Applied Science Periodical ISSN 0972-5504
Volume - XXIV, No. 3, August 2022
Refereed and Peer-Reviewed Quarterly Periodical
Journal website : www.internationaljournalsiwan.com

Premium Linear-Exponential Distribution

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(Received : June 15, 2022; Accepted : July 20, 2022;
Published Online : August 22, 2022)

Abstract :

In the present study, a one-parameter continuous probability distribution, which is a modified form of 'New Linear-exponential distribution' and 'Modified Linear-exponential distribution', has been proposed for statistical modeling of survival time data with better results. Several characteristics and descriptive measures of Statistics of the proposed distribution have been derived and discussed. To test validity of the theoretical work of this distribution, goodness of fit has been applied to some over-dispersed data-sets which were earlier used by other researchers. It is expected to be a better alternative of Lindley distribution and in some of the cases; it is expected to be a better alternative of New Linear-exponential distribution and Modified Linear-exponential distribution.

Keywords : Modified Linear-exponential distribution, New Linear-exponential distribution, Lindley distribution, Statistical moments, Estimation of Parameters, Goodness of fit.

1. Introduction :

Statistical literature is full of several concepts on countable and continuous probability distributions with varieties of applications in allied fields of social and physical sciences. Sah, (Sah, 2022), obtained 'New Linear-exponential distribution (NLED)' for statistical modeling of survival time data and given by its probability density function (*pdf*)

$$f_1(y) = \frac{\alpha^2}{(\pi\alpha+1)}(\pi + y)e^{-\alpha y}\,; y > 0, \alpha > 0 \qquad (1)$$

It has been observed that, in most of the cases, NLED (1) gives better fit to the over-dispersed data-sets than Lindley distribution, (Lindley, 1958), (LD) given by its *pdf*

$$f_2(y) = \frac{\alpha^2}{(1+\alpha)}(1 + y)e^{-\alpha y}\,; y > 0, \alpha > 0 \qquad (2)$$

Sah, (Sah, 2022), has recently obtained 'Modified Linear-exponential distribution (MLED)', which is a modified form of NLED (1), given by its *pdf*

$$f_3(y) = \frac{(\pi\alpha)^2}{(\pi^2\alpha^2+1)}(\pi\alpha + y)e^{-\pi\alpha y}\,; y > 0, \alpha > 0 \qquad (3)$$

It has been developed for a better alternative of LD and NLED for modeling of survival time data. It has been observed that MLED gives better fit to the over-dispersed data having low-degree of variation. It has been observed that NLED gives better fit to the data-sets having high-degree of variations. The proposed distribution is a premium quality Linear-exponential distribution, and hence named as 'Premium Linear-exponential distribution (PLED)' which is a very useful probability distribution for modeling of over-dispersed data of low as well as high-degrees of variations. Hence, PLED is expected to be a better alternative of LD, NLED and MLED.

In section-1, a brief introduction and literature review have been placed for the proposed distribution. In section-2, results obtained for the proposed distribution have been placed under different sub-headings in systematic manner. In section-2.1, probability density function, probability distribution function, moments generating function and mode of the distribution have been obtained. In section-2.2, statistical moments about origin as well as the mean and different characteristics such as dispersion, Skewness and Kurtosis have been obtained and discussed. In section-2.3, the reliability function, Hazard rate function and the mean residual life time have been formulated for the proposed distributions which are very useful tools for analyzing quality of a machinery system. In section-2.4, the estimation of parameters has been discussed by the method of moments as well as maximum likelihood method. In section-3, applications of the distribution have been explained by taking some over-dispersed secondary data which were previously used by others. In section- 4, conclusion of the proposed distribution is concisely placed.

2. Results :

The section contains findings about different characteristics and measure of PLED which are placed under following sub-headings.

2.1 Premium Linear-Exponential Distribution (PLED) and its important characteristics

2.2 Statistical Moments and related measures

2.3 The Reliability Function, Hazard Rate Function and the Mean Residual life time function, and

2.4 The Estimation of Parameters

2.1 Premium Linear-Exponential Distribution (PLED) and Its Important Characteristics :

Let Y denotes a continuous random variable which follows PLED with a single parameter 'α'. Its probability density function has been obtained as

$$f(y) = \frac{\alpha^2}{(1+\pi\alpha^2)}(\pi\alpha + y)e^{-\alpha y} \; ; y > 0, \alpha > 0 \tag{4}$$

The expression (4) is the pdf of PLED which follows all the basic criteria of probability distribution.

Probability distribution function : It is obtained as

$$F(Y) = P(Y \le y) = \int_0^y f(y)dy = \frac{\alpha^2}{(1+\pi\alpha^2)} \int_0^y (\pi\alpha + y)e^{-\alpha y}dy = 1 - \frac{(1+\alpha y+\pi\alpha^2)}{(1+\pi\alpha^2)} e^{-\alpha y} \quad (5)$$

The expression (5) is the probability distribution function of PLED (4). Graphical presentations of probability density function (pdf) and cumulative distribution function (cdf) are given below.

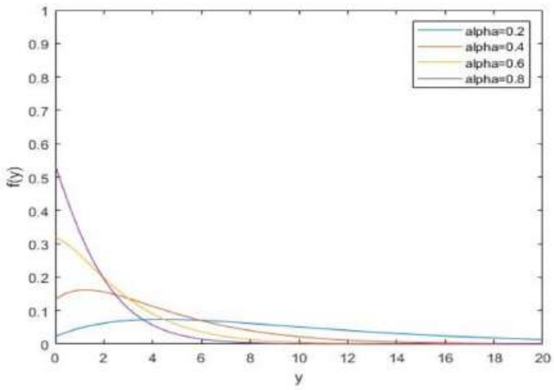

Fig. 1 : Graph of pdf of PLED at $\alpha = 0.2, 0.4, 0.6, 0.8$

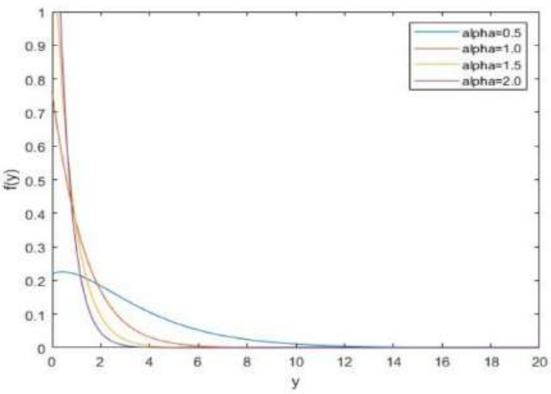

Fig. 2 : Graph of pdf of PLED at $\alpha = 0.5, 1.0, 1.5, 2.0$

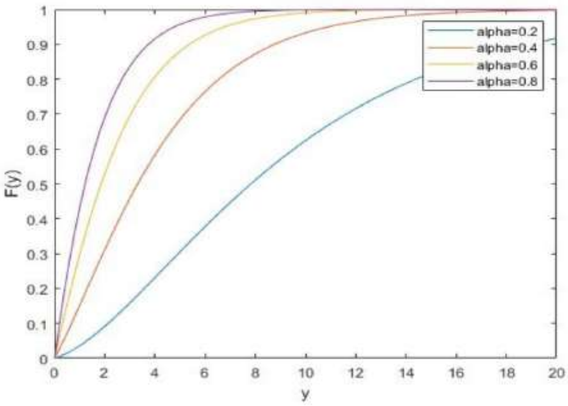

Fig. 3 : Graph of cdf of PLED at $\alpha = 0.2, 0.4, 0.6, 0.8$

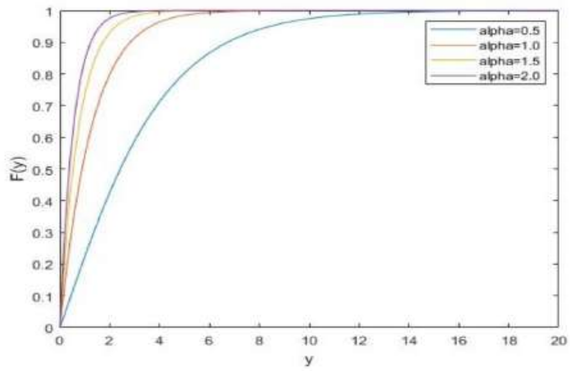

Fig. 4 : Graph of cdf of PLED at $\alpha = 0.5, 1.0, 1.5, 2.0$

Moment Generating Function (M.G.F.) : It can be obtained as

$$M_Y(t) = E(e^{ty}) = \int_0^\infty e^{ty} f(y)dy = \frac{\alpha^2}{(1+\pi\alpha^2)} \int_0^\infty (\pi\alpha + y)e^{-(\alpha-t)y}dy$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}\left[\pi\alpha \int_0^\infty e^{-(\alpha-t)y}\, dy + \int_0^\infty y e^{-(\alpha-t)y}\, dy\right]$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}\left[-\frac{(\pi\alpha)e^{-(\alpha-t)y}}{(\alpha-t)} - \frac{ye^{-(\alpha-t)y}}{(\alpha-t)} - \frac{e^{-(\alpha-t)y}}{(\alpha-t)^2}\right]_0^\infty$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}\left[\frac{(\pi\alpha)}{(\alpha-t)} + \frac{1}{(\alpha-t)^2}\right] = \frac{\alpha^2}{(1+\pi\alpha^2)}\frac{[\pi\alpha(\alpha-t)+1]}{(\alpha-t)^2} \tag{6}$$

The expression (6) is the M.G.F. of PLED (4).

Mode :

It is the value of the random variable Y, which follows PLED (4), at which probability density function is maximum. That is, $f'(y) = 0$ and $f''(y) < 0$.

Differentiate the expression (3) with respect to y, we get

$$\frac{\partial[f(y)]}{\partial y} = \frac{\alpha^2}{(1+\pi\alpha^2)}\frac{\partial[(\pi\alpha+y)e^{-\alpha y}]}{\partial y} = \frac{\alpha^2}{(1+\pi\alpha^2)}\frac{\partial[(\pi\alpha e^{-\alpha y}+ye^{-\alpha y})]}{\partial y}$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}\left[\pi\alpha(-\alpha)e^{-\alpha y} + e^{-\alpha y} - \alpha y e^{-\alpha y}\right]$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}[1 - \alpha y - \pi\alpha^2]e^{-\alpha y} \tag{7}$$

By using $f'(y) = 0$, we get $y = \dfrac{(1-\pi\alpha^2)}{\alpha}$ (8)

The expression (8) is the mode of PLED (4) because $f''(y) < 0$.

2.2 Statistical Moments and related measures :

Under this section, we have been obtained and discussed about

- The r^{th} moment about origin
- The first four moments about the mean, and
- Dispersion, Skewness and Kurtosis.

The r^{th} moment about origin : It can be obtained by

$$\mu_r' = E(Y^r) = \int_0^\infty y^r f(y)dy = \frac{\alpha^2}{(1+\pi\alpha^2)} \int_0^\infty y^r(\pi\phi + y)e^{-\alpha y}\, dy$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}\left[\pi\alpha \int_0^\infty y^r e^{-\alpha y}\, dy + \int_0^\infty y^{r+1}\, e^{-\alpha y} dy\right]$$

$$= \frac{\alpha^2}{(1+\pi\alpha^2)}\left[\frac{(\pi\alpha)\Gamma(r+1)}{\alpha^{r+1}} + \frac{\Gamma(r+2)}{\alpha^{r+2}}\right] = \frac{r!(1+r+\pi\alpha^2)}{\alpha^r(1+\pi\alpha^2)} \quad (9)$$

The expression (9) is the r^{th} moment about origin of the PLED (4). Substituting $r = 1, 2, 3, 4$ in the expression (9), we get the first four moments as follows

$$\mu_1' = \frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)} \quad (10)$$

$$\mu_2' = \frac{2!(3+\pi\alpha^2)}{\alpha^2(1+\pi\alpha^2)} \quad (11)$$

$$\mu_3' = \frac{3!(4+\pi\alpha^2)}{\alpha^3(1+\pi\alpha^2)} \quad (12)$$

$$\mu_4' = \frac{4!(5+\pi\alpha^2)}{\alpha^4(1+\pi\alpha^2)} \quad (13)$$

The first four moments about the mean : These are obtained as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = \frac{2!(3+\pi\alpha^2)}{\alpha^2(1+\pi\alpha^2)} - \left[\frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}\right]^2 \quad (14)$$

The expression (14) is the variance of the PLED (4). From the expressions (10) and (14), the following conclusions have been made

(a) If $\alpha = 1.132841495$, PLED (4) will be equi-dispersed i.e., $\mu_1' = \mu_2$ (15)

(b) If $\alpha < 1.132841495$, PLED (4) will be over-dispersed i.e., $\mu_1' < \mu_2$ (16)

(c) If $\alpha > 1.132841495$, PLED (4) will be under-dispersed i.e., $\mu_1' > \mu_2$ (17)

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= \frac{3!(4+\pi\alpha^2)}{\alpha^3(1+\pi\alpha^2)} - 3\left[\frac{2!(3+\pi\alpha^2)}{\alpha^2(1+\pi\alpha^2)}\right]\left[\frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}\right] - 2\left[\frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}\right]^3$$

$$= \frac{[6(4+\pi\alpha^2)(1+\pi\alpha^2)^2 - 6(3+\pi\alpha^2)(2+\pi\alpha^2)(1+\pi\alpha^2) + 2(2+\pi\alpha^2)^3]}{[\alpha(1+\pi\alpha^2)]^3}$$

After a little simplification, we get

$$\mu_3 = \frac{(4+12\pi\alpha^2+12\pi^2\alpha^4+2\pi^3\alpha^6)}{[\alpha(1+\pi\alpha)]^3} > 0 \tag{18}$$

Here, $\alpha > 0$ and hence the proposed distribution is positively skewed. The fourth moment about the mean of PLED (4) can be obtained as

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= \frac{4!(5+\pi\alpha^2)}{\alpha^4(1+\pi\alpha^2)} - 4\left[\frac{3!(4+\pi\alpha^2)}{\alpha^3(1+\pi\alpha^2)}\right]\left[\frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}\right]$$

$$+ 6\left[\frac{2!(3+\pi\alpha^2)}{\alpha^2(1+\pi\alpha^2)}\right]\left[\frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}\right]^2 - 3\left[\frac{1!(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}\right]^4$$

After a little simplification, we get

$$\mu_4 = \frac{3[8+32\pi\alpha^2+44\pi^2\alpha^4+24\pi^3\alpha^6+3\pi^4\alpha^8]}{[\alpha(1+\pi\alpha)]^3} \tag{19}$$

Index of dispersion : It is given by

$$I = \frac{\mu_2}{\mu_1'} = \frac{(2+4\pi\alpha^2+\pi^2\alpha^2)}{[\alpha(1+\pi\alpha^2)(2+\pi\alpha^2)]} \tag{20}$$

Co-efficient of Skewness : It can be obtained as

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(4+12\pi\alpha^2+12\pi^2\alpha^4+2\pi^3\alpha^6)^2}{(2+4\pi\alpha^2+\pi^2\alpha^2)^3} \tag{21}$$

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{(4+12\pi\alpha^2+12\pi^2\alpha^4+2\pi^3\alpha^6)}{(2+4\pi\alpha^2+\pi^2\alpha^2)^{3/2}} \tag{22}$$

From the expression (22), we can observe that $(\sqrt{2}) < \gamma_1 < \infty$. Hence, PLED (4) is positively skewed distribution. It can also be observed that γ_1 is an increasing function of α.

Co-efficient of Kurtosis : It can be obtained as

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3(8+32\pi\alpha^2+44\pi^2\alpha^4+24\pi^3\alpha^6+3\pi^4\alpha^8)}{(2+4\pi\alpha^2+\pi^2\alpha^2)^2} \tag{23}$$

Here, $\beta_2 > 3$ and hence, $\gamma_2 = (\beta_2 - 3) > 0$. So, PLED (4) is Leptokurtic. From the expression (23), it can also be observed that $6 < \beta_2 < \infty$.

Remarks : It has been found that

- PLED (4) is over-dispersed if $\alpha < 1.132841495$.
- It is positively skewed, and if $(\sqrt{2}) < \gamma_1 < \infty$.
- It is Leptokurtic if $6 < \beta_2 < \infty$.

2.3 The Reliability Function, Hazard Rate Function and Mean Residual Life Function of PLED :

The Reliability Function :

Reliability of a product or a system is directly proportional to the faith on the company where the product or system manufactured. A product is said to be reliable when it works without failure in the guaranty period provided by the company. We can observe that it is widely used in science and technology, in the sector of production engineering, and in the sectors related to business and economics. The complement of probability distribution function is called reliability function. It may also be called survival function. Reliability function of a random variable 'Y', which follows PLED (4) with parameter α, is denoted by $R(Y=t)$ and it can be given by

$$R(Y=t) = P(Y>t) = \int_t^{\infty} f(y)dy = 1 - F(Y=t) = \frac{(1+\alpha t+\pi\alpha^2)}{(1+\pi\alpha^2)} e^{-\alpha t} \tag{24}$$

Where, $F(Y=t) = \int_0^t f(y)dy = 1 - \frac{(1+\alpha t+\pi\alpha^2)}{(1+\pi\alpha^2)} e^{-\alpha t}$

The expression (24) is the reliability function of PLED (4). From the expression (24), it can be observed that

$$R(t=0) = 1 \text{ and } \lim_{t\to\infty} R(t) = 0$$

Hence, as time (t) increases, $R(Y=t)$ decreases. It indicates that $R(Y=t)$ is a monotone decreasing function of t.

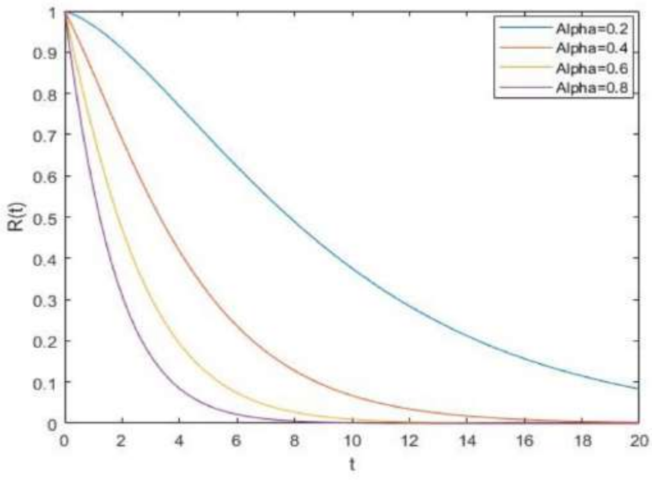

Fig. 5 : Graph of Reliability Function at ϕ = 0.2, 0.4, 0.6, 0.8

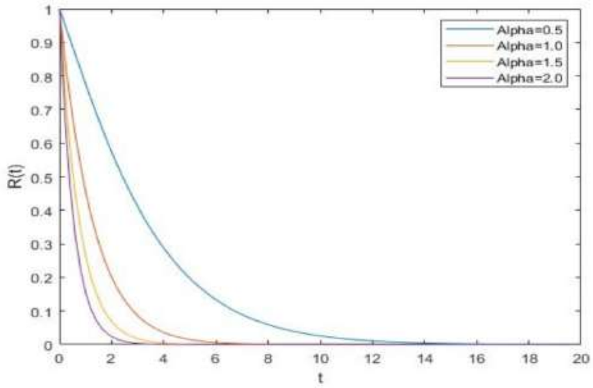

Fig. 6 : Graph of Reliability Function at $\phi = 0.5, 1.0, 1.5, 2.0$

The Hazard Rate Function :

It is also known as failure rate function. It is defined as the ratio of Probability density function to the reliability function. It is generally applied to obtain probability of failure of a system given that the system is functioning. Now the days, there are many sectors where it is used for reliability testing such as production engineering, quality control, business and economics.

Let $F(Y = t) = P(Y \leq t)$ denotes the probability that the system will fail in the interval 0 to t. Let $F[Y = (t + \Delta t)] = P[Y \leq (t + \Delta t)]$ denotes the probability that the system will fail in the interval 0 to $(t + \Delta t)$. Hence, $F(t + \Delta t)$ - $F(t)$ denotes probability that the system will fail in the interval t to $(t + \Delta t)$. The conditional probability of failure of the system in the interval t to $(t + \Delta t)$ given that the system will survive to time 't' has been obtained as

$$\frac{F(t + \Delta t) - F(t)}{R(t)} \tag{25}$$

The average rate of failure can be obtained by dividing the expression (25) by Δt

$$\frac{F(t + \Delta t) - F(t)}{\Delta t} \cdot \frac{1}{R(t)} \tag{26}$$

Taking the limit, $\Delta t \to 0$, we get failure rate of the system given by

$$h(y=t)=\frac{F'(y=t)}{R(t)}=\frac{f(y=t)}{1-F(y=t)} \tag{27}$$

The main reason for defining the $h(y)$ is that it is more convenient to work with than $f(y)$. Putting the value of $f(y)$ and $1-F(y)$ of PLED in the expression $h(y)$, we get the general expression of the Hazard rate function based on PLED (4) has been obtained as

$$h(y=t)=\frac{\alpha^2(\pi\alpha+t)}{[1+\alpha t+\pi\phi^2]} \tag{28}$$

$$\text{At } t=0,\ h(y=t=0)=\frac{\pi\alpha^3}{[1+\pi\alpha^2]} \tag{29}$$

It is also obvious that $h(t)$ is an increasing function of t and α.

Mean Residual Life Function :

In reliability studies, the expected additional life time given that a system has survived until time 't' is called mean residual life function. Let a random variable Y denotes the life of the system under study, the mean residual life function is given by

$$m(y)=E[Y-y/Y>y]=\frac{\int_y^\infty [1-F(t)]dt}{1-F(y)}=\frac{(2+\pi\alpha^2+\alpha y)}{\alpha(1+\pi\alpha^2+\alpha y)} \tag{30}$$

Where,

$$1-F(t)=\frac{(1+\alpha t+\pi\alpha^2)}{(1+\pi\alpha^2)}e^{-\alpha y} \text{ and}$$

$$\int_y^\infty \{1-F(t)\}dt=\int_y^\infty \frac{(1+\alpha t+\pi\alpha^2)}{[1+\pi\alpha^2]}e^{-\alpha t}\,dt=\frac{(2+\alpha y+\pi\alpha^2)e^{-\alpha y}}{\alpha(1+\pi\alpha^2)}$$

$$\text{At } y=0,\ m(y=0)=\frac{(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}=\mu_1' \tag{31}$$

It can also be seen that at $y = 0$, the mean residual life function (30) is the mean of PLED (4). It can also be seen that $m(y)$ is a decreasing function of y.

2.4 Estimation of Parameter of PLED :

An estimate of the parameter α of PLED (4) has been obtained by applying (a) Method of moments and (b) Method of maximum likelihood.

(a) Method of moments : To estimate value of the parameter, the first moment about the origin is required. We need the first moment about origin to get estimate of the parameter α. The population mean is replaced by respective sample moment and using the expression (4), an estimate of α can be obtained as follows.

$$\mu_1' = \frac{(2+\pi\alpha^2)}{\alpha(1+\pi\alpha^2)}$$

Or, $$\pi\mu_1'\alpha^3 - \pi\alpha^2 + \mu_1'\alpha - 2 = 0 \tag{32}$$

The expression (32) is polynomial in third degree which can be solved by Regula-Falsi method or Newton-Rapson method to estimate value of α.

(b) The method of maximum likelihood :

Let us consider a random sample, $(y_1, y_2,..., y_n)$, of size n from PLED (4). The likelihood function, L, of the PLED (4) can be obtained as

$$L = \prod_{i=1}^{n} f(y;\alpha) = \left(\frac{\alpha^2}{1+\pi\alpha^2}\right)^n \left[\prod_{i=1}^{n}(\pi\alpha + y_i)\right] e^{-\alpha\bar{y}} \tag{33}$$

Taking the natural log on both sides of the expression (32) and hence, the log likelihood function is given as

$$\ln L = n\ln(\alpha^2) - n\ln(1+\pi\alpha^2) + \sum_{y=1}^{n}\ln(\pi\alpha + y_i) - n\alpha\bar{y} \tag{34}$$

Differentiating the expression (34) with respect to α, we get

$$\frac{\partial \ln L}{\partial\alpha} = \frac{2n}{\alpha} - \frac{2n\pi\alpha}{(1+\pi\alpha^2)} + \sum_{i=1}^{n}\frac{\pi}{(\pi\alpha + y_i)} - n\bar{y} = 0 \tag{35}$$

We can obtain an estimate of α by solving the expression (35).

3. Applications and discussion :

To test validity of the theoretical work, the fitting of PLID (4) has been applied to the following to data.

Example (1) : Survival times (in days) of guinea pigs infected with virulent tubercle bacilli, reported by Bjerkedal (Bjerkedal, 1960).

Survival Time (in days)	0-80	80-160	160-240	240-320	320-400	400-480	480-560
Observed frequency	8	30	18	8	4	3	1

Example (2) : Mortality grouped data for blackbirds species reported by Paranjpe and Rajarshi (Paranjpe and Rajarshi, 1986).

Survival Time (in days)	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	>8
Observed frequency	192	60	50	20	12	7	6	3	2

The expected frequencies according to the LD, NLED, MLED have also been given, for ready comparison with those obtained by the PLED, in the following tables.

Table I : Expected Vs Observed of Example (1)

Survival Time (in days)	**Observed frequency**	**Expected frequency**			
		LD	**NLED**	**MLED**	**PLED**
0-80	8	16.1	16.4	15.9	15.9
80-160	30	21.9	21.6	22.0	22.0
160-240	18	15.4	15.3	15.5	15.5
240-320	8	9.0	9.0	9.0	9.0
320-400	4	5.5	4.9	4.8	4.8

400-480	3	1.8	2.5	2.5	2.5
480-560	1	2.3	2.3	2.3	2.3
Total	72	72.0	72.0	72.0	72.0
$\mu_1'=181.11111$	$\widehat{\alpha}$	0.011	0.010860	0.011394	0.0110408
$\mu_2'=43911.11111$	$d.f.$	3	3	3	3
	χ^2	7.77	8.43	7.61	7.61

Table II : Expected Vs Observed of Example (2)

Survival Time (in days)	Observed frequency	Expected frequency			
		LD	NLED	MLED	PLED
0-1	192	173.5	160.8	151.3	159.7
1-2	60	98.6	91.0	99.0	91.8
2-3	50	46.5	49.0	53.4	49.5
3-4	20	20.1	25.5	26.3	25.7
4-5	12	8.1	13.0	12.3	12.9
5-6	7	3.2	6.5	5.5	6.4
6-7	6	1.4	3.2	2.4	3.1
7-8	3	0.4	1.6	1.1	1.5
>8	2	0.3	1.4	0.9	1.4
Total	352	352.0	352.0	352.0	352.0
$\mu_1' = 1.568181$	$\widehat{\alpha}$	0.984	0.816541737	$\widehat{\alpha}=0.96716$	$\widehat{\alpha}=0.836925$
$\mu_2' = 5.005682$	$d.f.$	4	5	5	5
	χ^2	49.85(4)	20.10(5)	34.64(5)	21.41(5)

From table (I), we can observe that the value of Chi-square of PLED (4) is less than the LD as well as NLED with same degrees of freedom. From table (II), we can observe that the value of Chi-square of PLED (4) is less than LD and MLED. Hence, it may conclude that PLED (4) gives better fit to the same nature of over-dispersed data-sets than LD and MLED and in some cases, it gives better fit than NLED.

4. Conclusion :

Several structural and descriptive measures of PLED (4) have been obtained. It has been found that this distribution is positively skewed in shape and Leptokurtic in size. From table - I, it has been observed that it gives lesser value of Chi-Square than LD and NLED. From table - II, it can be observed that it gives lesser value of Chi-Square than LD and MLED. Hence, it is expected to give a better alternative for fitting of similar nature of survival time data than LD, NLED, MLED.

Conflict of Interest :

The author declares that there is no conflict of interest.

Acknowledgement :

The author expresses their gratitude to the referee and entire team for his valuable comments and suggestions which improved the quality of the research paper.

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