Warrant Pricing Analytics

1. Introduction

An equity warrant is an option on the equity of a firm issued by the same firm, which gives the holder the right to purchase shares at a fixed price from the firm at a future date. When a warrant is exercised, the firm typically issues new shares at the exercise price to fill the order. The resulting increase in shares outstanding dilutes the share value.

Warrants are in many ways similar to call options, but a few key differences distinguish them. Warrants tend to have longer durations than do exchange-traded call options. They are traded over the counter more often than on an exchange. Investors cannot write warrants like they can options. Warrants do not pay dividends or come with voting rights. When warrants are exercised, the company typically issues new shares at the exercise price to fill the order. The resulting increase in shares outstanding dilutes the share value.

Warrants frequently attached to bonds or preferred stock as a sweetener can be used to enhance the yield of the bond and make them more attractive to potential buyers.

Most commonly issued warrants are often detachable, meaning that they can be separated from the bond and sold on the secondary market before expiration. A detached warrant is a warrant that once was attached to a bond and is still quoted in bond units.

2. Payoff

If there were n shares outstanding and m warrants exercised, the dilution factor corresponding to the percentage of the firm value that is represented by the warrants is given by

$$\alpha = m/(m+n)$$

The payoff of the warrant at T is given by

$$payoff = \frac{m}{m+n}\max(A-K,0)$$

where

$A = V/m$ the asset price

V the firm value

3. Valuation

Warrants can be valued by the Black-Scholes model, but some modifications must be made to the parameters.

The price of the warrant under the diluted Black-Scholes model is given by

$$W = \frac{m}{m+n}\left(Ae^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2)\right)$$

where

$$d_{1,2} = \frac{\ln\left(\frac{A}{K}\right) + (r - q \pm 0.5\sigma T)}{\sigma\sqrt{T}}$$

Strictly speaking, *A* is the asset price of the firm and σ is the volatility of the firm (not stock). Both of them are not observable. For simplicity, some people use stock price and stock volatility to replace the firm value *A* and the firm volatility σ above, although this simplification generally underestimates the warrant's price.

To value a detached warrant, one needs to calculate the strike of the detached warrant from the bond's parameters:

$$K = \frac{Conversion\ xrate \cdot\ Face}{Conversion\ Ratio}$$

Then apply regular option model. Finally recast into standard bond units.

4. Assumption

There are several assumptions in this simplified warrant model (5): 1) the price process of the stock follows a geometric Brownian motion; 2) the stock provides a continuous proportional instantaneous pay-out (continuous dividend); 3) the risk-free interest rate is deterministic; 4) the volatility is constant; 5) the asset value per share is equal to the stock price; 6) the volatility of the firm is equal to the volatility of the stock.

Reference:

https://finpricing.com/lib/EqSpread.html