

# Capped Swap Explained

# Capped Swap

- ◆ A capped swap is an interest rate swap with a cap where the floating rate of the swap is capped at a certain level.
- ◆ A capped swap consists of a long position in a swap and a cap with a predetermined strike rate and maturity equal to swap's maturity.
- ◆ Given the optionality, an up-front fee or premium has to be paid by the floating rate payer.
- ◆ A capped swap can be decomposed into two positions: an interest rate swap and an interest rate cap.

# Capped Swap

- ◆ A floored swap is an interest rate swap with a floor where the floating rate of the swap is floored at a certain level.
- ◆ A floored swap consists of a long position in a swap and a floor with a predetermined strike rate and maturity equals to swap's maturity.
- ◆ Given the optionality, an up-front fee or premium has to be paid by the floating rate receiver.
- ◆ A floored swap can be decomposed into two positions: an interest rate swap and an interest rate floor.

# Capped Swap

- ◆ There are four types of capped or floored swaps.
  - ◆ Capped payer swap
  - ◆ Capped receiver swap
  - ◆ Floored payer swap
  - ◆ Floored receiver swap
- ◆ The present value of a capped payer swap is given by

$$PV_{CappedPayerSwap}(t) = PV_{float}(t) - PV_{fixed}(t) - PV_{cap}(t)$$

where

$PV_{float}$  is the present value of the floating leg of the underlying swap;

$PV_{fixed}$  is the present value of the fixed leg of the underlying swap;

$PV_{cap}$  is the present value of the embedded cap.

# Capped Swap

- ◆ The present value of a capped receiver swap can be expressed as

$$PV_{CappedReceiverSwap}(t) = PV_{fixed}(t) - PV_{float}(t) + PV_{cap}(t)$$

- ◆ The present value of a floored payer swap can be represented as

$$PV_{FlooredPayerSwap}(t) = PV_{float}(t) - PV_{fixed}(t) + PV_{floor}(t)$$

Where  $PV_{floor}$  is the present value of the embedded floor.

- ◆ The present value of a floored receiver swap can be computed as

$$PV_{FlooredReceiverSwap}(t) = PV_{fixed}(t) - PV_{float}(t) - PV_{floor}(t)$$

# Capped Swap

- ◆ The present value of the fixed leg is given by

$$PV_{fixed}(t) = RN \sum_{i=1}^n \tau_i D_i$$

where  $R$  - the fixed rate;  $N$  - the notional;  $\tau_i$  - the day count fraction for period  $[T_{i-1}, T_i]$ ;  $D_i = D(t, T_i)$  - the discount factor.

- ◆ The present value of the floating leg is given by

$$PV_{float}(t) = N \sum_{i=1}^n (F_i + s) \tau_i D_i$$

where  $s$  - the floating spread;  $F_i = F(t; T_{i-1}, T_i) = \frac{1}{\tau_i} \left( \frac{D_{i-1}}{D_i} - 1 \right)$  - the simply compounded forward rate

# Capped Swap

- ◆ The present value of the cap is given by

$$PV_{cap}(t) = N \sum_{i=1}^n \tau_i D_i (F_i \Phi(d_1) - K \Phi(d_2))$$

where  $d_{1,2} = \left( \ln \left( \frac{F_i}{K} \right) \pm 0.5 \sigma_i^2 T_i \right) / (\sigma_i \sqrt{T_i})$  and  $\Phi$  - the cumulative normal distribution function.

- ◆ The present value of the floor is given by

$$PV_{cap}(t) = N \sum_{i=1}^n \tau_i D_i (K \Phi(-d_2) - F_i \Phi(-d_1))$$



# Thanks!



Reference:

<https://finpricing.com/lib/EqLookback.html>

