, we can write

$$\frac{\partial I_1}{\partial H} = -\frac{2H \exp\left(-A(\sqrt{R^2 + H^2})^{\beta_{NLoS}}\right)}{R^2} + \frac{2H \exp\left(-A(H)^{\beta_{NLoS}}\right)}{R^2} \quad (14)$$

Remark 1. Since the derivative is in the form of exponential, there is no critical point. However, we can check the sign of the derivative to analyse the behavior of the function. Thus, we can write as follows,

$$\frac{2H \exp\left(-A(H)^{\beta_{NLoS}}\right)}{R^2} - \frac{2H \exp\left(-A(\sqrt{R^2 + H^2})^{\beta_{NLoS}}\right)}{R^2} \begin{matrix} > \\ < \end{matrix} 0 \quad (15)$$

$$\exp\left(-A(H)^{\beta_{NLoS}}\right) - \exp\left(-A(\sqrt{R^2 + H^2})^{\beta_{NLoS}}\right) \begin{matrix} > \\ < \end{matrix} 0 \quad (16)$$

Obviously, the above function is positive for all values of H . Since, R and β_{NLoS} are both larger than 1, the first exponential argument on the left hand side is smaller than the right one. Consequently, since there is a negative sign in the exponential, we can conclude that the first term is larger than the second, and this concludes the proof. This shows that the outage probability for the first case (I_1) is an increasing function of H . Thus, the optimal altitude is the minimum possible value of H .

Remark 2. As we can see, the only difference between I_1 and I_3 is the value of β , and since both of β_{NLoS} and β_{LoS} are larger than 1, we can conclude that the behavior of I_3 is similar to I_1 . Thus, the optimal value for the altitude of UAV is the minimum possible value of H . It should be

noted that, for the case of I_3 , to stratify the condition $D_h \geq \sqrt{R^2 + H^2}$, we should have $H \geq \frac{Rh_{blk}}{\sqrt{D^2 - h_{blk}^2}}$. Otherwise, if $D_h < \sqrt{R^2 + H^2}$, the considered scenario will be I_2 which we will investigate in the following. Therefore, it is concluded that the function of I_3 is increasing with respect to H , hence the optimal value of altitude is equal to $H = \frac{Rh_{blk}}{\sqrt{D^2 - h_{blk}^2}}$.

B. Optimal altitude for H_2

Similar to H_1 and H_3 , for H_2 we have,

$$\frac{\partial I_2}{\partial H} = \frac{2D^2 H}{R^2 h_{blk}^2} \left(\exp\left(-A\left(\frac{DH}{h_{blk}}\right)^{\beta_{NLoS}}\right) - \exp\left(-A\left(\frac{DH}{h_{blk}}\right)^{\beta_{LoS}}\right) \right) + \frac{2H}{R^2} \left(\exp\left(-AH^{\beta_{LoS}}\right) - \exp\left(-A(\sqrt{R^2 + H^2})^{\beta_{NLoS}}\right) \right) \quad (17)$$

Since the above equation is unsolvable, we solve it by bisection.

Remark 3. Based on the values of P_{tx} , τ , N and pathloss exponents of LoS and NLoS links, we may have two scenarios for optimal altitude. The first scenario can happen when the cell edge user can be covered by the UAV if it is in LoS state. This means that if the UAV increases its altitude to a degree which all users in the region are in LoS state (including the cell edge user). In this situation the optimal altitude is the value that the UAV can have a LoS link with all users.

The second scenario is when the UAV can not have a successful link with the cell edge user even if it is in the LoS state. This is due to the fact that the value of P_{tx} does not satisfy the SNR outage requirement. In this case, because of the impact of pathloss, and also since NLoS users are located further away from the center of the region, NLoS terms in (17) are equal to zero. Consequently, (17) can be simplified as below,

$$\begin{aligned} \frac{\partial I_2}{\partial H} &= \frac{2D^2H}{R^2h_{blk}^2} \left(-\exp \left(-A \left(\frac{DH}{h_{blk}} \right)^{\beta_{LoS}} \right) \right) \\ &+ \frac{2H}{R^2} \left(\exp \left(-AH^{\beta_{LoS}} \right) \right) \end{aligned} \quad (18)$$

Thus, the optimal altitude can be obtained in a closed-form expression as

$$\begin{aligned} H^* &= \left(\frac{\ln(D/h_{blk})}{A((D/h_{blk})^{\beta_{LoS}} - 1)} \right)^{(1/\beta_{LoS})} \\ &= \left(\frac{P_{tx} \ln(D/h_{blk})}{N\tau K((D/h_{blk})^{\beta_{LoS}} - 1)} \right)^{(1/\beta_{LoS})} \end{aligned} \quad (19)$$

V. PERFORMANCE ANALYSIS

The goal of this section is to examine the impact of most important system parameters i.e., P_{tx} , D and h_{blk} on the optimal altitude for the case of H_2 (19).

Proposition 1. Let us consider $\zeta = P_{tx}$. The following holds true: $H^*(\zeta)$ is monotonically increasing in ζ .

Proof. It follows by direct inspection of (19). \square

Proposition 2. Let us consider $\zeta = \frac{D}{h_{blk}}$. The following holds true: i) $H^*(\zeta)$ is monotonically increasing in ζ .

Proof. From (19), $H^*(\zeta) \leq 0$ holds true for all parameters values considered. We have,

$$H^{*'}(\zeta) = \frac{(\ln(\zeta))^{\frac{1-\beta}{\beta}} P_{tx}^{1/\beta} (\zeta^\beta - 1 - \beta \zeta^\beta \ln(\zeta))}{\zeta^\beta (N\tau K)^{1/\beta} (\zeta^\beta - 1)^{\frac{1+\beta}{\beta}}} \quad (20)$$

Since $(\zeta^\beta - 1 - \beta \zeta^\beta \ln(\zeta))$ is negative and $(\zeta^\beta - 1)^{\frac{1+\beta}{\beta}}$ is positive for all values, we can conclude that the derivative sign is negative. Thus H^* is decreasing in ζ which means it is decreasing in D and increasing in h_{blk} . \square

In Fig. 1, the outage probability is plotted versus the altitude of the UAV. In the interest of verifying analytical derivations, the simulation results and analytical results are compared with each other, which confirms the tightness of the solutions. Furthermore, as can be observed, with an increase in the altitude, the outage probability first reduces since higher altitude implies a higher probability of LoS, and then it increases due to a higher distance from the ground users which results in higher pathloss. As a result, there exists an optimal value for UAV's altitude which can maximize the performance of the system in terms of minimizing outage probability.

In Fig. 2, the UAV transmit power is plotted versus the optimal altitude (H_2^*). As we can see, the transmit power has a huge impact on the optimal altitude. Firstly, an increase in the transmit power increases the optimal altitude due to decrease in the pathloss which means that the UAV can have a higher altitude to achieve more LoS links that can be covered by the given transmit power. This relation continues until the UAV's

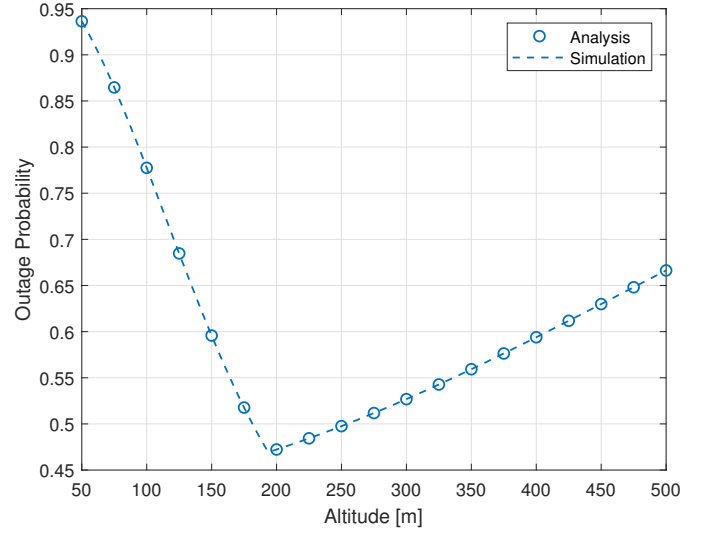


Fig. 1: Optimal altitude versus outage probability. Verifying analytical derivations

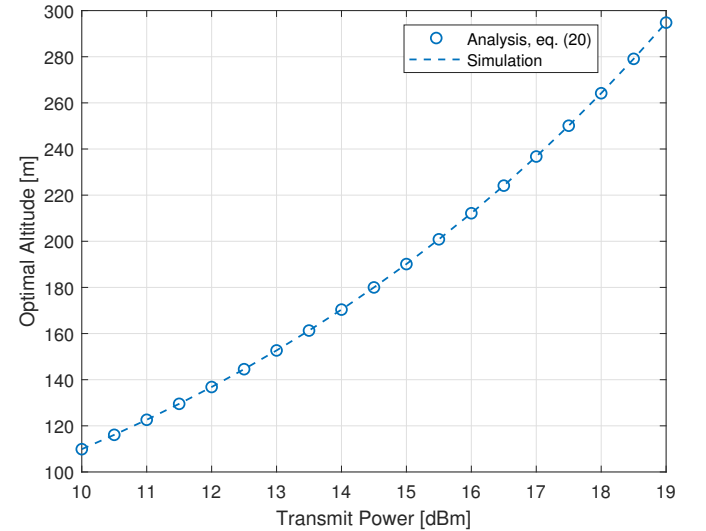


Fig. 2: Optimal altitude versus transmit power

altitude increases to a value that all ground users will have a LoS link with the UAV. In that case, increasing the transmit power does not impact the optimal altitude further.

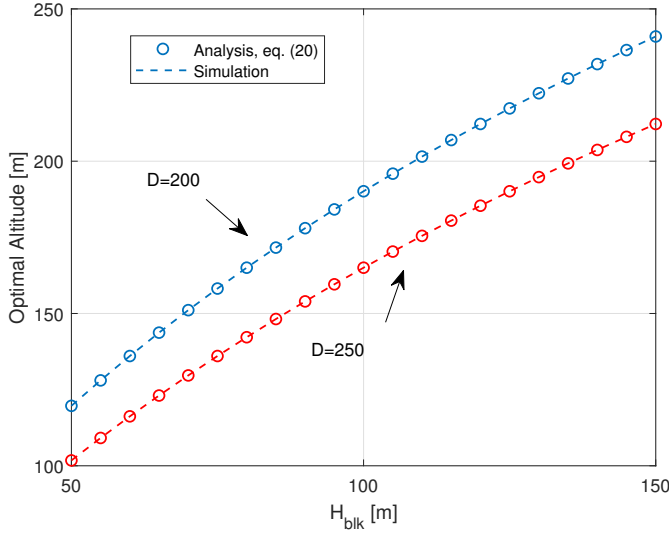
In Fig. 3, the blockage height h_{blk} is plotted versus the optimal altitude (H_2^*). It can be observed from this figure that with an increase in the blockage height, the optimal altitude increases since the higher blockage height means that the UAV needs to reach higher altitudes to archive LoS links with ground users.

VI. CONCLUSION

In this work, we utilised 3D LoS ball model for coverage probability analysis and derivation of optimal altitude in

Table II: Step by step derivation of outage probability (APPENDIX A)

$P_{(LoS)}(SNR < \tau) = \int_{d_{\min}}^{D_{H,h_{blk}}} H(d_{\max} - D_{H,h_{blk}}) H(D_{H,h_{blk}} - d_{\min}) (1 - \exp(-Ar^{\beta_{LoS}})) f_r dr$ $+ \int_{d_{\min}}^{d_{\max}} H(D_{H,h_{blk}} - d_{\max}) (1 - \exp(-Ar^{\beta_{LoS}})) f_r dr$	(I)
$P_{(NLoS)}(SNR < \tau) = \int_{d_{\min}}^{d_{\max}} H(d_{\max} - D_{H,h_{blk}}) H(d_{\min} - D_{H,h_{blk}}) (1 - \exp(-Ar^{\beta_{NLoS}})) f_r dr$ $+ \int_{D_{H,h_{blk}}}^{d_{\max}} H(d_{\max} - D_{H,h_{blk}}) H(D_{H,h_{blk}} - d_{\min}) (1 - \exp(-Ar^{\beta_{NLoS}})) f_r dr$	(II)
$I_1 = \int_{d_{\min}}^{d_{\max}} H(d_{\max} - D_{H,h_{BLK}}) H(d_{\min} - D_{H,h_{BLK}}) (1 - \exp(-Ar^{\beta_{NLoS}})) f_r dr$	(III)
$I_2 = \int_{d_{\min}}^{D_{H,h_{BLK}}} H(d_{\max} - D_{H,h_{BLK}}) H(D_{H,h_{BLK}} - d_{\min}) (1 - \exp(-Ar^{\beta_{LoS}}))$ $+ \int_D^{d_{\max}} H(d_{\max} - D_{H,h_{BLK}}) H(D_{H,h_{BLK}} - d_{\min}) (1 - \exp(-Ar^{\beta_{NLoS}})) f_r dr$	(IV)
$I_3 = \int_{d_{\min}}^{d_{\max}} H(D_{H,h_{BLK}} - d_{\max}) (1 - \exp(-Ar^{\beta_{LoS}})) f_r dr$	(V)


 Fig. 3: Optimal altitude versus h_{blk}

cellular-connected UAV networks. We developed an analytical framework which is more tractable than existing ones. In particular, based on the our analysis for optimal altitude, we derived a closed-form formula for the optimal altitude which maximizes the coverage probability for one scenario, and a lower bound for two other scenarios that corresponds to the existence of LoS and NoS users in the region. In future work, it will be interesting to study the impact of interference from nearby users on the optimal altitude and conduct a more detailed analysis of cellular-connected UAV networks in presence of interference.

VII. APPENDIX A

DERIVATION OF OUTAGE PROBABILITY

The step by step derivation of outage probability has been given in Table II. To remove the indicator functions from the outage probability expression (6), we use the Heaviside function. Thus, it is necessary to consider three scenarios,

$$\begin{aligned} d_{\max} &> D_{H,h_{BLK}} \text{ \& } d_{\min} > D_{H,h_{BLK}} \\ d_{\max} &> D_{H,h_{BLK}} \text{ \& } d_{\min} < D_{H,h_{BLK}} \\ D_{H,h_{BLK}} &> d_{\max}, d_{\min} \end{aligned} \quad (21)$$

Here, d_{\max} is the distance from the UAV to the cell edge user and d_{\min} is the distance from user located at the origin. By considering all possible scenarios regarding the values of d_{\max} and d_{\min} , we can rewrite the outage probability for LoS and NLoS links separately as in (I) and (II). As can be seen, there are overlapping regions in both LoS and NLoS terms. Consequently, we combine the overlapping regions in I_1 , I_2 and I_3 .

REFERENCES

- [1] I. Bor-Yaliniz and H. Yanikomeroglu, "The new frontier in ran heterogeneity: Multi-tier drone-cells," *IEEE Communications Magazine*, vol. 54, no. 11, pp. 48–55, 2016.
- [2] M. Mozaffari, W. Saad, M. Bennis, and M. Debbah, "Efficient deployment of multiple unmanned aerial vehicles for optimal wireless coverage," *IEEE Communications Letters*, vol. 20, no. 8, pp. 1647–1650, 2016.
- [3] J. Lyu, Y. Zeng, R. Zhang, and T. J. Lim, "Placement optimization of uav-mounted mobile base stations," *IEEE Communications Letters*, vol. 21, no. 3, pp. 604–607, 2017.
- [4] M. Mozaffari, W. Saad, M. Bennis, and M. Debbah, "Unmanned aerial vehicle with underlaid device-to-device communications: Performance and tradeoffs," *IEEE Transactions on Wireless Communications*, vol. 15, no. 6, pp. 3949–3963, 2016.
- [5] M. M. Azari, F. Rosas, and S. Pollin, "Cellular connectivity for uavs: Network modeling, performance analysis, and design guidelines," *IEEE Transactions on Wireless Communications*, vol. 18, no. 7, pp. 3366–3381, 2019.
- [6] H. He, S. Zhang, Y. Zeng, and R. Zhang, "Joint altitude and beamwidth optimization for uav-enabled multiuser communications," *IEEE Communications Letters*, vol. 22, no. 2, pp. 344–347, 2018.
- [7] X. Yu, J. Zhang, R. Schober, and K. B. Letaief, "A tractable framework for coverage analysis of cellular-connected uav networks," in *2019 IEEE International Conference on Communications Workshops (ICC Workshops)*, pp. 1–6, 2019.
- [8] J. You, S. Jung, J. Seo, and J. Kang, "Energy-efficient 3-d placement of an unmanned aerial vehicle base station with antenna tilting," *IEEE Communications Letters*, vol. 24, no. 6, pp. 1323–1327, 2020.
- [9] T. Bai and R. W. Heath, "Coverage and rate analysis for millimeter-wave cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 1100–1114, 2014.