Bond Futures Analytics

Bond futures are exchange-traded instruments, with an underlying that is a basket of deliverable bonds. For most bond futures, the seller has the option to deliver any of the instruments in the basket.

The basket is composed of government bonds from a unique issuer (country) with rules on remaining maturity, initial maturity and issue size to be eligible.

The bonds in the basket are transformed to be comparable through a conversion factor mechanism. The factor is such that in a certain reference yield environment all the bonds have the same price. The reference yield acts somewhat like a strike for the delivery process.

The seller usually picks up the cheapest bond in the basket to deliver, called the cheapest-to-deliver (CTD). The CTD bond is normally delivered on the last delivery day of the month.

Suppose there are N bonds in the basket. Each of them (1 _ i _ N) has ni coupons after the common delivery date t0; the cash flows amount are ci;j and are paid on ti;j . Let Accrued Interest i(t) denote the accrued interest of bond i for delivery date t. The conversion factor associated with each bond is denoted Ki.

The bond future notice takes place on t0. The time t futures price is denoted by Ft. At delivery, the short party can choose the bond to be delivered (i) and receives the amount F. Ki + Accrued Interest i_ (t0) on the delivery date.

The present value of a bond future contract is represented as:

$$PV(t) = nN\left(\frac{F_B(t,T)}{CF} - K\right)\exp(-t_T T)/100$$

where

t the valuation date

K the delivery price

n the number of contracts

N the amount value for the bond future

$F_B(t,T) = (P - C_\Sigma)\exp(r_T T) - A$ the forward clean price of the delivered bond (CTD) at t

P the bond dirty price at t

T the future maturity date

CF the conversion factor for a bond to deliver in a bond futures contract

r_T the continuously compounded interest rate between t and T

$C_\Sigma = \sum_{t_i \leq T} Cexp(-r_i t_i)$ the present value sum of all coupons of the underlying bond between t and T

A the accrual interest before T.

In a typical bond future, the number of bonds entering into the valuation formula (k) is two to five, even if the basket is larger. There are one to four non-trivial k's to estimate. Once the estimation of the interval ends is done, the pricing of the futures is similar to the one of a swaption.

To estimate those ends, a numerical estimate of the potential CTD bonds is done through a procedure similar to a numerical integration. This can be done with few points (like 100). The goal is not to find the intersection points precisely but only to find their existence. Once the bonds (mi) and a rough estimate of interval ends are available, a numerical solution of the intersection between two curves is done.

You can find more details at

https://finpricing.com/lib/EqRangeAccrual.html