FX Forward Contract Valuation

This article discusses the valuation of Foreign Exchange Rate Forward Contract (FEF). An FX forward contract is an obligation to exchange a particular amount of one currency for a certain amount of another currency at a future time.

Notations used as follows.

t	Valuation Date
T	Maturity Date
T_S	Settlement Date
X_i	Spot Exchange Rate at time t_i
r^{Ccy}	Risk-free discount rate of currency Ccy
K	Strike price
F_i	Forward exchange rate for time interval (t, t_i), where $t_i > t$
N	Notional amount

The forward exchange rate is computed as is computed as $F_T = E_t\left[X_T\right] = X_t \cdot e^{(r^{Ccy1} - r^{Ccy2})(T-t)}$, where $[F_i] = \dfrac{Ccy1}{Ccy2}$.

The actual pricing (Mark-to-Market) on FEF is done at the inventory level and is in the base currency, which is usually *USD*.

<u>Direct Quote</u>

The rates are quoted as $[K]=[X_i]=[F_i]=\frac{USD}{EUR}$. Payoff currency is always USD and notional currency is either USD or EUR.

1. Notional currency = EUR

The payoff at the maturity is defined as $\beta(X_T - K)N_{EUR}$ in *USD*. The expected payoff is then calculated as

$$E_t\left[\beta(X_T - K)\right]\cdot N_{EUR} = \beta(F_T - K)N_{EUR}.$$

where β (1 or –1) is the long / short indicator. The price of the contract in *USD* is obtained by discounting the expected payoff with *USD.*

$$V = \beta(F_T - K)N_{EUR}\cdot DF^{USD}(t, T_S) \tag{1}$$

where $DF^{USD}(t, T_S)$ is the *USD* discount factor between the settlement date and the valuation date.

2. Notional currency = USD

The payoff at the maturity is defined as $\beta(X_T - K)\cdot\frac{N_{USD}}{K}$ in *USD.* Similarly, the price of the contract in *USD* is given by

$$V = \beta(F_T - K)\cdot\frac{N_{USD}}{K}\cdot DF^{USD}(t, T_S) \tag{2}$$

For all of the above cases, the dollar value *Delta in USD* is computed as

$$USD\ Delta = \frac{V|_{X_t-\Delta} - V|_{X_t+\Delta}}{2\Delta} \cdot X_t \tag{3}$$

where X_t is the spot exchange rate and the perturbation on the spot rate Δ is set to be 0.00005. This dollar value delta is used for daily hedging.

<u>Indirect Quote</u>

The rates are quoted as $[K]=[X_i]=[F_i]=\frac{CAD}{USD}$. Payoff currency is always CAD and notional currency is either USD or CAD.

1. Notional currency = USD

The payoff at the maturity is defined as $\beta(K - X_T)N_{USD}$ in *CAD*. The price of the contract in *USD* is obtained by dividing the expected payoff by the forward rate, F_T, and by discounting it with *USD*

$$V = \beta(K - F_T)N_{USD} \cdot \frac{1}{F_T} \cdot DF^{USD}(t, T_S) \tag{4}$$

2. Notional currency = CAD

The payoff at the maturity is defined as $\beta(K - X_T) \cdot \frac{N_{CAD}}{K}$ in *CAD*. The price of the contract in *USD* is obtained by dividing the expected payoff by the forward rate, F_T, and by discounting it with *USD*

$$V = \beta(K - F_T) \cdot \frac{N_{CAD}}{K} \cdot \frac{1}{F_T} \cdot DF^{USD}(t, T_S) \tag{5}$$

The price of the contract and the perturbation of the spot rate in delta calculation is in USD/CAD. Thus, the perturbation of the spot is done as $\frac{1}{X_t^{+}} = \frac{1}{X_t} + \Delta$ and $\frac{1}{X_t^{-}} = \frac{1}{X_t} - \Delta$ where $\Delta = 0.00005$. Thus, the USD Delta is defined by

$$USD\ Delta = \frac{V_{ANR}\big|_{X_t^-} - V_{ANR}\big|_{X_t^+}}{2\Delta} \cdot \frac{1}{X_t} \tag{6}$$

The lag between the maturity date and settlement date is usually 2 business days for direct quote and 1 business day for indirect quote. For instance, the forward rate, F_T, is observed at the maturity date, *T*, and settled on the settlement date, $T_S = T + 1$ or $T + 2$. When FEF deals are entered directly into Inventory, T_S is, by default, set equal to *T*. Since there is no lag between *T* and T_S for FEF, the forward rate in the pricing formulae needs to be specified: it is the rate settled on T_S. It should also be noted that the discount factor is calculated from the maturity date to the valuation date, i.e. $DF^{USD}(t,T)$. In other words, the pricing formulae, equations (1), (2), (4) and (5), become

$$V = \beta\left(F_{T_S-2} - K\right)N_{EUR} \cdot DF^{USD}(t,T) \tag{1'}$$

$$V = \beta\left(F_{T_S-2} - K\right) \cdot \frac{N_{USD}}{K} \cdot DF^{USD}(t,T) \tag{2'}$$

$$V = \beta\left(K - F_{T_S-1}\right)N_{USD} \cdot \frac{1}{F_T} \cdot DF^{USD}(t,T) \tag{4'}$$

$$V = \beta\left(K - F_{T_S-1}\right) \cdot \frac{N_{CAD}}{K} \cdot \frac{1}{F_T} \cdot DF^{USD}(t,T) \tag{5'}$$

As long as $T = T_S$, which is the case for FEF, the valuation is done based on forward rate settled on T_S and the discount factor between the settlement date (since $T = T_S$) and the valuation date. It agrees with the pricing discussed in the previous section.

Testing results for direct quote are summarized in table 1 (pricing) and table 2 (USD delta). The first case in tables 1 and 2 deals with a matured FEF and the pricing in equations (1) and (2) are modified as the following.

$$V = \beta(X_t - K)N_{EUR} \tag{7}$$

$$V = \beta(X_t - K) \cdot \frac{N_{USD}}{K} \tag{8}$$

Similarly, testing results for indirect quote are summarized in table 3 (pricing) and table 4 (USD delta). The pricing modification for a matured FEF (first case in tables 3 and 4) is done as following.

$$V = \beta(K - X_t)N_{USD} \cdot \frac{1}{X_t} \tag{9}$$

$$V = \beta(K - X_t) \cdot \frac{N_{CAD}}{K} \cdot \frac{1}{X_t} \tag{10}$$

The forward rates in equations (1), (2), (4) and (5) become the spot rate X_t in the above equations. Since the pricing for these matured FEF involves the spot price, which varies day-to-day, there is non-zero delta values on these cases. You can find similar valuation topics at https://finpricing.com/lib/FiZeroBond.html