FX Reciprocal Average Rate Contract Valuation

This article presents a valuation model for Noon Average Rate Contract (NRC) and Reciprocal Average Rate Contract (NRC 1). NRC is a forward contract on average foreign exchange rate and NRC 1 is a forward contract on reciprocal of average foreign exchange rate.

Consider a forward contract on the average foreign exchange rate. Notations used as follows.

t	Valuation Date
T	Maturity Date
T_S	Settlement Date
X_i	Spot Exchange Rate at time t_i
r^{Ccy}	Risk-free discount rate of currency Ccy
t_A	Averaging start date
K	Strike price
H_i	Historical exchange rate at time $t_i \leq t$
F_i	Forward exchange rate for time interval (t, t_i), where $t_i > t$
N	Notional amount

The average exchange rate, X_A, with *m* historical rate averaging points and *n* spot rate averaging points, is computed as

$$X_A = \frac{\sum_{i=1}^{m} H_i + \sum_{i=m+1}^{m+n} X_i}{m+n} \quad \text{if } t_A \le t, \text{ and } \quad X_A = \frac{\sum_{i=1}^{n} X_i}{n} \quad \text{if } t_A > t.$$

where $t_1 = t_A$, $t_m = t$ and $t_{m+n} = T$ if $t_A \le t$, and $t_n = T$ if $t_A > t$.

The average forward rate, $F_A = E_t[X_A]$, is then computed as

$$F_A = \frac{\sum_{i=1}^{m} H_i + \sum_{i=m+1}^{m+n} F_i}{m+n} \quad \text{if } t_A \le t, \text{ and } \quad F_A = \frac{\sum_{i=1}^{n} F_i}{n} \quad \text{if } t_A > t$$

and the forward exchange rate is computed as $F_i = X_t \cdot e^{(r^{Ccy1} - r^{Ccy2})(t_i - t)}$, where $[F_i] = \frac{Ccy1}{Ccy2}$.

The actual pricing (Mark-to-Market) is done at the inventory level and is in the base currency, which is usually *USD*.

Direct quote

The rates are quoted as $[K] = [X_i] = [F_i] = \frac{USD}{EUR}$**.** There are 4 possible cases depending on the notional currency and payoff currency**.**

1. $[N] = EUR$ and payoff currency = USD : **NRC**

The payoff at the maturity is defined as $\beta(X_A - K)N_{EUR}$ in *USD*. Thus, the expected payoff is calculated as $E_t[\beta(X_A - K)] \cdot N_{EUR} = \beta(F_A - K)N_{EUR}$. The arbitrage-free price of the contract in *USD* is then obtained by discounting the expected payoff from the settlement date to the valuation date.

$$V = \beta(F_A - K)N_{EUR} \cdot DF^{USD}(t, T_S) \qquad (1)$$

where $DF^{USD}(t, T_S)$ is the *USD* discount factor between the settlement date to the valuation date.

2. $[N] = EUR$ and payoff currency = *EUR* : **NRC 1**

The payoff at the maturity is defined as $\beta\left(\frac{1}{K} - \frac{1}{X_A}\right)N_{EUR}K$ in *EUR* and the expected payoff is then calculated as $E_t\left[\beta\left(\frac{1}{K} - \frac{1}{X_A}\right)\right]N_{EUR}K \cong \beta\left(\frac{1}{K} - \frac{1}{F_A}\right)N_{EUR}K$. Here, the approximation $E_t\left[\frac{1}{X_A}\right] = \frac{1}{F_A}$ used in this relation, in general, does not hold.

However, this is a good approximation when the volatility of X_i is relatively small, which is usually the case for exchange rates. The arbitrage-free price of the contract in *USD* is then obtained by multiplying the expected payoff by the forward rate, F_T, and by discounting it with *USD* from the settlement date to the valuation date.

$$V = \beta\left(\frac{1}{K} - \frac{1}{F_A}\right)N_{EUR}K \cdot F_T \cdot DF^{USD}(t, T_S) \qquad (2)$$

3. $[N] = USD$ and payoff currency = *USD* : **NRC**

The payoff at the maturity is defined as $\beta(X_A - K) \cdot \frac{N_{USD}}{K}$ in *USD* and the expected payoff is $\beta(F_A - K) \cdot \frac{N_{USD}}{K}$. The arbitrage-free price of the contract in *USD* is then given by

$$V = \beta(F_A - K) \cdot \frac{N_{USD}}{K} \cdot DF^{USD}(t, T_S) \qquad (3)$$

4. $[N] = USD$ and payoff currency = EUR : **NRC 1**

The payoff at the maturity is defined as $\beta\left(\frac{1}{K} - \frac{1}{X_A}\right)N_{USD}$ in *EUR* and the expected payoff is approximated as $\beta\left(\frac{1}{K} - \frac{1}{F_A}\right)N_{USD}$. Again, the arbitrage-free price of the contract in *USD* is obtained by multiplying the expected payoff by the forward rate, F_T, and by discounting it with *USD*. Thus,

$$V = \beta\left(\frac{1}{K} - \frac{1}{F_A}\right)N_{USD}F_T \cdot DF(t, T_S) \tag{4}$$

where β (1 or –1) is the long/short indicator. For all of the above cases, the dollar value *Delta in USD* is computed as

$$USD\ Delta = \frac{V\big|_{X_t - \Delta} - V\big|_{X_t + \Delta}}{2\Delta} \cdot X_t \tag{5}$$

where X_t is the spot exchange rate and the perturbation on the spot rate Δ is set to be 0.00005. This dollar value delta is used for daily hedging.

Indirect quote

The rates are quoted as $[K] = [X_i] = [F_i] = \frac{CAD}{USD}$**.** Again, there are 4 possible cases depending on the notional currency and payoff currency**.**

1. $[N] = USD$ and payoff currency = CAD : **NRC**

The payoff at the maturity is defined as $\beta(K - X_A)N_{USD}$ in *CAD* and the expected payoff is then calculated as $\beta(K - F_A)N_{USD}$. The price of the contract in *USD* is obtained by dividing the expected payoff by the forward rate, F_T, and by discounting it with *USD*

$$V = \beta(K - F_A)N_{USD} \cdot \frac{1}{F_T} \cdot DF^{USD}(t, T_S) \qquad (6)$$

2. $[N] = USD$ and payoff currency = *USD* : **<u>NRC 1</u>**

The payoff at the maturity is defined as $\beta\left(\frac{1}{X_A} - \frac{1}{K}\right)N_{USD}K$ in *USD* and the expected payoff is approximated as $\beta\left(\frac{1}{F_A} - \frac{1}{K}\right)N_{USD}K$. The price of the contract in *USD* is then given by

$$V = \beta\left(\frac{1}{F_A} - \frac{1}{K}\right)N_{USD}K \cdot DF^{USD}(t, T_S) \qquad (7)$$

3. $[N] = CAD$ and payoff currency = *CAD* : **<u>NRC</u>**

The payoff at the maturity is defined as $\beta(K - X_A) \cdot \frac{N_{CAD}}{K}$ in *CAD*. Similar to case 1, the price of the contract in *USD* is then given by

$$V = \beta(K - F_A) \cdot \frac{N_{CAD}}{K} \cdot \frac{1}{F_T} \cdot DF^{USD}(t, T_S) \qquad (8)$$

4. $[N] = CAD$ and payoff currency = USD : **NRC 1**

The payoff at the maturity is defined as $\beta\left(\frac{1}{X_A} - \frac{1}{K}\right)N_{CAD}$ in *USD*. Similar to case 2, the price of the contract in *USD* is then given by

$$V = \beta\left(\frac{1}{F_A} - \frac{1}{K}\right)N_{CAD} \cdot DF^{USD}(t, T_S) \tag{9}$$

The price of the contract and the perturbation of the spot rate in delta calculation is in USD/CAD. Thus, the perturbation of the spot is done as $\frac{1}{X_t^{+}} = \frac{1}{X_t} + \Delta$ and $\frac{1}{X_t^{-}} = \frac{1}{X_t} - \Delta$ where $\Delta = 0.00005$. Thus, for all of the above cases, the USD Delta is defined by

$$USD\ Delta = \frac{V|_{X_t^-} - V|_{X_t^+}}{2\Delta} \cdot \frac{1}{X_t} \tag{10}$$

It is possible that matured NRC could be in the system (not paid out to clients) and, thus, daily Mark-to-Market and Delta calculation are also done on those matured NRCs. The first case in table 1 and 2 deals with a matured NRC and the pricing of 4 sub cases need to be modified. The equations (1) to (4) are modified for a matured NRC as follows.

$$V = \beta(F_A - K)N_{EUR} \quad , \qquad \text{for Payoff Currency} = USD \tag{11}$$

$$V = \beta\left(\frac{1}{K} - \frac{1}{F_A}\right)N_{EUR}K \cdot X_t \quad , \qquad \text{for Payoff Currency} = EUR \tag{12}$$

$$V = \beta(F_A - K) \cdot \frac{N_{USD}}{K} \quad , \qquad \text{for Payoff Currency} = USD \tag{13}$$

$$V = \beta\left(\frac{1}{K} - \frac{1}{F_A}\right)N_{USD}X_t \quad , \qquad \text{for Payoff Currency} = EUR \tag{14}$$

where F_A is calculated only from historical rates. Note that the forward rate in equation (2) and (4) becomes the spot rate X_t in the above equations (12) and (14). Since the pricing in equations (12) and (14) involves the spot price which varies day-to-day, there is non-zero delta values on these cases (see 2nd and 4th sub cases of case 1 in table 2).

Similarly, testing results for indirect quote are summarized in table 3 (pricing) and table 4 (USD delta). The pricing modification for a matured NRC is done as following.

$$V = \beta(K - F_A)N_{USD} \cdot \frac{1}{X_t}, \qquad \text{for Payoff Currency} = CAD \qquad (15)$$

$$V = \beta\left(\frac{1}{F_A} - \frac{1}{K}\right)N_{USD}K, \qquad \text{for Payoff Currency} = USD \qquad (16)$$

$$V = \beta(K - F_A) \cdot \frac{N_{CAD}}{K} \cdot \frac{1}{X_t}, \qquad \text{for Payoff Currency} = CAD \qquad (17)$$

$$V = \beta\left(\frac{1}{F_A} - \frac{1}{K}\right)N_{CAD}, \qquad \text{for Payoff Currency} = USD \qquad (18)$$

Again, F_A is calculated only from historical rates and the forward rate in equation (6) and (8) becomes the spot rate X_t in the above equations (15) and (17). Since the pricing in equations (15) and (17) involves the spot price which varies day-to-day, there is non-zero delta values on these cases (see https://finpricing.com/lib/EqBarrier.html).

The 2nd case for both direct and indirect quote examines the case where the average rate includes historical rates as well as forward rates. The last 3 cases test with various maturity dates and strike prices. The results indicate that the QA model calculates consistently to FX NRC and NRC 1.