

Lemma on Combinatorial Geometric Series with Binomial Coefficients

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Abstract: This paper presents a lemma and its corollaries on the combinatorial geometric series and summation of series of binomial coefficients. Also, the coefficient for each term in combinatorial geometric series refers to a binomial coefficient. These ideas can enable the scientific researchers to solve the real life problems.

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1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea stimulated his mind to create a combinatorial geometric series [1-12]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r . In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-12] is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r .

$$\sum_{i_1=0}^n \sum_{i_2=i_1}^n \sum_{i_3=i_2}^n \cdots \sum_{i_r=i_{r-1}}^n x^{i_r} = \sum_{i=0}^n V_i^r x^i \quad \& \quad V_n^r = \frac{(n+1)(n+2)(n+3) \cdots (n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1$ and $n, r \in N = \{0, 1, 2, 3, \dots\}$.

Here, $\sum_{i=0}^n V_i^r x^i$ refers to the combinatorial geometric series and

V_n^r is the binomial coefficient for combinatorial geometric series.

3. Lemma on Binomial Coefficient

Lemma 3.1: $\sum_{i=1}^n V_i^{n-i} = \sum_{i=0}^{n-1} 2^i = 2^n - 1$

Proof. Let us prove this lemma using the summation of series of binomial coefficients and sum of combinatorial geometric series

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1 \Rightarrow \sum_{i=0}^{n-1} 2^i = 2^n - 1.$$

$$\sum_{i=0}^n V_i^{n-i} = 2^n \Rightarrow \sum_{i=1}^n V_i^{n-i} = 2^n - 1.$$

Hence, the lemma is proved.

Corollary 3.1: $\sum_{i=k}^{n-1} 2^i = 2^n - 2^k.$

Corollary 3.2: $\sum_{i=k+1}^n V_i^{n-i} = 2^n - \frac{1}{(n-k)!} \sum_{i=k}^{n-k} (k+i).$

Corollary 3.3: $\sum_{i=0}^r V_i^n = V_r^{n+1} \Rightarrow \sum_{i=k+1}^r V_i^n = V_r^{n+1} - V_k^{n+1}.$

4. Conclusion

In this article, a lemma and its corollaries were introduced on the combinatorial geometric series and series of binomial coefficients. This idea can enable the scientific researchers to solve the real life problems.

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