Yield Curve Explained

Yield Curve

- Yield curve is defined as the relationship between the yield-to-maturity on a zero coupon bond and the bond's maturity.
- Zero yield curves play an essential role in financial valuation as one needs t forecasting future cashflows using appropriate forward rates and discounting the flows.
- Yield curve bootstrapping is based on these underlying instruments: cash rates, futures and swaps.
- The underlying instruments are divided into two groups based on the maturities. Those with shorter maturities are classified as short to medium term instruments, including cash rates, futures, and short term swaps

Yield Curve

- Those with longer maturities are long term instruments: such as, medium to long term swaps.
- The bootstrapping method is employed to calculate discount factors at maturities of cash rates and futures first.
- If there is a gap, the following calculated continuous compounding forward rate is used as the forward rate for this gap.
- For Libor futures, the expiration of the instrument (third Wednesday of the month) plus the tenor of the rate is used as the maturity of the underlying cash Libor.

Yield Curve

- These maturity dates are adjusted based on the day roll convention, and recorded in the generated curve as anchor dates, together with the generated discount factors.
- If there are short-term Libor swaps with maturities shorter than the longest Libor maturity of the corresponding Libor futures, additional information is taken from these Libor swaps such that additional anchors and discount factors are inserted in the generated curve.

Yield Curve

- Having obtained short to medium term discount factors as described above, the discount factors at the remaining Libor swap maturity dates, unadjusted, are solved simultaneously by setting equal numbers of equations.
- Any unknowns, such that the input spot Libor swaps with adjusted coupon dates and adjusted maturity dates, are at par when the calculated discount factors are applied, though some of them may stand at non-business dates
- Convert the discount factors to continuous compounding zero rates

Yield Curve

- Yield curve bootstrapping needs to handle gaps and overlaps properly.
- Each underlying instrument may have its own day count basis and business day convention. Those conventions are taken into account when deriving discount factors.
- As long as the same day count convention is used both for converting discount factors to zero rates and for interpolating zero rates, the results should be consistent.

Yield Curve

- Given a Future price, the yield or zero rate can be directly calculated as

$$r = \frac{(100 - P)}{100} - \frac{CvxAdj}{10000}$$

where

P	the quoted interest rate Future price
r	the derived yield or zero rate
CvxAdj	the Future convexity adjustment quoted in basis points (bps)

Yield Curve

- Assuming that we have all yields up to 3 years and now need to derive up to 4 years.
  - Let x be the yield at 4 years.
  - Use an interpolation methd to get yields at 3.25, 3.5 and 3.75 years as Ax, Bx, Cx,Dx.
  - Given the 4 year market swap rate, we can use a root-finding algorithm to solve the x that makes the value of the 4 year inception swap equal to zero.
  - After that, we get all yields or equivalent discount factors up to 4 years

Yield Curve

- Repeat the above procedure till the longest swap maturity.
- There are two keys in yield curve construction: interpolation and root finding.
- Most popular interpolation algorithms in curve bootstrapping are linear, log-linear and cubic spline.

Yield Curve

- One needs an optimization solution to match the prices of curve-generated instruments to their market quotes.
- Popular optimization algorithms include Levenberg-Marquardt, Newton, Brent, etc.

Thank You

You can find more details at

https://finpricing.com/lib/EqVariance.html