

Construction of Magic Squares With Magic Rectangles Blocks

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Abstract

*In this work, magic squares are written with **equal and unequal sums** blocks of **magic rectangles**. The magic squares studied are of orders 12, 15, 20, 21, 24, 27, 28, 30, 33, 35, 29 and 45. The **equal sum blocks magic rectangles** are for the magic squares of orders 12, 20, 24, 28 and 30. This lead us to **semi-magic squares**. After making necessary changes, these **semi-magic squares** are brought to **magic squares**. The **unequal sum blocks magic rectangles** are for the magic squares of orders 15, 21, 27, 33, 39 and 45. Order 45 is written two different ways. One as different sum blocks of order 5×9 and second as different sum blocks of order 3×15 .*

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1 Magic Rectangles

Magic rectangles are well known in literature. Below are few examples of magic rectangles.

1.1 Multiples of 3

1.1.1 Magic Rectangle of Order 3×5

Example 1. Let's consider a magic rectangle of order 3×5 given by

| | | | | | |
|----|----|----|----|----|----|
| 14 | 10 | 4 | 5 | 7 | 40 |
| 1 | 3 | 8 | 13 | 15 | 40 |
| 9 | 11 | 12 | 6 | 2 | 40 |
| 24 | 24 | 24 | 24 | 24 | |

1.1.2 Magic Rectangle of Order 3×7

Example 2. Let's consider a magic rectangle of order 3×7 given by

| | | | | | | | |
|----|----|----|----|----|----|----|----|
| 1 | 12 | 13 | 6 | 17 | 20 | 8 | 77 |
| 18 | 19 | 15 | 11 | 7 | 3 | 4 | 77 |
| 14 | 2 | 5 | 16 | 9 | 10 | 21 | 77 |
| 33 | 33 | 33 | 33 | 33 | 33 | 33 | |

1.1.3 Magic Rectangle of Order 3×9

Example 3. Let's consider a magic rectangle of order 3×9 given by

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 15 | 5 | 16 | 21 | 22 | 9 | 26 | 11 | 126 |
| 24 | 25 | 18 | 20 | 14 | 8 | 10 | 3 | 4 | 126 |
| 17 | 2 | 19 | 6 | 7 | 12 | 23 | 13 | 27 | 126 |
| 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 | |

1.1.4 Magic Rectangle of Order 3×11

Example 4. Let's consider a magic rectangle of order 3×11 given by

| | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|-----|
| 22 | 29 | 3 | 7 | 24 | 9 | 26 | 13 | 16 | 32 | 6 | 187 |
| 1 | 20 | 30 | 23 | 19 | 17 | 15 | 11 | 4 | 14 | 33 | 187 |
| 28 | 2 | 18 | 21 | 8 | 25 | 10 | 27 | 31 | 5 | 12 | 187 |
| 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | |

1.1.5 Magic Rectangle of Order 3×13

Example 5. Let's consider a magic rectangle of order 3×13 given by

| | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| 34 | 2 | 21 | 26 | 8 | 22 | 10 | 31 | 16 | 33 | 37 | 5 | 15 | 260 |
| 1 | 23 | 36 | 27 | 28 | 29 | 20 | 11 | 12 | 13 | 4 | 17 | 39 | 260 |
| 25 | 35 | 3 | 7 | 24 | 9 | 30 | 18 | 32 | 14 | 19 | 38 | 6 | 260 |
| 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | |

1.1.6 Magic Rectangle of Order 3×15

Example 6. Let's consider a magic rectangle of order 3×15 given by

| | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| 1 | 28 | 26 | 4 | 9 | 27 | 25 | 12 | 35 | 36 | 17 | 22 | 43 | 44 | 16 | 345 |
| 38 | 39 | 40 | 41 | 31 | 32 | 33 | 23 | 13 | 14 | 15 | 5 | 6 | 7 | 8 | 345 |
| 30 | 2 | 3 | 24 | 29 | 10 | 11 | 34 | 21 | 19 | 37 | 42 | 20 | 18 | 45 | 345 |
| 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | |

1.1.7 Magic Rectangle of Order 3×17

Example 7. Let's consider a magic rectangle of order 3×17 given by

| | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| 1 | 31 | 29 | 4 | 9 | 32 | 11 | 28 | 39 | 40 | 15 | 42 | 18 | 25 | 49 | 50 | 19 | 442 |
| 44 | 45 | 46 | 47 | 35 | 36 | 30 | 38 | 26 | 14 | 22 | 16 | 17 | 5 | 6 | 7 | 8 | 442 |
| 33 | 2 | 3 | 27 | 34 | 10 | 37 | 12 | 13 | 24 | 41 | 20 | 43 | 48 | 23 | 21 | 51 | 442 |
| 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | |

1.1.8 Magic Rectangle of Order 3×19

Example 8. Let's consider a magic rectangle of order 3×19 given by

| | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| 38 | 49 | 3 | 32 | 5 | 11 | 35 | 13 | 42 | 15 | 44 | 25 | 46 | 21 | 28 | 54 | 24 | 56 | 10 | 551 |
| 1 | 36 | 50 | 51 | 52 | 39 | 40 | 41 | 31 | 29 | 27 | 17 | 18 | 19 | 6 | 7 | 8 | 22 | 57 | 551 |
| 48 | 2 | 34 | 4 | 30 | 37 | 12 | 33 | 14 | 43 | 16 | 45 | 23 | 47 | 53 | 26 | 55 | 9 | 20 | 551 |
| 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | |

In a similar way we can write further cases.

1.2 Multiples of 5

1.2.1 Magic Rectangle of Order 5×7

Example 9. Let's consider a magic rectangle of order 5×7 given by

| | | | | | | | |
|----|----|----|----|----|----|----|-----|
| 15 | 26 | 13 | 6 | 20 | 24 | 22 | 126 |
| 1 | 33 | 27 | 11 | 31 | 19 | 4 | 126 |
| 28 | 2 | 29 | 18 | 7 | 34 | 8 | 126 |
| 32 | 17 | 5 | 25 | 9 | 3 | 35 | 126 |
| 14 | 12 | 16 | 30 | 23 | 10 | 21 | 126 |
| 90 | 90 | 90 | 90 | 90 | 90 | 90 | |

1.2.2 Magic Rectangle of Order 5×9

Example 10. Let's consider a magic rectangle of order 5×9 given by

| | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 20 | 43 | 19 | 21 | 7 | 12 | 9 | 31 | 45 | 207 |
| 17 | 22 | 18 | 38 | 14 | 40 | 10 | 44 | 4 | 207 |
| 35 | 33 | 5 | 16 | 23 | 30 | 41 | 13 | 11 | 207 |
| 42 | 2 | 36 | 6 | 32 | 8 | 28 | 24 | 29 | 207 |
| 1 | 15 | 37 | 34 | 39 | 25 | 27 | 3 | 26 | 207 |
| 115 | 115 | 115 | 115 | 115 | 115 | 115 | 115 | 115 | |

1.2.3 Magic Rectangle of Order 5×11

Example 11. Let's consider a magic rectangle of order 5×11 given by

| | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 23 | 51 | 27 | 7 | 19 | 9 | 15 | 13 | 53 | 36 | 55 | 308 |
| 50 | 25 | 18 | 21 | 26 | 17 | 48 | 11 | 4 | 54 | 34 | 308 |
| 44 | 42 | 40 | 24 | 46 | 28 | 10 | 32 | 16 | 14 | 12 | 308 |
| 22 | 2 | 52 | 45 | 8 | 39 | 30 | 35 | 38 | 31 | 6 | 308 |
| 1 | 20 | 3 | 43 | 41 | 47 | 37 | 49 | 29 | 5 | 33 | 308 |
| 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | |

1.2.4 Magic Rectangle of Order 5×13

Example 12. Let's consider a magic rectangle of order 5×13 given by

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 30 | 62 | 52 | 54 | 9 | 10 | 11 | 16 | 40 | 63 | 43 | 38 | 429 |
| 25 | 61 | 32 | 27 | 8 | 22 | 20 | 18 | 42 | 59 | 45 | 64 | 6 | 429 |
| 51 | 49 | 47 | 53 | 29 | 31 | 33 | 35 | 37 | 13 | 19 | 17 | 15 | 429 |
| 60 | 2 | 21 | 7 | 24 | 48 | 46 | 44 | 58 | 39 | 34 | 5 | 41 | 429 |
| 28 | 23 | 3 | 26 | 50 | 55 | 56 | 57 | 12 | 14 | 4 | 36 | 65 | 429 |
| 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | |

1.2.5 Magic Rectangle of Order 5×15

Example 13. Let's consider a magic rectangle of order 5×15 given by

| | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 31 | 33 | 3 | 71 | 61 | 27 | 11 | 12 | 13 | 19 | 47 | 52 | 41 | 74 | 75 | 570 |
| 30 | 28 | 70 | 37 | 32 | 34 | 25 | 23 | 21 | 66 | 67 | 72 | 50 | 7 | 8 | 570 |
| 60 | 58 | 56 | 54 | 59 | 62 | 36 | 38 | 40 | 14 | 17 | 22 | 20 | 18 | 16 | 570 |
| 68 | 69 | 26 | 4 | 9 | 10 | 55 | 53 | 51 | 42 | 44 | 39 | 6 | 48 | 46 | 570 |
| 1 | 2 | 35 | 24 | 29 | 57 | 63 | 64 | 65 | 49 | 15 | 5 | 73 | 43 | 45 | 570 |
| 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | |

1.2.6 Magic Rectangle of Order 5×17

Example 14. Let's consider a magic rectangle of order 5×17 given by

| | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 36 | 79 | 3 | 81 | 69 | 10 | 39 | 41 | 13 | 24 | 15 | 16 | 77 | 82 | 6 | 55 | 85 | 731 |
| 33 | 38 | 29 | 42 | 35 | 32 | 30 | 72 | 26 | 74 | 22 | 49 | 52 | 59 | 46 | 84 | 8 | 731 |
| 67 | 65 | 63 | 61 | 68 | 66 | 11 | 28 | 43 | 58 | 75 | 20 | 18 | 25 | 23 | 21 | 19 | 731 |
| 78 | 2 | 40 | 27 | 34 | 37 | 64 | 12 | 60 | 14 | 56 | 54 | 51 | 44 | 57 | 48 | 53 | 731 |
| 1 | 31 | 80 | 4 | 9 | 70 | 71 | 62 | 73 | 45 | 47 | 76 | 17 | 5 | 83 | 7 | 50 | 731 |
| 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | |

1.2.7 Magic Rectangle of Order 5×19

Example 15. Let's consider a magic rectangle of order 5×19 given by

| | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 39 | 87 | 43 | 4 | 90 | 77 | 12 | 13 | 31 | 15 | 27 | 25 | 18 | 85 | 91 | 7 | 93 | 60 | 95 | 912 |
| 86 | 41 | 34 | 32 | 47 | 40 | 35 | 33 | 46 | 29 | 82 | 17 | 54 | 59 | 66 | 51 | 8 | 94 | 58 | 912 |
| 76 | 74 | 72 | 70 | 68 | 75 | 73 | 44 | 80 | 48 | 16 | 52 | 23 | 21 | 28 | 26 | 24 | 22 | 20 | 912 |
| 38 | 2 | 88 | 45 | 30 | 37 | 42 | 79 | 14 | 67 | 50 | 63 | 61 | 56 | 49 | 64 | 62 | 55 | 10 | 912 |
| 1 | 36 | 3 | 89 | 5 | 11 | 78 | 71 | 69 | 81 | 65 | 83 | 84 | 19 | 6 | 92 | 53 | 9 | 57 | 912 |
| 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | |

1.3 Multiples of 7

1.3.1 Magic Rectangle of Order 7×9

Example 16. Let's consider a magic rectangle of order 7×9 given by

| | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 58 | 7 | 63 | 2 | 14 | 8 | 55 | 21 | 60 | 288 |
| 42 | 48 | 10 | 53 | 5 | 47 | 18 | 52 | 13 | 288 |
| 19 | 33 | 23 | 34 | 39 | 40 | 27 | 44 | 29 | 288 |
| 15 | 61 | 36 | 38 | 32 | 26 | 28 | 3 | 49 | 288 |
| 35 | 20 | 37 | 24 | 25 | 30 | 41 | 31 | 45 | 288 |
| 51 | 12 | 46 | 17 | 59 | 11 | 54 | 16 | 22 | 288 |
| 4 | 43 | 9 | 56 | 50 | 62 | 1 | 57 | 6 | 288 |
| 224 | 224 | 224 | 224 | 224 | 224 | 224 | 224 | 224 | |

1.3.2 Magic Rectangle of Order 7×11

Example 17. Let's consider a magic rectangle of order 7×11 given by

| | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 75 | 73 | 4 | 67 | 10 | 17 | 2 | 33 | 8 | 71 | 69 | 429 |
| 58 | 16 | 52 | 22 | 57 | 6 | 65 | 12 | 59 | 18 | 64 | 429 |
| 44 | 51 | 25 | 29 | 46 | 31 | 48 | 35 | 38 | 54 | 28 | 429 |
| 23 | 42 | 63 | 1 | 41 | 39 | 37 | 77 | 15 | 36 | 55 | 429 |
| 50 | 24 | 40 | 43 | 30 | 47 | 32 | 49 | 53 | 27 | 34 | 429 |
| 14 | 60 | 19 | 66 | 13 | 72 | 21 | 56 | 26 | 62 | 20 | 429 |
| 9 | 7 | 70 | 45 | 76 | 61 | 68 | 11 | 74 | 5 | 3 | 429 |
| 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | |

1.3.3 Magic Rectangle of Order 7×13

Example 18. Let's consider a magic rectangle of order 7×13 given by

| | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--|
| 84 | 9 | 82 | 1 | 90 | 3 | 20 | 11 | 80 | 39 | 88 | 5 | 86 | 598 | |
| 21 | 70 | 62 | 78 | 15 | 76 | 7 | 68 | 25 | 66 | 17 | 74 | 19 | 598 | |
| 60 | 28 | 47 | 52 | 34 | 48 | 36 | 57 | 42 | 59 | 63 | 31 | 41 | 598 | |
| 27 | 49 | 23 | 79 | 54 | 55 | 46 | 37 | 38 | 13 | 69 | 43 | 65 | 598 | |
| 51 | 61 | 29 | 33 | 50 | 35 | 56 | 44 | 58 | 40 | 45 | 64 | 32 | 598 | |
| 73 | 18 | 75 | 26 | 67 | 24 | 85 | 16 | 77 | 14 | 30 | 22 | 71 | 598 | |
| 6 | 87 | 4 | 53 | 12 | 81 | 72 | 89 | 2 | 91 | 10 | 83 | 8 | 598 | |
| 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | |

1.3.4 Magic Rectangle of Order 7×15

Example 19. Let's consider a magic rectangle of order 7×15 given by

| | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 102 | 99 | 6 | 101 | 15 | 92 | 13 | 23 | 3 | 104 | 1 | 35 | 10 | 97 | 94 | 795 |
| 79 | 22 | 70 | 20 | 76 | 29 | 78 | 8 | 88 | 17 | 90 | 26 | 81 | 24 | 87 | 795 |
| 31 | 58 | 56 | 34 | 39 | 57 | 55 | 42 | 65 | 66 | 47 | 52 | 73 | 74 | 46 | 795 |
| 68 | 69 | 85 | 11 | 61 | 62 | 63 | 53 | 43 | 44 | 45 | 95 | 21 | 37 | 38 | 795 |
| 60 | 32 | 33 | 54 | 59 | 40 | 41 | 64 | 51 | 49 | 67 | 72 | 50 | 48 | 75 | 795 |
| 19 | 82 | 25 | 80 | 16 | 89 | 18 | 98 | 28 | 77 | 30 | 86 | 36 | 84 | 27 | 795 |
| 12 | 9 | 96 | 71 | 105 | 2 | 103 | 83 | 93 | 14 | 91 | 5 | 100 | 7 | 4 | 795 |
| 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | |

1.3.5 Magic Rectangle of Order 7×17

Example 20. Let's consider a magic rectangle of order 7×17 given by

| | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| 110 | 11 | 108 | 13 | 119 | 2 | 117 | 4 | 26 | 14 | 105 | 16 | 103 | 39 | 114 | 7 | 112 | 1020 |
| 27 | 92 | 80 | 90 | 18 | 101 | 20 | 99 | 9 | 89 | 32 | 87 | 34 | 98 | 23 | 96 | 25 | 1020 |
| 35 | 65 | 63 | 38 | 43 | 66 | 45 | 62 | 73 | 74 | 49 | 76 | 52 | 59 | 83 | 84 | 53 | 1020 |
| 78 | 79 | 29 | 115 | 69 | 70 | 64 | 72 | 60 | 48 | 56 | 50 | 51 | 5 | 91 | 41 | 42 | 1020 |
| 67 | 36 | 37 | 61 | 68 | 44 | 71 | 46 | 47 | 58 | 75 | 54 | 77 | 82 | 57 | 55 | 85 | 1020 |
| 95 | 24 | 97 | 22 | 86 | 33 | 88 | 31 | 111 | 21 | 100 | 19 | 102 | 30 | 40 | 28 | 93 | 1020 |
| 8 | 113 | 6 | 81 | 17 | 104 | 15 | 106 | 94 | 116 | 3 | 118 | 1 | 107 | 12 | 109 | 10 | 1020 |
| 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | |

1.3.6 Magic Rectangle of Order 7×19

Example 21. Let's consider a magic rectangle of order 7×19 given by

| | | | | | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| 129 | 125 | 8 | 127 | 6 | 115 | 18 | 117 | 16 | 29 | 4 | 131 | 2 | 133 | 14 | 45 | 12 | 123 | 119 | 1273 |
| 100 | 28 | 88 | 26 | 109 | 38 | 97 | 36 | 99 | 10 | 111 | 22 | 113 | 20 | 101 | 32 | 103 | 30 | 110 | 1273 |
| 76 | 87 | 41 | 70 | 43 | 49 | 73 | 51 | 80 | 53 | 82 | 63 | 84 | 59 | 66 | 92 | 62 | 94 | 48 | 1273 |
| 39 | 74 | 107 | 13 | 90 | 77 | 78 | 79 | 69 | 67 | 65 | 55 | 56 | 57 | 44 | 121 | 27 | 60 | 95 | 1273 |
| 86 | 40 | 72 | 42 | 68 | 75 | 50 | 71 | 52 | 81 | 54 | 83 | 61 | 85 | 91 | 64 | 93 | 47 | 58 | 1273 |
| 24 | 104 | 31 | 102 | 33 | 114 | 21 | 112 | 23 | 124 | 35 | 98 | 37 | 96 | 25 | 108 | 46 | 106 | 34 | 1273 |
| 15 | 11 | 122 | 89 | 120 | 1 | 132 | 3 | 130 | 105 | 118 | 17 | 116 | 19 | 128 | 7 | 126 | 9 | 5 | 1273 |
| 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | |

1.4 Multiples of 4

1.4.1 Magic Rectangle of Order 4×6

Example 22. Let's consider a magic rectangle of order 4×6 given by

| | | | | | | |
|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 22 | 23 | 24 | 75 |
| 19 | 20 | 21 | 4 | 5 | 6 | 75 |
| 18 | 17 | 16 | 9 | 8 | 7 | 75 |
| 12 | 11 | 10 | 15 | 14 | 13 | 75 |
| 50 | 50 | 50 | 50 | 50 | 50 | |

1.4.2 Magic Rectangle of Order 4×8

Example 23. Let's consider a magic rectangle of order 4×8 given by

| | | | | | | | | |
|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 29 | 30 | 31 | 32 | 132 |
| 25 | 26 | 27 | 28 | 5 | 6 | 7 | 8 | 132 |
| 24 | 23 | 22 | 21 | 12 | 11 | 10 | 9 | 132 |
| 16 | 15 | 14 | 13 | 20 | 19 | 18 | 17 | 132 |
| 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 | |

This magic rectangle can also be written as two pandiagonal equal sum magic squares of order 4. See below

Example 24. Let's consider a magic rectangle of order 4×8 given by

| | | | | | | | | |
|----|----|----|----|----|----|----|----|-----|
| 7 | 28 | 1 | 30 | 15 | 20 | 9 | 22 | 132 |
| 2 | 29 | 8 | 27 | 10 | 21 | 16 | 19 | 132 |
| 32 | 3 | 26 | 5 | 24 | 11 | 18 | 13 | 132 |
| 25 | 6 | 31 | 4 | 17 | 14 | 23 | 12 | 132 |
| 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 | |

1.4.3 Magic Rectangle of Order 4×10

Example 25. Let's consider a magic rectangle of order 4×10 given by

| | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 36 | 37 | 38 | 39 | 40 | 205 |
| 31 | 32 | 33 | 34 | 35 | 6 | 7 | 8 | 9 | 10 | 205 |
| 30 | 29 | 28 | 27 | 26 | 15 | 14 | 13 | 12 | 11 | 205 |
| 20 | 19 | 18 | 17 | 16 | 25 | 24 | 23 | 22 | 21 | 205 |
| 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | |

1.4.4 Magic Rectangle of Order 4×14

Example 26. Let's consider a magic rectangle of order 4×14 given by

| | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 399 |
| 43 | 44 | 45 | 46 | 47 | 48 | 49 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 399 |
| 42 | 41 | 40 | 39 | 38 | 37 | 36 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 399 |
| 28 | 27 | 26 | 25 | 24 | 23 | 22 | 35 | 34 | 33 | 32 | 31 | 30 | 29 | 399 |
| 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | |

1.5 Multiples of 6

Below are few examples of magic rectangles multiples of 6.

1.5.1 Magic Rectangle of Order 6×10

Example 27. Let's consider a magic rectangle of order 6×10 given by

| | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 55 | 54 | 12 | 13 | 47 | 46 | 45 | 14 | 18 | 305 |
| 2 | 56 | 53 | 11 | 42 | 20 | 40 | 21 | 23 | 37 | 305 |
| 3 | 57 | 52 | 10 | 36 | 35 | 27 | 28 | 32 | 25 | 305 |
| 58 | 4 | 9 | 51 | 30 | 26 | 33 | 34 | 29 | 31 | 305 |
| 59 | 5 | 8 | 50 | 19 | 38 | 22 | 39 | 41 | 24 | 305 |
| 60 | 6 | 7 | 49 | 43 | 17 | 15 | 16 | 44 | 48 | 305 |
| 183 | 183 | 183 | 183 | 183 | 183 | 183 | 183 | 183 | 183 | |

1.5.2 Magic Rectangle of Order 6×14

Example 28. Let's consider a magic rectangle of order 6×14 given by

| | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 79 | 78 | 12 | 13 | 67 | 66 | 24 | 25 | 59 | 58 | 57 | 26 | 30 | 595 |
| 2 | 80 | 77 | 11 | 14 | 68 | 65 | 23 | 54 | 32 | 52 | 33 | 35 | 49 | 595 |
| 3 | 81 | 76 | 10 | 15 | 69 | 64 | 22 | 48 | 47 | 39 | 40 | 44 | 37 | 595 |
| 82 | 4 | 9 | 75 | 70 | 16 | 21 | 63 | 42 | 38 | 45 | 46 | 41 | 43 | 595 |
| 83 | 5 | 8 | 74 | 71 | 17 | 20 | 62 | 31 | 50 | 34 | 51 | 53 | 36 | 595 |
| 84 | 6 | 7 | 73 | 72 | 18 | 19 | 61 | 55 | 29 | 27 | 28 | 56 | 60 | 595 |
| 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | |

2 Odd Order Magic Squares

Below are few odd magic squares constructed with blocks of unequal sums of **magic rectangles**.

2.1 Magic Square of Order 15×15

Example 29. A magic square of order 15×15 constructed based on blocks of magic rectangles of order 3×5 is given by

| | | | | | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | | | | | | | | | | | | | | 1695 |
| 209 | 205 | 199 | 200 | 202 | 14 | 10 | 4 | 5 | 7 | 134 | 130 | 124 | 125 | 127 | 1695 |
| 196 | 198 | 203 | 208 | 210 | 1 | 3 | 8 | 13 | 15 | 121 | 123 | 128 | 133 | 135 | 1695 |
| 204 | 206 | 207 | 201 | 197 | 9 | 11 | 12 | 6 | 2 | 129 | 131 | 132 | 126 | 122 | 1695 |
| 149 | 145 | 139 | 140 | 142 | 44 | 40 | 34 | 35 | 37 | 164 | 160 | 154 | 155 | 157 | 1695 |
| 136 | 138 | 143 | 148 | 150 | 31 | 33 | 38 | 43 | 45 | 151 | 153 | 158 | 163 | 165 | 1695 |
| 144 | 146 | 147 | 141 | 137 | 39 | 41 | 42 | 36 | 32 | 159 | 161 | 162 | 156 | 152 | 1695 |
| 59 | 55 | 49 | 50 | 52 | 119 | 115 | 109 | 110 | 112 | 179 | 175 | 169 | 170 | 172 | 1695 |
| 46 | 48 | 53 | 58 | 60 | 106 | 108 | 113 | 118 | 120 | 166 | 168 | 173 | 178 | 180 | 1695 |
| 54 | 56 | 57 | 51 | 47 | 114 | 116 | 117 | 111 | 107 | 174 | 176 | 177 | 171 | 167 | 1695 |
| 74 | 70 | 64 | 65 | 67 | 194 | 190 | 184 | 185 | 187 | 89 | 85 | 79 | 80 | 82 | 1695 |
| 61 | 63 | 68 | 73 | 75 | 181 | 183 | 188 | 193 | 195 | 76 | 78 | 83 | 88 | 90 | 1695 |
| 69 | 71 | 72 | 66 | 62 | 189 | 191 | 192 | 186 | 182 | 84 | 86 | 87 | 81 | 77 | 1695 |
| 104 | 100 | 94 | 95 | 97 | 224 | 220 | 214 | 215 | 217 | 29 | 25 | 19 | 20 | 22 | 1695 |
| 91 | 93 | 98 | 103 | 105 | 211 | 213 | 218 | 223 | 225 | 16 | 18 | 23 | 28 | 30 | 1695 |
| 99 | 101 | 102 | 96 | 92 | 219 | 221 | 222 | 216 | 212 | 24 | 26 | 27 | 21 | 17 | 1695 |
| 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 |

In this case the magic sum is $S_{15 \times 15} := 1695$. The magic rectangles of order 3×5 are of **different magic sums**.

2.2 Magic Square of Order 21×21

Example 30. A magic square of order 21×21 constructed based on blocks of magic rectangles of order 3×7 is given by

| | | | | | | | | | | | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | | | | | | | | | | | | | | | | | | | 4641 | |
| 1 | 12 | 13 | 6 | 17 | 20 | 8 | 358 | 369 | 370 | 363 | 374 | 377 | 365 | 274 | 285 | 286 | 279 | 290 | 293 | 281 | 4641 |
| 18 | 19 | 15 | 11 | 7 | 3 | 4 | 375 | 376 | 372 | 368 | 364 | 360 | 361 | 291 | 292 | 288 | 284 | 280 | 276 | 277 | 4641 |
| 14 | 2 | 5 | 16 | 9 | 10 | 21 | 371 | 359 | 362 | 373 | 366 | 367 | 378 | 287 | 275 | 278 | 289 | 282 | 283 | 294 | 4641 |
| 232 | 243 | 244 | 237 | 248 | 251 | 239 | 379 | 390 | 391 | 384 | 395 | 398 | 386 | 22 | 33 | 34 | 27 | 38 | 41 | 29 | 4641 |
| 249 | 250 | 246 | 242 | 238 | 234 | 235 | 396 | 397 | 393 | 389 | 385 | 381 | 382 | 39 | 40 | 36 | 32 | 28 | 24 | 25 | 4641 |
| 245 | 233 | 236 | 247 | 240 | 241 | 252 | 392 | 380 | 383 | 394 | 387 | 388 | 399 | 35 | 23 | 26 | 37 | 30 | 31 | 42 | 4641 |
| 253 | 264 | 265 | 258 | 269 | 272 | 260 | 295 | 306 | 307 | 300 | 311 | 314 | 302 | 85 | 96 | 97 | 90 | 101 | 104 | 92 | 4641 |
| 270 | 271 | 267 | 263 | 259 | 255 | 256 | 312 | 313 | 309 | 305 | 301 | 297 | 298 | 102 | 103 | 99 | 95 | 91 | 87 | 88 | 4641 |
| 266 | 254 | 257 | 268 | 261 | 262 | 273 | 308 | 296 | 299 | 310 | 303 | 304 | 315 | 98 | 86 | 89 | 100 | 93 | 94 | 105 | 4641 |
| 106 | 117 | 118 | 111 | 122 | 125 | 113 | 211 | 222 | 223 | 216 | 227 | 230 | 218 | 316 | 327 | 328 | 321 | 332 | 335 | 323 | 4641 |
| 123 | 124 | 120 | 116 | 112 | 108 | 109 | 228 | 229 | 225 | 221 | 217 | 213 | 214 | 333 | 334 | 330 | 326 | 322 | 318 | 319 | 4641 |
| 119 | 107 | 110 | 121 | 114 | 115 | 126 | 224 | 212 | 215 | 226 | 219 | 220 | 231 | 329 | 317 | 320 | 331 | 324 | 325 | 336 | 4641 |
| 337 | 348 | 349 | 342 | 353 | 356 | 344 | 127 | 138 | 139 | 132 | 143 | 146 | 134 | 169 | 180 | 181 | 174 | 185 | 188 | 176 | 4641 |
| 354 | 355 | 351 | 347 | 343 | 339 | 340 | 144 | 145 | 141 | 137 | 133 | 129 | 130 | 186 | 187 | 183 | 179 | 175 | 171 | 172 | 4641 |
| 350 | 338 | 341 | 352 | 345 | 346 | 357 | 140 | 128 | 131 | 142 | 135 | 136 | 147 | 182 | 170 | 173 | 184 | 177 | 178 | 189 | 4641 |
| 400 | 411 | 412 | 405 | 416 | 419 | 407 | 43 | 54 | 55 | 48 | 59 | 62 | 50 | 190 | 201 | 202 | 195 | 206 | 209 | 197 | 4641 |
| 417 | 418 | 414 | 410 | 406 | 402 | 403 | 60 | 61 | 57 | 53 | 49 | 45 | 46 | 207 | 208 | 204 | 200 | 196 | 192 | 193 | 4641 |
| 413 | 401 | 404 | 415 | 408 | 409 | 420 | 56 | 44 | 47 | 58 | 51 | 52 | 63 | 203 | 191 | 194 | 205 | 198 | 199 | 210 | 4641 |
| 148 | 159 | 160 | 153 | 164 | 167 | 155 | 64 | 75 | 76 | 69 | 80 | 83 | 71 | 421 | 432 | 433 | 426 | 437 | 440 | 428 | 4641 |
| 165 | 166 | 162 | 158 | 154 | 150 | 151 | 81 | 82 | 78 | 74 | 70 | 66 | 67 | 438 | 439 | 435 | 431 | 427 | 423 | 424 | 4641 |
| 161 | 149 | 152 | 163 | 156 | 157 | 168 | 77 | 65 | 68 | 79 | 72 | 73 | 84 | 434 | 422 | 425 | 436 | 429 | 430 | 441 | 4641 |
| 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | |

In this case the magic sum is $S_{21 \times 21} := 4641$. The magic rectangles of order 3×7 are of *different magic sums*.

2.3 Magic Square of Order 27×27

Example 31. A magic square of order 27×27 constructed based on blocks of magic rectangles of order 3×9 is given by

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 505 | 516 | 503 | 496 | 510 | 514 | 512 | 15 | 26 | 13 | 6 | 20 | 24 | 22 | 960 | 971 | 958 | 951 | 965 | 969 | 967 | 1100 | 1111 | 1098 | 1091 | 1105 | 1109 | 1107 | 470 | 481 | 468 | 461 | 475 | 479 | 477 | 21455 |
| 491 | 523 | 517 | 501 | 521 | 509 | 494 | 1 | 33 | 27 | 11 | 31 | 19 | 4 | 946 | 978 | 972 | 956 | 976 | 964 | 949 | 1086 | 1118 | 1112 | 1096 | 1116 | 1104 | 1089 | 456 | 488 | 482 | 466 | 486 | 474 | 459 | 21455 |
| 518 | 492 | 519 | 508 | 497 | 524 | 498 | 28 | 2 | 29 | 18 | 7 | 34 | 8 | 973 | 947 | 974 | 963 | 952 | 979 | 953 | 1113 | 1087 | 1114 | 1103 | 1092 | 1119 | 1093 | 483 | 457 | 484 | 473 | 462 | 489 | 463 | 21455 |
| 522 | 507 | 495 | 515 | 499 | 493 | 525 | 32 | 17 | 5 | 25 | 9 | 3 | 35 | 977 | 962 | 950 | 970 | 954 | 948 | 980 | 1117 | 1102 | 1090 | 1110 | 1094 | 1088 | 1120 | 487 | 472 | 460 | 480 | 464 | 458 | 490 | 21455 |
| 504 | 502 | 506 | 520 | 513 | 500 | 511 | 14 | 12 | 16 | 30 | 23 | 10 | 21 | 959 | 957 | 961 | 975 | 968 | 955 | 966 | 1099 | 1097 | 1101 | 1115 | 1108 | 1095 | 1106 | 469 | 467 | 471 | 485 | 478 | 465 | 476 | 21455 |
| 890 | 901 | 888 | 881 | 895 | 899 | 897 | 1135 | 1146 | 1133 | 1126 | 1140 | 1144 | 1142 | 50 | 61 | 48 | 41 | 55 | 59 | 57 | 575 | 586 | 573 | 566 | 580 | 584 | 582 | 400 | 411 | 398 | 391 | 405 | 409 | 407 | 21455 |
| 876 | 908 | 902 | 886 | 906 | 894 | 879 | 1121 | 1153 | 1147 | 1131 | 1151 | 1139 | 1124 | 36 | 68 | 62 | 46 | 66 | 54 | 39 | 561 | 593 | 587 | 571 | 591 | 579 | 564 | 386 | 418 | 412 | 396 | 416 | 404 | 389 | 21455 |
| 903 | 877 | 904 | 893 | 882 | 909 | 883 | 1148 | 1122 | 1149 | 1138 | 1127 | 1154 | 1128 | 63 | 37 | 64 | 53 | 42 | 69 | 43 | 588 | 562 | 589 | 578 | 567 | 594 | 568 | 413 | 387 | 414 | 403 | 392 | 419 | 393 | 21455 |
| 907 | 892 | 880 | 900 | 884 | 878 | 910 | 1152 | 1137 | 1125 | 1145 | 1129 | 1123 | 1155 | 67 | 52 | 40 | 60 | 44 | 38 | 70 | 592 | 577 | 565 | 585 | 569 | 563 | 595 | 417 | 402 | 390 | 410 | 394 | 388 | 420 | 21455 |
| 889 | 887 | 891 | 905 | 898 | 885 | 896 | 1134 | 1132 | 1136 | 1150 | 1143 | 1130 | 1141 | 49 | 47 | 51 | 65 | 58 | 45 | 56 | 574 | 572 | 576 | 590 | 583 | 570 | 581 | 399 | 397 | 401 | 415 | 408 | 395 | 406 | 21455 |
| 435 | 446 | 433 | 426 | 440 | 444 | 442 | 925 | 936 | 923 | 916 | 930 | 934 | 932 | 995 | 1006 | 993 | 986 | 1000 | 1004 | 1002 | 155 | 166 | 153 | 146 | 160 | 164 | 162 | 540 | 551 | 538 | 531 | 545 | 549 | 547 | 21455 |
| 421 | 453 | 447 | 431 | 451 | 439 | 424 | 911 | 943 | 937 | 921 | 941 | 929 | 914 | 981 | 1013 | 1007 | 991 | 1011 | 999 | 984 | 141 | 173 | 167 | 151 | 171 | 159 | 144 | 526 | 558 | 552 | 536 | 556 | 544 | 529 | 21455 |
| 448 | 422 | 449 | 438 | 427 | 454 | 428 | 938 | 912 | 939 | 928 | 917 | 944 | 918 | 1008 | 982 | 1009 | 998 | 987 | 1014 | 988 | 168 | 142 | 169 | 158 | 147 | 174 | 148 | 553 | 527 | 554 | 543 | 532 | 559 | 533 | 21455 |
| 452 | 437 | 425 | 445 | 429 | 423 | 455 | 942 | 927 | 915 | 935 | 919 | 913 | 945 | 1012 | 997 | 985 | 1005 | 989 | 983 | 1015 | 172 | 157 | 145 | 165 | 149 | 143 | 175 | 557 | 542 | 530 | 550 | 534 | 528 | 560 | 21455 |
| 434 | 432 | 436 | 450 | 443 | 430 | 441 | 924 | 922 | 926 | 940 | 933 | 920 | 931 | 994 | 992 | 996 | 1010 | 1003 | 990 | 1001 | 154 | 152 | 156 | 170 | 163 | 150 | 161 | 539 | 537 | 541 | 555 | 548 | 535 | 546 | 21455 |
| 190 | 201 | 188 | 181 | 195 | 199 | 197 | 365 | 376 | 363 | 356 | 370 | 374 | 372 | 610 | 621 | 608 | 601 | 615 | 619 | 617 | 855 | 866 | 853 | 846 | 860 | 864 | 862 | 1030 | 1041 | 1028 | 1021 | 1035 | 1039 | 1037 | 21455 |
| 176 | 208 | 202 | 186 | 206 | 194 | 179 | 351 | 383 | 377 | 361 | 381 | 369 | 354 | 596 | 628 | 622 | 606 | 626 | 614 | 599 | 841 | 873 | 867 | 851 | 871 | 859 | 844 | 1016 | 1048 | 1042 | 1026 | 1046 | 1034 | 1019 | 21455 |
| 203 | 177 | 204 | 193 | 182 | 209 | 183 | 378 | 352 | 379 | 368 | 357 | 384 | 358 | 623 | 597 | 624 | 613 | 602 | 629 | 603 | 868 | 842 | 869 | 858 | 847 | 874 | 848 | 1043 | 1017 | 1044 | 1033 | 1022 | 1049 | 1023 | 21455 |
| 207 | 192 | 180 | 200 | 184 | 178 | 210 | 382 | 367 | 355 | 375 | 359 | 353 | 385 | 627 | 612 | 600 | 620 | 604 | 598 | 630 | 872 | 857 | 845 | 865 | 849 | 843 | 875 | 1047 | 1032 | 1020 | 1040 | 1024 | 1018 | 1050 | 21455 |
| 189 | 187 | 191 | 205 | 198 | 185 | 196 | 364 | 362 | 366 | 380 | 373 | 360 | 371 | 609 | 607 | 611 | 625 | 618 | 605 | 616 | 854 | 852 | 856 | 870 | 863 | 850 | 861 | 1029 | 1027 | 1031 | 1045 | 1038 | 1025 | 1036 | 21455 |
| 680 | 691 | 678 | 671 | 685 | 689 | 687 | 1065 | 1076 | 1063 | 1056 | 1070 | 1074 | 1072 | 225 | 236 | 223 | 216 | 230 | 234 | 232 | 295 | 306 | 293 | 286 | 300 | 304 | 302 | 785 | 796 | 783 | 776 | 790 | 794 | 792 | 21455 |
| 666 | 698 | 692 | 676 | 696 | 684 | 669 | 1051 | 1083 | 1077 | 1061 | 1081 | 1069 | 1054 | 211 | 243 | 237 | 221 | 241 | 229 | 214 | 281 | 313 | 307 | 291 | 311 | 299 | 284 | 771 | 803 | 797 | 781 | 801 | 789 | 774 | 21455 |
| 693 | 667 | 694 | 683 | 672 | 699 | 673 | 1078 | 1052 | 1079 | 1068 | 1057 | 1084 | 1058 | 238 | 212 | 239 | 228 | 217 | 244 | 218 | 308 | 282 | 309 | 298 | 287 | 314 | 288 | 798 | 772 | 799 | 788 | 777 | 804 | 778 | 21455 |
| 697 | 682 | 670 | 690 | 674 | 668 | 700 | 1082 | 1067 | 1055 | 1075 | 1059 | 1053 | 1085 | 242 | 227 | 215 | 235 | 219 | 213 | 245 | 312 | 297 | 285 | 305 | 289 | 283 | 315 | 802 | 787 | 775 | 795 | 779 | 773 | 805 | 21455 |
| 679 | 677 | 681 | 695 | 688 | 675 | 686 | 1064 | 1062 | 1066 | 1080 | 1073 | 1060 | 1071 | 224 | 222 | 226 | 240 | 233 | 220 | 231 | 294 | 292 | 296 | 310 | 303 | 290 | 301 | 784 | 782 | 786 | 800 | 793 | 780 | 791 | 21455 |
| 820 | 831 | 818 | 811 | 825 | 829 | 827 | 645 | 656 | 643 | 636 | 650 | 654 | 652 | 1170 | 1181 | 1168 | 1161 | 1175 | 1179 | 1177 | 85 | 96 | 83 | 76 | 90 | 94 | 92 | 330 | 341 | 328 | 321 | 335 | 339 | 337 | 21455 |
| 806 | 838 | 832 | 816 | 836 | 824 | 809 | 631 | 663 | 657 | 641 | 661 | 649 | 634 | 1156 | 1188 | 1182 | 1166 | 1186 | 1174 | 1159 | 71 | 103 | 97 | 81 | 101 | 89 | 74 | 316 | 348 | 342 | 326 | 346 | 334 | 319 | 21455 |
| 833 | 807 | 834 | 823 | 812 | 839 | 813 | 658 | 632 | 659 | 648 | 637 | 664 | 638 | 1183 | 1157 | 1184 | 1173 | 1162 | 1189 | 1163 | 98 | 72 | 99 | 88 | 77 | 104 | 78 | 343 | 317 | 344 | 333 | 322 | 349 | 323 | 21455 |
| 837 | 822 | 810 | 830 | 814 | 808 | 840 | 662 | 647 | 635 | 655 | 639 | 633 | 665 | 1187 | 1172 | 1160 | 1180 | 1164 | 1158 | 1190 | 102 | 87 | 75 | 95 | 79 | 73 | 105 | 347 | 332 | 320 | 340 | 324 | 318 | 350 | 21455 |
| 819 | 817 | 821 | 835 | 828 | 815 | 826 | 644 | 642 | 646 | 660 | 653 | 640 | 651 | 1169 | 1167 | 1171 | 1185 | 1178 | 1165 | 1176 | 84 | 82 | 86 | 100 | 93 | 80 | 91 | 329 | 327 | 331 | 345 | 338 | 325 | 336 | 21455 |
| 750 | 761 | 748 | 741 | 755 | 759 | 757 | 120 | 131 | 118 | 111 | 125 | 129 | 127 | 260 | 271 | 258 | 251 | 265 | 269 | 267 | 1205 | 1216 | 1203 | 1196 | 1210 | 1214 | 1212 | 715 | 726 | 713 | 706 | 720 | 724 | 722 | 21455 |
| 736 | 768 | 762 | 746 | 766 | 754 | 739 | 106 | 138 | 132 | 116 | 136 | 124 | 109 | 246 | 278 | 272 | 256 | 276 | 264 | 249 | 1191 | 1223 | 1217 | 1201 | 1221 | 1209 | 1194 | 701 | 733 | 727 | 711 | 731 | 719 | 704 | 21455 |
| 763 | 737 | 764 | 753 | 742 | 769 | 743 | 133 | 107 | 134 | 123 | 112 | 139 | 113 | 273 | 247 | 274 | 263 | 252 | 279 | 253 | 1218 | 1192 | 1219 | 1208 | 1197 | 1224 | 1198 | 728 | 702 | 729 | 718 | 707 | 734 | 708 | 21455 |
| 767 | 752 | 740 | 760 | 744 | 738 | 770 | 137 | 122 | 110 | 130 | 114 | 108 | 140 | 277 | 262 | 250 | 270 | 254 | 248 | 280 | 1222 | 1207 | 1195 | 1215 | 1199 | 1193 | 1225 | 732 | 717 | 705 | 725 | 709 | 703 | 735 | 21455 |
| 749 | 747 | 751 | 765 | 758 | 745 | 756 | 119 | 117 | 121 | 135 | 128 | 115 | 126 | 259 | 257 | 261 | 275 | 268 | 255 | 266 | 1204 | 1202 | 1206 | 1220 | 1213 | 1200 | 1211 | 714 | 712 | 716 | 730 | 723 | 710 | 721 | 21455 |
| 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 |

In this case the magic sum is $S_{35 \times 35} := 21455$. The magic rectangles of order 5×7 are of different magic sums.

2.6 Magic Square of Order 39×39

Example 34. A magic square of order 39×39 constructed based on blocks of magic rectangles of order 3×13 is given by

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|
| 875 | 898 | 874 | 876 | 862 | 867 | 864 | 886 | 900 | 740 | 763 | 739 | 741 | 727 | 732 | 729 | 751 | 765 | 1550 | 1573 | 1549 | 1551 | 1537 | 1542 | 1539 | 1561 | 1575 | 1865 | 1888 | 1864 | 1866 | 1852 | 1857 | 1854 | 1876 | 1890 | 20 | 43 | 19 | 21 | 7 | 12 | 9 | 31 | 45 | 45585 |
| 872 | 877 | 873 | 893 | 869 | 895 | 865 | 899 | 859 | 737 | 742 | 738 | 758 | 734 | 760 | 730 | 764 | 724 | 1547 | 1552 | 1548 | 1568 | 1544 | 1570 | 1540 | 1574 | 1534 | 1862 | 1867 | 1863 | 1883 | 1859 | 1885 | 1855 | 1889 | 1849 | 17 | 22 | 18 | 38 | 14 | 40 | 10 | 44 | 4 | 45585 |
| 890 | 888 | 860 | 871 | 878 | 885 | 896 | 868 | 866 | 755 | 753 | 725 | 736 | 743 | 750 | 761 | 733 | 731 | 1565 | 1563 | 1535 | 1546 | 1553 | 1560 | 1571 | 1543 | 1541 | 1880 | 1878 | 1850 | 1861 | 1868 | 1875 | 1886 | 1858 | 1856 | 35 | 33 | 5 | 16 | 23 | 30 | 41 | 13 | 11 | 45585 |
| 897 | 857 | 891 | 861 | 887 | 863 | 883 | 879 | 884 | 762 | 722 | 756 | 726 | 752 | 728 | 748 | 744 | 749 | 1572 | 1532 | 1566 | 1536 | 1562 | 1538 | 1558 | 1554 | 1559 | 1887 | 1847 | 1881 | 1851 | 1877 | 1853 | 1873 | 1869 | 1874 | 42 | 2 | 36 | 6 | 32 | 8 | 28 | 24 | 29 | 45585 |
| 856 | 870 | 892 | 889 | 894 | 880 | 882 | 858 | 881 | 721 | 735 | 757 | 754 | 759 | 745 | 747 | 723 | 746 | 1531 | 1545 | 1567 | 1564 | 1569 | 1555 | 1557 | 1533 | 1556 | 1846 | 1860 | 1882 | 1879 | 1884 | 1870 | 1872 | 1848 | 1871 | 1 | 15 | 37 | 34 | 39 | 25 | 27 | 3 | 26 | 45585 |
| 1910 | 1933 | 1909 | 1911 | 1897 | 1902 | 1899 | 1921 | 1935 | 965 | 988 | 964 | 966 | 952 | 957 | 954 | 976 | 990 | 1460 | 1483 | 1459 | 1461 | 1447 | 1452 | 1449 | 1471 | 1485 | 65 | 88 | 64 | 66 | 52 | 57 | 54 | 76 | 90 | 650 | 673 | 649 | 651 | 637 | 642 | 639 | 661 | 675 | 45585 |
| 1907 | 1912 | 1908 | 1928 | 1904 | 1930 | 1900 | 1934 | 1894 | 962 | 967 | 963 | 983 | 959 | 985 | 955 | 989 | 949 | 1457 | 1462 | 1458 | 1478 | 1454 | 1480 | 1450 | 1484 | 1444 | 62 | 67 | 63 | 83 | 59 | 85 | 55 | 89 | 49 | 647 | 652 | 648 | 668 | 644 | 670 | 640 | 674 | 634 | 45585 |
| 1925 | 1923 | 1895 | 1906 | 1913 | 1920 | 1931 | 1903 | 1901 | 980 | 978 | 950 | 961 | 968 | 975 | 986 | 958 | 956 | 1475 | 1473 | 1445 | 1456 | 1463 | 1470 | 1481 | 1453 | 1451 | 80 | 78 | 50 | 61 | 68 | 75 | 86 | 58 | 56 | 665 | 663 | 635 | 646 | 653 | 660 | 671 | 643 | 641 | 45585 |
| 1932 | 1892 | 1926 | 1896 | 1922 | 1898 | 1918 | 1914 | 1919 | 987 | 947 | 981 | 951 | 977 | 953 | 973 | 969 | 974 | 1482 | 1442 | 1476 | 1446 | 1472 | 1448 | 1468 | 1464 | 1469 | 87 | 47 | 81 | 51 | 77 | 53 | 73 | 69 | 74 | 672 | 632 | 666 | 636 | 662 | 638 | 658 | 654 | 659 | 45585 |
| 1891 | 1905 | 1927 | 1924 | 1929 | 1915 | 1917 | 1893 | 1916 | 946 | 960 | 982 | 979 | 984 | 970 | 972 | 948 | 971 | 1441 | 1455 | 1477 | 1474 | 1479 | 1465 | 1467 | 1443 | 1466 | 46 | 60 | 82 | 79 | 84 | 70 | 72 | 48 | 71 | 631 | 645 | 667 | 664 | 669 | 655 | 657 | 633 | 656 | 45585 |
| 830 | 853 | 829 | 831 | 817 | 822 | 819 | 841 | 855 | 785 | 808 | 784 | 786 | 772 | 777 | 774 | 796 | 810 | 200 | 223 | 199 | 201 | 187 | 192 | 189 | 211 | 225 | 1595 | 1618 | 1594 | 1596 | 1582 | 1587 | 1584 | 1606 | 1620 | 1640 | 1663 | 1639 | 1641 | 1627 | 1632 | 1629 | 1651 | 1665 | 45585 |
| 827 | 832 | 828 | 848 | 824 | 850 | 820 | 854 | 814 | 782 | 787 | 783 | 803 | 779 | 805 | 775 | 809 | 769 | 197 | 202 | 198 | 218 | 194 | 220 | 190 | 224 | 184 | 1592 | 1597 | 1593 | 1613 | 1589 | 1615 | 1585 | 1619 | 1579 | 1637 | 1642 | 1638 | 1658 | 1634 | 1660 | 1630 | 1664 | 1624 | 45585 |
| 845 | 843 | 815 | 826 | 833 | 840 | 851 | 823 | 821 | 800 | 798 | 770 | 781 | 788 | 795 | 806 | 778 | 776 | 215 | 213 | 185 | 196 | 203 | 210 | 221 | 193 | 191 | 1610 | 1608 | 1580 | 1591 | 1598 | 1605 | 1616 | 1588 | 1586 | 1655 | 1653 | 1625 | 1636 | 1643 | 1650 | 1661 | 1633 | 1631 | 45585 |
| 852 | 812 | 846 | 816 | 842 | 818 | 838 | 834 | 839 | 807 | 767 | 801 | 771 | 797 | 773 | 793 | 789 | 794 | 222 | 182 | 216 | 186 | 212 | 188 | 208 | 204 | 209 | 1617 | 1577 | 1611 | 1581 | 1607 | 1583 | 1603 | 1599 | 1604 | 1662 | 1622 | 1656 | 1626 | 1652 | 1628 | 1648 | 1644 | 1649 | 45585 |
| 811 | 825 | 847 | 844 | 849 | 835 | 837 | 813 | 836 | 766 | 780 | 802 | 799 | 804 | 790 | 792 | 768 | 791 | 181 | 195 | 217 | 214 | 219 | 205 | 207 | 183 | 206 | 1576 | 1590 | 1612 | 1609 | 1614 | 1600 | 1602 | 1578 | 1601 | 1621 | 1635 | 1657 | 1654 | 1659 | 1645 | 1647 | 1623 | 1646 | 45585 |
| 920 | 943 | 919 | 921 | 907 | 912 | 909 | 931 | 945 | 1685 | 1708 | 1684 | 1686 | 1672 | 1677 | 1674 | 1696 | 1710 | 695 | 718 | 694 | 696 | 682 | 687 | 684 | 706 | 720 | 245 | 268 | 244 | 246 | 232 | 237 | 234 | 256 | 270 | 1505 | 1528 | 1504 | 1506 | 1492 | 1497 | 1494 | 1516 | 1530 | 45585 |
| 917 | 922 | 918 | 938 | 914 | 940 | 910 | 944 | 904 | 1682 | 1687 | 1683 | 1703 | 1679 | 1705 | 1675 | 1709 | 1669 | 692 | 697 | 693 | 713 | 689 | 715 | 685 | 719 | 679 | 242 | 247 | 243 | 263 | 239 | 265 | 235 | 269 | 229 | 1502 | 1507 | 1503 | 1523 | 1499 | 1525 | 1495 | 1529 | 1489 | 45585 |
| 935 | 933 | 905 | 916 | 923 | 930 | 941 | 913 | 911 | 1700 | 1698 | 1670 | 1681 | 1688 | 1695 | 1706 | 1678 | 1676 | 710 | 708 | 680 | 691 | 698 | 705 | 716 | 688 | 686 | 260 | 258 | 230 | 241 | 248 | 255 | 266 | 238 | 236 | 1520 | 1518 | 1490 | 1501 | 1508 | 1515 | 1526 | 1498 | 1496 | 45585 |
| 942 | 902 | 936 | 906 | 932 | 908 | 928 | 924 | 929 | 1707 | 1667 | 1701 | 1671 | 1697 | 1673 | 1693 | 1689 | 1694 | 717 | 677 | 711 | 681 | 707 | 683 | 703 | 699 | 704 | 267 | 227 | 261 | 231 | 257 | 233 | 253 | 249 | 254 | 1527 | 1487 | 1521 | 1491 | 1517 | 1493 | 1513 | 1509 | 1514 | 45585 |
| 901 | 915 | 937 | 934 | 939 | 925 | 927 | 903 | 926 | 1666 | 1680 | 1702 | 1699 | 1704 | 1690 | 1692 | 1668 | 1691 | 676 | 690 | 712 | 709 | 714 | 700 | 702 | 678 | 701 | 226 | 240 | 262 | 259 | 264 | 250 | 252 | 228 | 251 | 1486 | 1500 | 1522 | 1519 | 1524 | 1510 | 1512 | 1488 | 1511 | 45585 |
| 290 | 313 | 289 | 291 | 277 | 282 | 279 | 301 | 315 | 605 | 628 | 604 | 606 | 592 | 597 | 594 | 616 | 630 | 1010 | 1033 | 1009 | 1011 | 997 | 1002 | 999 | 1021 | 1035 | 1415 | 1438 | 1414 | 1416 | 1402 | 1407 | 1404 | 1426 | 1440 | 1730 | 1753 | 1729 | 1731 | 1717 | 1722 | 1719 | 1741 | 1755 | 45585 |
| 287 | 292 | 288 | 308 | 284 | 310 | 280 | 314 | 274 | 602 | 607 | 603 | 623 | 599 | 625 | 595 | 629 | 589 | 1007 | 1012 | 1008 | 1028 | 1004 | 1030 | 1000 | 1034 | 994 | 1412 | 1417 | 1413 | 1433 | 1409 | 1435 | 1405 | 1439 | 1399 | 1727 | 1732 | 1728 | 1748 | 1724 | 1750 | 1720 | 1754 | 1714 | 45585 |
| 305 | 303 | 275 | 286 | 293 | 300 | 311 | 283 | 281 | 620 | 618 | 590 | 601 | 608 | 615 | 626 | 598 | 596 | 1025 | 1023 | 995 | 1006 | 1013 | 1020 | 1031 | 1003 | 1001 | 1430 | 1428 | 1400 | 1411 | 1418 | 1425 | 1436 | 1408 | 1406 | 1745 | 1743 | 1715 | 1726 | 1733 | 1740 | 1751 | 1723 | 1721 | 45585 |
| 312 | 272 | 306 | 276 | 302 | 278 | 298 | 294 | 299 | 627 | 587 | 621 | 591 | 617 | 593 | 613 | 609 | 614 | 1032 | 992 | 1026 | 996 | 1022 | 998 | 1018 | 1014 | 1019 | 1437 | 1397 | 1431 | 1401 | 1427 | 1403 | 1423 | 1419 | 1424 | 1752 | 1712 | 1746 | 1716 | 1742 | 1718 | 1738 | 1734 | 1739 | 45585 |
| 271 | 285 | 307 | 304 | 309 | 295 | 297 | 273 | 296 | 586 | 600 | 622 | 619 | 624 | 610 | 612 | 588 | 611 | 991 | 1005 | 1027 | 1024 | 1029 | 1015 | 1017 | 993 | 1016 | 1396 | 1410 | 1432 | 1429 | 1434 | 1420 | 1422 | 1398 | 1421 | 1711 | 1725 | 1747 | 1744 | 1749 | 1735 | 1737 | 1713 | 1736 | 45585 |
| 515 | 538 | 514 | 516 | 502 | 507 | 504 | 526 | 540 | 1775 | 1798 | 1774 | 1776 | 1762 | 1767 | 1764 | 1786 | 1800 | 1325 | 1348 | 1324 | 1326 | 1312 | 1317 | 1314 | 1336 | 1350 | 335 | 358 | 334 | 336 | 322 | 327 | 324 | 346 | 360 | 1100 | 1123 | 1099 | 1101 | 1087 | 1092 | 1089 | 1111 | 1125 | 45585 |
| 512 | 517 | 513 | 533 | 509 | 535 | 505 | 539 | 499 | 1772 | 1777 | 1773 | 1793 | 1769 | 1795 | 1765 | 1799 | 1759 | 1322 | 1327 | 1323 | 1343 | 1319 | 1345 | 1315 | 1349 | 1309 | 332 | 337 | 333 | 353 | 329 | 355 | 325 | 359 | 319 | 1097 | 1102 | 1098 | 1118 | 1094 | 1120 | 1090 | 1124 | 1084 | 45585 |
| 530 | 528 | 500 | 511 | 518 | 525 | 536 | 508 | 506 | 1790 | 1788 | 1760 | 1771 | 1778 | 1785 | 1796 | 1768 | 1766 | 1340 | 1338 | 1310 | 1321 | 1328 | 1335 | 1346 | 1318 | 1316 | 350 | 348 | 320 | 331 | 338 | 345 | 356 | 328 | 326 | 1115 | 1113 | 1085 | 1096 | 1103 | 1110 | 1121 | 1093 | 1091 | 45585 |
| 537 | 497 | 531 | 501 | 527 | 503 | 523 | 519 | 524 | 1797 | 1757 | 1791 | 1761 | 1787 | 1763 | 1783 | 1779 | 1784 | 1347 | 1307 | 1341 | 1311 | 1337 | 1313 | 1333 | 1329 | 1334 | 357 | 317 | 351 | 321 | 347 | 323 | 343 | 339 | 344 | 1122 | 1082 | 1116 | 1086 | 1112 | 1088 | 1108 | 1104 | 1109 | 45585 |
| 496 | 510 | 532 | 529 | 534 | 520 | 522 | 498 | 521 | 1756 | 1770 | 1792 | 1789 | 1794 | 1780 | 1782 | 1758 | 1781 | 1306 | 1320 | 1342 | 1339 | 1344 | 1330 | 1332 | 1308 | 1331 | 316 | 330 | 352 | 349 | 354 | 340 | 342 | 318 | 341 | 1081 | 1095 | 1117 | 1114 | 1119 | 1105 | 1107 | 1083 | 1106 | 45585 |
| 380 | 403 | 379 | 381 | 367 | 372 | 369 | 391 | 405 | 425 | 448 | 424 | 426 | 412 | 417 | 414 | 436 | 450 | 1820 | 1843 | 1819 | 1821 | 1807 | 1812 | 1809 | 1831 | 1845 | 1235 | 1258 | 1234 | 1236 | 1222 | 1227 | 1224 | 1246 | 1260 | 1190 | 1213 | 1189 | 1191 | 1177 | 1182 | 117 | | | |

3 Even Order Magic Squares

Below are few even magic squares constructed with blocks of equal sums of **magic rectangles**.

3.1 Magic Square of Order 12×12

Example 37. A *semi-magic square* of order 12×12 constructed based on blocks of magic rectangles of order 4×6 is given by

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | | | | | | | | | | | | | 810 |
| 1 | 2 | 3 | 142 | 143 | 144 | 13 | 14 | 15 | 130 | 131 | 132 | 870 | |
| 139 | 140 | 141 | 4 | 5 | 6 | 127 | 128 | 129 | 16 | 17 | 18 | 870 | |
| 138 | 137 | 136 | 9 | 8 | 7 | 126 | 125 | 124 | 21 | 20 | 19 | 870 | |
| 12 | 11 | 10 | 135 | 134 | 133 | 24 | 23 | 22 | 123 | 122 | 121 | 870 | |
| 25 | 26 | 27 | 118 | 119 | 120 | 37 | 38 | 39 | 106 | 107 | 108 | 870 | |
| 115 | 116 | 117 | 28 | 29 | 30 | 103 | 104 | 105 | 40 | 41 | 42 | 870 | |
| 114 | 113 | 112 | 33 | 32 | 31 | 102 | 101 | 100 | 45 | 44 | 43 | 870 | |
| 36 | 35 | 34 | 111 | 110 | 109 | 48 | 47 | 46 | 99 | 98 | 97 | 870 | |
| 49 | 50 | 51 | 94 | 95 | 96 | 61 | 62 | 63 | 82 | 83 | 84 | 870 | |
| 91 | 92 | 93 | 52 | 53 | 54 | 79 | 80 | 81 | 64 | 65 | 66 | 870 | |
| 90 | 89 | 88 | 57 | 56 | 55 | 78 | 77 | 76 | 69 | 68 | 67 | 870 | |
| 60 | 59 | 58 | 87 | 86 | 85 | 72 | 71 | 70 | 75 | 74 | 73 | 870 | |
| 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 978 | |

In this case the *semi-magic sum* is $S_{12 \times 12} := 870$. The magic rectangles of order 4×6 are of *equal magic sums*.

Example 38. After making few changes in the entries of above Example 37, below is a magic square of order 12×12 constructed based on blocks of magic rectangles of order 4×6 is given by

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--|-----|
| | | | | | | | | | | | | | 870 |
| 1 | 140 | 3 | 4 | 143 | 144 | 48 | 38 | 39 | 106 | 107 | 97 | | 870 |
| 139 | 2 | 141 | 142 | 5 | 6 | 102 | 101 | 100 | 45 | 44 | 43 | | 870 |
| 138 | 137 | 136 | 9 | 8 | 7 | 103 | 104 | 105 | 40 | 41 | 42 | | 870 |
| 12 | 11 | 10 | 135 | 134 | 133 | 37 | 47 | 46 | 99 | 98 | 108 | | 870 |
| 25 | 26 | 27 | 118 | 119 | 120 | 13 | 14 | 15 | 130 | 131 | 132 | | 870 |
| 115 | 116 | 117 | 28 | 29 | 30 | 127 | 128 | 129 | 16 | 17 | 18 | | 870 |
| 114 | 113 | 112 | 33 | 32 | 31 | 126 | 125 | 124 | 21 | 20 | 19 | | 870 |
| 36 | 35 | 34 | 111 | 110 | 109 | 24 | 23 | 22 | 123 | 122 | 121 | | 870 |
| 49 | 50 | 51 | 94 | 95 | 96 | 61 | 62 | 81 | 64 | 83 | 84 | | 870 |
| 91 | 92 | 88 | 57 | 53 | 54 | 79 | 80 | 63 | 82 | 65 | 66 | | 870 |
| 60 | 89 | 93 | 52 | 56 | 85 | 72 | 77 | 76 | 69 | 68 | 73 | | 870 |
| 90 | 59 | 58 | 87 | 86 | 55 | 78 | 71 | 70 | 75 | 74 | 67 | | 870 |
| 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | | 870 |

The magic square sum is $S_{12 \times 12} := 870$. The magic rectangles of order 4×6 are of equal magic sums.

3.2 Magic Square of Order 20×20

Example 39. A semi-magic square of order 20×20 constructed based on blocks of magic rectangles of order 4×10 is given by

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10990 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 778 | 779 | 780 | 781 | 782 | 783 | 784 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 750 | 751 | 752 | 753 | 754 | 755 | 756 | 10990 |
| 771 | 772 | 773 | 774 | 775 | 776 | 777 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 743 | 744 | 745 | 746 | 747 | 748 | 749 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 10990 |
| 770 | 769 | 768 | 767 | 766 | 765 | 764 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 742 | 741 | 740 | 739 | 738 | 737 | 736 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | 10990 |
| 28 | 27 | 26 | 25 | 24 | 23 | 22 | 763 | 762 | 761 | 760 | 759 | 758 | 757 | 56 | 55 | 54 | 53 | 52 | 51 | 50 | 735 | 734 | 733 | 732 | 731 | 730 | 729 | 10990 |
| 57 | 58 | 59 | 60 | 61 | 62 | 63 | 722 | 723 | 724 | 725 | 726 | 727 | 728 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 694 | 695 | 696 | 697 | 698 | 699 | 700 | 10990 |
| 715 | 716 | 717 | 718 | 719 | 720 | 721 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 687 | 688 | 689 | 690 | 691 | 692 | 693 | 98 | 93 | 94 | 95 | 96 | 97 | 92 | 10990 |
| 714 | 713 | 712 | 711 | 710 | 709 | 708 | 77 | 76 | 75 | 74 | 73 | 72 | 71 | 686 | 685 | 684 | 683 | 682 | 681 | 680 | 99 | 104 | 103 | 102 | 101 | 100 | 105 | 10990 |
| 84 | 83 | 82 | 81 | 80 | 79 | 78 | 707 | 706 | 705 | 704 | 703 | 702 | 701 | 112 | 111 | 110 | 109 | 108 | 107 | 106 | 679 | 678 | 677 | 676 | 675 | 674 | 673 | 10990 |
| 659 | 114 | 115 | 116 | 117 | 118 | 119 | 666 | 121 | 668 | 669 | 670 | 671 | 672 | 141 | 142 | 143 | 144 | 145 | 146 | 147 | 638 | 639 | 640 | 641 | 642 | 643 | 644 | 10990 |
| 113 | 660 | 661 | 662 | 663 | 664 | 665 | 120 | 667 | 122 | 123 | 124 | 125 | 126 | 631 | 632 | 633 | 634 | 635 | 636 | 637 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 10990 |
| 658 | 657 | 656 | 655 | 654 | 653 | 652 | 133 | 132 | 131 | 130 | 129 | 128 | 127 | 630 | 629 | 628 | 627 | 626 | 625 | 624 | 161 | 160 | 159 | 158 | 157 | 156 | 155 | 10990 |
| 140 | 139 | 138 | 137 | 136 | 135 | 134 | 651 | 650 | 649 | 648 | 647 | 646 | 645 | 168 | 167 | 166 | 165 | 164 | 163 | 162 | 623 | 622 | 621 | 620 | 619 | 618 | 617 | 10990 |
| 169 | 170 | 171 | 172 | 173 | 174 | 175 | 610 | 611 | 612 | 613 | 614 | 615 | 616 | 197 | 576 | 199 | 200 | 201 | 202 | 203 | 582 | 583 | 584 | 585 | 586 | 587 | 210 | 10990 |
| 603 | 604 | 605 | 606 | 607 | 608 | 609 | 176 | 177 | 178 | 179 | 180 | 181 | 182 | 575 | 198 | 577 | 578 | 579 | 580 | 581 | 204 | 205 | 206 | 207 | 208 | 209 | 588 | 10990 |
| 602 | 601 | 600 | 599 | 598 | 597 | 596 | 189 | 188 | 187 | 186 | 185 | 184 | 183 | 574 | 573 | 572 | 571 | 570 | 569 | 568 | 217 | 216 | 215 | 214 | 213 | 212 | 211 | 10990 |
| 196 | 195 | 194 | 193 | 192 | 191 | 190 | 595 | 594 | 593 | 592 | 591 | 590 | 589 | 224 | 223 | 222 | 221 | 220 | 219 | 218 | 567 | 566 | 565 | 564 | 563 | 562 | 561 | 10990 |
| 225 | 226 | 227 | 228 | 229 | 230 | 231 | 558 | 555 | 556 | 557 | 554 | 559 | 560 | 253 | 254 | 255 | 256 | 257 | 258 | 259 | 526 | 527 | 528 | 529 | 530 | 531 | 532 | 10990 |
| 547 | 548 | 549 | 550 | 551 | 552 | 553 | 232 | 233 | 234 | 235 | 236 | 237 | 238 | 519 | 520 | 521 | 522 | 523 | 524 | 525 | 260 | 261 | 262 | 263 | 264 | 265 | 266 | 10990 |
| 546 | 545 | 544 | 543 | 542 | 541 | 540 | 245 | 244 | 243 | 242 | 241 | 240 | 239 | 518 | 517 | 516 | 515 | 514 | 513 | 512 | 273 | 272 | 271 | 270 | 269 | 268 | 267 | 10990 |
| 252 | 251 | 250 | 249 | 248 | 247 | 246 | 535 | 538 | 537 | 536 | 539 | 534 | 533 | 280 | 279 | 278 | 277 | 276 | 275 | 274 | 511 | 510 | 509 | 508 | 507 | 506 | 505 | 10990 |
| 281 | 282 | 283 | 284 | 285 | 286 | 287 | 498 | 499 | 500 | 501 | 502 | 503 | 504 | 309 | 310 | 311 | 312 | 313 | 314 | 315 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 10990 |
| 491 | 492 | 497 | 494 | 495 | 496 | 493 | 288 | 289 | 290 | 291 | 292 | 293 | 294 | 463 | 464 | 465 | 466 | 467 | 468 | 469 | 316 | 317 | 318 | 319 | 320 | 321 | 322 | 10990 |
| 490 | 489 | 484 | 487 | 486 | 485 | 488 | 301 | 300 | 299 | 298 | 297 | 296 | 295 | 336 | 461 | 460 | 459 | 458 | 457 | 456 | 329 | 454 | 327 | 326 | 325 | 324 | 323 | 10990 |
| 308 | 307 | 306 | 305 | 304 | 303 | 302 | 483 | 482 | 481 | 480 | 479 | 478 | 477 | 462 | 335 | 334 | 333 | 332 | 331 | 330 | 455 | 328 | 453 | 452 | 451 | 450 | 449 | 10990 |
| 337 | 338 | 339 | 340 | 341 | 342 | 343 | 442 | 443 | 444 | 445 | 446 | 447 | 448 | 407 | 366 | 367 | 368 | 369 | 370 | 371 | 414 | 415 | 416 | 375 | 418 | 419 | 420 | 10990 |
| 435 | 436 | 437 | 438 | 439 | 440 | 441 | 344 | 345 | 346 | 347 | 348 | 349 | 350 | 365 | 408 | 409 | 410 | 411 | 412 | 413 | 372 | 373 | 374 | 417 | 376 | 377 | 378 | 10990 |
| 434 | 433 | 432 | 431 | 430 | 429 | 428 | 357 | 356 | 355 | 354 | 353 | 352 | 351 | 392 | 405 | 404 | 403 | 402 | 401 | 400 | 385 | 384 | 383 | 382 | 381 | 380 | 393 | 10990 |
| 364 | 363 | 362 | 361 | 360 | 359 | 358 | 427 | 426 | 425 | 424 | 423 | 422 | 421 | 406 | 391 | 390 | 389 | 388 | 387 | 386 | 399 | 398 | 397 | 396 | 395 | 394 | 379 | 10990 |
| 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |

The magic square sum is $S_{28 \times 28} := 10990$. The magic rectangles of order 4×14 are of equal magic sums.

3.5 Magic Square of Order 30×30

Example 45. A semi-magic square of order 30×30 constructed based on blocks of magic rectangles of order 6×10 is given by

- Remark 3.1.** 1. *By making little changes in entries, the semi-magic squares are written as magic squares with equal sums blocks of magic rectangles.*
2. *Magic rectangles multiples 4, such as 4×8 , 4×8 , 4×12 , etc. are not considered here. These types can easily be written as equal sums pandiagonal magic squares of order 4 as shown in Example 24. The magic squares with equal sums blocks of order 4 are already studied by author [2, 6, 7, 10].*
3. *Some applications of magic triangles to magic crosses can be seen in author's work [10].*

4 Author's Contributions to Magic Squares and Recreating Numbers

For author's contribution to **magic squares** and **recreation numbers** please see the links below:

- Inder J. Taneja, Magic Squares, <https://inderjtaneja.com/2019/06/27/publications-magic-squares/>
- Inder J. Taneja, Recreation of Numbers, <https://inderjtaneja.com/2019/06/27/publications-recreation-of-numbers/>

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