Zero Rate Curve Construction

Yield Curve

- Zero rate curve is the relationship between the yield-to-maturity on a zero coupon bond and the bond's maturity.
- Zero yield curve is bootstrapped from a series of specified instruments.
- The basis idea of bootstrapping is:
- Start with a cash or bond whose spot price is known and that has a single future payment of a known amount.
- The discount factor for this instrument can be readily obtained from ratio of the spot price to the amount of its future payments.

Yield Curve

- The next group consists of FRA or futures. For fixed maturity FRA, the market value is zero. The spot price is equal to the strike price.
- The final group consists of swaps.
- The maturity of each instrument represents a unique node or term on the resulting discount curve.
- The first instrument establishes the first node on the discount curve.
- The discount factor from continuously compounded rate can be expressed as

Yield Curve

$$d(t, t, t_1) = e^{-r(t, t, t_1, CTS, DC) \cdot TF(t, t_1, DC)}$$

- Assuming that we have constructed the first j-1 nodes on the discount curve, the next jth node in the sequence is assessed in terms of how many of its cash flows occur between the maturity of the last instrument in the sequence and the maturity of the present instrument.
- If there is only one cash flow, that payment falls exactly on t_j and it establishes the next node on the discount curve directly.

Yield Curve

- If there are multiple cash flows, the next instrument establishes the next few nodes corresponding to all cash flows on the discount curve.
- This case is more difficult than the previous one as one instrument must provide information to produce more than one discount factor.
- It is, therefore, necessary to interpolate. Interpolation normally is performed at the level of continuously compounded rates as

$$r_{j,k} = r_{j-1} \cdot \left(\frac{t_j - t_{j,k}}{t_j - t_{j-1}} \right) + r_j \cdot \left(\frac{t_{j,k} - t_{j-1}}{t_j - t_{j-1}} \right)$$

Yield Curve

- Substituting the imposed linear relation into the valuation model of the instrument.
- A root finding algorithm, such as Newton-Raphson, is used to solve the equation. The algorithm thus utilises the curve interpolation to calculate the discount factor.
- Repeat the above process until the last instrument in the sequence has been incorporated.
- That yield the desired bootstrapped discount curve.
- The discount curve is then converted into continuously compounded zero rate curve.

Thank You

You can find more details at

https://finpricing.com/lib/EqBarrier.html