FX Average Noon Rate Agreement Pricing Model

Foreign Exchange Rate Average Noon Rate Agreement (ANR) is an agreement to buy or sell USD dollars on a future value date at a rate equal to the average rate for a specified period and adjusted by forward points agreed at the inception.

Notations used as follows.

t	Valuation Date
T	Maturity Date
T_S	Settlement Date
X_i	Spot Exchange Rate at time t_i
r^{Ccy}	Risk-free discount rate of currency Ccy
t_A	Averaging start date
K	Strike price = Fixed Forward Points
H_i	Historical exchange rate at time $t_i \le t$
F_i	Forward exchange rate for time interval (t, t_i), where $t_i > t$
N	Notional amount

The average exchange rate, X_A, with m historical rate averaging points and n spot rate averaging points, is computed as

$$X_A = \frac{\sum_{i=1}^{m} H_i + \sum_{i=m+1}^{m+n} X_i}{m+n} \quad \text{if } t_A \le t, \text{ and } \quad X_A = \frac{\sum_{i=1}^{n} X_i}{n} \quad \text{if } t_A > t.$$

where $t_1 = t_A$, $t_m = t$ and $t_{m+n} = T$ if $t_A \le t$, and $t_n = T$ if $t_A > t$.

The average forward rate, $F_A = E_t[X_A]$, is then computed as

$$F_A = \frac{\sum_{i=1}^{m} H_i + \sum_{i=m+1}^{m+n} F_i}{m+n} \text{ if } t_A \le t, \text{ and } F_A = \frac{\sum_{i=1}^{n} F_i}{n} \text{ if } t_A > t$$

and the forward exchange rate is computed as $F_i = X_t \cdot e^{(r^{Ccy1} - r^{Ccy2})(t_i - t)}$, where $[F_i] = \frac{Ccy1}{Ccy2}$.

The actual pricing (Mark-to-Market) is done at the inventory level and is in the base currency, which is usually *USD*. Currently, this product can only be used by Indirect Currencies, and is normally used in CADUSD only. Therefore, this report examines indirect quote only.

The rates are quoted as $[K] = [X_i] = [F_i] = \frac{CAD}{USD}$**.** The notional currency is in USD only and the payoff currency is CAD only**.** The payoff at the maturity is defined as $\beta(X_A + K - X_T)N_{USD}$ in *CAD*. The expected payoff is then calculated as

$$E_t[\beta(X_A + K - X_T)] \cdot N_{USD} = \beta(F_A + K - F_T)N_{USD}.$$

where β (1 or –1) is the long / short indicator. The price of the contract in *USD* is obtained by dividing the expected payoff by the forward rate, F_T, and by discounting it with *USD*

$$V_{ANR} = \beta(F_A + K - F_T)N_{USD} \cdot \frac{1}{F_T} \cdot DF^{USD}(t, T_S) \quad (1)$$

The price of the contract and the perturbation of the spot rate in delta calculation is in USD/CAD. Thus, the perturbation of the spot is done as $\frac{1}{X_t^+} = \frac{1}{X_t} + \Delta$ and $\frac{1}{X_t^-} = \frac{1}{X_t} - \Delta$ where $\Delta = 0.00005$. Thus, the USD Delta is defined by

$$USD\ Delta = \frac{V_{ANR}\big|_{X_t^-} - V_{ANR}\big|_{X_t^+}}{2\Delta} \cdot \frac{1}{X_t} \tag{2}$$

ANR payoff can de decomposed into two NRCs as the following.

$$\beta\left(X_A + K - X_T\right)N_{USD} = -\beta\left((-K) - X_A\right)N_{USD} + \beta \cdot \left(-X_T\right)N_{USD}$$

where the first term on the right hand side is the payoff of a NRC with strike price $-K$, and the second term is the payoff of a one day NRC ($t_A = T$) with zero strike price[1]. Thus, a long position of ANR is decomposed into a short of the first NRC and a long of the second NRC. The ANR pricing can be written as

$$V_{ANR} = V_{NRC}(1) + V_{NRC}(2) \tag{3}$$

where

$$V_{NRC}(1) = \beta\left(F_A + K\right)N_{USD} \cdot \frac{1}{F_T} \cdot DF^{USD}(t, T_S) \tag{4}$$

and

$$V_{NRC}(2) = -\beta \cdot N_{USD} \cdot DF^{USD}(t, T_S) \tag{5}$$

The second NRC, equation (5), is actually a cash instrument and, therefore, it has zero delta value.

We examine the pricing and delta calculation with 5 test cases (for each of these test cases, ANR decomposition into 2 NRCs are also tested). The valuation date (called *Spot Date* in Atlas) is August 31, 2004. Actual/365 for Day Count Base, Daily Averaging frequency, and N=1,000,000 USD is used in all test cases.

It is possible that matured ANR could be in the system (not paid out to clients) and, thus, daily Mark-to-Market and Delta calculations are also done on those matured ANRs. For a matured ANR, the pricing in equations (1), (4) and (5) are modified as the following.

$$V_{ANR} = \beta(F_A + K - X_t)N_{USD} \cdot \frac{1}{X_t} \tag{6}$$

$$V_{NRC}(1) = \beta(F_A + K)N_{USD} \cdot \frac{1}{X_t} \tag{7}$$

$$V_{NRC}(2) = -\beta N_{USD} \tag{8}$$

Here, F_A is calculated only from historical rates and the forward rate in equation (1) and (4) becomes the spot rate X_t in the above equations (6) and (7). Since the pricing in equations (6) and (7) involve the spot price, which varies day-to-day, there are non-zero delta values on these cases (see case 1 in table 2). Note that the 2nd NRC, equation (8), should have the maturity date same as valuation date (i.e., $t = t_A = T$). You can find other pricing models at https://finpricing.com/lib/FiBond.html

Appendix 1. Test Cases

Base Currency: USD

Underlying Currency: CAD

Principal Amount: 1,000,000 USD

Spot Rate: 1.31895 CAD/USD

Case No.	Position (Buy / Sell)	Start Date	Maturity Date	Settlement Date	Strike Price
1	Sell	01-Jun-2004	30-Jun-2004	01-Jul-2004	0.0013
2	Sell	03-Aug-2004	03-Sep-2004	07-Sep-2004	-0.0075

[1] For detailed discussion on NRC pricing, see reference [1].

3	Sell	18-Sep-2007	17-Oct-2007	18-Oct-2007	0.0156
4	Buy	25-Feb-2005	30-Mar-2005	31-Mar-2005	0.0105
5	Buy	10-Aug-2004	09-Sep-2004	10-Spe-2004	-0.0025

Appendix 2. Forward Foreign Exchange Rate

Number of Days from Valuation Date	CAD/USD Forward Points (bps)	CAD/USD Forward Outright
7	1.1500	1.319065
14	2.4500	1.319195
30	5.4500	1.319495
59	11.2000	1.320070
91	17.5000	1.320700
122	23.0000	1.321250
153	29.0000	1.321850
181	34.5000	1.322400
273	52.0000	1.324150
365	67.5000	1.325700
546	94.7500	1.328425
730	122.0000	1.331150

Appendix 3. USD Discount Factor

Number of Days from Valuation Date	USD DF
7	0.99969190
14	0.99938399
30	0.99868091

61	0.99716984
91	0.99558334
181	0.99031568
273	0.98421443
365	0.97755095
730	0.94537551
1,098	0.90750025
1,462	0.86651370
1,826	0.82442859

Appendix 4. Historical Rate

CAD FX SPOT HISTORY							
Date	**Rate**	**Date**	**Rate**	**Date**	**Rate**	**Date**	**Rate**
01-Jun-04	1.36850	24-Jun-04	1.34300	20-Jul-04	1.30980	12-Aug-04	1.33260
02-Jun-04	1.36160	25-Jun-04	1.34870	21-Jul-04	1.32420	13-Aug-04	1.30980
03-Jun-04	1.36070	28-Jun-04	1.34330	22-Jul-04	1.31310	16-Aug-04	1.30760
04-Jun-04	1.35020	29-Jun-04	1.34600	23-Jul-04	1.32170	17-Aug-04	1.30780
07-Jun-04	1.34480	30-Jun-04	1.34040	26-Jul-04	1.33200	18-Aug-04	1.30730
08-Jun-04	1.34680	01-Jul-04	1.31895	27-Jul-04	1.33480	19-Aug-04	1.29630
09-Jun-04	1.35410	02-Jul-04	1.32520	28-Jul-04	1.33050	20-Aug-04	1.29770
10-Jun-04	1.35680	06-Jul-04	1.32640	29-Jul-04	1.32480	23-Aug-04	1.30630
11-Jun-04	1.36470	07-Jul-04	1.31960	30-Jul-04	1.32920	24-Aug-04	1.30500
14-Jun-04	1.36830	08-Jul-04	1.31630	02-Aug-04	1.31895	25-Aug-04	1.30410
15-Jun-04	1.36900	09-Jul-04	1.32090	03-Aug-04	1.31910	26-Aug-04	1.31190
16-Jun-04	1.37730	12-Jul-04	1.31870	04-Aug-04	1.31550	27-Aug-04	1.31030
17-Jun-04	1.37530	13-Jul-04	1.32590	05-Aug-04	1.31780	30-Aug-04	1.31730
18-Jun-04	1.36440	14-Jul-04	1.32210	06-Aug-04	1.30980		
21-Jun-04	1.36400	15-Jul-04	1.32350	09-Aug-04	1.31650		
22-Jun-04	1.35900	16-Jul-04	1.30860	10-Aug-04	1.31390		
23-Jun-04	1.36160	19-Jul-04	1.30790	11-Aug-04	1.32390		