Digital Option Analytics

A digital option is an option that pays a fixed amount only if the underlying price meets the strike price. Digital call pays a fixed amount if the underlying price ends up above the strike price, while digital put pays a fixed amount if the underlying price is below the strike price at option maturity.

The payoff of a digital option is either a fixed amount or nothing at all. The two main types of digital options are the cash-or-nothing digital option and the asset-or-nothing digital option. The cash-or-nothing digital option pays some fixed amount of cash if the option expires in-the-money while the asset-or-nothing pays the value of the underlying security.

For a call, the underlying price must be higher than the strike at expiry to trigger the rebate or receipt of the underlying. For a put, the underlying price must be lower than the strike at expiry to trigger the rebate or receipt of the underlying.

The risk sensitivities, especially Gamma, may be unstable when at the money or near the exercise date due to the discontinuous property inherent in digital option. Risk sensitivities become less meaningful near discontinuities and kinks. As Vega may change signs near the strike, it is probably difficult to create a conservative estimate of the volatility.

These are also commonly referred to as “all or nothing” or “digital options”. Digital options can be valued using the Black-Scholes models. The PV is calculated as the product of fixed payment times call price (c) or put price (p) where:

The value of a cash-or-nothing call option is

$$c = e^{-rt}\Phi(d_2)$$

The Delta of a cash-or-nothing call option is

$$\Delta_c = \frac{\partial C}{\partial S} = Ke^{-r_p t_p} n(d_2) \frac{\partial d_2}{\partial S}$$

The Gamma of a cash-or-nothing call option is

$$\Gamma_c = \frac{\partial^2 C}{\partial S^2} = \frac{\partial}{\partial S} \frac{Ke^{-r_p t_p} n(d_2)}{S\sigma\sqrt{t_e}}$$

The Vega of a cash-or-nothing call option is

$$\nu_c = \frac{\partial C}{\partial \sigma} = e^{-r_p t_p} Kn(d_2) \frac{\partial d_2}{\partial \sigma}$$

Similarly we can get value and Greeks of a cash-or-nothing put option:

$$p = e^{-rt}\Phi(-d_2)$$

Asset-or-nothing:

$$c = Se^{-r_f t}\Phi(d_1)$$

$$p = Se^{-r_f t}\Phi(-d_1)$$

where

$$d_1 = \frac{ln(s/x) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\ \sqrt{t}$$

and

s is the current spot price

r is the risk free interest rate

t is the expiry date in years

σ is the implied volatility

Φ is the standard normal cumulative distribution function

The risk sensitivities, especially Gamma, is very unstable under the Black-Scholes. Therefore, the better solution is to represent a digital option as a tight option spread.

Reference:

https://finpricing.com/lib/EqWarrant.html