Bermudan Swaption Explained

Bermudan Swaption

- A Bermudan swaption is an option that gives the owner the right to enter a swap at each predetermined date in the exercise schedule.
- Bermudan swaptions give the holders some flexibility to be exercised into a swap on any of a number of dates.
- A comparison of European, American and Bermudan swaptions
  - European swaption has only one exercise date at the maturity.
  - American swaption has multiple exercise dates (daily)
  - Bermudan swaption has multiple exercise dates (but not daily): such as quarterly, monthly, etc.

Bermudan Swaption

- At the maturity T, the payoff of a Bermudan swaption is given by

$$Payoff(T) = \max(0, V_{swap}(T))$$

where $V_{swap}(T)$ is the value of the underlying swap at T.

- At any exercise date T_i, the payoff of the Bermudan swaption is given by

$$Payoff(T_i) = max\left(V_{swap}(T_i), I(T_i)\right)$$

where $V_{swap}(T_i)$ is the exercise value of the Bermudan swap and $I(T_i)$ is the intrinsic value.

Bermudan Swaption

- We assume the notional amounts for each swap period for floating leg and fixed leg are the same. We also assume period start and end dates for floating and fixed leg are the same.
- We assume that swap rate follows the SDE below

$$dS_{k_j}(t) = \sigma_{k_j} \cdot S_{k_j}(t) \cdot dW_{k_j}(t),$$

- where W is a standard Wiener process and σ is the volatility
- The swap rate is

$$S_{k_j}(t) = S_{k_j}(0) \cdot \exp\left(-\frac{1}{2}\sigma_{k_j}^2 \cdot t + \sigma_{k_j} W_{k_j}(t) \right),$$

Bermudan Swaption

- The volatility of swap rate can be calculated as

$$\sigma_i = \left| \frac{\gamma_N \bar{F}_{i,N}^0(0)\bar{\sigma}_{i,N} + \sum_{k=i+1}^{N-1} (\gamma_k - \gamma_{k+1}) \bar{F}_{i,k}^0(0)\bar{\sigma}_{i,k}}{\gamma_N \bar{F}_{i,N}^0(0) + \sum_{k=i+1}^{N-1} (\gamma_k - \gamma_{k+1}) \bar{F}_{i,k}^0(0)} \right|$$

- The payoff of Bermudan swap is

$$[V(T,\beta)]^+ = [\beta \cdot [F_{i,j}(T) - X_{i,j}(T)]]^+$$

Thanks!

Reference:

https://finpricing.com/lib/EqCppi.html