Credit Valuation Adjustment (CVA) Analytics

Counterparty credit risk is the risk that a counterparty to a financial contract will default prior to expiry, failing to make future contracts and resulting in a credit exposure to the financial institution. Traditional methods of managing counterparty credit risk include netting agreements and collateralization to reduce the overall exposure to a single counterparty. Over the past decade, institutions have also used Credit Default Swap (CDS) protection to manage exposures.

Credit Valuation Adjustment (CVA) is the difference in value of an OTC derivatives position due to counterparty credit risk. More informally, think of CVA as the fair value of buying protection against the counterparty's potential failure to meet contractual obligations. From this perspective, calculation of CVA is a pricing exercise similar to option pricing.

In particular, because we are interested in the fair value of protection we can express the problem as the cost of hedging against default, allowing us to work in a risk-neutral measure in which the expected return of the hedged portfolio is the risk-free rate.

Consider a position subject to both market risk (due to adverse moves in risk factors) and counterparty risk (due to potential failure of counterparty to meet contractual obligations). We assume only the counterparty can default (later sections will extend to bilateral CVA in which both counterparties can default). Denote the credit risk-free value of the position at time t and maturing at time T by $V(t,T)$, and the value under credit risk by $\tilde{V}(t,T)$.

We expect the value under credit risk to be less than the risk-free value, in order to compensate the holder for taking on the risk of the counterparty's default. Then the counterparty price of risk (i.e. the CVA) is then a positive number.

$$\delta V(t,T) \equiv V(t,T) - \tilde{V}(t,T)$$

The counterparty price of risk for a long position must be positive, since the short position holds an implicit option to default, funded by the holder of the long position.

Let's derive some basic properties of CVA by assuming very general forms for the risk-free and risky prices above. Assume the contract pays $V(T,T) = X_T$ at maturity $t=T$. Standard arbitrage pricing theory gives the value of the credit risk-free position as the expected value of the discounted payoff in the risk neutral measure:

$$V(t,T) = E^P\left[\frac{\beta(t)}{\beta(T)} X_T \;|F_t\right]$$

in the usual notation. That is, the risk-free price is simply the expected discounted payoff under the risk-neutral measure.

To determine the value under credit risk we model the expected amount lost due to default. Introduce a Bernoulli default process χ and Loss Given Default (LGD) $\Lambda = (1-R)$. We consider unilateral CVA, so we are only exposed to default risk if the contract has positive value. Then

$$\tilde{V}(t,T) = V(t,T) - E^P\left[(1-R)\chi_{t\le\tau\le T}\frac{\beta(t)}{\beta(\tau)} V^+(\tau,T) \;|F_t\right]$$

Again, this is simply the expected discounted payoff under the risk-neutral measure, conditional on default occurring before maturity and the position having positive value to the financial institution. Note this expectation is under the risk-neutral (i.e. pricing) measure, since the ultimate objective is to hedge the credit risk due to the counterparty's potential to default

CVA follows as

$$\delta V(t,T)=E^{P}\left[(1-R)\chi_{t\le\tau\le T}\,\frac{\beta(t)}{\beta(\tau)}V^{+}(\tau,T)\,|F_{t}\right].$$

It is convenient to view the CVA at *t=0* as an integral over default states occurring over the life of the contract. We have

$$\delta V(0,T)=E^{P}\left[(1-R)\chi_{0\le\tau\le T}\,\frac{1}{\beta(\tau)}V^{+}(\tau,T)\,|F_{0}\right]$$

$$=\int_{0}^{T}E^{P}\left[\frac{(1-R)}{\beta(\tau)}V^{+}(\tau,T)\,|t=\tau\right]\psi_{\tau}(t)dt$$

Notice that the expectation under the integral is conditional on default occurring at time τ. If the exposure and the money market account are independent of the counterparty's credit state (i.e. there is no *wrong-way risk*) the conditioning is immaterial and the expression simplifies. If LGD is constant and the market and credit risk components are independent then CVA is

$$\delta V(0,T)=\int_{0}^{T}E^{P}\left[\frac{(1-R)}{\beta(\tau)}V^{+}(\tau,T)\,|t=\tau\right]\psi_{\tau}(t)dt$$

$$=(1-R)\int_{0}^{T}E^{P}\left[\frac{V^{+}(t,T)}{\beta(\tau)}\right]\psi_{\tau}(t)dt$$

You can find more details at

https://finpricing.com/lib/EqCppi.html