FX Asian Option Explained

Asian Option Overview

- Asian options are European style options where the payoff depends on the average price during at least some part of the life of the option.
- The payoff is different from the case of a European option or American option, where the payoff of the option contract depends on the rate of the underlying asset at exercise date.
- Asian options are less expensive than regular options and are arguably more appropriate than regular options for meeting some of investment needs.
- Asian options have relatively low volatility due to the averaging mechanism. They are used by traders who are exposed to the underlying asset over a period of time.

Forex Market Convention

- A Currency Pair is specified by a *primary currency* and a *quoting currency.* The terminology indicates that FX rates for the pair are given in terms of amount of quoting currency per one unit of primary currency. Then for any amount of primary currency, one converts to quoting currency by multiplication:

$$\text{QuotingCurrency Amount} = (\text{FX Rate}) * (\text{Primary Currency Amount})$$

- Example: For a EUR/USD pair with EUR defined as the primary currency and USD as the quoting currency, a quote of 1.20 indicates 1.0 EUR can be exchanged for 1.20 USD.

Forex Market Convention (Cont.)

- A Currency Pair also requires a specification of the number of days between the quotation date (or trade date) and the Spot Date on which the exchange is to take place at that quote. Typically this is two business days.
- The Spot Days can be specified in two ways. It can be established directly for each Currency Pair. Or it can be derived from the default Spot Days assigned to each individual currency, in which case the Spot Days for the pair is defined to be the maximum of the Spot Days of the component currencies.
- The date on which the exchange takes place is termed the *Value Date* in the FX market. The Value Date is typically the Spot Date, but can also be one business day after the quotation date (for O/N quotes).

Valuation

- The payoff of an Asian option depends on the average of the underling stock price over certain time interval. Since no general analytical solutions for the price of the Asian option is known, a variety of techniques have been proposed to analyze the arithmetic average Asian options.
- Let S(t) be a process of a foreign currency exchange rate, measured in units of domestic currency as per one unit of foreign currency. Let $\{t_1 < \cdots < t_m \leq t_{m+1} < \cdots t_n\}$ be a set of average dates and T be a payoff settlement date.
- The price of the underlying exchange rate is recorded on the set of average dates to obtain the price arithmetic average of A, where

$$A = \frac{1}{n}\sum_{i=1}^{n} S(t_i).$$

Valuation (Cont.)

- The payoff of this Asian-European type option at the settlement date is given by

$$N \times [\beta(A - K)]^{+},$$

- where N is the notional principal in domestic currency, and K is the option strike, and (1 or –1) is the call/put indicator.
- Let t = 0 be the value date and assume $t_m \leq \tau \leq t_{m+1}$. Thus the underlying prices are known at t_i. Using an approximation, the value of call option is given by

$$V_{call}(\tau) = \frac{n-m}{n} C'(\tau)$$

Valuation (Cont.)

- Where

$$C'(\tau) = \exp(-r_n t_n)\left\{\frac{1}{n}\sum_{i=m+1}^{n}\frac{1}{n-m}\exp(\mu_i + \frac{\sigma_i^2}{2})\cdot\Phi\left(\frac{\mu - \ln K'}{\sigma} + \frac{\sigma_i}{\sigma}\right) - K'\cdot\Phi\left(\frac{\mu - \ln K'}{\sigma}\right)\right\}$$

- And

$$K' = \frac{n}{n-m}\left(K - \frac{1}{n}\sum_{i=1}^{m} S(t_i)\right).$$

- Assuming that S follows a geometric Brownian motion,

$$dS = (r - r_f)Sdt + vSdW$$

Valuation (Cont.)

- The sifted lognormal parameters are

$$y_1 = \frac{M_2 - M_1^2}{z - (M_2 - M_1^2)/z}$$

$$y_{11} = M_2 - M_1^2 + y_1^2$$

$$\delta = M_1 - y_1$$

$$z = \left(\frac{\mu_3 + \sqrt{\mu_3^2 + 4\mu_2^3}}{2}\right)^{\frac{1}{3}}$$

$$\mu_3 = M_3 - 3M_1(M_2 - M_1^2) - M_1^3$$

- By assuming that the average asset price is lognormal, you can use Black's model to price an Asian option.

Valuation (Cont.)

- The present value of an Asian call option is given by

$$PV_C = (y_1 N(d_1) - (K - \psi A - \delta) N(d_2)) D$$

$$d_1 = \frac{ln\left(\frac{\sqrt{y_{11}}}{K - \psi A - \delta}\right)}{\sqrt{\ln(y_{11}/y_1^2)}}$$

$$d_2 = d_1 - \sqrt{\ln(\frac{y_{11}}{y_1^2})}$$

where

$D = D(0,T)$ the discount factor

Valuation (Cont.)

$D = D(0, T)$	the discount factor
N	the cumulative standard normal distribution function
T	the maturity date
ψ	the sum of weights corresponding to spent fixing periods
A	the spent average
K	the strike

- The present value of an Asian put option is given by

$$PV_p = \big((K - \psi A - \delta)N(-d_2) - y_1 N(-d_1)\big)D$$

Thank You

Reference:

http://localhost/lib/EqCppi.html