Correlation Swap Analytics

A Correlation Swap is a contract whose payoff at maturity is the difference between the realized correlation and the strike. The realized correlation has two main components: historical correlation part and future realized correlation part.

One may to use the current correlation swap strike level on the market as the expectation of future realized correlation part. This way the essential part needed for the mark-to-market model becomes a direct input. The market level should be verified independently by middle office. Capping (flooring) can be included in the definition of the realized correlation.

A correlation swap is a forward contract on realized correlation of a basket made of a fixed number of stock indices. The payoff at maturity date to the holder of a correlation swap is the notional amount of dollar times the difference between the basket's realized correlation and the strike price. Valuation of the correlation swap involves decomposition of the contract into two periods, one that has become historical, and the other with stock prices still unknown.

The unknown correlation from the value date to the swap maturity can be estimated using the current correlation swap strike level on the market for exactly the remaining swap period, if this is available. Otherwise, a replication method might be attempted.

Suppose there are *n* consecutive trading days $\{t_i, i = 1, \cdots, n\}$, and there are *k* underlying stocks $S_i, i = 1,2,..., k$. At the maturity of a correlation swap, the realized correlation is defined as

$$Corr_R = \max\left(F, \min\left(C, \frac{2}{k \cdot (k-1)} \cdot \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \rho_{ij}\right)\right),,$$

where $\rho_{ij} = corr(S_i, S_j)$ is the correlation between stocks S_i and S_j (see definition below), and C and F are contractual cap and floor levels. The payoff of the correlation swap at maturity T is given by

$$V(T) = N \times (Corr_R - K),$$

where N is the notional amount and K is the delivery (strike) price for correlation swap.

Denote by $R(t,T)$ the continuously compounded interest rate applying on [t, T], with day count convention (DCC) function τ, and $df(t,T) = \exp(-\tau(t,T)R(t,T))$ the discount factor at t for maturity T.

Let t be the valuation date, $t < T$. If $t_{m+1} > t \geq t_m$, the stock prices at $\{t_i, i = 1, \cdots, m\}$ have become historical. We then have

$$Corr_R = \max\left(F, \min\left(C, \frac{m}{n} Corr_{hist} + \frac{n-m}{n} Corr_{fut}\right)\right).$$

Reference:

https://finpricing.com/lib/EqQuanto.html