Puttable Bond Explained

1. Puttable Bond Definition

- A puttable bond is a bond in which the investor has the right to sell the bond back to the issuer at specified times (puttable dates) for a specified price (put price).
- At each puttable date prior to the bond maturity, the investor may sell the bond back to its issuer and get the investment money back.
- The underlying bonds can be fixed rate bonds or floating rate bonds.
- A puttable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.
- Puttable bonds protect investors. Therefore, a puttable bond normally pays investors a lower coupon than a non-callable bond.

2. The Advantage of Puttable Bonds

- Although a puttable bond is a higher cost to the investor and an uncertainty to the issuer comparing to a regular bond, it is actually quite attractive to both issuers and investors.
- For investors, puttable bonds allow them to reduce interest costs at a future date should rate increase.
- For issuers, puttable bonds allow them to pay a lower interest rate of return until the bonds are sold back.
- If interest rates have increased since the issuer first issues the bond, the investor is like to call its current bond and reinvest it at a higher coupon.

3. Puttable Bond Payoffs

- At the bond maturity T, the payoff of a puttable bond is given by

$$V_p(T) = \begin{cases} F + C & if\ not\ ptted \\ \max\,(P_p, F + C) & if\ putted \end{cases}$$

where

F – the principal or face value;

C – the coupon;

P_p – the put price;

max (*x*, *y*) – the maximum of *x* and *y*

T - the maturity date;

- The payoff of the puttable bond at any call date T_i can be expressed as

$$V_p(T_i) = \begin{cases} \overline{V}_{T_i} & if\ not\ putted \\ \max\left(P_p, \overline{V}_{T_i}\right) & if\ putted \end{cases}$$

where

$\overline{V}_{T_i}$ – continuation value at T_i

P_p – the put price;

max (*x*, *y*) – the maximum of *x* and *y*

T_i - the i-th call date;

4. Model Selection Criteria

- Given the valuation complexity of a callable bond (e.g., embedded Bermudan option), there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price the callable bond.
- The selection of interest rate term structure models
  - Popular IR term structure models:

    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
    - HJM and LMM are too complex.
    - Hull-White is inaccurate for computing sensitivities.
    - Therefore, we choose either LGM or QGM.
- The selection of numeric approaches
    - After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.
    - Commonly used numeric approaches are tree, partial differential equation (PDE), lattice, and Monte Carlo simulation.
    - Tree and Monte Carlo are notorious for inaccuracy in sensitivity calculation.
    - Therefore, we choose either PDE or lattice.
- We decide to use LGM plus lattice.

5. LGM Model

- The dynamics

$$dX(t) = \alpha(t)dW$$

Where X is the single state variable; W is the Wiener process.

- The numeraire is given by

$$N(t,X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

- The zero coupon bond price is

$$B(t,X;T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

6. LGM Assumption

- The LGM model is mathematically equivalent to the Hull-White model but offers
    - Significant improvements in calibration stability and accuracy.

   - More accurate and stable in sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.

7. LGM calibration

- Match today's curve

  At time t, X(0)=0 and H(0)=0. Thus Z(0,0;T)=D(T). In other words, the LGM automatically fits today's discount curve.
- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

You can find more details at

https://finpricing.com/lib/EqRangeAccrual.html