FX Volatility Surface Research

FX Volatility

- An implied volatility is the volatility implied by the market price of an option based on the Black-Scholes option pricing model.
- One can extract the implied volatility from the market volatility smile corresponding to the option's moneyness.
- FX volatility surface is a three-dimensional plot of the implied volatility as a function of term and Delta smile.
- FX volatility comes in Delta convention of market quotes for at-the-money, 10%, and 25% deltas.
- One needs to computes moneyness for each delta and term to generate volatility smile surface with moneyness and term structure.

FX Volatility

- Unlike in other markets that quote volatility versus strike directly.
- The volatility surface in FX market is constructed based on the sticky delta rule.
- The assumption is that options are valued depending on their delta, so that when the FX spot rate moves and the delta of an option changes accordingly.
- Commonly, linear interpolation and flat extrapolation are applied to extract volatility from the volatility smile surface.

FX Volatility

- Let S be the FX spot rate of a currency pair in Cd/Cf at time t, where Cd is domestic currency and Cf is foreign currency.
- For example, consider the currency pair USD/EUR. USD is the domestic currency and EUR is the foreign currency. The spot exchange rate, in USD/EUR, will represent the USD price of one unit of EUR (for instance, 1.20 USD per 1 EUR).
- For direct quote volatility surface, At-The-Money (ATM) is defined by 0-Delta Straddle, where the related put and call have the same delta

$$F_{t,T}^{D} / K_{T}^{D} = \exp\left[-\tfrac{1}{2}\sigma^{2}T\right]$$

FX Volatility

- Let S (indirect quote) be the FX spot rate of a currency pair in Cf/Cd at time t, where Cd is domestic currency and Cf is foreign currency.
- Note indirect quote is in reverse form of a direct quote.
- For example, consider the currency pair CAD/USD. USD is the domestic currency ( ) and CAD is the foreign currency ( ). The spot exchange rate, in CAD/USD, will represent the CAD price of one unit of USD (for instance, 0.85 CAD per 1 USD).

FX Volatility

- For indirect quote, the moneyness is calculated as

$$\frac{F_{t,T}^{I}}{K_{T}^{I}} = \exp\left[\tfrac{1}{2}\sigma^{2}T\right]$$

- Market provides few entries of the smile matrix: the volatilities for at-the-money, 25% and 10% deltas in Strangle/Risk Reversal Delta convention

Thank You

You can find more details at

https://finpricing.com/lib/EqLookback.html