Capped FRN Swap Model

A model is presented for pricing European/Bermudan type callable capped floating rate note (FRN) swaps. The capped FRN swap is a contract to swap cash-flows between a vanilla floating rate leg and a capped floating rate leg. The option gives the right to call the swap back in favor of the option owner.

Pricing the capped FRN swap is relatively simple as the value can be expressed in a close-form analytical formula. To price the option with Bermudan type, the capped FRN swap has been reduced to a contract to swap cash-flows between a fixed leg and a caplet leg. To make the reduction valid, we need to assume that the date-setting in both legs of the capped FRN swap are almost consistent.

With the help of this simplification, each intrinsic value of the option is the one of a corresponding caption. Therefore, the option becomes Bermudan type caption. Under the model of a single-factor dynamics of a pseudo-forward rate, this caption an be priced by using the tree approach.

Let 1*;* 2 be index of cash-flow legs. Let *Li* (*t*) be a forward interest rate seen at a time of *t* for the forward accrual period.

$$L_i^{(\iota)}(t) = \frac{1}{\beta_i^{(\iota)}} \left(\frac{D_{tt_{i-1}^{(\iota)}}}{D_{tt_i^{(\iota)}}} - 1 \right)$$

A capped floating rate cash-flow is

$$\hat{V}_i(t_i) = \alpha_i \times \max\{\hat{f}_i,\, L_i(t_{i-1})\} = \alpha_i \times \left(L_i(t_{i-1}) + [\hat{f}_i - L_i(t_{i-1})]^+ \right) \ ,$$

A floored floating rate cash flow is

$$\hat{V}_i(t_i) = \alpha_i \times L_i(t_{i-1}) \ ,$$

The present value (PV) at the time of *t* of the cash-flow, *Vi*(*ti*), can be given by

$$\mathrm{PV}_t(V_i(t_i)) \quad = \quad \alpha_i \cdot D_{tt_i} \cdot \varepsilon_i + D_{tt_i} \cdot \mathrm{E}_t[\hat{V}_i(t_i)]$$

It is clear to see that, on the right side of equation above, the first term is the value of the floating coupon, the second term is value of the *i*th caplet and the last term is the value of the *i*th floorlet. The caplet and the floorlet of the rate, *Li*, are reset at *ti*¡1.

In this case, the *Leg 2* is composed of vanilla cash-flows which is generated by floating rates with spreads and cash-flows in *Leg 1* are capped floating rate with rates spreads.

$$\begin{aligned}
\mathrm{PV}_t\left(V_{(i,n_1)}^{\mathrm{cappedFRN}}\right) &= \sum_{k=i}^{n_1} \alpha_k^{(1)} \cdot D_{tt_k^{(1)}} \cdot \left(L_k^{(1)}(t) + \varepsilon_k^{(1)}\right) - \sum_{k=i}^{n_1} \alpha_k^{(1)} \cdot D_{tt_k^{(1)}} \cdot C_k\left(t, L_k^{(1)}(t), \hat{c}_k\right) \ , \\
\mathrm{PV}_t\left(V_{(j,n_2)}^{\mathrm{vanillaFloating}}\right) &= \sum_{k=j}^{n_2} \alpha_k^{(2)} \cdot D_{tt_k^{(2)}} \cdot \left(L_k^{(2)}(t) + \varepsilon_k^{(2)}\right) \ , \\
\mathrm{PV}_{(i,n_1),(j,n_2)}^{\mathrm{cappedFRNswap}}(t,\beta) &= \beta\left\{\sum_{k=i}^{n_1} \alpha_k^{(1)} \cdot D_{tt_k^{(1)}} \cdot \left(L_k^{(1)}(t) + \varepsilon_k^{(1)}\right) - \sum_{k=j}^{n_2} \alpha_k^{(2)} \cdot D_{tt_k^{(2)}} \cdot \left(L_k^{(2)}(t) + \varepsilon_k^{(2)}\right)\right. \\
&\qquad \left. - \sum_{k=i}^{n_1} \alpha_k^{(1)} \cdot D_{tt_k^{(1)}} \cdot C_k\left(t, L_k^{(1)}(t), \hat{c}_k\right)\right\} \ .
\end{aligned}$$

Let us introduce two piece-wise defined functions

$$\begin{aligned}
L^*(t) &= \mathrm{Interpolation}\left\{t;\ (t_0, L_1^*), \cdots, (t_{n-1}, L_n^*)\right\} \ , \\
\sigma(t) &= \mathrm{Interpolation}\left\{t;\ (t_0, \sigma_1), \cdots, (t_{n-1}, \sigma_n)\right\} \ ,
\end{aligned}$$

Usually, the flat extrapolation may be applied to the range of [0; t0]. Then, we define a process L under a propriate measure as

$$L(t) = L^*(t) \cdot \exp\left[-\frac{1}{2}\sigma^2(t) \cdot t + \sigma(t) \cdot W_t\right] \ ,$$

where W is a standard Wiener process

We have

$$\begin{aligned} L(t_{k-1}) &= L^*(t_{k-1}) \cdot \exp\left[-\frac{1}{2}\sigma^2(t_{k-1}) \cdot t_{k-1} + \sigma(t_{k-1}) \cdot W_{t_{k-1}}\right] \\ &= L^*(t_{k-1}) \cdot \exp\left[-\frac{1}{2}\sigma^2(t_{k-1}) \cdot t + \sigma(t_{k-1}) \cdot W_t\right] \times \\ &\quad \exp\left[-\frac{1}{2}\sigma^2(t_{k-1}) \cdot (t_{k-1} - t) + \sigma(t_{k-1})\sqrt{t_{k-1} - t} \cdot z\right] \ , \end{aligned}$$

Reference:

https://finpricing.com/lib/EqCppi.html