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This worksheet contains the Maple codes used for the population models with the strong Allee effect, of the following paper.

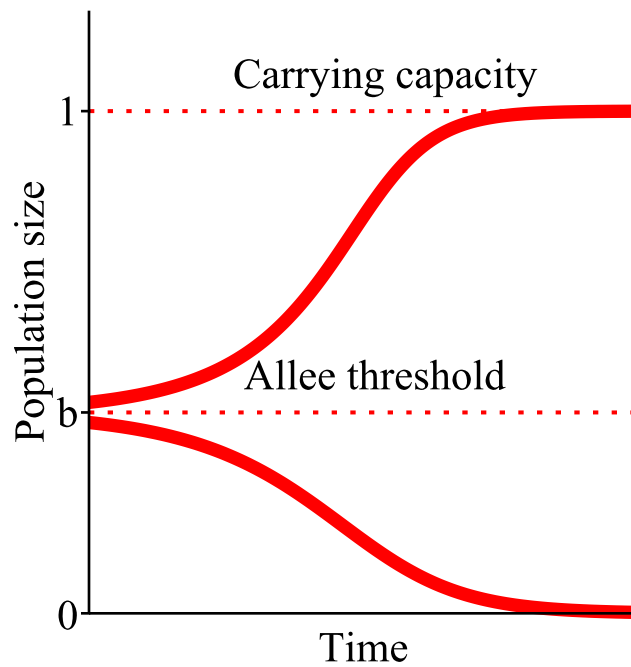
AmirHosein Sadeghimanesh, Matthew England, Resultant Tools for Parametric Polynomial Systems with Application to Population Models, 2022.

## Single population with the strong Allee effect

Plotting Figure 1.

Consider 1 population model with the strong Allee effect, normalized such that the carrying capacity is equal to 1. Let the Allee threshold be equal to  $2/5$ .

```
> survivingTrajectory := dsolve({diff(x(t),t) = x(t)*(1-x(t))*(x(t)-2/5), x(0)=21/50}, x(t), type=numeric):
survivingCurve := plots:-odeplot(survivingTrajectory, [t, x(t)], 0..25, color=red, thickness=5):
extinctingTrajectory := dsolve({diff(x(t),t) = x(t)*(1-x(t))*(x(t)-2/5), x(0)=19/50}, x(t), type=numeric):
extinctingCurve := plots:-odeplot(extinctingTrajectory, [t, x(t)], 0..25, color=red, thickness=5):
text1 := plots:-textplot([13.5,1.02,"Carrying capacity", font=["TimesNewRoman",26]], align=above):
dottedLine1 := plottools:-line([0,1], [25,1], color=red, linestyle=dot):
text2 := plots:-textplot([13,21/50,"Allee threshold", font=["TimesNewRoman",26]], align=above):
dottedLine2 := plottools:-line([0,2/5], [25,2/5], color=red, linestyle=dot):
strongAlleeEffect := plots:-display(survivingCurve, extinctingCurve, dottedLine1, text1, dottedLine2, text2, view=[0..25,0..6/5], labels=["Time", "Population size"], labelfont=["TimesNewRoman",26], labeldirections=[horizontal,vertical], tickmarks=[[], [0="0", 2/5="b", 1="1"]], axesfont=["TimesNewRoman",26]):
```



## Symbolic-Numeric method

Plotting only part a of Figure 4. The rest are taken from the following reference.

Gergely Rost, AmirHosein Sadeghimanesh, Exotic bifurcations in three connected populations with Allee effect, International Journal of Bifurcation and Chaos, 31(13): Article No. 2150202, 2021, <https://doi.org/10.1142/S0218127421502023>

The steady state equations for the three populations.

```
> vars := [seq(cat(x__,i), i=1..3)]: # variables
params := [a,b]: # parameters
F := [seq(vars[i]*(1-vars[i])*(vars[i]-b)-2*a*vars[i]+add(a*
vars[j], j in {1,2,3} minus {i}), i=1..3)]:
<seq(polynomial, polynomial in F)>;
```

$$\begin{bmatrix} x_1(1-x_1)(x_1-b) - 2ax_1 + ax_2 + ax_3 \\ x_2(1-x_2)(x_2-b) - 2ax_2 + ax_1 + ax_3 \\ x_3(1-x_3)(x_3-b) - 2ax_3 + ax_1 + ax_2 \end{bmatrix} \quad (2.1)$$

```
> bSampleValues := [seq(0+(i-1)*(1/2)/10, i=1..11)];
bSampleValues := [0, 1/20, 1/10, 3/20, 1/5, 1/4, 3/10, 7/20, 2/5, 9/20, 1/2] \quad (2.2)
```

```
> with(RootFinding:-Parametric):
> CellDecomposition([seq(eval(polynomial = 0, [b=0]), polynomial
in F), seq(var > 0, var in vars), a > 0], vars, [a]);
```

<i>Equations</i>	=	$[x_1^2 (1 - x_1) - 2 a x_1 + a x_2 + a x_3, x_2^2 (1 - x_2) - 2 a x_2 + a x_1 + a x_3, x_3^2 (1 - x_3) - 2 a x_3 + a x_1 + a x_2]$
<i>Inequalities</i>	=	$[x_1, x_2, x_3, a]$
<i>Filter</i>	=	$0 \neq 1$
<i>Variables</i>	=	$[x_1, x_2, x_3]$
<i>Parameters</i>	=	$[a]$
<i>DiscriminantVariety</i>	=	$[[a], [3 a + 1], [27 a^2 - 9 a + 1], [8748 a^5 - 972 a^4 - 648 a^3 + 214 a^2 - 24 a + 1]]$
<i>ProjectionPolynomials</i>	=	$[[a, 3 a + 1, 8748 a^5 - 972 a^4 - 648 a^3 + 214 a^2 - 24 a + 1]]$
<i>SamplePoints</i>	=	$[[a = 1]]$

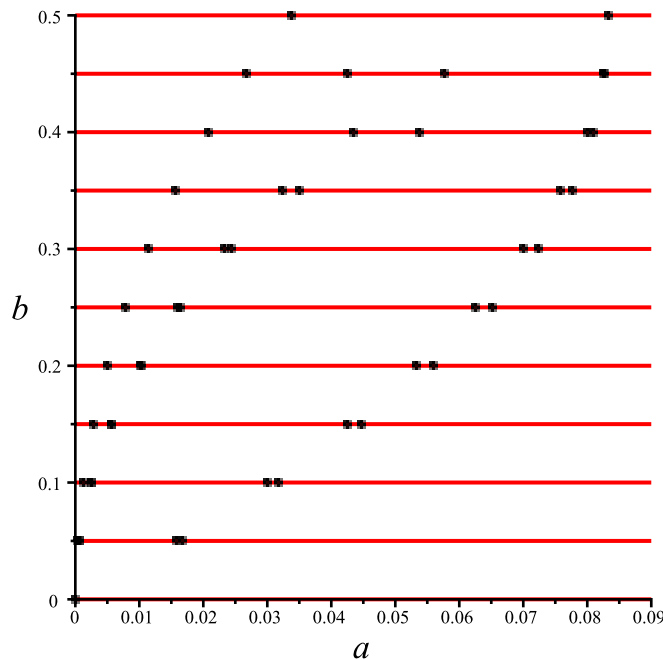
```
> intersections := Array([]):
ArrayTools:-Append(intersections, [0,0]):
# The only point for b = 0, is a = 0, because of the
discriminantVariety is 'a' as you can see above in the summary
of the CellDecomposition's result.
```

```
1 for i1 from 2 by 1 to 11 do
2   bValue := bSampleValue[i1]:
3   tempCAD := CellDecomposition(seq(eval(polynomial= 0, [b=bValue]), polynomial in F), seq(var > 0, var in vars), a
> 0], vars, [a]):
4   tempAValues := Array(seq(0, i=1..numelems(tempCAD-SamplePoint)-1)): # pre-allocation
5   for i2 from 1 by 1 to numelems(tempCAD-SamplePoint)-1 do
6     tempCell := CellDescription(tempCAD, i2):
7     tempAValue[i2] := evalf(RootOf(tempCell[1][4], tempCell[1][3], index=real[tempCell[1][5]])):
8   end do:
9   ArrayTools:-Extend(intersections, [seq([i3, bValue], i3 in tempAValue)]):
```

Now we have all the intersection points of the discriminant variety with the 11 horizontal lines.

```
> intersectionPoints := plots:-pointplot(intersections, symbol=
solidcircle, color=black, view=[0..0.09,0..0.5]):
lineArray := Array([seq(plot(bSampleValues[i], a=0..0.09,
color=red, thickness=1), i=1..11)]):
horizontalLines := plots:-display(seq(horizontalLine,
horizontalLine in lineArray)):
numericApproach := plots:-display(horizontalLines,
```

```
intersectionPoints, labels=[a,b], labelfont=["TimesNewRoman",
20], view=[0..0.09,0..0.5]);
```



## Attempting to use the CAD with respect to the discriminant variety when it is computed by GB

One can uncomment the following part and test if RootFinding:-Parametric computations terminate on their computer without hitting a memory limit or any other problem.

```
> # st := time[real](): # st stands for 'start time'.
# cadDifficult := CellDecomposition([seq(polynomial = 0,
polynomial in F), seq(var > 0, var in vars), seq(param > 0,
param in params)], vars, params):
# time[real]() - st; # to also see how long it took time to
have the computation completed, in case your computer was lucky
enough ^_^
```

One can uncomment the following part and test if Grobner basis computations terminate on their computer without hitting a memory limit or any other problem. Note that changing the order of the variables will not change the computation time since there is a symmetry in the system with respect to the variables. And in the monomial order for Grobner basis computation, the variables must be greater than the parameters. The only option is to change the order of the parameters, or to use elimination-type block monomial orders. The block order made by tdeg (grevlex) on each two blocks of the variables and parameters have been tested as well with no success.

We first need to compute the determinant of the Jacobian matrix. Note that in this example the number of nonnegative real solutions changes only when the number of real solutions changes. That is the reason we only check the discriminant variety coming from the determinant of Jacobian matrix.

```

> J := Matrix([seq([seq(diff(polynomial, var), var in vars)],
polynomial in F)]):
d := LinearAlgebra:-Determinant(J):
equations := [seq(polynomial, polynomial in F), d]:
> # st := time[real]():
# GrobnerDifficult := Groebner:-Basis(equations, plex(seq(var,
var in vars), seq(param, param in params))):
# time[real]() - st;

```

## Using resultant techniques

Using Dixon Resultant implementation in the Maple package DR to get the discriminant variety.

```

> read("C:\\Home\\Packages\\dixon-master\\dr.mpl"): # replace the
directory with the one that dr.mpl file is located there on
your own computer.
> st:=time[real]():
DV := DR:-DixonResultant(equations,vars):
time[real]()-st;

```

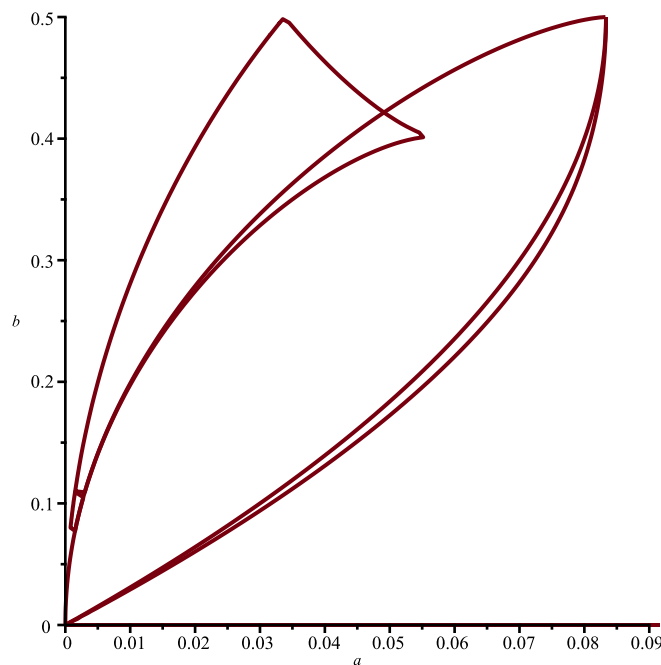
337.5430000000

(4.1)

```

> plots:-implicitplot(DV,a=0..0.15,b=0..0.5,view=[0..0.09,0..0.5]
);

```



Some tiny places where the plot looks ugly is because of the plot function, not because of the result. One can ask a finer plot.

The Dixon resultant computed is the following.

```

> DV;
42391158275216203514294433201 b2 (3 a + b)8 (2 a + b)24 (4 a b4 - 36 a2 b2 - 8 a b3) (4.2)

```

$$\begin{aligned}
& -b^4 + 108a^3 + 36a^2b + 12ab^2 + 2b^3 - 36a^2 - 8ab - b^2 + 4a) \\
& ^9 (256a^4b^{10} - 32a^2b^{12} - 6144a^5b^8 - 1280a^4b^9 + 768a^3b^{10} + 192a^2b^{11} \\
& + 12ab^{12} + 54784a^6b^6 + 24576a^5b^7 - 4416a^4b^8 - 3840a^3b^9 - 776a^2b^{10} \\
& - 72ab^{11} - b^{12} - 165888a^7b^4 - 164352a^6b^5 - 16512a^5b^6 + 25344a^4b^7 \\
& + 10848a^3b^8 + 2120a^2b^9 + 204ab^{10} + 6b^{11} - 248832a^8b^2 + 331776a^7b^3 \\
& + 207744a^6b^4 - 36480a^5b^5 - 54528a^4b^6 - 20352a^3b^7 - 3800a^2b^8 - 360ab^9 \\
& - 15b^{10} + 2239488a^9 + 248832a^8b - 497664a^7b^2 - 141568a^6b^3 + 62976a^5b^4 \\
& + 69504a^4b^5 + 25152a^3b^6 + 4592a^2b^7 + 432ab^8 + 20b^9 - 248832a^8 \\
& + 331776a^7b + 207744a^6b^2 - 36480a^5b^3 - 54528a^4b^4 - 20352a^3b^5 \\
& - 3800a^2b^6 - 360ab^7 - 15b^8 - 165888a^7 - 164352a^6b - 16512a^5b^2 \\
& + 25344a^4b^3 + 10848a^3b^4 + 2120a^2b^5 + 204ab^6 + 6b^7 + 54784a^6 + 24576a^5b \\
& - 4416a^4b^2 - 3840a^3b^3 - 776a^2b^4 - 72ab^5 - b^6 - 6144a^5 - 1280a^4b \\
& + 768a^3b^2 + 192a^2b^3 + 12ab^4 + 256a^4 - 32a^2b^2)^3 (3a + 1 - b)^8 (b^2 \\
& + 3a - b)^8 (b - 1)^2
\end{aligned}$$

> **factor\_list: factors(DV):**

**factors\_list:= [seq(j[1], j in factor\_list[2])];**

$$\begin{aligned}
factors\_list := & \left[ a + \frac{b}{3}, \frac{1}{186624} ab^4 - \frac{1}{69984} a^2b^2 + \frac{1}{8748} a^4b^{10} - \frac{1}{69984} a^2b^{12} \right. \\
& - \frac{2}{729} a^5b^8 - \frac{5}{8748} a^4b^9 + \frac{1}{2916} a^3b^{10} + \frac{1}{11664} a^2b^{11} + \frac{1}{186624} ab^{12} \\
& + \frac{107}{4374} a^6b^6 + \frac{8}{729} a^5b^7 - \frac{23}{11664} a^4b^8 - \frac{5}{2916} a^3b^9 - \frac{97}{279936} a^2b^{10} \\
& - \frac{1}{31104} ab^{11} - \frac{2}{27} a^7b^4 - \frac{107}{1458} a^6b^5 - \frac{43}{5832} a^5b^6 + \frac{11}{972} a^4b^7 \\
& + \frac{113}{23328} a^3b^8 + \frac{265}{279936} a^2b^9 + \frac{17}{186624} ab^{10} - \frac{1}{9} a^8b^2 + \frac{4}{27} a^7b^3 \\
& + \frac{541}{5832} a^6b^4 - \frac{95}{5832} a^5b^5 - \frac{71}{2916} a^4b^6 - \frac{53}{5832} a^3b^7 - \frac{475}{279936} a^2b^8 \\
& - \frac{5}{31104} ab^9 + \frac{1}{9} a^8b - \frac{2}{9} a^7b^2 - \frac{553}{8748} a^6b^3 + \frac{41}{1458} a^5b^4 + \frac{181}{5832} a^4b^5 \\
& + \frac{131}{11664} a^3b^6 + \frac{287}{139968} a^2b^7 + \frac{1}{5184} ab^8 + \frac{4}{27} a^7b + \frac{541}{5832} a^6b^2 \\
& - \frac{95}{5832} a^5b^3 - \frac{71}{2916} a^4b^4 - \frac{53}{5832} a^3b^5 + a^9 - \frac{1}{2239488} b^{12} + \frac{1}{373248} b^{11} \\
& \left. - \frac{5}{746496} b^{10} + \frac{5}{559872} b^9 - \frac{1}{9} a^8 - \frac{5}{746496} b^8 - \frac{2}{27} a^7 + \frac{1}{373248} b^7 \right] \quad (4.3)
\end{aligned}$$

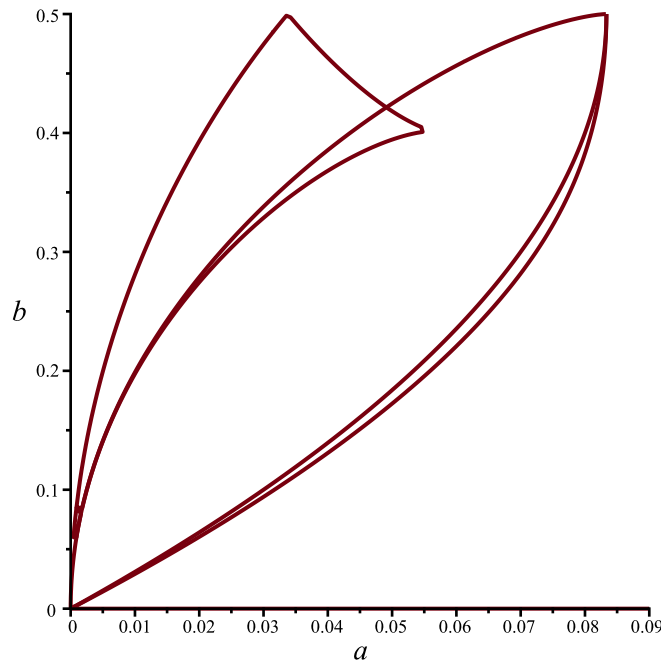
$$\begin{aligned}
& + \frac{107}{4374} a^6 - \frac{1}{2239488} b^6 - \frac{2}{729} a^5 + \frac{1}{8748} a^4 - \frac{475}{279936} a^2 b^6 - \frac{5}{31104} a b^7 \\
& - \frac{107}{1458} a^6 b - \frac{43}{5832} a^5 b^2 + \frac{11}{972} a^4 b^3 + \frac{113}{23328} a^3 b^4 + \frac{265}{279936} a^2 b^5 \\
& + \frac{17}{186624} a b^6 + \frac{8}{729} a^5 b - \frac{23}{11664} a^4 b^2 - \frac{5}{2916} a^3 b^3 - \frac{97}{279936} a^2 b^4 \\
& - \frac{1}{31104} a b^5 - \frac{5}{8748} a^4 b + \frac{1}{2916} a^3 b^2 + \frac{1}{11664} a^2 b^3, a + \frac{b}{2}, b, \frac{1}{27} a b^4 \\
& - \frac{1}{3} a^2 b^2 - \frac{2}{27} a b^3 - \frac{1}{108} b^4 + a^3 + \frac{1}{3} a^2 b + \frac{1}{9} a b^2 + \frac{1}{54} b^3 - \frac{1}{3} a^2 \\
& - \frac{2}{27} a b - \frac{1}{108} b^2 + \frac{1}{27} a, \frac{1}{3} b^2 + a - \frac{1}{3} b, a + \frac{1}{3} - \frac{b}{3}, b - 1 \Big]
\end{aligned}$$

```
> numelems(factors_list);
```

8

(4.4)

```
> st := time[real]():
curvesPP:=Array(1..numelems(factors_list)):
for i from 1 by 1 to numelems(factors_list) do
    curvesPP[i]:=plots:-implicitplot(factors_list[i]=0, a=0.
.0.09, b=0..0.5):
end do:
finalPP := plots:-display(seq(curvesPP[i], i=1..numelems
(factors_list)), view=[0..0.09,0..0.5], labelfont=
["TimesNewRoman",18]);
time[real]()-st;
```



0.0610000000

(4.5)

The plot got a bit better, but still not that much.

It has 8 irreducible factors two of which has no solutions in the positive orthant, namely  $3a+b$  and  $2a+b$ .

The interesting region where the number of steady states temporarily increases is related to the longest irreducible factor.

**> longest\_polynomial := factors\_list[2];**

$$\text{longest\_polynomial} := -\frac{95}{5832} a^5 b^3 + \frac{287}{139968} a^2 b^7 + \frac{181}{5832} a^4 b^5 + \frac{131}{11664} a^3 b^6 \quad (4.6)$$

$$- \frac{553}{8748} a^6 b^3 + \frac{41}{1458} a^5 b^4 + \frac{1}{9} a^8 b - \frac{475}{279936} a^2 b^8 - \frac{5}{31104} a b^9 - \frac{2}{9} a^7 b^2$$

$$- \frac{53}{5832} a^3 b^7 - \frac{71}{2916} a^4 b^6 - \frac{95}{5832} a^5 b^5 + \frac{541}{5832} a^6 b^4 - \frac{1}{9} a^8 b^2$$

$$+ \frac{17}{186624} a b^{10} + \frac{265}{279936} a^2 b^9 - \frac{107}{1458} a^6 b^5 + \frac{113}{23328} a^3 b^8 - \frac{43}{5832} a^5 b^6$$

$$- \frac{2}{27} a^7 b^4 - \frac{1}{31104} a b^{11} + \frac{11}{972} a^4 b^7 + \frac{8}{729} a^5 b^7 + \frac{4}{27} a^7 b^3$$

$$- \frac{97}{279936} a^2 b^{10} - \frac{5}{2916} a^3 b^9 + \frac{1}{186624} a b^{12} - \frac{23}{11664} a^4 b^8 + \frac{107}{4374} a^6 b^6$$

$$- \frac{1}{69984} a^2 b^{12} - \frac{2}{729} a^5 b^8 + \frac{1}{2916} a^3 b^{10} - \frac{5}{8748} a^4 b^9 + \frac{1}{11664} a^2 b^{11}$$

$$+ \frac{1}{8748} a^4 b^{10} + \frac{1}{186624} a b^4 - \frac{1}{69984} a^2 b^2 + \frac{1}{2916} a^3 b^2 + \frac{1}{11664} a^2 b^3$$

$$- \frac{5}{8748} a^4 b - \frac{5}{2916} a^3 b^3 + \frac{17}{186624} a b^6 - \frac{23}{11664} a^4 b^2 - \frac{1}{31104} a b^5$$

$$+ \frac{265}{279936} a^2 b^5 + \frac{8}{729} a^5 b - \frac{97}{279936} a^2 b^4 + \frac{113}{23328} a^3 b^4 - \frac{107}{1458} a^6 b$$

$$+ \frac{11}{972} a^4 b^3 - \frac{5}{31104} a b^7 - \frac{43}{5832} a^5 b^2 - \frac{475}{279936} a^2 b^6 - \frac{53}{5832} a^3 b^5$$

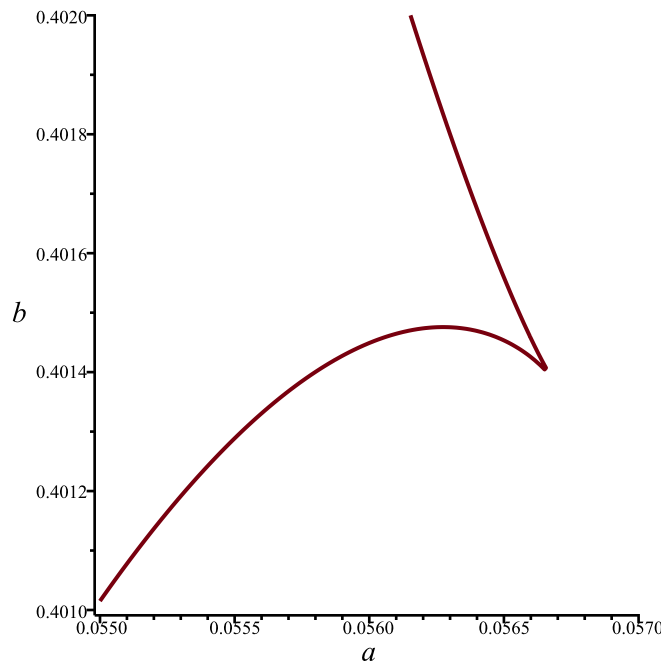
$$- \frac{71}{2916} a^4 b^4 + \frac{541}{5832} a^6 b^2 + \frac{4}{27} a^7 b + \frac{1}{5184} a b^8 + a^9 - \frac{1}{9} a^8 - \frac{2}{27} a^7$$

$$+ \frac{1}{373248} b^7 + \frac{107}{4374} a^6 - \frac{1}{2239488} b^6 + \frac{1}{373248} b^{11} + \frac{5}{559872} b^9$$

$$- \frac{1}{2239488} b^{12} - \frac{2}{729} a^5 - \frac{5}{746496} b^8 + \frac{1}{8748} a^4 - \frac{5}{746496} b^{10}$$

**> plots:-implicitplot(longest\_polynomial, a=0.0550..0.0570, b=0.4010..0.4020, view=[0.0550..0.0570, 0.4010..0.4020], labelfont=["TimesNewRoman",18]);**





Now using our ResChain package.

```
> libname ,= "C:\\Home\\Packages\\ResChain": # replace the
    directory with the one that ResChain.mpl file is located there
    on your own computer.
```

First trying the simple resultant chain.

```
> st := time[real]():
    RC1 := ResChain:-ResChainSimple(equations, vars):
    time[real]()-st;
```

0.5170000000 (4.7)

```
> RC1;
```

$$\begin{aligned}
 & \left[ a, a + \frac{b}{3}, \frac{1}{27} a b^4 - \frac{1}{3} a^2 b^2 - \frac{2}{27} a b^3 - \frac{1}{108} b^4 + a^3 + \frac{1}{3} a^2 b + \frac{1}{9} a b^2 \right. \\
 & + \frac{1}{54} b^3 - \frac{1}{3} a^2 - \frac{2}{27} a b - \frac{1}{108} b^2 + \frac{1}{27} a, \frac{1}{3} b^2 + a - \frac{1}{3} b, a + \frac{1}{3} - \frac{b}{3}, \\
 & - \frac{2}{295245} b^5 + \frac{16094752}{7381125} a^5 b^3 - \frac{5305468}{36905625} a^2 b^7 - \frac{92002262}{12301875} a^4 b^5 \\
 & - \frac{59871236}{36905625} a^3 b^6 + \frac{112092782}{7381125} a^6 b^3 - \frac{431663996}{36905625} a^5 b^4 + \frac{660483196}{4100625} a^8 b \\
 & + \frac{3785144}{7381125} a^2 b^8 + \frac{2768}{91125} a b^9 + \frac{1277227352}{12301875} a^7 b^2 + \frac{56745616}{12301875} a^3 b^7 \\
 & + \frac{823300432}{36905625} a^4 b^6 + \frac{1717835408}{36905625} a^5 b^5 - \frac{618746998}{36905625} a^6 b^4 - \frac{59141914}{164025} a^8 b^2 \\
 & \left. - \frac{31456}{492075} a b^{10} - \frac{35984204}{36905625} a^2 b^9 - \frac{178654852}{7381125} a^6 b^5 - \frac{303963908}{36905625} a^3 b^8 \right]
 \end{aligned}
 \quad (4.8)$$

$$\begin{aligned}
& - \frac{1209280604}{12301875} a^5 b^6 + \frac{1573007972}{4100625} a^7 b^4 + \frac{117656}{1476225} a b^{11} - \frac{1482310358}{36905625} a^4 b^7 \\
& + \frac{1526698304}{12301875} a^5 b^7 - \frac{2909391448}{12301875} a^7 b^3 + \frac{44014624}{36905625} a^2 b^{10} + \frac{14679752}{1476225} a^3 b^9 \\
& - \frac{31456}{492075} a b^{12} + \frac{198758368}{4100625} a^4 b^8 + \frac{2126387128}{36905625} a^6 b^6 + \frac{3785144}{7381125} a^2 b^{12} \\
& - \frac{1209280604}{12301875} a^5 b^8 - \frac{303963908}{36905625} a^3 b^{10} - \frac{1482310358}{36905625} a^4 b^9 \\
& - \frac{35984204}{36905625} a^2 b^{11} + \frac{823300432}{36905625} a^4 b^{10} - \frac{788}{7381125} a b^4 - \frac{1844}{7381125} a^2 b^2 \\
& + \frac{16}{295245} a b^3 - \frac{8}{1476225} a b^2 + \frac{268372}{36905625} a^3 b^2 + \frac{1844}{820125} a^2 b^3 \\
& - \frac{123136}{7381125} a^4 b - \frac{1820576}{36905625} a^3 b^3 + \frac{2564}{820125} a b^6 + \frac{451606}{12301875} a^4 b^2 \\
& - \frac{1436}{2460375} a b^5 + \frac{242128}{12301875} a^2 b^5 + \frac{12469576}{36905625} a^5 b - \frac{36212}{4100625} a^2 b^4 \\
& + \frac{211672}{2460375} a^3 b^4 - \frac{166024}{12301875} a^6 b + \frac{1290674}{36905625} a^4 b^3 - \frac{784}{164025} a b^7 \\
& - \frac{40429708}{36905625} a^5 b^2 - \frac{117596}{36905625} a^2 b^6 + \frac{1698424}{7381125} a^3 b^5 + \frac{38535764}{36905625} a^4 b^4 \\
& - \frac{4447186}{1476225} a^6 b^2 - \frac{219951796}{7381125} a^7 b - \frac{9856}{2460375} a b^8 + \frac{1}{1476225} b^{20} \\
& - \frac{2}{295245} b^{19} + \frac{127}{7381125} b^{18} + \frac{94}{2460375} b^{17} - \frac{9839}{36905625} b^{16} + \frac{14452}{36905625} b^{15} \\
& + \frac{1502}{4100625} b^{14} - \frac{74176}{36905625} b^{13} - \frac{1904}{45} a^{11} + a^{12} - \frac{64}{164025} a^3 b \\
& + \frac{2402468}{273375} a^9 - \frac{2446654621}{4100625} a^8 b^4 - \frac{5513864252}{12301875} a^7 b^5 - \frac{36586784}{91125} a^9 b^2 \\
& + \frac{2123952364}{4100625} a^8 b^3 - \frac{2402468}{91125} a^9 b + \frac{219951796}{36905625} a^7 b^{10} - \frac{166024}{12301875} a^6 b^{11} \\
& - \frac{40429708}{36905625} a^5 b^{12} + \frac{1290674}{36905625} a^4 b^{13} + \frac{211672}{2460375} a^3 b^{14} + \frac{242128}{12301875} a^2 b^{15} \\
& + \frac{2564}{820125} a b^{16} - \frac{165120799}{4100625} a^8 b^8 - \frac{219951796}{7381125} a^7 b^9 - \frac{4447186}{1476225} a^6 b^{10} \\
& + \frac{16094752}{7381125} a^5 b^{11} + \frac{38535764}{36905625} a^4 b^{12} + \frac{1698424}{7381125} a^3 b^{13} - \frac{117596}{36905625} a^2 b^{14} \\
& - \frac{784}{164025} a b^{15} + \frac{2402468}{273375} a^9 b^6 + \frac{660483196}{4100625} a^8 b^7 + \frac{1277227352}{12301875} a^7 b^8 \\
& + \frac{112092782}{7381125} a^6 b^9 - \frac{431663996}{36905625} a^5 b^{10} - \frac{92002262}{12301875} a^4 b^{11} - \frac{59871236}{36905625} a^3 b^{12}
\end{aligned}$$

$$\begin{aligned}
& - \frac{5305468}{36905625} a^2 b^{13} - \frac{9856}{2460375} a b^{14} + \frac{304022}{675} a^{10} b^4 - \frac{2402468}{91125} a^9 b^5 \\
& - \frac{59141914}{164025} a^8 b^6 - \frac{2909391448}{12301875} a^7 b^7 - \frac{618746998}{36905625} a^6 b^8 + \frac{1717835408}{36905625} a^5 b^9 \\
& + \frac{56745616}{12301875} a^3 b^{11} + \frac{2768}{91125} a b^{13} - \frac{1904}{45} a^{11} b^2 - \frac{608044}{675} a^{10} b^3 \\
& - \frac{36586784}{91125} a^9 b^4 + \frac{2123952364}{4100625} a^8 b^5 + \frac{1573007972}{4100625} a^7 b^6 - \frac{178654852}{7381125} a^6 b^7 \\
& + \frac{1904}{45} a^{11} b + \frac{304022}{225} a^{10} b^2 + \frac{231533044}{273375} a^9 b^3 - \frac{608044}{675} a^{10} b \\
& + \frac{64}{1476225} a^3 b^{18} - \frac{8}{1476225} a b^{20} + \frac{15392}{7381125} a^4 b^{16} - \frac{64}{164025} a^3 b^{17} \\
& - \frac{1844}{7381125} a^2 b^{18} + \frac{16}{295245} a b^{19} - \frac{1781368}{36905625} a^5 b^{14} - \frac{123136}{7381125} a^4 b^{15} \\
& + \frac{268372}{36905625} a^3 b^{16} + \frac{1844}{820125} a^2 b^{17} - \frac{788}{7381125} a b^{18} + \frac{83012}{36905625} a^6 b^{12} \\
& + \frac{12469576}{36905625} a^5 b^{13} + \frac{451606}{12301875} a^4 b^{14} - \frac{1820576}{36905625} a^3 b^{15} - \frac{36212}{4100625} a^2 b^{16} \\
& - \frac{1436}{2460375} a b^{17} + \frac{304022}{675} a^{10} - \frac{165120799}{4100625} a^8 + \frac{219951796}{36905625} a^7 \\
& + \frac{94}{2460375} b^7 + \frac{83012}{36905625} a^6 + \frac{127}{7381125} b^6 + \frac{64}{1476225} a^3 - \frac{74176}{36905625} b^{11} \\
& + \frac{14452}{36905625} b^9 + \frac{482}{164025} b^{12} - \frac{1781368}{36905625} a^5 - \frac{9839}{36905625} b^8 + \frac{15392}{7381125} a^4 \\
& + \frac{1502}{4100625} b^{10} + \frac{1}{1476225} b^4, - \frac{95}{5832} a^5 b^3 + \frac{287}{139968} a^2 b^7 + \frac{181}{5832} a^4 b^5 \\
& + \frac{131}{11664} a^3 b^6 - \frac{553}{8748} a^6 b^3 + \frac{41}{1458} a^5 b^4 + \frac{1}{9} a^8 b - \frac{475}{279936} a^2 b^8 \\
& - \frac{5}{31104} a b^9 - \frac{2}{9} a^7 b^2 - \frac{53}{5832} a^3 b^7 - \frac{71}{2916} a^4 b^6 - \frac{95}{5832} a^5 b^5 + \frac{541}{5832} a^6 b^4 \\
& - \frac{1}{9} a^8 b^2 + \frac{17}{186624} a b^{10} + \frac{265}{279936} a^2 b^9 - \frac{107}{1458} a^6 b^5 + \frac{113}{23328} a^3 b^8 \\
& - \frac{43}{5832} a^5 b^6 - \frac{2}{27} a^7 b^4 - \frac{1}{31104} a b^{11} + \frac{11}{972} a^4 b^7 + \frac{8}{729} a^5 b^7 + \frac{4}{27} a^7 b^3 \\
& - \frac{97}{279936} a^2 b^{10} - \frac{5}{2916} a^3 b^9 + \frac{1}{186624} a b^{12} - \frac{23}{11664} a^4 b^8 + \frac{107}{4374} a^6 b^6 \\
& - \frac{1}{69984} a^2 b^{12} - \frac{2}{729} a^5 b^8 + \frac{1}{2916} a^3 b^{10} - \frac{5}{8748} a^4 b^9 + \frac{1}{11664} a^2 b^{11} \\
& + \frac{1}{8748} a^4 b^{10} + \frac{1}{186624} a b^4 - \frac{1}{69984} a^2 b^2 + \frac{1}{2916} a^3 b^2 + \frac{1}{11664} a^2 b^3
\end{aligned}$$

$$\begin{aligned}
& -\frac{5}{8748} a^4 b - \frac{5}{2916} a^3 b^3 + \frac{17}{186624} a b^6 - \frac{23}{11664} a^4 b^2 - \frac{1}{31104} a b^5 \\
& + \frac{265}{279936} a^2 b^5 + \frac{8}{729} a^5 b - \frac{97}{279936} a^2 b^4 + \frac{113}{23328} a^3 b^4 - \frac{107}{1458} a^6 b \\
& + \frac{11}{972} a^4 b^3 - \frac{5}{31104} a b^7 - \frac{43}{5832} a^5 b^2 - \frac{475}{279936} a^2 b^6 - \frac{53}{5832} a^3 b^5 \\
& - \frac{71}{2916} a^4 b^4 + \frac{541}{5832} a^6 b^2 + \frac{4}{27} a^7 b + \frac{1}{5184} a b^8 + a^9 - \frac{1}{9} a^8 - \frac{2}{27} a^7 \\
& + \frac{1}{373248} b^7 + \frac{107}{4374} a^6 - \frac{1}{2239488} b^6 + \frac{1}{373248} b^{11} + \frac{5}{559872} b^9 \\
& - \frac{1}{2239488} b^{12} - \frac{2}{729} a^5 - \frac{5}{746496} b^8 + \frac{1}{8748} a^4 - \frac{5}{746496} b^{10}, - \frac{1}{3645} a \\
& + \frac{73}{36450} b^5 + \frac{56}{3645} a^2 b^7 + \frac{1586}{18225} a^3 b^6 - \frac{14}{3645} a^2 b^8 + \frac{1}{729} a b^9 \\
& - \frac{1}{3645} a b^{10} + \frac{169}{6075} a b^4 - \frac{1411}{18225} a^2 b^2 - \frac{8}{675} a b^3 + \frac{56}{3645} a^2 b \\
& + \frac{11}{12150} a b^2 + \frac{1}{729} a b + \frac{3622}{6075} a^3 b^2 + \frac{3253}{18225} a^2 b^3 + \frac{134}{135} a^4 b \\
& - \frac{13802}{18225} a^3 b^3 + \frac{169}{6075} a b^6 - \frac{67}{45} a^4 b^2 - \frac{44}{1215} a b^5 + \frac{3253}{18225} a^2 b^5 - a^5 b \\
& - \frac{4174}{18225} a^2 b^4 + \frac{3622}{6075} a^3 b^4 + \frac{134}{135} a^4 b^3 - \frac{8}{675} a b^7 + a^5 b^2 - \frac{1411}{18225} a^2 b^6 \\
& - \frac{1586}{6075} a^3 b^5 - \frac{67}{135} a^4 b^4 + \frac{11}{12150} a b^8 - \frac{1586}{6075} a^3 b - \frac{1}{2916} b^3 + \frac{73}{36450} b^7 \\
& - \frac{127}{36450} b^6 + \frac{1586}{18225} a^3 - \frac{1}{2916} b^9 + a^5 + \frac{1}{72900} b^8 - \frac{67}{135} a^4 + \frac{1}{14580} b^{10} \\
& + \frac{1}{72900} b^4 - \frac{14}{3645} a^2 + \frac{1}{14580} b^2, \frac{8}{675} a b^4 - \frac{1}{5} a^2 b^2 - \frac{16}{675} a b^3 - \frac{1}{675} b^4 \\
& + a^3 + \frac{1}{5} a^2 b + \frac{8}{225} a b^2 + \frac{2}{675} b^3 - \frac{1}{5} a^2 - \frac{16}{675} a b - \frac{1}{675} b^2 + \frac{8}{675} a, \\
& - \frac{7}{3383532} b^5 + \frac{1012651}{56392200} a^5 b^3 + \frac{92561}{42294150} a^2 b^7 + \frac{1943761}{101505960} a^4 b^5 \\
& + \frac{31261471}{3045178800} a^3 b^6 - \frac{4076027}{38064735} a^6 b^3 - \frac{9229}{626580} a^5 b^4 + \frac{847}{2655} a^8 b \\
& - \frac{37427}{21147075} a^2 b^8 - \frac{26633}{135341280} a b^9 - \frac{75793}{783225} a^7 b^2 - \frac{12385849}{1522589400} a^3 b^7 \\
& - \frac{942302}{63441225} a^4 b^6 + \frac{1012651}{56392200} a^5 b^5 + \frac{1573912}{12688245} a^6 b^4 - \frac{847}{2655} a^8 b^2 \\
& + \frac{341881}{4060238400} a b^{10} + \frac{30701}{33835320} a^2 b^9 - \frac{1073489}{12688245} a^6 b^5 + \frac{25631269}{6090357600} a^3 b^8
\end{aligned}$$

$$\begin{aligned}
& - \frac{458911}{18797400} a^5 b^6 - \frac{75793}{2349675} a^7 b^4 - \frac{1993}{676706400} a b^{11} + \frac{1673329}{253764900} a^4 b^7 \\
& + \frac{111341}{7049025} a^5 b^7 + \frac{151586}{2349675} a^7 b^3 - \frac{34727}{126882450} a^2 b^{10} - \frac{244517}{152258940} a^3 b^9 \\
& - \frac{70807}{4060238400} a b^{12} - \frac{828229}{1015059600} a^4 b^8 + \frac{1073489}{38064735} a^6 b^6 - \frac{851}{101505960} a^2 b^{12} \\
& - \frac{111341}{28196100} a^5 b^8 + \frac{363317}{761294700} a^3 b^{10} - \frac{313}{563922} a^4 b^9 + \frac{851}{16917660} a^2 b^{11} \\
& + \frac{313}{2819610} a^4 b^{10} - \frac{70807}{4060238400} a b^4 - \frac{851}{101505960} a^2 b^2 + \frac{7}{845883} a b^3 \\
& - \frac{1}{845883} a b^2 + \frac{363317}{761294700} a^3 b^2 + \frac{851}{16917660} a^2 b^3 - \frac{313}{563922} a^4 b \\
& - \frac{244517}{152258940} a^3 b^3 + \frac{341881}{4060238400} a b^6 - \frac{828229}{1015059600} a^4 b^2 - \frac{1993}{676706400} a b^5 \\
& + \frac{30701}{33835320} a^2 b^5 + \frac{111341}{7049025} a^5 b - \frac{34727}{126882450} a^2 b^4 + \frac{25631269}{6090357600} a^3 b^4 \\
& - \frac{1073489}{12688245} a^6 b + \frac{1673329}{253764900} a^4 b^3 - \frac{26633}{135341280} a b^7 - \frac{458911}{18797400} a^5 b^2 \\
& - \frac{37427}{21147075} a^2 b^6 - \frac{12385849}{1522589400} a^3 b^5 - \frac{942302}{63441225} a^4 b^4 + \frac{1573912}{12688245} a^6 b^2 \\
& + \frac{151586}{2349675} a^7 b + \frac{28393}{112784400} a b^8 + \frac{1}{3383532} b^{14} - \frac{7}{3383532} b^{13} \\
& - \frac{8}{93987} a^3 b + a^9 + \frac{4}{281961} a^3 b^{12} - \frac{1}{845883} a b^{14} - \frac{8}{93987} a^3 b^{11} \\
& + \frac{7}{845883} a b^{13} - \frac{847}{2655} a^8 - \frac{75793}{2349675} a^7 - \frac{9551}{8120476800} b^7 + \frac{1073489}{38064735} a^6 \\
& + \frac{227951}{48722860800} b^6 + \frac{4}{281961} a^3 - \frac{9551}{8120476800} b^{11} + \frac{48529}{2436143040} b^9 \\
& + \frac{227951}{48722860800} b^{12} - \frac{111341}{28196100} a^5 - \frac{37969}{3248190720} b^8 + \frac{313}{2819610} a^4 \\
& - \frac{37969}{3248190720} b^{10} + \frac{1}{3383532} b^4, - \frac{700}{1608739947} b^5 - \frac{76200389}{6434959788} a^5 b^3 \\
& + \frac{7678951}{38609758728} a^2 b^7 - \frac{10075541}{3217479894} a^4 b^5 - \frac{33049303}{115829276184} a^3 b^6 \\
& + \frac{828081737}{14478659523} a^6 b^3 + \frac{22911107}{1608739947} a^5 b^4 - \frac{2515}{23157} a^8 b - \frac{7863659}{77219517456} a^2 b^8 \\
& - \frac{49510}{1608739947} a b^9 - \frac{7759030}{59582961} a^7 b^2 + \frac{35005105}{57914638092} a^3 b^7 + \frac{9461158}{4826219841} a^4 b^6 \\
& - \frac{76200389}{6434959788} a^5 b^5 - \frac{204146489}{9652439682} a^6 b^4 + \frac{2515}{23157} a^8 b^2 + \frac{28727}{4826219841} a b^{10}
\end{aligned}$$

$$\begin{aligned}
& - \frac{5023195}{77219517456} a^2 b^9 - \frac{43128454}{4826219841} a^6 b^5 - \frac{176908045}{231658552368} a^3 b^8 \\
& + \frac{45312311}{6434959788} a^5 b^6 - \frac{7759030}{178748883} a^7 b^4 + \frac{14738}{1608739947} a b^{11} \\
& - \frac{35059}{4826219841} a^4 b^7 - \frac{4266896}{1608739947} a^5 b^7 + \frac{15518060}{178748883} a^7 b^3 \\
& + \frac{967847}{8579946384} a^2 b^{10} + \frac{9431645}{28957319046} a^3 b^9 - \frac{43769}{4826219841} a b^{12} \\
& - \frac{6249007}{6434959788} a^4 b^8 + \frac{43128454}{14478659523} a^6 b^6 + \frac{43784}{4826219841} a^2 b^{12} \\
& + \frac{1066724}{1608739947} a^5 b^8 + \frac{507271}{28957319046} a^3 b^{10} + \frac{9391040}{14478659523} a^4 b^9 \\
& - \frac{87568}{1608739947} a^2 b^{11} - \frac{1878208}{14478659523} a^4 b^{10} - \frac{43769}{4826219841} a b^4 \\
& + \frac{43784}{4826219841} a^2 b^2 + \frac{5600}{1608739947} a b^3 - \frac{800}{1608739947} a b^2 \\
& + \frac{507271}{28957319046} a^3 b^2 - \frac{87568}{1608739947} a^2 b^3 + \frac{9391040}{14478659523} a^4 b \\
& + \frac{9431645}{28957319046} a^3 b^3 + \frac{28727}{4826219841} a b^6 - \frac{6249007}{6434959788} a^4 b^2 \\
& + \frac{14738}{1608739947} a b^5 - \frac{5023195}{77219517456} a^2 b^5 - \frac{4266896}{1608739947} a^5 b \\
& + \frac{967847}{8579946384} a^2 b^4 - \frac{176908045}{231658552368} a^3 b^4 - \frac{43128454}{4826219841} a^6 b \\
& - \frac{35059}{4826219841} a^4 b^3 - \frac{49510}{1608739947} a b^7 + \frac{45312311}{6434959788} a^5 b^2 \\
& - \frac{7863659}{77219517456} a^2 b^6 + \frac{35005105}{57914638092} a^3 b^5 + \frac{9461158}{4826219841} a^4 b^4 \\
& - \frac{204146489}{9652439682} a^6 b^2 + \frac{15518060}{178748883} a^7 b + \frac{23324}{536246649} a b^8 + \frac{100}{1608739947} b^{14} \\
& - \frac{700}{1608739947} b^{13} - \frac{217600}{4826219841} a^3 b + a^9 + \frac{108800}{14478659523} a^3 b^{12} \\
& - \frac{800}{1608739947} a b^{14} - \frac{217600}{4826219841} a^3 b^{11} + \frac{5600}{1608739947} a b^{13} + \frac{2515}{23157} a^8 \\
& - \frac{7759030}{178748883} a^7 - \frac{9551}{38609758728} b^7 + \frac{43128454}{14478659523} a^6 + \frac{227951}{231658552368} b^6 \\
& + \frac{108800}{14478659523} a^3 - \frac{9551}{38609758728} b^{11} + \frac{242645}{57914638092} b^9 + \frac{227951}{231658552368} b^{12} \\
& + \frac{1066724}{1608739947} a^5 - \frac{189845}{77219517456} b^8 - \frac{1878208}{14478659523} a^4 - \frac{189845}{77219517456} b^{10}
\end{aligned}$$

$$\begin{aligned}
& + \frac{100}{1608739947} b^4, \frac{155}{324} a b^3 - \frac{1}{324} b^5 + \frac{187}{324} a^2 b^3 + \frac{1}{9} a b^4 + \frac{25}{36} a^3 b^2 \\
& - \frac{5}{432} b^6 - \frac{35}{648} a b^5 - \frac{5}{81} a^2 b^4 - \frac{5}{432} b^2 - \frac{35}{648} a b + \frac{187}{324} a^2 b - \frac{1}{324} b^3 \\
& + \frac{1}{9} a b^2 + \frac{19}{648} b^4 + \frac{35}{18} a^2 b^2 + \frac{49}{18} a^3 b + a^4 + \frac{25}{36} a^3 - \frac{5}{81} a^2, b, \frac{16}{27} a b^6 \\
& + \frac{241}{81} a^2 b^4 - \frac{16}{9} a b^5 - \frac{4}{81} b^6 + \frac{37}{9} a^3 b^2 - \frac{482}{81} a^2 b^3 + \frac{271}{162} a b^4 + \frac{4}{27} b^5 \\
& + a^4 - \frac{37}{9} a^3 b + \frac{119}{36} a^2 b^2 - \frac{31}{81} a b^3 - \frac{23}{144} b^4 + \frac{25}{36} a^3 - \frac{107}{324} a^2 b \\
& - \frac{103}{648} a b^2 + \frac{47}{648} b^3 - \frac{5}{81} a^2 + \frac{35}{648} a b - \frac{5}{432} b^2, b - 1, - \frac{5}{81} a^2 b^4 \\
& + \frac{35}{648} a b^5 - \frac{5}{432} b^6 + \frac{25}{36} a^3 b^2 - \frac{107}{324} a^2 b^3 - \frac{103}{648} a b^4 + \frac{47}{648} b^5 + a^4 \\
& - \frac{37}{9} a^3 b + \frac{119}{36} a^2 b^2 - \frac{31}{81} a b^3 - \frac{23}{144} b^4 + \frac{37}{9} a^3 - \frac{482}{81} a^2 b + \frac{271}{162} a b^2 \\
& + \frac{4}{27} b^3 + \frac{241}{81} a^2 - \frac{16}{9} a b - \frac{4}{81} b^2 + \frac{16}{27} a \Big]
\end{aligned}$$

> numelems(RC1);

16

(4.9)

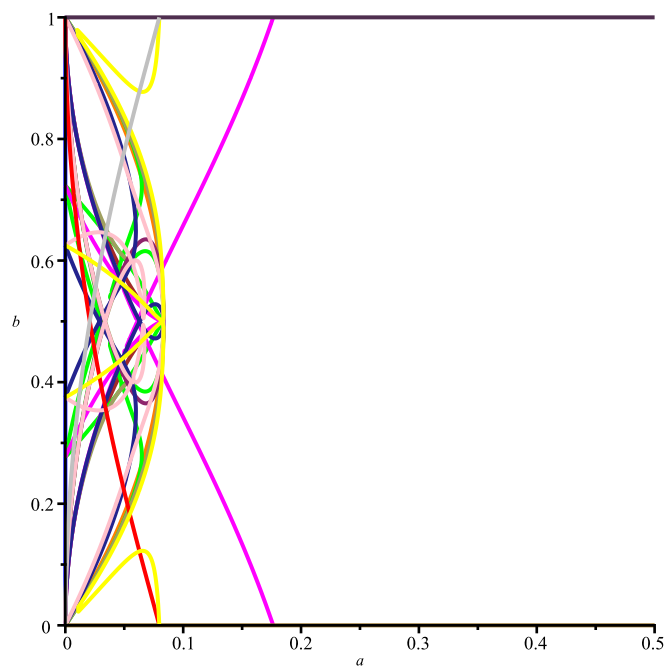
> color\_list := ['blue', 'aquamarine', 'brown', 'coral', 'cyan',  
'green', 'khaki', 'magenta', 'maroon', 'navy', 'pink', 'red',  
'sienna', 'yellow', 'violet', 'gray'];  
numelems(color\_list);

*color\_list := [blue, aquamarine, brown, coral, cyan, green, khaki, magenta, maroon, navy,  
pink, red, sienna, yellow, violet, gray]*

16

(4.10)

> NN := numelems(RC1):  
st := time[real]():  
curvesRC1 := Array(1..NN):  
for i from 1 by 1 to NN do  
    curvesRC1[i] := plots:-implicitplot(RC1[i]=0, a=0..0.5, b=  
0..1, color=color\_list[i]):  
end do:  
plots:-display(seq(curvesRC1[i], i=1..NN), view=[0..0.5, 0..1]);  
time[real]()-st;

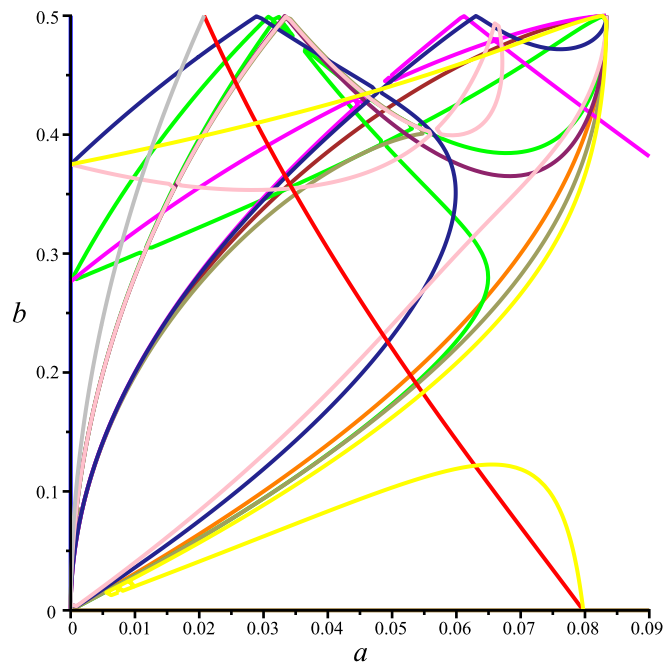


0.9030000000

(4.11)

Let look at it in the same region as in the previous work.

```
> st := time[real]():
curvesRC1:=Array(1..NN):
for i from 1 by 1 to NN do
    curvesRC1[i]:= plots:-implicitplot(RC1[i]=0, a=0..0.09, b=
    0..0.5, color=color_list[i]):
end do:
finalCV1 := plots:-display(seq(curvesRC1[i], i=1..NN), view=[0.
.0.09,0..0.5], labelfont=["TimesNewRoman",18]);
time[real]()-st;
```



(4.12)



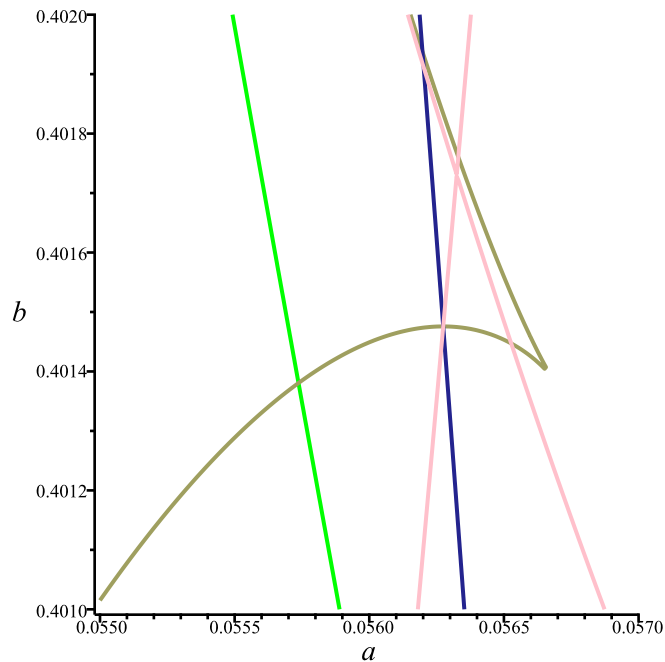
0.3190000000

(4.12)

```

> st := time[real]():
curvesRC1:=Array(1..NN):
for i from 1 by 1 to NN do
  curvesRC1[i]:=plots:-implicitplot(RC1[i]=0, a=0.0550.
.0.0570, b=0.4010..0.4020, color=color_list[i]):
end do:
zoomedRC1 := plots:-display(seq(curvesRC1[i], i=1..NN), view=
[0.0550..0.0570,0.4010..0.4020], labelfont=["TimesNewRoman",18]
);
time[real]()-st;

```



0.3340000000

(4.13)

As it can be seen. ResChainSimple generates the boundaries that Dixon resultant generated plus some extra components.

Now trying ResChainBranching which must generate less components.

```

> st := time[real]():
RC2 := ResChain:-ResChainBranching(equations, vars):
time[real]()-st;

```

0.0050000000

(4.14)

```
> RC2;
```

$$\begin{aligned}
 & \left[ a, a + \frac{b}{3}, \frac{1}{27} a b^4 - \frac{1}{3} a^2 b^2 - \frac{2}{27} a b^3 - \frac{1}{108} b^4 + a^3 + \frac{1}{3} a^2 b + \frac{1}{9} a b^2 \right. \\
 & \quad + \frac{1}{54} b^3 - \frac{1}{3} a^2 - \frac{2}{27} a b - \frac{1}{108} b^2 + \frac{1}{27} a, \frac{1}{3} b^2 + a - \frac{1}{3} b, a + \frac{1}{3} - \frac{b}{3}, \\
 & \quad \left. - \frac{2}{295245} b^5 + \frac{16094752}{7381125} a^5 b^3 - \frac{5305468}{36905625} a^2 b^7 - \frac{92002262}{12301875} a^4 b^5 \right]
 \end{aligned}$$

(4.15)

$$\begin{aligned}
& - \frac{59871236}{36905625} a^3 b^6 + \frac{112092782}{7381125} a^6 b^3 - \frac{431663996}{36905625} a^5 b^4 + \frac{660483196}{4100625} a^8 b \\
& + \frac{3785144}{7381125} a^2 b^8 + \frac{2768}{91125} a b^9 + \frac{1277227352}{12301875} a^7 b^2 + \frac{56745616}{12301875} a^3 b^7 \\
& + \frac{823300432}{36905625} a^4 b^6 + \frac{1717835408}{36905625} a^5 b^5 - \frac{618746998}{36905625} a^6 b^4 - \frac{59141914}{164025} a^8 b^2 \\
& - \frac{31456}{492075} a b^{10} - \frac{35984204}{36905625} a^2 b^9 - \frac{178654852}{7381125} a^6 b^5 - \frac{303963908}{36905625} a^3 b^8 \\
& - \frac{1209280604}{12301875} a^5 b^6 + \frac{1573007972}{4100625} a^7 b^4 + \frac{117656}{1476225} a b^{11} - \frac{1482310358}{36905625} a^4 b^7 \\
& + \frac{1526698304}{12301875} a^5 b^7 - \frac{2909391448}{12301875} a^7 b^3 + \frac{44014624}{36905625} a^2 b^{10} + \frac{14679752}{1476225} a^3 b^9 \\
& - \frac{31456}{492075} a b^{12} + \frac{198758368}{4100625} a^4 b^8 + \frac{2126387128}{36905625} a^6 b^6 + \frac{3785144}{7381125} a^2 b^{12} \\
& - \frac{1209280604}{12301875} a^5 b^8 - \frac{303963908}{36905625} a^3 b^{10} - \frac{1482310358}{36905625} a^4 b^9 \\
& - \frac{35984204}{36905625} a^2 b^{11} + \frac{823300432}{36905625} a^4 b^{10} - \frac{788}{7381125} a b^4 - \frac{1844}{7381125} a^2 b^2 \\
& + \frac{16}{295245} a b^3 - \frac{8}{1476225} a b^2 + \frac{268372}{36905625} a^3 b^2 + \frac{1844}{820125} a^2 b^3 \\
& - \frac{123136}{7381125} a^4 b - \frac{1820576}{36905625} a^3 b^3 + \frac{2564}{820125} a b^6 + \frac{451606}{12301875} a^4 b^2 \\
& - \frac{1436}{2460375} a b^5 + \frac{242128}{12301875} a^2 b^5 + \frac{12469576}{36905625} a^5 b - \frac{36212}{4100625} a^2 b^4 \\
& + \frac{211672}{2460375} a^3 b^4 - \frac{166024}{12301875} a^6 b + \frac{1290674}{36905625} a^4 b^3 - \frac{784}{164025} a b^7 \\
& - \frac{40429708}{36905625} a^5 b^2 - \frac{117596}{36905625} a^2 b^6 + \frac{1698424}{7381125} a^3 b^5 + \frac{38535764}{36905625} a^4 b^4 \\
& - \frac{4447186}{1476225} a^6 b^2 - \frac{219951796}{7381125} a^7 b - \frac{9856}{2460375} a b^8 + \frac{1}{1476225} b^{20} \\
& - \frac{2}{295245} b^{19} + \frac{127}{7381125} b^{18} + \frac{94}{2460375} b^{17} - \frac{9839}{36905625} b^{16} + \frac{14452}{36905625} b^{15} \\
& + \frac{1502}{4100625} b^{14} - \frac{74176}{36905625} b^{13} - \frac{1904}{45} a^{11} + a^{12} - \frac{64}{164025} a^3 b \\
& + \frac{2402468}{273375} a^9 - \frac{2446654621}{4100625} a^8 b^4 - \frac{5513864252}{12301875} a^7 b^5 - \frac{36586784}{91125} a^9 b^2 \\
& + \frac{2123952364}{4100625} a^8 b^3 - \frac{2402468}{91125} a^9 b + \frac{219951796}{36905625} a^7 b^{10} - \frac{166024}{12301875} a^6 b^{11} \\
& - \frac{40429708}{36905625} a^5 b^{12} + \frac{1290674}{36905625} a^4 b^{13} + \frac{211672}{2460375} a^3 b^{14} + \frac{242128}{12301875} a^2 b^{15}
\end{aligned}$$

$$\begin{aligned}
& + \frac{2564}{820125} a b^{16} - \frac{165120799}{4100625} a^8 b^8 - \frac{219951796}{7381125} a^7 b^9 - \frac{4447186}{1476225} a^6 b^{10} \\
& + \frac{16094752}{7381125} a^5 b^{11} + \frac{38535764}{36905625} a^4 b^{12} + \frac{1698424}{7381125} a^3 b^{13} - \frac{117596}{36905625} a^2 b^{14} \\
& - \frac{784}{164025} a b^{15} + \frac{2402468}{273375} a^9 b^6 + \frac{660483196}{4100625} a^8 b^7 + \frac{1277227352}{12301875} a^7 b^8 \\
& + \frac{112092782}{7381125} a^6 b^9 - \frac{431663996}{36905625} a^5 b^{10} - \frac{92002262}{12301875} a^4 b^{11} - \frac{59871236}{36905625} a^3 b^{12} \\
& - \frac{5305468}{36905625} a^2 b^{13} - \frac{9856}{2460375} a b^{14} + \frac{304022}{675} a^{10} b^4 - \frac{2402468}{91125} a^9 b^5 \\
& - \frac{59141914}{164025} a^8 b^6 - \frac{2909391448}{12301875} a^7 b^7 - \frac{618746998}{36905625} a^6 b^8 + \frac{1717835408}{36905625} a^5 b^9 \\
& + \frac{56745616}{12301875} a^3 b^{11} + \frac{2768}{91125} a b^{13} - \frac{1904}{45} a^{11} b^2 - \frac{608044}{675} a^{10} b^3 \\
& - \frac{36586784}{91125} a^9 b^4 + \frac{2123952364}{4100625} a^8 b^5 + \frac{1573007972}{4100625} a^7 b^6 - \frac{178654852}{7381125} a^6 b^7 \\
& + \frac{1904}{45} a^{11} b + \frac{304022}{225} a^{10} b^2 + \frac{231533044}{273375} a^9 b^3 - \frac{608044}{675} a^{10} b \\
& + \frac{64}{1476225} a^3 b^{18} - \frac{8}{1476225} a b^{20} + \frac{15392}{7381125} a^4 b^{16} - \frac{64}{164025} a^3 b^{17} \\
& - \frac{1844}{7381125} a^2 b^{18} + \frac{16}{295245} a b^{19} - \frac{1781368}{36905625} a^5 b^{14} - \frac{123136}{7381125} a^4 b^{15} \\
& + \frac{268372}{36905625} a^3 b^{16} + \frac{1844}{820125} a^2 b^{17} - \frac{788}{7381125} a b^{18} + \frac{83012}{36905625} a^6 b^{12} \\
& + \frac{12469576}{36905625} a^5 b^{13} + \frac{451606}{12301875} a^4 b^{14} - \frac{1820576}{36905625} a^3 b^{15} - \frac{36212}{4100625} a^2 b^{16} \\
& - \frac{1436}{2460375} a b^{17} + \frac{304022}{675} a^{10} - \frac{165120799}{4100625} a^8 + \frac{219951796}{36905625} a^7 \\
& + \frac{94}{2460375} b^7 + \frac{83012}{36905625} a^6 + \frac{127}{7381125} b^6 + \frac{64}{1476225} a^3 - \frac{74176}{36905625} b^{11} \\
& + \frac{14452}{36905625} b^9 + \frac{482}{164025} b^{12} - \frac{1781368}{36905625} a^5 - \frac{9839}{36905625} b^8 + \frac{15392}{7381125} a^4 \\
& + \frac{1502}{4100625} b^{10} + \frac{1}{1476225} b^4, - \frac{95}{5832} a^5 b^3 + \frac{287}{139968} a^2 b^7 + \frac{181}{5832} a^4 b^5 \\
& + \frac{131}{11664} a^3 b^6 - \frac{553}{8748} a^6 b^3 + \frac{41}{1458} a^5 b^4 + \frac{1}{9} a^8 b - \frac{475}{279936} a^2 b^8 \\
& - \frac{5}{31104} a b^9 - \frac{2}{9} a^7 b^2 - \frac{53}{5832} a^3 b^7 - \frac{71}{2916} a^4 b^6 - \frac{95}{5832} a^5 b^5 + \frac{541}{5832} a^6 b^4 \\
& - \frac{1}{9} a^8 b^2 + \frac{17}{186624} a b^{10} + \frac{265}{279936} a^2 b^9 - \frac{107}{1458} a^6 b^5 + \frac{113}{23328} a^3 b^8
\end{aligned}$$

$$\begin{aligned}
& -\frac{43}{5832} a^5 b^6 - \frac{2}{27} a^7 b^4 - \frac{1}{31104} a b^{11} + \frac{11}{972} a^4 b^7 + \frac{8}{729} a^5 b^7 + \frac{4}{27} a^7 b^3 \\
& -\frac{97}{279936} a^2 b^{10} - \frac{5}{2916} a^3 b^9 + \frac{1}{186624} a b^{12} - \frac{23}{11664} a^4 b^8 + \frac{107}{4374} a^6 b^6 \\
& -\frac{1}{69984} a^2 b^{12} - \frac{2}{729} a^5 b^8 + \frac{1}{2916} a^3 b^{10} - \frac{5}{8748} a^4 b^9 + \frac{1}{11664} a^2 b^{11} \\
& + \frac{1}{8748} a^4 b^{10} + \frac{1}{186624} a b^4 - \frac{1}{69984} a^2 b^2 + \frac{1}{2916} a^3 b^2 + \frac{1}{11664} a^2 b^3 \\
& -\frac{5}{8748} a^4 b - \frac{5}{2916} a^3 b^3 + \frac{17}{186624} a b^6 - \frac{23}{11664} a^4 b^2 - \frac{1}{31104} a b^5 \\
& + \frac{265}{279936} a^2 b^5 + \frac{8}{729} a^5 b - \frac{97}{279936} a^2 b^4 + \frac{113}{23328} a^3 b^4 - \frac{107}{1458} a^6 b \\
& + \frac{11}{972} a^4 b^3 - \frac{5}{31104} a b^7 - \frac{43}{5832} a^5 b^2 - \frac{475}{279936} a^2 b^6 - \frac{53}{5832} a^3 b^5 \\
& -\frac{71}{2916} a^4 b^4 + \frac{541}{5832} a^6 b^2 + \frac{4}{27} a^7 b + \frac{1}{5184} a b^8 + a^9 - \frac{1}{9} a^8 - \frac{2}{27} a^7 \\
& + \frac{1}{373248} b^7 + \frac{107}{4374} a^6 - \frac{1}{2239488} b^6 + \frac{1}{373248} b^{11} + \frac{5}{559872} b^9 \\
& -\frac{1}{2239488} b^{12} - \frac{2}{729} a^5 - \frac{5}{746496} b^8 + \frac{1}{8748} a^4 - \frac{5}{746496} b^{10}, \frac{8}{675} a b^4 \\
& -\frac{1}{5} a^2 b^2 - \frac{16}{675} a b^3 - \frac{1}{675} b^4 + a^3 + \frac{1}{5} a^2 b + \frac{8}{225} a b^2 + \frac{2}{675} b^3 - \frac{1}{5} a^2 \\
& -\frac{16}{675} a b - \frac{1}{675} b^2 + \frac{8}{675} a, -\frac{7}{3383532} b^5 + \frac{1012651}{56392200} a^5 b^3 \\
& + \frac{92561}{42294150} a^2 b^7 + \frac{1943761}{101505960} a^4 b^5 + \frac{31261471}{3045178800} a^3 b^6 - \frac{4076027}{38064735} a^6 b^3 \\
& -\frac{9229}{626580} a^5 b^4 + \frac{847}{2655} a^8 b - \frac{37427}{21147075} a^2 b^8 - \frac{26633}{135341280} a b^9 \\
& -\frac{75793}{783225} a^7 b^2 - \frac{12385849}{1522589400} a^3 b^7 - \frac{942302}{63441225} a^4 b^6 + \frac{1012651}{56392200} a^5 b^5 \\
& + \frac{1573912}{12688245} a^6 b^4 - \frac{847}{2655} a^8 b^2 + \frac{341881}{4060238400} a b^{10} + \frac{30701}{33835320} a^2 b^9 \\
& -\frac{1073489}{12688245} a^6 b^5 + \frac{25631269}{6090357600} a^3 b^8 - \frac{458911}{18797400} a^5 b^6 - \frac{75793}{2349675} a^7 b^4 \\
& -\frac{1993}{676706400} a b^{11} + \frac{1673329}{253764900} a^4 b^7 + \frac{111341}{7049025} a^5 b^7 + \frac{151586}{2349675} a^7 b^3 \\
& -\frac{34727}{126882450} a^2 b^{10} - \frac{244517}{152258940} a^3 b^9 - \frac{70807}{4060238400} a b^{12} \\
& -\frac{828229}{1015059600} a^4 b^8 + \frac{1073489}{38064735} a^6 b^6 - \frac{851}{101505960} a^2 b^{12} - \frac{111341}{28196100} a^5 b^8
\end{aligned}$$

$$\begin{aligned}
& + \frac{363317}{761294700} a^3 b^{10} - \frac{313}{563922} a^4 b^9 + \frac{851}{16917660} a^2 b^{11} + \frac{313}{2819610} a^4 b^{10} \\
& - \frac{70807}{4060238400} a b^4 - \frac{851}{101505960} a^2 b^2 + \frac{7}{845883} a b^3 - \frac{1}{845883} a b^2 \\
& + \frac{363317}{761294700} a^3 b^2 + \frac{851}{16917660} a^2 b^3 - \frac{313}{563922} a^4 b - \frac{244517}{152258940} a^3 b^3 \\
& + \frac{341881}{4060238400} a b^6 - \frac{828229}{1015059600} a^4 b^2 - \frac{1993}{676706400} a b^5 + \frac{30701}{33835320} a^2 b^5 \\
& + \frac{111341}{7049025} a^5 b - \frac{34727}{126882450} a^2 b^4 + \frac{25631269}{6090357600} a^3 b^4 - \frac{1073489}{12688245} a^6 b \\
& + \frac{1673329}{253764900} a^4 b^3 - \frac{26633}{135341280} a b^7 - \frac{458911}{18797400} a^5 b^2 - \frac{37427}{21147075} a^2 b^6 \\
& - \frac{12385849}{1522589400} a^3 b^5 - \frac{942302}{63441225} a^4 b^4 + \frac{1573912}{12688245} a^6 b^2 + \frac{151586}{2349675} a^7 b \\
& + \frac{28393}{112784400} a b^8 + \frac{1}{3383532} b^{14} - \frac{7}{3383532} b^{13} - \frac{8}{93987} a^3 b + a^9 \\
& + \frac{4}{281961} a^3 b^{12} - \frac{1}{845883} a b^{14} - \frac{8}{93987} a^3 b^{11} + \frac{7}{845883} a b^{13} - \frac{847}{2655} a^8 \\
& - \frac{75793}{2349675} a^7 - \frac{9551}{8120476800} b^7 + \frac{1073489}{38064735} a^6 + \frac{227951}{48722860800} b^6 \\
& + \frac{4}{281961} a^3 - \frac{9551}{8120476800} b^{11} + \frac{48529}{2436143040} b^9 + \frac{227951}{48722860800} b^{12} \\
& - \frac{111341}{28196100} a^5 - \frac{37969}{3248190720} b^8 + \frac{313}{2819610} a^4 - \frac{37969}{3248190720} b^{10} \\
& + \frac{1}{3383532} b^4, b, b-1 \Big]
\end{aligned}$$

```
> numelems(RC2);
```

11

(4.16)

```
> evalb(`and`(seq(polynomial in RC1, polynomial in RC2)));
```

true

(4.17)

This shows that the branching algorithm computes a smaller subset of RC1, which means less extra components.

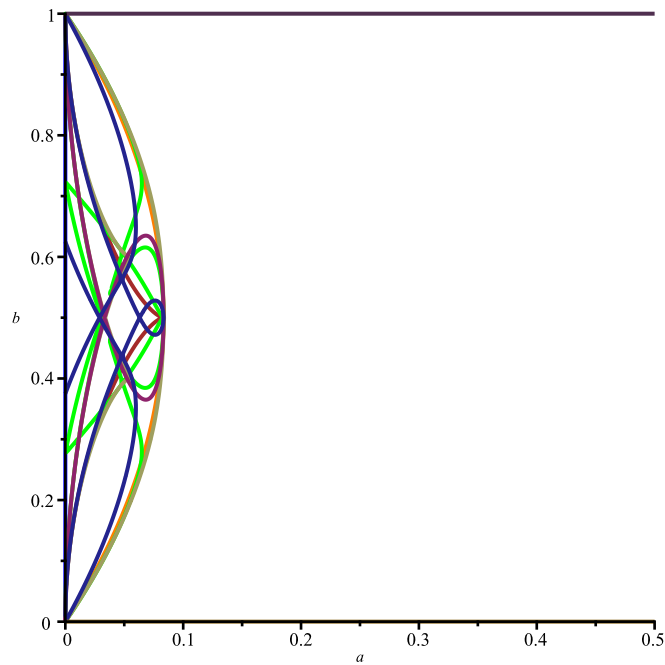
```
> # getting index of the polynomials of RC2 from RC1 to use the
  same colors in both plots for the same curves. Then it is
  easier to see which component is removed.
NN2 := numelems(RC2):
index_list := Array([seq(0, i=1..NN2)]):
for i from 1 by 1 to NN2 do
  member(RC2[i], RC1, 'indice'):
  index_list[i] := indice:
end do:
```

```
index_list;
```

```
[ 1 2 3 4 5 6 7 9 10 13 15 ]
```

(4.18)

```
> st := time[real]():  
curvesRC2 := Array(1..NN2):  
for i from 1 by 1 to NN2 do  
    curvesRC2[i] := plots:-implicitplot(RC2[i]=0, a=0..0.5, b=  
    0..1, color=color_list[ index_list[i] ]):  
end do:  
plots:-display(seq(curvesRC2[i], i=1..NN2), view=[0..0.5, 0..1]);  
time[real]()-st;
```



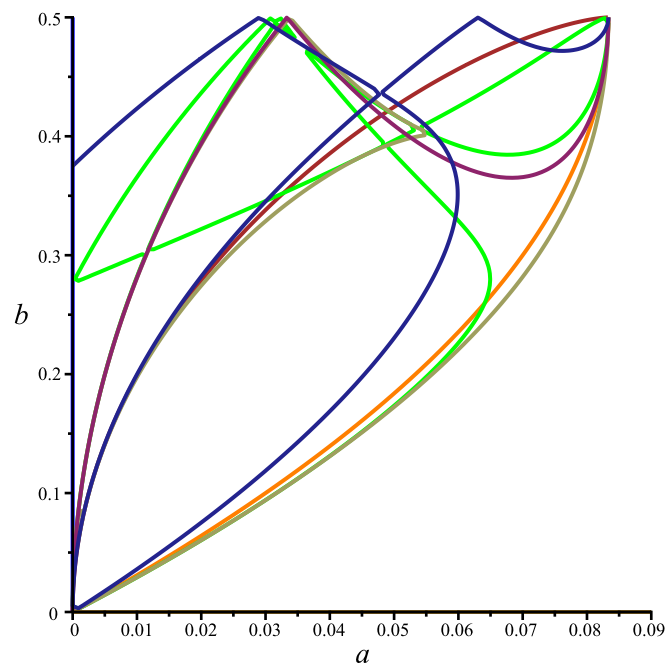
0.4720000000

(4.19)

Looks better than RC1.

Again looking at it in the same region as in the previous work.

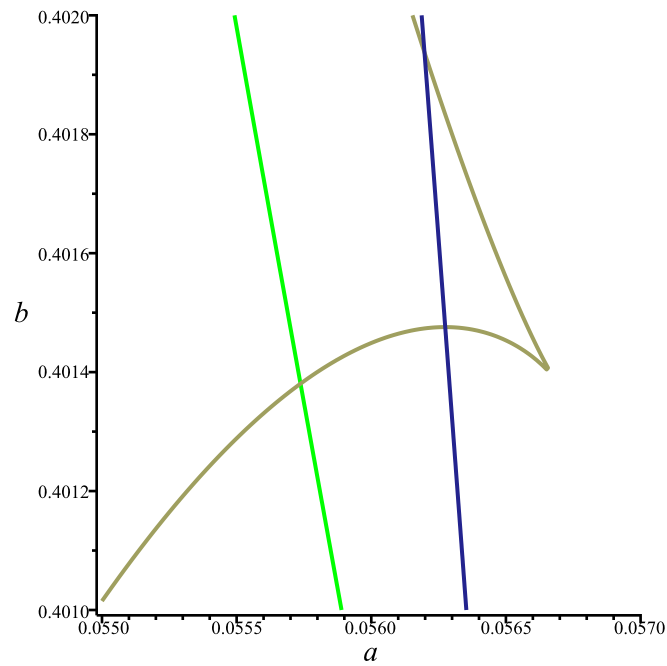
```
> st := time[real]():  
curvesRC2:=Array(1..NN2):  
for i from 1 by 1 to NN2 do  
    curvesRC2[i]:= plots:-implicitplot(RC2[i]=0, a=0..0.09, b=  
    0..0.5, color=color_list[ index_list[i] ]):  
end do:  
finalCV2 := plots:-display(seq(curvesRC2[i], i=1..NN2), view=  
[0..0.09, 0..0.5], labelfont=["TimesNewRoman", 18]);  
time[real]()-st;
```



0.1880000000

(4.20)

```
> st := time[real]():
curvesRC2:=Array(1..NN2):
for i from 1 by 1 to NN2 do
  curvesRC2[i]:= plots:-implicitplot(RC2[i]=0, a=0.0550.
    .0.0570, b=0.4010..0.4020, color=color_list[ index_list[i] ]):
end do:
zoomedRC1 := plots:-display(seq(curvesRC2[i], i=1..NN2), view=
  [0.0550..0.0570,0.4010..0.4020], labelfont=["TimesNewRoman",18]
);
time[real]()-st;
```



0.2160000000

(4.21)

The output of the Dixon resultant is already proving that what have been found with the algebraic-numeric method in the previous work was a complete classification for the three population model. Therefore we do not spend more time on computing the open CAD with respect to any of the three results above. But just to show that the problem with the RootFinding[Parametric] algorithm was in the Discriminant variety computation. We introduce an auxiliary variable 'y' and an auxiliary equation  $y-1=0$  which no matter what conditions we put on the parameters always has 1 real solutions. Then we ask the RootFinding:-Parametric:-CellDecomposition for the system generated by this auxiliary equation and inequations of the form ' $p \neq 0$ ' where p's are polynomials in RC2. Technically this is the same as asking open CAD with respect to the curves in RC2.

```
> equations_list := [ y-1 ]: # an auxiliary variable y and an
auxiliary equation y-1 which always has 1 real solutions no
matter what happens on the parameters.
inequations_list := [ seq(p <> 0, p in RC2) ]:
# Technically we are saying that the parameters must not be on
the curves defined by these equations.
positivity_list := [a, b]:
> st := time[real]():
RC2openCAD := RootFinding:-Parametric:-CellDecomposition( [seq
(p = 0, p in equations_list), seq(p > 0, p in positivity_list),
seq(p <> 0, p in inequations_list)], [y], [a, b]);
time[real]()-st;
```

112.8410000000 (4.22)

```
> numelems(RC2openCAD:-SamplePoints);
5016 (4.23)
```

It takes about 2 minutes to finish this open CAD.

If one is interested, can take the sample points and solve the initial system at these sample points, assign the number of solutions to each cell and then make a plot which is not our intention in this work.

Using the same colors for the curves in the result of the Dixon resultant approach as in the ResChain results.

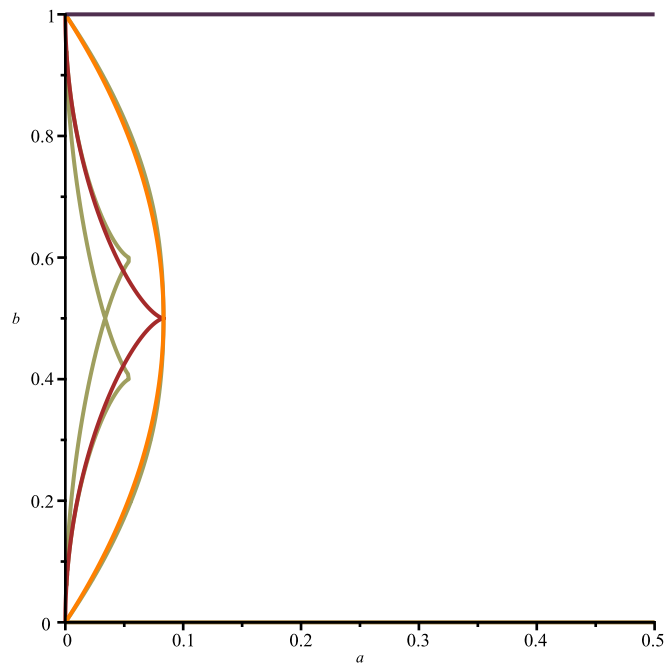
```
> # getting index of the factors of DV from RC1 to use the same
colors in the plots for the same curves.
NN3 := numelems(factors_list):
index_list2 := Array([seq(0, i=1..NN3)]):
for i from 1 by 1 to NN3 do
member(factors_list[i], RC1, 'indice'):
index_list2[i] := indice:
end do:
index_list2;
```

$[ 2 \ 7 \ 7 \ 13 \ 3 \ 4 \ 5 \ 15 ]$  (4.24)

```
> st := time[real]():
curvesDV := Array(1..NN3):
for i from 1 by 1 to NN3 do
curvesDV[i] := plots:-implicitplot(factors_list[i]=0, a=0.
```



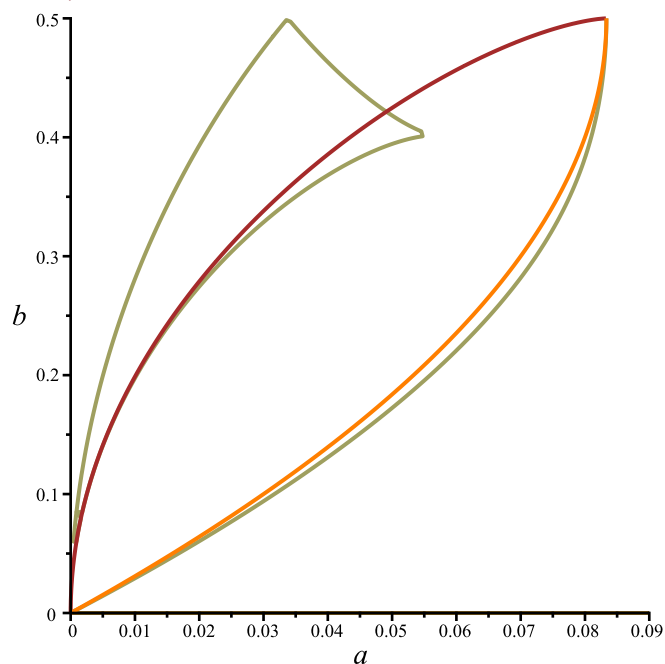
```
.0.5, b=0..1, color=color_list[ index_list2[i] ]):
end do:
plots:-display(seq(curvesDV[i],i=1..NN3), view=[0..0.5,0..1]);
time[real]()-st;
```



0.2300000000

(4.25)

```
> st := time[real]():
curvesDV:=Array(1..NN3):
for i from 1 by 1 to NN3 do
    curvesDV[i]:= plots:-implicitplot(factors_list[i]=0, a=0.
    .0.09, b=0..0.5, color=color_list[ index_list2[i] ]):
end do:
finalDV := plots:-display(seq(curvesDV[i], i=1..NN3), view=[0.
.0.09,0..0.5], labelfont=["TimesNewRoman",18]);
time[real]()-st;
```

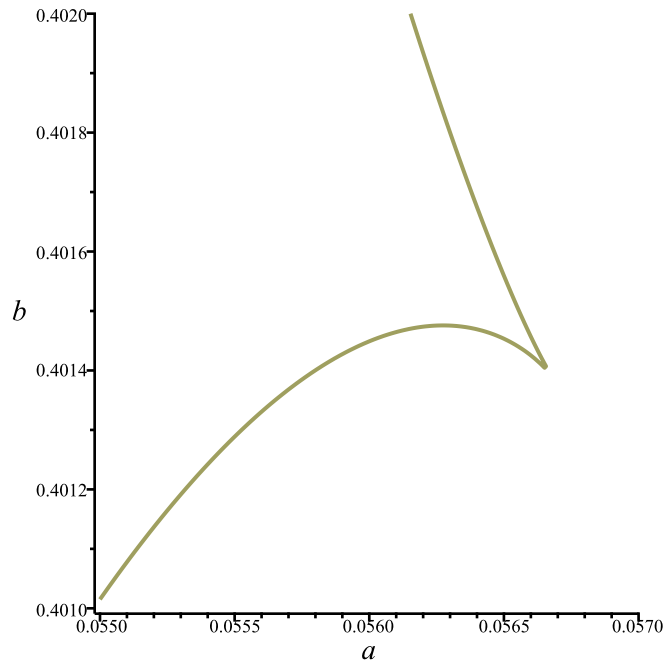


(4.26)

0.0710000000

(4.26)

```
> st := time[real]():
curvesDV:=Array(1..NN3):
for i from 1 by 1 to NN3 do
  curvesDV[i]:= plots:-implicitplot(factors_list[i]=0, a=
0.0550..0.0570, b=0.4010..0.4020, color=color_list[ index_list2
[i] ]):
end do:
zoomedDV := plots:-display(seq(curvesDV[i], i=1..NN3), view=
[0.0550..0.0570,0.4010..0.4020], labelfont=["TimesNewRoman",18]
);
time[real]()-st;
```



0.0950000000

(4.27)

Counting the maximum number of terms in the polynomials appearing in each of the three methods.

```
> max([seq(nops(polynomial), polynomial in RC1)]);
153
```

(4.28)

```
> max([seq(nops(polynomial), polynomial in RC2)]);
153
```

(4.29)

```
> max([seq(nops(polynomial), polynomial in factors_list)]);
72
```

(4.30)

The largest total degree of the polynomials in these three approaches.

```
> max([seq(degree(polynomial), polynomial in RC1)]);
21
```

(4.31)

```
> max([seq(degree(polynomial), polynomial in RC2)]);
21
```

(4.32)

```
> max([seq(degree(polynomial), polynomial in factors_list)]);
```

## Appendix 1: An example where ResChainBranching does not produce the extra components of ResChainSimple

An example where ResChainSimple produces an extra term, but ResChainBranching does not.

Let us say that we are looking for a condition on values of a parameter 'a' so that the union of circle and parabola, intersects the alpha curve and the horizontal line  $y=a$ . So we have the following parametric system of equations with 3 polynomials and 2 variables.

```
> exF := [(x^2+y^2-1)*(y-x^2), x^2+x^3-y^2, y-a]:
    exVars := [x, y]:
```

Trying ResChainSimple:

```
> exRC1 := ResChain:-ResChainSimple(exF, exVars);
    exRC1 := [a, a + 1, a - 1, a^2 - 3 a + 1, a^4 + a^2 - 1] (5.1)
```

We get 5 polynomials.

Trying ResChainBranching:

```
> exRC1 := ResChain:-ResChainBranching(exF, exVars);
    exRC1 := [a, a^2 - 3 a + 1, a^4 + a^2 - 1] (5.2)
```

We get 3 polynomials.

Now let's use elimination via Grobner basis computation to get the exact conditions.

```
> G := PolynomialIdeals:-EliminationIdeal(PolynomialIdeals:-
    PolynomialIdeal(exF), {a});
    G := <a^8 - 3 a^7 + 2 a^6 - 3 a^5 + 3 a^3 - a^2> (5.3)
```

```
> g := PolynomialIdeals:-Generators(G)[1]:
    g_factors := [seq(pair[1], pair in factors(g)[2])];
    g_factors := [a, a^2 - 3 a + 1, a^4 + a^2 - 1] (5.4)
```

As one can see, we got the three polynomials of the result of ResChainBranching so ResChainBranching here had no extra component, whereas ResChainSimple had two extra components.

## Appendix 2: An example where ResChainBranching produces extra components.

An example where ResChainBranching produces extra terms, but there are still possibility for coming up with more optimized version of ResChain algorithms avoiding computation of these extra terms.

Consider the following five irreducible polynomials in two variables  $x$  and  $y$ , and three parameters  $a$ ,  $b$  and  $c$ .

```
> f := x^2 + y^2 - 1:
   g := x^2 - y^2:
   h := y - x^2 + a:
   p := y - b:
   q := x - c:
```

We want to find conditions on the parameters for which the system of equations consisted of the following four polynomials have a solution.

```
> F := [ f, g*q, h*q, p]:
   <seq(polynomial, polynomial in F)>;
```

$$\begin{bmatrix} x^2 + y^2 - 1 \\ (x^2 - y^2)(x - c) \\ (-x^2 + a + y)(x - c) \\ y - b \end{bmatrix} \quad (6.1)$$

First we use ResChainBranching.

```
> conditions1 := ResChainBranching(F, [x, y]);
```

$$conditions1 := \left[ c^4 - 2ac^2 + a^2 + c^2 - 1, b^2 + c^2 - 1, c^2 - \frac{1}{2}, a^2 - a - \frac{1}{4}, b^2 - \frac{1}{2} \right] \quad (6.2)$$

```
> numelems(conditions1);
```

$$5 \quad (6.3)$$

It gave us 5 polynomials.

This time we follow the idea introduced at the end of section 5 in the paper. Namely using the information that we have, the second and the third input polynomials have a common factor to start the chain of resultants with two branches. One branch with  $f$ ,  $g$  and  $p$ , and the other with  $f$ ,  $q$  and  $p$ .

```
> conditions2_branch1 := ResChainBranching([f, g, h, p], [x, y]);
```

$$conditions2\_branch1 := \left[ a^2 - a - \frac{1}{4}, b^2 - \frac{1}{2} \right] \quad (6.4)$$

```
> conditions2_branch2 := ResChainBranching([f, q, p], [x, y]);
```

$$conditions2\_branch2 := [b^2 + c^2 - 1] \quad (6.5)$$

```
> conditions2 := [seq(polynomial, polynomial in
conditions2_branch1), seq(polynomial, polynomial in
conditions2_branch2)];
```

$$conditions2 := \left[ a^2 - a - \frac{1}{4}, b^2 - \frac{1}{2}, b^2 + c^2 - 1 \right] \quad (6.6)$$

```
> numelems(conditions2);
```

$$3 \quad (6.7)$$

```
> for polynomial in conditions1 do
  if not polynomial in conditions2 then
    print(polynomial);
  end if;
end do;
```

$$c^4 - 2 a c^2 + a^2 + c^2 - 1$$

$$c^2 - \frac{1}{2} \quad (6.8)$$

This version produces two polynomials less than ResChainBranching which the two extra polynomials are shown above. In the text it is mentioned that these two extra components are coming from  $\text{res}(\text{res}(f,g), \text{res}(f,q))$  and  $\text{res}(\text{res}(f,q), \text{res}(f,h))$ . Let us check this.

```
> ResChainBranching([f, g, q], [x, y]);
```

$$\left[ c^2 - \frac{1}{2} \right] \quad (6.9)$$

```
> ResChainBranching([f, q, h], [x, y]);
```

$$[c^4 - 2 a c^2 + a^2 + c^2 - 1] \quad (6.10)$$

As expected.

Now let us compute the discriminant variety using elimination theory via Grobner basis computation.

```
> ideal := PolynomialIdeals[EliminationIdeal](PolynomialIdeals
[PolynomialIdeal](F), {a, b, c});
```

$$ideal := \langle 2 b^4 + 2 b^2 c^2 - 3 b^2 - c^2 + 1, 2 a b^2 + 2 a c^2 + 2 b^3 + 2 b c^2 - b^2 - c^2 - 2 a - 2 b + 1 \rangle \quad (6.11)$$

```
> gen := PolynomialIdeals[Generators](ideal);
```

$$gen := \{2 b^4 + 2 b^2 c^2 - 3 b^2 - c^2 + 1, 2 a b^2 + 2 a c^2 + 2 b^3 + 2 b c^2 - b^2 - c^2 - 2 a - 2 b + 1\} \quad (6.12)$$

```
> numelems(gen);
```

$$2 \quad (6.13)$$

```
> seq(element[1], element in factors(gen[1])[2]);
```

$$b^2 + c^2 - 1, b^2 - \frac{1}{2} \quad (6.14)$$

```
> seq(element[1], element in factors(gen[2])[2]);
```

$$a + b - \frac{1}{2}, b^2 + c^2 - 1 \quad (6.15)$$

It is clear that  $V(\text{gen})$  consists of  $V(b^2+c^2-1)$  union with  $V(b^2-1/2, a+b-1/2)$ . These two components are contained in the union of solution sets of the three polynomials in the modified version and the two extra components of the result of `ResChainBranching` indeed are extra.

## Appendix 3: Attempting the four connected population model

Checking why `ResChainBranching` or `ResChainSimple` end up with `[0]` for the 4 population model example.

```
> vars := [seq(cat(x__,i), i = 1 .. 4)]:
> params := [a, b]:
> F := [seq(vars[i] * (1 - vars[i]) * (vars[i] - b) - 2 * a *
  vars[i] + add(a * vars[j], j in {1, 2, 3, 4} minus {i}), i = 1
  .. 4)]:
> <seq(polynomial, polynomial in F)>;
```

$$\begin{bmatrix} x_1(1-x_1)(x_1-b) - 2ax_1 + ax_2 + ax_3 + ax_4 \\ x_2(1-x_2)(x_2-b) - 2ax_2 + ax_1 + ax_3 + ax_4 \\ x_3(1-x_3)(x_3-b) - 2ax_3 + ax_1 + ax_2 + ax_4 \\ x_4(1-x_4)(x_4-b) - 2ax_4 + ax_1 + ax_2 + ax_3 \end{bmatrix} \quad (7.1)$$

```
> J := Matrix([seq([seq(diff(polynomial, var), var in vars)],
  polynomial in F)]):
> d := LinearAlgebra:-Determinant(J):
> equations := [seq(polynomial, polynomial in F), d]:
> st := time[real]():
  timelimit(120, ResChainSimple(equations, vars)):
  time[real]() - st;
```

0.22900000000 (7.2)

```
> RCS := ResChainSimple(equations, vars);
  RCS := [0] (7.3)
```

Some extra info from inner steps of the algorithm can be seen by using 'infolevel'.

```
> infolevel[ResChainSimple] := 5:
> ResChainSimple(equations, vars):
```

The following subsection contains several polynomials appearing during the computation of `ResChainSimple`. We just assign some names to them for convenience.

## Saving polynomials by names.

$$g_l := -\frac{1}{3} x_l b - \frac{1}{3} x_2 b + \frac{1}{3} x_l^2 + \frac{1}{3} x_l x_2 + \frac{1}{3} x_2^2 + a + \frac{1}{3} b - \frac{1}{3} x_l - \frac{1}{3} x_2 \quad (7.1.1)$$

$$g_2 := -\frac{1}{3} x_l b - \frac{1}{3} x_3 b + \frac{1}{3} x_l^2 + \frac{1}{3} x_l x_3 + \frac{1}{3} x_3^2 + a + \frac{1}{3} b - \frac{1}{3} x_l - \frac{1}{3} x_3 \quad (7.1.2)$$

$$g_3 := -bx_l^2 + x_l^3 + ax_l - ax_2 - ax_3 + x_l b - x_l^2 \quad (7.13)$$

$$g_4 := -\frac{2}{3} a x_l^3 x_2 - \frac{2}{3} a x_l^3 x_3 + \frac{1}{3} x_l^4 - \frac{2}{3} x_l^5 + \frac{1}{3} x_l^6 + a^3 - \frac{2}{3} b^2 x_l^3 + \frac{4}{3} b x_l^4 \quad (7.1.4)$$

$$\begin{aligned} & -\frac{2}{3} b x_l^3 + \frac{1}{3} b^2 x_l^4 - \frac{2}{3} b x_l^5 + \frac{5}{3} a x_l^4 - 2 a x_l^3 + \frac{1}{3} a x_l^2 + \frac{7}{3} a^2 x_l^2 \\ & + \frac{1}{3} b^2 x_l^2 - a^2 x_l - 2 a b x_l^3 - \frac{5}{3} a^2 x_l x_3 + \frac{2}{3} a x_l^2 x_2 - \frac{5}{3} a^2 x_l x_2 - a^2 b x_l \\ & - \frac{1}{3} a b^2 x_l + \frac{7}{3} a b x_l^2 + \frac{2}{3} a x_l^2 x_3 - \frac{1}{3} a b x_l + \frac{1}{3} a b^2 x_l^2 + \frac{2}{3} a^2 x_2 x_3 \\ & + \frac{1}{3} a^2 b x_2 + \frac{1}{3} a^2 b x_3 + \frac{1}{3} a^2 x_2^2 + \frac{1}{3} a^2 x_3^2 + \frac{1}{3} a^2 b + \frac{1}{3} a^2 x_2 + \frac{1}{3} a^2 x_3 \\ & + \frac{2}{3} a b x_l^2 x_2 - \frac{2}{3} a b x_l x_2 + \frac{2}{3} a b x_l^2 x_3 - \frac{2}{3} a b x_l x_3 \end{aligned}$$

```
> g_5 := 2/3*a^2*b^3*x_1^5-7/3*a^2*b^2*x_1^6+8/3*a^2*b*
x_1^7-9*a^2*x_1^6*x_2^2-9*a^2*x_1^6*x_3^2+4*a^2*x_1^5*
x_2^3+4*a^2*x_1^5*x_3^3+8/9*a*b^4*x_1^5-28/9*a*b^3*
x_1^6+32/9*a*b^2*x_1^7-4/3*a*b*x_1^8+4/3*a*x_1^8*
x_2+4/3*a*x_1^8*x_3+16/3*a*x_1^7*x_2^2+16/3*a*x_1^7*
x_3^2+10/9*b^5*x_1^4*x_2+10/9*b^5*x_1^4*x_3-40/9*b^4*
x_1^5*x_2-40/9*b^4*x_1^5*x_3-5/3*b^4*x_1^4*x_2^2-5/3*
b^4*x_1^4*x_3^2+56/9*b^3*x_1^6*x_2+56/9*b^3*x_1^6*
x_3+6*b^3*x_1^5*x_2^2+6*b^3*x_1^5*x_3^2-32/9*b^2*x_1^7*
x_2-32/9*b^2*x_1^7*x_3-7*b^2*x_1^6*x_2^2-7*b^2*x_1^6*
x_3^2+2/3*b*x_1^8*x_2+2/3*b*x_1^8*x_3+8/3*b*x_1^7*
x_2^2+8/3*b*x_1^7*x_3^2-4/3*x_1^8*x_2*x_3-16/3*x_1^7*
x_2^2*x_3-16/3*x_1^7*x_2*x_3^2-7*x_1^6*x_2^2*
x_3^2+4/9*a^3*b^3*x_1^3-34/9*a^3*b^2*x_1^4+22/3*a^3*b*
```

$$\begin{aligned}
& x \frac{1^5+2^5a^3x}{2^2-12a^3x} \frac{1^5x}{1^4x} \frac{2+2a^3x}{3^2+10a^3x} \frac{1^5x}{1^3x} \frac{3-12a^3x}{2^3+10a^3x} \frac{1^4x}{2^4-2a^3x} \\
& x \frac{3^3-2a^3x}{2^4-4a^3x} \frac{1^2x}{2^3x} \frac{2^4-2a^3x}{3^3-2a^3x} \frac{1^2x}{2^2x} \frac{3^4-2a^3x}{3^4+16/27} \\
& x \frac{a^2b^4x}{1^3-199/27a^2b^3x} \frac{1^4+154/9a^2b^2x}{1^4+154/9a^2b^2x} \frac{1^5-13}{1^6+6a^2x} \\
& x \frac{1^6x}{2^2+14a^2x} \frac{1^5x}{3^2-20/3a^2x} \frac{2+6a^2x}{1^4x} \frac{1^6x}{2^3} \frac{3+14a^2x}{-20/3a^2x} \\
& x \frac{1^4x}{3^3+4/27ab^5x} \frac{1^3-100/27ab^4x}{1^4+98/9ab^3x} \frac{1^5-98/9ab^2x}{1^6+32/9abx} \frac{1^7}{-32/9ax} \\
& x \frac{1^7x}{2-32/9ax} \frac{1^7x}{3-14/3ax} \frac{1^6x}{2^2} \frac{-14/3ax}{1^6x} \frac{3^2-8/9b^5x}{1^3x} \frac{2-8/9b^5x}{1^3x} \\
& x \frac{3+40/9b^4x}{2^2-2ax} \frac{1^8x}{3^2-4/9b^5x} \frac{1^5x}{2-4/9b^5x} \frac{1^5x}{3+14/9b^4x} \frac{1^6x}{2+14/9b^4x} \\
& x \frac{1^6x}{3+2/3b^4x} \frac{1^5x}{2^2+2/3b^4x} \frac{1^5x}{3^2-16/9b^3x} \frac{1^7x}{-16/9b^3x} \frac{2}{3-7/3b^3x} \\
& x \frac{1^6x}{3^2+2/3b^2x} \frac{1^8x}{2+2/3b^2x} \frac{1^8x}{3+8/3b^2x} \frac{1^7x}{2^2+8/3b^2x} \frac{1^7x}{3^2-bx} \\
& x \frac{1^8x}{2^2-bx} \frac{1^8x}{3^2+2x} \frac{1^8x}{2^2x} \frac{3+2x}{1^8x} \frac{2x}{3^2+8x} \frac{1^7x}{2^2x} \\
& x \frac{3^2+10/9b^2x}{1^4x} \frac{2+10/9b^2x}{1^4x} \frac{3-4/9b^2x}{1^5x} \frac{2-4/9b^2x}{1^5x} \frac{3-2/3a^2b^2x}{2^3x} \frac{3-a^2b^2x}{2^2x} \\
& x \frac{3^2-2/3a^2b^2x}{2x} \frac{3^3+230/9a^2b^2x}{2+230/9a^2b^2x} \frac{1^4x}{3+82/9a^2b^2x} \frac{1^3x}{2^2+82/9a^2b^2x} \frac{1^3x}{3^2-8a^2bx} \\
& x \frac{1^2x}{2^3-8a^2bx} \frac{1^2x}{3^3+2/3a^2bx} \frac{1x}{2^4+2/3a^2bx} \frac{1x}{3^4+2/3a^2bx} \frac{1x}{2^4x} \\
& x \frac{3+4/3a^2bx}{2^3x} \frac{3^2+4/3a^2bx}{2^2x} \frac{3^3+2/3a^2bx}{2x} \frac{3^4-40/9a^2x}{1^3x} \frac{2^2x}{-40/9a^2x} \\
& x \frac{3^2+8a^2x}{1^2x} \frac{2^3x}{3+12a^2x} \frac{1^2x}{2^2x} \frac{3^2+8a^2x}{1^2x} \frac{2x}{3^3-4/3a^2x} \\
& x \frac{1^2x}{2^4x} \frac{3-8/3a^2x}{1x} \frac{2^3x}{3^2-8/3a^2x} \frac{1x}{2^2x} \frac{3^3-4/3a^2x}{1x} \frac{2x}{3^4-4/27ab^5x} \\
& x \frac{1x}{3+22/9ab^4x} \frac{1^2x}{2+22/9ab^4x} \frac{1^2x}{3-2/9ab^4x} \frac{1x}{2^2-2/9ab^4x} \frac{1x}{3^2-320/27ab^3x} \\
& x \frac{1^3x}{2-320/27ab^3x} \frac{1^3x}{3-4/3ab^3x} \frac{1^2x}{-4/3ab^3x} \frac{1^2x}{3^2+2/3ab^3x} \frac{1x}{2^3+2/3ab^3x} \\
& x \frac{1x}{3^3+52/9ab^2x} \frac{1^3x}{2^2+52/9ab^2x} \frac{1^3x}{2^2-2ab^2x} \frac{1^2x}{2^3-2ab^2x} \frac{1^2x}{3^3-50/9abx} \\
& x \frac{1^4x}{2^2-50/9abx} \frac{1^4x}{3^2+4/3abx} \frac{1^3x}{2^3+4/3abx} \frac{1^3x}{3^3+104/9ax} \frac{1^5x}{2x} \\
& x \frac{3+40/9ax}{1^4x} \frac{2x}{3^2-8/3ax} \frac{1^3x}{2^3x} \frac{3-4ax}{1^3x} \frac{2^2x}{2^2x} \frac{3^2-8/3ax}{1^3x} \\
& x \frac{2x}{3^3-8/9b^4x} \frac{1^2x}{2x} \frac{3+16/3b^3x}{1^3x} \frac{2x}{3+2/3b^3x} \frac{1^2x}{2^2x} \frac{2^2x}{3^2-100/9b^2x} \\
& x \frac{1^4x}{-8/3b^2x} \frac{1^3x}{2^2x} \frac{3-b^3x}{1^2x} \frac{2^2x}{3^2+224/9b^2x} \frac{1^5x}{2x} \\
& x \frac{3+40/3b^2x}{1^4x} \frac{2^2x}{3+40/3b^2x} \frac{1^4x}{2^2x} \frac{3^2-140/9b^2x}{1^6x} \frac{2x}{3-40/3b^2x} \\
& x \frac{1^5x}{2^2x} \frac{3-40/3b^2x}{1^5x} \frac{2x}{3^2-8/9a^4b^2x} \frac{1x}{2-8/9a^4b^2x} \frac{1x}{3+4/9a^4b^2x} \\
& x \frac{2x}{3+2a^4b^2x} \frac{2^2x}{3+2a^4b^2x} \frac{2x}{3^2-2a^4x} \frac{1^2x}{2x} \frac{3+4a^4x}{1x} \frac{2^2x}{2^2x} \\
& x \frac{3+4a^4x}{1x} \frac{2x}{3^2-44/27a^3b^3x} \frac{1x}{2-44/27a^3b^3x} \frac{1x}{3+16/27a^3b^3x} \frac{2x}{3+38/9a^3b^2x} \\
& x \frac{1^2x}{2+38/9a^3b^2x} \frac{1^2x}{3-22/9a^3b^2x} \frac{1x}{2^2-22/9a^3b^2x} \frac{1x}{3^2+8/9a^3b^2x} \frac{2^2x}{3+8/9a^3b^2x} \\
& x \frac{2x}{3^2-44/9a^3b^2x} \frac{1^3x}{2-44/9a^3b^2x} \frac{1^3x}{3-16/3a^3b^2x} \frac{1^2x}{2^2-16/3a^3b^2x} \frac{1^2x}{3^2+22/3a^3b^2x} \\
& x \frac{1x}{2^3+22/3a^3b^2x} \frac{1x}{3^3-8/3a^3b^2x} \frac{2^3x}{3-4a^3b^2x}
\end{aligned}$$



$$\begin{aligned}
& x^2_2 x^3_2 - 8/3 a^3 b^2 x^3_2 - 40/3 a^3 x^1_3 x^2_2 \\
& x^3_2 + 2 a^3 x^1_2 x^2_2 + 2 a^3 x^1_2 x^2_2 - 8/3 a^3 x^1_2 x^2_2 - 8/3 a^3 x^1_2 x^2_2 \\
& x^2_2 x^3_2 - 8/9 a^2 b^4 x^1_2 x^2_2 - 8/9 a^2 b^4 x^1_2 x^2_2 - 8/3 a^3 x^1_2 x^2_2 \\
& a^2 b^4 x^2_2 x^3_2 + 58/9 a^2 b^3 x^1_2 x^2_2 + 58/9 a^2 b^3 x^1_2 x^2_2 \\
& x^1_2 x^3_2 - 16/9 a^2 b^3 x^1_2 x^2_2 - 16/9 a^2 b^3 x^1_2 x^2_2 - 536/27 a^2 b^2 x^1_3 x^2_2 \\
& - 2/9 a^2 b^3 x^2_2 x^3_2 - 2/9 a^2 b^3 x^2_2 x^3_2 - 536/27 a^2 b^2 x^1_3 x^2_2 \\
& b^2 x^1_3 x^2_2 - 536/27 a^2 b^2 x^1_3 x^2_2 - 19/9 a^2 b^2 x^1_3 x^2_2 \\
& x^1_2 x^2_2 - 19/9 a^2 b^2 x^1_2 x^2_2 - 19/9 a^2 b^2 x^1_2 x^2_2 \\
& x^2_3 + 4 a^2 b^2 x^1_2 x^3_2 + 16 a^2 b^2 x^1_3 x^3_2 - a^2 b^2 x^1_3 x^3_2 \\
& x^1_2 x^2_4 - a^2 b^2 x^1_2 x^3_4 - a^2 b^2 x^2_4 x^3_2 - 2 a^2 b^2 x^2_3 x^3_2 \\
& b^2 x^2_3 x^3_2 - a^2 b^2 x^2_2 x^3_4 - 16/3 a^2 x^1_5 x^2_2 \\
& x^3_2 + 20/3 a^2 x^1_4 x^2_2 x^3_2 + 20/3 a^2 x^1_4 x^2_2 x^3_2 - 32/3 a^2 x^1_3 x^2_2 x^3_2 \\
& - 32/3 a^2 x^1_3 x^2_2 x^3_2 - 32/3 a^2 x^1_3 x^2_2 x^3_2 - 8 a^2 x^1_2 x^3_2 - 8 a^2 x^1_2 x^3_2 \\
& x^1_2 x^2_2 x^3_2 + 2 a^2 x^1_2 x^3_2 - 2 a^2 x^1_2 x^3_2 - 2 a^2 x^1_2 x^3_2 - 2 a^2 x^1_2 x^3_2 \\
& x^2_4 x^3_2 + 4 a^2 x^1_2 x^3_2 + 2 a^2 x^1_2 x^3_2 - 2 a^2 x^1_2 x^3_2 - 2 a^2 x^1_2 x^3_2 \\
& x^3_4 + 4/9 a^2 b^5 x^1_2 x^2_4 + 4/9 a^2 b^5 x^1_2 x^2_4 - 20/3 a^2 b^4 x^1_3 x^2_4 \\
& b^4 x^1_3 x^2_4 - 20/3 a^2 b^4 x^1_3 x^2_4 - 20/3 a^2 b^4 x^1_3 x^2_4 - 20/3 a^2 b^4 x^1_3 x^2_4 \\
& x^2_2 + 2/3 a^2 b^4 x^1_2 x^2_4 + 190/9 a^2 b^3 x^1_4 x^2_4 + 190/9 a^2 b^3 x^1_4 x^2_4 \\
& a^2 b^3 x^1_4 x^2_4 + 52/9 a^2 b^3 x^1_3 x^2_4 + 52/9 a^2 b^3 x^1_3 x^2_4 + 52/9 a^2 b^3 x^1_3 x^2_4 \\
& x^3_2 - 2 a^2 b^3 x^1_2 x^2_3 - 2 a^2 b^3 x^1_2 x^2_3 - 2 a^2 b^3 x^1_2 x^2_3 - 2 a^2 b^3 x^1_2 x^2_3 \\
& b^2 x^1_5 x^2_2 - 242/9 a^2 b^2 x^1_5 x^2_2 - 190/9 a^2 b^2 x^1_4 x^2_2 - 190/9 a^2 b^2 x^1_4 x^2_2 \\
& x^2_2 - 190/9 a^2 b^2 x^1_4 x^2_2 - 16/3 a^2 b^2 x^1_3 x^2_2 - 16/3 a^2 b^2 x^1_3 x^2_2 \\
& x^2_3 + 16/3 a^2 b^2 x^1_3 x^2_2 - 140/9 a^2 b^2 x^1_6 x^2_2 + 140/9 a^2 b^2 x^1_6 x^2_2 \\
& a^2 b^2 x^1_6 x^2_2 + 58/3 a^2 b^2 x^1_5 x^2_2 + 58/3 a^2 b^2 x^1_5 x^2_2 + 58/3 a^2 b^2 x^1_5 x^2_2 \\
& - 10/3 a^2 b^2 x^1_4 x^2_3 - 10/3 a^2 b^2 x^1_4 x^2_3 - 10/3 a^2 b^2 x^1_4 x^2_3 - 10/3 a^2 b^2 x^1_4 x^2_3 \\
& x^2_2 x^3_2 - 44/3 a^2 x^1_5 x^2_2 x^3_2 - 44/3 a^2 x^1_5 x^2_2 x^3_2 - 44/3 a^2 x^1_5 x^2_2 x^3_2 \\
& x^3_2 + 20/3 a^2 x^1_4 x^2_3 x^3_2 + 20/3 a^2 x^1_4 x^2_3 x^3_2 + 20/3 a^2 x^1_4 x^2_3 x^3_2 \\
& a^2 x^1_3 x^2_3 x^3_2 + 4 a^2 x^1_3 x^2_3 x^3_2 + 4 a^2 x^1_3 x^2_3 x^3_2 + 4 a^2 x^1_3 x^2_3 x^3_2 \\
& x^1_2 x^2_2 x^3_2 + 16/3 b^4 x^1_3 x^2_2 x^3_2 + 16/3 b^4 x^1_3 x^2_2 x^3_2 + 16/3 b^4 x^1_3 x^2_2 x^3_2 \\
& x^2_2 x^3_2 + 2/3 b^4 x^1_2 x^2_2 x^3_2 - 160/9 b^3 x^1_4 x^2_2 x^3_2 - 160/9 b^3 x^1_4 x^2_2 x^3_2 \\
& x^3_2 - 16/3 b^3 x^1_3 x^2_2 x^3_2 - 16/3 b^3 x^1_3 x^2_2 x^3_2 - 16/3 b^3 x^1_3 x^2_2 x^3_2 \\
& x^3_2 + 44/27 a^3 b^3 x^1_2 x^2_4 + 44/27 a^3 b^3 x^1_2 x^2_4 - 16/27 a^3 b^3 x^1_2 x^2_4 \\
& - 16/27 a^3 b^3 x^1_2 x^2_4 - 16/27 a^3 b^3 x^1_2 x^2_4 - 16/27 a^3 b^3 x^1_2 x^2_4 \\
& a^3 b^3 x^2_2 x^3_2 - 16/27 a^3 b^3 x^2_2 x^3_2 - 64/9 a^3 b^2 x^1_3 x^2_2 x^3_2 - 64/9 a^3 b^2 x^1_3 x^2_2 x^3_2 \\
& x^1_3 x^2_2 - 64/9 a^3 b^2 x^1_3 x^2_2 - 10/3 a^3 b^2 x^1_2 x^2_2 x^3_2 - 10/3 a^3 b^2 x^1_2 x^2_2 x^3_2 \\
& x^2_2 + 10/3 a^3 b^2 x^1_2 x^2_2 x^3_2 + 14/3 a^3 b^2 x^1_4 x^2_2 x^3_2 + 14/3 a^3 b^2 x^1_4 x^2_2 x^3_2 \\
& x^2_4 + 14/3 a^3 b^2 x^1_4 x^2_2 x^3_2 + 6 a^3 b^2 x^1_3 x^2_2 x^3_2 + 6 a^3 b^2 x^1_3 x^2_2 x^3_2 \\
& x^1_3 x^2_2 - 22/3 a^3 b^2 x^1_2 x^2_2 x^3_2 - 22/3 a^3 b^2 x^1_2 x^2_2 x^3_2 - 22/3 a^3 b^2 x^1_2 x^2_2 x^3_2 \\
& x^3_2 + 4/3 a^3 b^2 x^1_2 x^2_4 + 4/3 a^3 b^2 x^1_2 x^2_4 - 4/3 a^3 b^2 x^1_2 x^2_4 - 4/3 a^3 b^2 x^1_2 x^2_4 \\
& x^2_4 x^3_2 + 8/3 a^3 b^2 x^2_3 x^2_2 + 8/3 a^3 b^2 x^2_3 x^2_2 - 2 a^3 b^2 x^2_2 x^3_2 - 2 a^3 b^2 x^2_2 x^3_2 \\
& x^3_2 + 4/3 a^3 b^2 x^2_2 x^3_4 + 10 a^3 b^2 x^1_3 x^2_2 x^3_2 + 10 a^3 b^2 x^1_3 x^2_2 x^3_2 \\
& a^3 b^2 x^1_3 x^2_2 x^3_2 - 4 a^3 b^2 x^1_2 x^2_2 x^3_2 - 14 a^3 b^2 x^1_2 x^2_2 x^3_2 - 14 a^3 b^2 x^1_2 x^2_2 x^3_2 \\
& x^1_2 x^2_2 x^3_2 - 4 a^3 b^2 x^1_2 x^2_2 x^3_2 - 4 a^3 b^2 x^1_2 x^2_2 x^3_2 - 4 a^3 b^2 x^1_2 x^2_2 x^3_2 \\
& x^2_3 x^2_2 + 8 a^3 b^2 x^1_2 x^2_2 x^3_2 + 16/9 a^2 b^4 x^1_2 x^2_2 x^3_2 + 16/9 a^2 b^4 x^1_2 x^2_2 x^3_2 \\
& x^2_2 + 16/9 a^2 b^4 x^1_2 x^2_2 x^3_2 - 8/27 a^2 b^4 x^1_2 x^2_2 x^3_2 - 8/27 a^2 b^4 x^1_2 x^2_2 x^3_2 \\
& a^2 b^4 x^1_2 x^2_2 x^3_2 - 8/27 a^2 b^4 x^1_2 x^2_2 x^3_2 - 356/27 a^2 b^3 x^1_3 x^2_2 x^3_2 - 356/27 a^2 b^3 x^1_3 x^2_2 x^3_2 \\
& x^2_2 x^3_2 + 28/9 a^2 b^3 x^1_2 x^2_2 x^3_2 + 28/9 a^2 b^3 x^1_2 x^2_2 x^3_2 + 28/9 a^2 b^3 x^1_2 x^2_2 x^3_2 \\
& x^3_2 + 230/9 a^2 b^2 x^1_4 x^2_2 x^3_2 + 230/9 a^2 b^2 x^1_4 x^2_2 x^3_2 + 230/9 a^2 b^2 x^1_4 x^2_2 x^3_2 \\
& x^3_2 + 82/9 a^2 b^2 x^1_3 x^2_2 x^3_2 + 82/9 a^2 b^2 x^1_3 x^2_2 x^3_2 + 82/9 a^2 b^2 x^1_3 x^2_2 x^3_2 \\
& a^2 b^2 x^1_2 x^2_2 x^3_2 - 8 a^2 b^2 x^1_2 x^2_2 x^3_2 - 8 a^2 b^2 x^1_2 x^2_2 x^3_2 - 8 a^2 b^2 x^1_2 x^2_2 x^3_2 \\
& x^1_2 x^2_4 + 2/3 a^2 b^2 x^1_2 x^2_2 x^3_2 + 2/3 a^2 b^2 x^1_2 x^2_2 x^3_2 + 2/3 a^2 b^2 x^1_2 x^2_2 x^3_2 \\
& x^3_2 + 4/3 a^2 b^2 x^2_3 x^2_2 + 4/3 a^2 b^2 x^2_3 x^2_2 - 2 a^2 b^2 x^2_2 x^3_2 - 2 a^2 b^2 x^2_2 x^3_2 \\
& a^2 b^2 x^2_2 x^3_2 - 176/9 a^2 b^2 x^1_5 x^2_2 x^3_2 - 176/9 a^2 b^2 x^1_5 x^2_2 x^3_2 - 176/9 a^2 b^2 x^1_5 x^2_2 x^3_2 \\
& x^1_5 x^2_2 - 248/9 a^2 b^2 x^1_4 x^2_2 x^3_2 - 248/9 a^2 b^2 x^1_4 x^2_2 x^3_2 - 248/9 a^2 b^2 x^1_4 x^2_2 x^3_2
\end{aligned}$$

x\_3^2+16\*a^2\*b\*x\_1^3\*x\_2^3-20/3\*a^2\*b\*x\_1^4\*x\_3^3+4\*a^2\*x\_1^5\*x\_2^2\*x\_3+4\*a^2\*x\_1^5\*x\_2\*x\_3^2-20\*a^2\*x\_1^4\*x\_2^2\*x\_3^2+16\*a^2\*x\_1^3\*x\_2^3\*x\_3^2+16\*a^2\*x\_1^3\*x\_2^2\*x\_3^3-3\*a^2\*x\_1^2\*x\_2^4\*x\_3^2-6\*a^2\*x\_1^2\*x\_2^3\*x\_3^3-3\*a^2\*x\_1^2\*x\_2^2\*x\_3^4-8/27\*a\*b^5\*x\_1^3\*x\_2^-8/27\*a\*b^5\*x\_1^3\*x\_3+140/27\*a\*b^4\*x\_1^4\*x\_2+140/27\*a\*b^4\*x\_1^4\*x\_3-4/9\*a\*b^4\*x\_1^3\*x\_2^2-4/9\*a\*b^4\*x\_1^3\*x\_3^2-44/3\*a\*b^3\*x\_1^5\*x\_2-44/3\*a\*b^3\*x\_1^5\*x\_3-50/9\*a\*b^3\*x\_1^4\*x\_2^2-50/9\*a\*b^3\*x\_1^4\*x\_3^2+4/3\*a\*b^3\*x\_1^3\*x\_2^3+4/3\*a\*b^3\*x\_1^3\*x\_3^3+140/9\*a\*b^2\*x\_1^6\*x\_2+140/9\*a\*b^2\*x\_1^6\*x\_3+58/3\*a\*b^2\*x\_1^5\*x\_2^2+58/3\*a\*b^2\*x\_1^5\*x\_3^2-10/3\*a\*b^2\*x\_1^4\*x\_2^3-10/3\*a\*b^2\*x\_1^4\*x\_3^3-64/9\*a\*b\*x\_1^7\*x\_2-64/9\*a\*b\*x\_1^7\*x\_3-56/3\*a\*b\*x\_1^6\*x\_2^2-56/3\*a\*b\*x\_1^6\*x\_3^2+2\*a\*b\*x\_1^5\*x\_2^3+2\*a\*b\*x\_1^5\*x\_3^3+28/3\*a\*x\_1^6\*x\_2^2\*x\_3+28/3\*a\*x\_1^6\*x\_2\*x\_3^2-4\*a\*x\_1^5\*x\_2^3\*x\_3+18\*a\*x\_1^5\*x\_2^2\*x\_3^2-4\*a\*x\_1^5\*x\_2\*x\_3^3-10\*a\*x\_1^4\*x\_2^3\*x\_3^2-10\*a\*x\_1^4\*x\_2^2\*x\_3^3+16/9\*b^5\*x\_1^3\*x\_2\*x\_3-100/9\*b^4\*x\_1^4\*x\_2\*x\_3-8/3\*b^4\*x\_1^3\*x\_2^2\*x\_3-8/3\*b^4\*x\_1^3\*x\_2\*x\_3^2+224/9\*b^3\*x\_1^5\*x\_2\*x\_3+40/3\*b^3\*x\_1^4\*x\_2^2\*x\_3+40/3\*b^3\*x\_1^4\*x\_2\*x\_3^2+4\*b^3\*x\_1^3\*x\_2^2\*x\_3^2-224/9\*b^2\*x\_1^6\*x\_2\*x\_3-24\*b^2\*x\_1^5\*x\_2^2\*x\_3-24\*b^2\*x\_1^5\*x\_2\*x\_3^2-15\*b^2\*x\_1^4\*x\_2^2\*x\_3^2+32/3\*b\*x\_1^7\*x\_2\*x\_3+56/3\*b\*x\_1^6\*x\_2^2\*x\_3+56/3\*b\*x\_1^6\*x\_2\*x\_3^2+18\*b\*x\_1^5\*x\_2^2\*x\_3^2-16/3\*b^2\*x\_1^7\*x\_2\*x\_3-7\*b^2\*x\_1^6\*x\_2^2\*x\_3+2\*b^2\*x\_1^6\*x\_2\*x\_3^2+8\*b^2\*x\_1^8\*x\_2^2\*x\_3+2\*b^2\*x\_1^8\*x\_2\*x\_3^2+8\*b^2\*x\_1^7\*x\_2^2\*x\_3^2-8/9\*a\*b^4\*x\_1^5\*x\_2-8/9\*a\*b^4\*x\_1^5\*x\_3+28/9\*a\*b^3\*x\_1^6\*x\_2+28/9\*a\*b^3\*x\_1^6\*x\_3+4/3\*a\*b^3\*x\_1^5\*x\_2^2+4/3\*a\*b^3\*x\_1^5\*x\_3^2-32/9\*a\*b^2\*x\_1^7\*x\_2-32/9\*a\*b^2\*x\_1^7\*x\_3-14/3\*a\*b^2\*x\_1^6\*x\_2^2-14/3\*a\*b^2\*x\_1^6\*x\_3^2+4/3\*a\*b\*x\_1^8\*x\_2+4/3\*a\*b\*x\_1^8\*x\_3+16/3\*a\*b\*x\_1^7\*x\_2^2+16/3\*a\*b\*x\_1^7\*x\_3^2-14\*a\*x\_1^6\*x\_2^2-x\_1^6\*x\_3^2+6\*a\*x\_1^5\*x\_2^3+x\_1^5\*x\_3^3-20/9\*b^5\*x\_1^4\*x\_2\*x\_3+88/9\*b^4\*x\_1^5\*x\_2\*x\_3+10/3\*b^4\*x\_1^4\*x\_2^2+x\_1^4\*x\_3^2-140/9\*b^3\*x\_1^6\*x\_2\*x\_3-40/3\*b^3\*x\_1^5\*x\_2^2+x\_1^5\*x\_3^2-40/3\*b^3\*x\_1^5\*x\_2\*x\_3^2-5\*b^3\*x\_1^4\*x\_2^2+x\_1^4\*x\_3^2+32/3\*b^2\*x\_1^7\*x\_2-x\_1^7\*x\_3^2+56/3\*b^2\*x\_1^6\*x\_2^2+x\_1^6\*x\_3^2+56/3\*b^2\*x\_1^6\*x\_2\*x\_3^2-18\*b^2\*x\_1^8\*x\_2\*x\_3-32/3\*b\*x\_1^7\*x\_2^2-x\_1^7\*x\_3^2-32/3\*b\*x\_1^7\*x\_2\*x\_3^2-16/27\*a^2\*b^4\*x\_1^3\*x\_2-16/27\*a^2\*b^4\*x\_1^3\*x\_3+154/27\*a^2\*b^3\*x\_1^4\*x\_2+154/27\*a^2\*b^3\*x\_1^4\*x\_3-8/9\*a^2\*b^3\*x\_1^3\*x\_2^2-8/9\*a^2\*b^3\*x\_1^3\*x\_2\*x\_3^2-100/9\*a^2\*b^2\*x\_1^5\*x\_2-100/9\*a^2\*b^2\*x\_1^5\*x\_3-37/9\*a^2\*b^2\*x\_1^4\*x\_2^2-37/9\*a^2\*b^2\*x\_1^4\*x\_3^2+8/3\*a^2\*b^2\*x\_1^3\*x\_2^3+8/3\*a^2\*b^2\*x\_1^3\*x\_2\*x\_3^2+6\*a^2\*b\*x\_1^6\*x\_2+6\*a^2\*b\*x\_1^6\*x\_3+14\*a^2\*b\*x\_1^5\*x\_2^2+14\*a^2\*b\*x\_1^5\*x\_3^2-20/3\*a^2\*b\*x\_1^4\*x\_2^3+8/9\*b^5\*x\_1^5\*x\_2^2+8/9\*b^5\*x\_1^5\*x\_3^2-28/9\*b^4\*x\_1^6\*x\_2-x\_1^6\*x\_3^2-4/3\*b^4\*x\_1^5\*x\_2^2+x\_1^5\*x\_3^2+32/9\*b^3\*x\_1^7\*x\_2-x\_1^7\*x\_3^2+14/3\*b^3\*x\_1^6\*x\_2^2+x\_1^6\*x\_3^2+14/3\*b^3\*x\_1^6\*x\_2\*x\_3^2-4/3\*b^2\*x\_1^8\*x\_2-x\_1^8\*x\_3^2-20/3\*a\*b^2\*x\_1^3\*x\_2+140/27\*a\*b\*x\_1^4\*x\_2+140/27\*a\*b\*x\_1^4\*x\_3-44/27\*a^3\*b\*x\_1\*x\_2-44/27\*a^3\*b\*x\_1\*x\_3+16/27\*a^3\*b\*x\_1^2\*x\_2+x\_1^2\*x\_3-8/27\*a\*b\*x\_1^3\*x\_2-8/27\*a\*b\*x\_1^3\*x\_3+16/27\*a\*x\_1^3\*x\_2^2+x\_1^3\*x\_3^2

$$\begin{aligned}
& x^2 x^3 + 190/9 a^3 b^2 x^4 + 190/9 a^2 b^3 x^5 + 190/9 a^4 x^6 - 44/3 a^3 b x^7 + 15 a^2 b^2 x^8 - 44/3 a^2 b^3 x^9 + 58/9 a^2 b^2 x^{10} \\
& x^{12} + 58/9 a^2 b^2 x^{13} + 12 x^{14} - 356/27 a^2 b^2 x^{15} + 13 x^{16} + 28/9 a^2 b^2 x^{17} + 12 x^{18} + 28/9 a^2 b^2 x^{19} \\
& b^2 x^{20} + 12 x^{21} + 3^2 + 160/27 a^2 x^{22} + 13 x^{23} + 20/3 a^2 b^2 x^{24} + 13 x^{25} - 4/9 b^3 x^{26} + 12 x^{27} + 3 - 7/3 b^3 x^{28} + 16 + 8/9 b^2 x^{29} \\
& x^{30} + 1^4 + 2 b^3 x^{31} + 15 - 7/9 b^2 x^{32} + 16 + 2/3 a^2 x^{33} + 15 - 5/9 b^3 x^{34} + 1^4 + 2/9 b^2 x^{35} + 15 + 4/9 a^3 x^{36} + 13 - 6 a^2 b^2 x^{37} + 1 x^{38} + 2^2 x^{39} \\
& x^{40} + 3^2 - 4 a^2 b^2 x^{41} + 1 x^{42} + 2 x^{43} + 3^3 - 16/3 a^2 b^3 x^{44} + 12 x^{45} + 2 x^{46} + 3 + 14/9 a^2 b^3 x^{47} + 1 x^{48} + 2^2 x^{49} + 14/9 a^2 b^3 x^{50} + 1 x^{51} + 2^2 x^{52} \\
& x^{53} + 592/27 a^2 b^2 x^{54} + 13 x^{55} + 2 x^{56} + 3 - 2 a^2 b^2 x^{57} + 12 x^{58} + 2^2 x^{59} + 3 - 2 a^2 b^2 x^{60} + 12 x^{61} + 2 x^{62} + 3^2 - 4/3 a^2 b^2 x^{63} + 1 x^{64} + 2^3 x^{65} + 3 - 2 a^2 b^2 x^{66} + 1 x^{67} + 2^2 x^{68} + 3^2 - 4/3 a^2 b^2 x^{69} + 1 x^{70} + 2 x^{71} + 3^3 - 760/27 a^2 b^2 x^{72} + 1^4 x^{73} + 2 x^{74} + 3 - 4 a^2 b^2 x^{75} + 13 x^{76} + 2^2 x^{77} + 3 - 4 a^2 b^2 x^{78} + 13 x^{79} + 2 x^{80} + 3^2 + 4 a^2 b^2 x^{81} + 1^2 x^{82} + 2^3 x^{83} + 3 + 6 a^2 b^2 x^{84} + 1^2 x^{85} + 2^2 x^{86} + 3^2 + 4 a^2 b^2 x^{87} + 1^2 x^{88} + 2 x^{89} + 3^3 - 56/9 a^3 b^2 x^{90} + 1 x^{91} + 2 x^{92} + 3 - 20/9 a^2 b^2 x^{93} + 1 x^{94} + 2 x^{95} + 3 + 76/27 a^2 b^2 x^{96} + 1 x^{97} + 2^2 x^{98} + 3 + 76/27 a^2 b^2 x^{99} + 1 x^{100} + 2 x^{101} + 3^2 + 256/27 a^2 b^2 x^{102} + 1^3 x^{103} + 2 x^{104} + 3 + 14 a^2 b^2 x^{105} + 1^2 x^{106} + 2^3 x^{107} + 3 + 9 a^2 b^2 x^{108} + 1^2 x^{109} + 2^2 x^{110} + 3^2 + 14 a^2 b^2 x^{111} + 1^2 x^{112} + 2 x^{113} + 3^3 - 8/3 a^2 b^2 x^{114} + 1 x^{115} + 2^4 x^{116} + 3 + 8/3 a^2 b^2 x^{117} + 1 x^{118} + 2^3 x^{119} + 3^2 + 8/3 a^2 b^2 x^{120} + 1 x^{121} + 2^2 x^{122} + 3^3 - 8/3 a^2 b^2 x^{123} + 1 x^{124} + 2 x^{125} + 3^4 + 8/27 a^2 b^5 x^{126} + 1 x^{127} + 2 x^{128} + 3 - 32/9 a^2 b^4 x^{129} + 1^2 x^{130} + 2 x^{131} + 3 + 4/9 a^2 b^4 x^{132} + 1 x^{133} + 2^2 x^{134} + 3 + 4/9 a^2 b^4 x^{135} + 1 x^{136} + 2 x^{137} + 3^2 + 592/27 a^2 b^3 x^{138} + 1^3 x^{139} + 2 x^{140} + 3 - 2 a^2 b^3 x^{141} + 1^2 x^{142} + 2^2 x^{143} + 3 - 2 a^2 b^3 x^{144} + 1 x^{145} + 2^2 x^{146} + 3^2 - 4/3 a^2 b^3 x^{147} + 1 x^{148} + 2^3 x^{149} + 3 - 2 a^2 b^3 x^{150} + 1 x^{151} + 2^2 x^{152} + 3^2 - 4/3 a^2 b^3 x^{153} + 1 x^{154} + 2 x^{155} + 3^3 - 400/9 a^2 b^2 x^{156} + 1^4 x^{157} + 2 x^{158} + 3 - 64/9 a^2 b^2 x^{159} + 1^3 x^{160} + 2^2 x^{161} + 3 - 64/9 a^2 b^2 x^{162} + 1^3 x^{163} + 2 x^{164} + 3^2 + 8 a^2 b^2 x^{165} + 1^2 x^{166} + 2^3 x^{167} + 3 + 6 a^2 b^2 x^{168} + 1^2 x^{169} + 2^2 x^{170} + 3^2 + 8 a^2 b^2 x^{171} + 1^2 x^{172} + 2 x^{173} + 3^3 + 2 a^2 b^2 x^{174} + 1 x^{175} + 2^3 x^{176} + 3^2 + 2 a^2 b^2 x^{177} + 1 x^{178} + 2^2 x^{179} + 3^3 + 32 a^2 b^2 x^{180} + 1^5 x^{181} + 2 x^{182} + 3 + 70/3 a^2 b^2 x^{183} + 1^4 x^{184} + 2^2 x^{185} + 3 + 70/3 a^2 b^2 x^{186} + 1^4 x^{187} + 2 x^{188} + 3^2 - 40/3 a^2 b^2 x^{189} + 1^3 x^{190} + 2^3 x^{191} + 3 - 4 a^2 b^2 x^{192} + 1^3 x^{193} + 2^2 x^{194} + 3^2 - 40/3 a^2 b^2 x^{195} + 1^3 x^{196} + 2 x^{197} + 3^3 - 6 a^2 b^2 x^{198} + 1^2 x^{199} + 2^3 x^{200} + 3^2 - 6 a^2 b^2 x^{201} + 1^2 x^{202} + 2^2 x^{203} + 3^3 - 4 a^4 b^2 x^{204} + 1 x^{205} + 2 x^{206} + 3 - 56/9 a^3 b^2 x^{207} + 1 x^{208} + 2 x^{209} + 3 + 104/9 a^3 b^2 x^{210} + 1^2 x^{211} + 2 x^{212} + 3 + 50/9 a^3 b^2 x^{213} + 1 x^{214} + 2^2 x^{215} + 3 + 50/9 a^3 b^2 x^{216} + 1 x^{217} + 2 x^{218} + 3^2 - 20/9 a^2 b^3 x^{219} + 1 x^{220} + 2 x^{221} + 3 + 2/9 a^2 b^2 x^{222} + 1^2 x^{223} + 2 x^{224} + 3 + 76/9 a^2 b^2 x^{225} + 1 x^{226} + 2^2 x^{227} + 3 + 76/9 a^2 b^2 x^{228} + 1 x^{229} + 2 x^{230} + 3^2 + 32/9 a^2 b^2 x^{231} + 1^3 x^{232} + 2 x^{233} + 3 - 26/3 a^2 b^2 x^{234} + 1^2 x^{235} + 2^2 x^{236} + 3 - 26/3 a^2 b^2 x^{237} + 1^2 x^{238} + 2 x^{239} + 3^2 - 4 a^2 b^2 x^{240} + 1 x^{241} + 2^3 x^{242} + 3 - 4/3 a^2 b^4 x^{243} + 1^2 x^{244} + 2^2 x^{245} + 3 - 4/3 a^2 b^4 x^{246} + 1^2 x^{247} + 2^2 x^{248} + 3 - 4/3 a^2 b^4 x^{249} + 1^2 x^{250} + 2 x^{251} + 3^2 - 760/27 a^2 b^3 x^{252} + 1^4 x^{253} + 2 x^{254} + 3 - 4 a^2 b^3 x^{255} + 1^3 x^{256} + 2^2 x^{257} + 3 - 4 a^2 b^3 x^{258} + 1^3 x^{259} + 2 x^{260} + 3^2 + 4 a^2 b^3 x^{261} + 1^2 x^{262} + 2^3 x^{263} + 3 + 6 a^2 b^3 x^{264} + 1^2 x^{265} + 2^2 x^{266} + 3^2 + 4 a^2 b^3 x^{267} + 1^2 x^{268} + 2 x^{269} + 3^3 + 32 a^2 b^2 x^{270} + 1^5 x^{271} + 2 x^{272} + 3 + 70/3 a^2 b^2 x^{273} + 1^4 x^{274} + 2^2 x^{275} + 3 + 70/3 a^2 b^2 x^{276} + 1^4 x^{277} + 2 x^{278} + 3^2 - 40/3 a^2 b^2 x^{279} + 1^3 x^{280} + 2^3 x^{281} + 3 - 4 a^2 b^2 x^{282} + 1^3 x^{283} + 2^2 x^{284} + 3^2 - 40/3 a^2 b^2 x^{285} + 1^3 x^{286} + 2 x^{287} + 3^3 - 6 a^2 b^2 x^{288} + 1^2 x^{289} + 2^3 x^{290} + 3^2 - 6 a^2 b^2 x^{291} + 1^2 x^{292} + 2^2 x^{293} + 3^3 - 112/9 a^2 b^2 x^{294} + 1^6 x^{295} + 2 x^{296} + 3 - 82/3 a^2 b^2 x^{297} + 1^5 x^{298} + 2^2 x^{299} + 3 - 82/3 a^2 b^2 x^{300} + 1^5 x^{301} + 2 x^{302} + 3^2 + 40/3 a^2 b^2 x^{303} + 1^4 x^{304} + 2^3 x^{305} + 3 - 20 a^2 b^2 x^{306} + 1^4 x^{307} + 2^2 x^{308} + 3^2 + 40/3 a^2 b^2 x^{309} + 1^4 x^{310} + 2 x^{311} + 3^3 + 16 a^2 b^2 x^{312} + 1^3 x^{313} + 2^3 x^{314} + 3^2 + 16 a^2 b^2 x^{315} + 1^3 x^{316} + 2^2 x^{317} + 3^3 + 16/27 a^3 b^3 x^{318} + 1 x^{319} + 2 x^{320} + 3 + 64/9 a^3 b^2 x^{321} + 1^2 x^{322} + 2 x^{323} + 3 - 8/9 a^3 b^2 x^{324} + 1 x^{325} + 2^2 x^{326} + 3^2 - 40/3 a^3 b^2 x^{327} + 1^3 x^{328} + 2 x^{329} + 3 + 2 a^3 b^2 x^{330} + 1^2 x^{331} + 2^2 x^{332} + 3 + 2 a^3 b^2 x^{333} + 1^2 x^{334} + 2^2 x^{335} + 3^2 - 8/3 a^3 b^2 x^{336} + 1 x^{337} + 2^3 x^{338} + 3 - 4 a^3 b^2 x^{339} + 1 x^{340} + 2^2 x^{341} + 3^2 - 8/3 a^3 b^2 x^{342} + 1 x^{343} + 2 x^{344} + 3^3 + 8/9 a^2 b^4 x^{345} + 1 x^{346} + 2 x^{347} + 3
\end{aligned}$$

$$\begin{aligned} & -4/3*a^2*b^3*x \quad 1^2*x \quad 2*x \quad 3+76/27*a^2*b^3*x \quad 1*x \quad 2^2* \\ & x \quad 3+76/27*a^2*b^3*x \quad 1*x \quad 2*x \quad 3^2+32/9*a^2*b^2*x \quad 1^3*x \quad 2* \\ & x \quad 3-26/3*a^2*b^2*x \quad 1^2*x \quad 2^2*x \quad 3-26/3*a^2*b^2*x \quad 1^2* \\ & x \quad 2*x \quad 3^2-4*a^2*b^2*x \quad 1*x \quad 2^3*x \quad 3-6*a^2*b^2*x \quad 1*x \quad 2^2* \\ & x \quad 3^2-4*a^2*b^2*x \quad 1*x \quad 2*x \quad 3^3+16/9*a^2*b*x \quad 1^3*x \quad 2^2* \\ & x \quad 3+16/9*a^2*b*x \quad 1^3*x \quad 2*x \quad 3^2-8/3*a*b^3*x \quad 1^3*x \quad 2^3* \\ & x \quad 3-4*a*b^3*x \quad 1^3*x \quad 2^2*x \quad 3^2-8/3*a*b^3*x \quad 1^3*x \quad 2* \\ & x \quad 3^3-56/9*a*b^2*x \quad 1^6*x \quad 2*x \quad 3-44/3*a*b^2*x \quad 1^5*x \quad 2^2* \\ & x \quad 3-44/3*a*b^2*x \quad 1^5*x \quad 2*x \quad 3^2+20/3*a*b^2*x \quad 1^4*x \quad 2^3* \\ & x \quad 3+20/3*a*b^2*x \quad 1^4*x \quad 2*x \quad 3^3+4*a*b^2*x \quad 1^3*x \quad 2^3* \\ & x \quad 3^2+4*a*b^2*x \quad 1^3*x \quad 2^2*x \quad 3^3+28/3*a*b*x \quad 1^6*x \quad 2^2* \\ & x \quad 3+28/3*a*b*x \quad 1^6*x \quad 2*x \quad 3^2-4*a*b*x \quad 1^5*x \quad 2^3*x \quad 3+18* \\ & a*b*x \quad 1^5*x \quad 2^2*x \quad 3^2-4*a*b*x \quad 1^5*x \quad 2*x \quad 3^3-10*a*b* \\ & x \quad 1^4*x \quad 2^3*x \quad 3^2-10*a*b*x \quad 1^4*x \quad 2^2*x \quad 3^3-16/9*a^2* \\ & b^4*x \quad 1^2*x \quad 2*x \quad 3+16/27*a^2*b^4*x \quad 1*x \quad 2^2*x \quad 3+16/27* \\ & a^2*b^4*x \quad 1*x \quad 2*x \quad 3^2+160/27*a^2*b^3*x \quad 1^3*x \quad 2*x \quad 3 \\ & -32/9*a^2*b^3*x \quad 1^2*x \quad 2^2*x \quad 3-32/9*a^2*b^3*x \quad 1^2*x \quad 2* \\ & x \quad 3^2-40/9*a^2*b^2*x \quad 1^3*x \quad 2^2*x \quad 3-40/9*a^2*b^2*x \quad 1^3* \\ & x \quad 2*x \quad 3^2+8*a^2*b^2*x \quad 1^2*x \quad 2^3*x \quad 3+12*a^2*b^2*x \quad 1^2* \\ & x \quad 2^2*x \quad 3^2+8*a^2*b^2*x \quad 1^2*x \quad 2*x \quad 3^3-4/3*a^2*b^2*x \quad 1* \\ & x \quad 2^4*x \quad 3-8/3*a^2*b^2*x \quad 1*x \quad 2^3*x \quad 3^2-8/3*a^2*b^2*x \quad 1* \\ & x \quad 2^2*x \quad 3^3-4/3*a^2*b^2*x \quad 1*x \quad 2*x \quad 3^4-16/3*a^2*b*x \quad 1^5* \\ & x \quad 2*x \quad 3+20/3*a^2*b*x \quad 1^4*x \quad 2^2*x \quad 3+20/3*a^2*b*x \quad 1^4* \\ & x \quad 2*x \quad 3^2-32/3*a^2*b*x \quad 1^3*x \quad 2^3*x \quad 3-32/3*a^2*b*x \quad 1^3* \\ & x \quad 2*x \quad 3^3+2*a^2*b*x \quad 1^2*x \quad 2^4*x \quad 3-8*a^2*b*x \quad 1^2*x \quad 2^3* \\ & x \quad 3^2-8*a^2*b*x \quad 1^2*x \quad 2^2*x \quad 3^3+2*a^2*b*x \quad 1^2*x \quad 2* \\ & x \quad 3^4+2*a^2*b*x \quad 1*x \quad 2^4*x \quad 3^2+4*a^2*b*x \quad 1*x \quad 2^3* \\ & x \quad 3^3+2*a^2*b*x \quad 1*x \quad 2^2*x \quad 3^4-8/9*a*b^5*x \quad 1^2*x \quad 2* \\ & x \quad 3+256/27*a*b^4*x \quad 1^3*x \quad 2*x \quad 3+16/27*a*b^5*x \quad 1^3*x \quad 2* \\ & x \quad 3-160/27*a*b^4*x \quad 1^4*x \quad 2*x \quad 3+8/9*a*b^4*x \quad 1^3*x \quad 2^2* \\ & x \quad 3+8/9*a*b^4*x \quad 1^3*x \quad 2*x \quad 3^2+104/9*a*b^3*x \quad 1^5*x \quad 2* \\ & x \quad 3+40/9*a*b^3*x \quad 1^4*x \quad 2^2*x \quad 3+40/9*a*b^3*x \quad 1^4*x \quad 2* \\ & x \quad 3^2-4/3*a^2*b*x \quad 1^2*x \quad 2*x \quad 3+8/9*a^2*b*x \quad 1*x \quad 2*x \quad 3 \\ & -32/9*a*b^2*x \quad 1^2*x \quad 2*x \quad 3+4/9*a*b^2*x \quad 1*x \quad 2^2*x \quad 3+4/9* \\ & a*b^2*x \quad 1*x \quad 2*x \quad 3^2-4/3*a*b*x \quad 1^2*x \quad 2^2*x \quad 3-4/3*a*b* \\ & x \quad 1^2*x \quad 2*x \quad 3^2+8/27*a*b^2*x \quad 1*x \quad 2*x \quad 3-8/9*a*b*x \quad 1^2* \\ & x \quad 2*x \quad 3-320/27*a*b^3*x \quad 1^4+98/9*a*b^2*x \quad 1^5-28/9*a*b* \\ & x \quad 1^6+28/9*a*x \quad 1^6*x \quad 2+28/9*a*x \quad 1^6*x \quad 3+40/3*a^2*b^2* \\ & x \quad 1^3-199/27*a^2*b*x \quad 1^4+154/27*a^2*x \quad 1^4*x \quad 2+154/27*a^2* \\ & x \quad 1^4*x \quad 3-8/9*a^2*x \quad 1^3*x \quad 2^2-8/9*a^2*x \quad 1^3* \\ & x \quad 3^2+116/27*a*b^3*x \quad 1^3-100/27*a*b^2*x \quad 1^4+8/9*a*b*x \quad 1^5 \\ & -8/9*a*x \quad 1^5*x \quad 2-8/9*a*x \quad 1^5*x \quad 3-62/27*a^3*b* \\ & x \quad 1^2+16/27*a^3*b*x \quad 2^2+16/27*a^3*b*x \quad 3^2+44/27*a^3* \\ & x \quad 1^2*x \quad 2+44/27*a^3*x \quad 1^2*x \quad 3-16/27*a^3*x \quad 1*x \quad 2^2 \\ & -16/27*a^3*x \quad 1*x \quad 3^2-16/27*a^3*x \quad 2^2*x \quad 3-16/27*a^3*x \quad 2* \\ & x \quad 3^2-4/3*a^2*b^2*x \quad 1^2+16/27*a^2*b*x \quad 1^3-16/27*a^2* \\ & x \quad 1^3*x \quad 2-16/27*a^2*x \quad 1^3*x \quad 3-2/9*a*b^3*x \quad 1^2+4/27*a* \\ & b^2*x \quad 1^3-8/9*a^2*b^2*x \quad 1*x \quad 2+16/9*a^2*b*x \quad 1^2*x \quad 2-8/27* \\ & a^2*b*x \quad 1*x \quad 2^2-4/27*a*b^3*x \quad 1*x \quad 2+4/9*a*b^2*x \quad 1^2* \\ & x \quad 2+16/9*a^2*b*x \quad 1^2*x \quad 3-8/27*a^2*b*x \quad 1*x \quad 3^2-4/27*a* \\ & b^3*x \quad 1*x \quad 3+4/9*a*b^2*x \quad 1^2*x \quad 3-8/9*a^2*b^2*x \quad 1*x \quad 3 \\ & -8/27*a^2*b*x \quad 2^2*x \quad 3-8/27*a^2*b*x \quad 2*x \quad 3^2+4/27*a^2*b^2* \\ & x \quad 2*x \quad 3-8/3*b^2*x \quad 1^3*x \quad 2*x \quad 3^2+88/9*b*x \quad 1^5*x \quad 2* \\ & x \quad 3+10/3*b*x \quad 1^4*x \quad 2^2*x \quad 3+10/3*b*x \quad 1^4*x \quad 2* \\ & x \quad 3^2+20/9*a^4*b*x \quad 1*x \quad 2+20/9*a^4*b*x \quad 1*x \quad 3-10/9*a^4*b* \\ & x \quad 2*x \quad 3-4*a^4*x \quad 1*x \quad 2*x \quad 3+38/27*a^3*b^2*x \quad 1*x \quad 2+38/27* \\ & a^3*b^2*x \quad 1*x \quad 3-4/27*a^3*b^2*x \quad 2*x \quad 3+38/9*a^3*b*x \quad 1^2* \end{aligned}$$

x 2+38/9\*a^3\*b\*x 1^2\*x 3-22/9\*a^3\*b\*x 1\*x 2^2-22/9\*a^3\*b\*x 1\*x 3^2+8/9\*a^3\*b\*x 2^2\*x 3+8/9\*a^3\*b\*x 2\*x 3^2+64/9\*a^3\*x 1^2\*x 2\*x 3-8/9\*a^3\*x 1\*x 2^2\*x 3-8/9\*a^3\*x 1\*x 2\*x 3^2+2/27\*a^2\*b^3\*x 2\*x 3-16/9\*a^2\*b^2\*x 1\*x 2^2-16/9\*a^2\*b^2\*x 1\*x 3^2-2/9\*a^2\*b^2\*x 2^2\*x 3-2/9\*a^2\*b^2\*x 2\*x 3^2-32/9\*a^2\*x 1^2\*x 2^2\*x 3-32/9\*a^2\*x 1^2\*x 2\*x 3^2-2/27\*a\*b^4\*x 1\*x 2-2/27\*a\*b^4\*x 1\*x 3+22/9\*a\*b^3\*x 1^2\*x 2+22/9\*a\*b^3\*x 1^2\*x 3-2/9\*a\*b^3\*x 1\*x 2^2-2/9\*a\*b^3\*x 1\*x 3^2+2/3\*a\*b^2\*x 1^2\*x 2^2+2/3\*a\*b^2\*x 1^2\*x 3^2-4/9\*a\*b\*x 1^3\*x 2^2-4/9\*a\*b\*x 1^3\*x 3^2-160/27\*a\*x 1^4\*x 2\*x 3+8/9\*a\*x 1^3\*x 2^2\*x 3+8/9\*a\*x 1^3\*x 2\*x 3^2+16/9\*b^2\*x 1^3\*x 2\*x 3-20/9\*b\*x 1^4\*x 2\*x 3+16/27\*a^2\*x 1\*x 2^2\*x 3+16/27\*a^2\*x 1\*x 2\*x 3^2-1/3\*b^4\*x 1^2\*x 2^2-1/3\*b^4\*x 1^2\*x 3^2+40/9\*b^3\*x 1^4\*x 2+40/9\*b^3\*x 1^4\*x 3+4/3\*b^3\*x 1^3\*x 2^2+4/3\*b^3\*x 1^3\*x 3^2-40/9\*b^2\*x 1^5\*x 2-40/9\*b^2\*x 1^5\*x 3-5/3\*b^2\*x 1^4\*x 2^2-5/3\*b^2\*x 1^4\*x 3^2+14/9\*b\*x 1^6\*x 2+14/9\*b\*x 1^6\*x 3+2/3\*b\*x 1^5\*x 2^2+2/3\*b\*x 1^5\*x 3^2-28/9\*x 1^6\*x 2\*x 3-4/3\*x 1^5\*x 2^2\*x 3-4/3\*x 1^5\*x 2\*x 3^2+2\*a^4\*b^2\*x 1-61/9\*a^4\*b\*x 1^2-10/9\*a^4\*b\*x 2^2-10/9\*a^4\*b\*x 3^2+2\*a^4\*x 2^2\*x 3+2\*a^4\*x 2\*x 3^2+2\*a^3\*b^3\*x 1-322/27\*a^3\*b^2\*x 1^2-4/27\*a^3\*b^2\*x 2^2-4/27\*a^3\*b^2\*x 3^2+44/3\*a^3\*b\*x 1^3-64/9\*a^3\*x 1^3\*x 2-64/9\*a^3\*x 1^3\*x 3+10/3\*a^3\*x 1^2\*x 2^2+10/3\*a^3\*x 1^2\*x 3^2+2/3\*a^2\*b^4\*x 1-62/9\*a^2\*b^3\*x 1^2+2/27\*a^2\*b^3\*x 2^2+2/27\*a^2\*b^3\*x 3^2+2/27\*a\*b^5\*x 1-14/9\*a\*b^4\*x 1^2+2/9\*b^4\*x 1^2\*x 2+2/9\*b^4\*x 1^2\*x 3-8/9\*b^3\*x 1^3\*x 2-8/9\*b^3\*x 1^3\*x 3+8/9\*x 1^5\*x 2\*x 3+2\*a^4\*b\*x 1-8/9\*a^4\*x 1\*x 2-8/9\*a^4\*x 1\*x 3+4/9\*a^4\*x 2\*x 3+2\*a^3\*b^2\*x 1+2/3\*a^2\*b^3\*x 1+4/27\*a^2\*b^2\*x 2^2+4/27\*a^2\*b^2\*x 3^2+2/27\*a\*b^4\*x 1+40/9\*b^4\*x 1^4\*x 3+4/3\*b^4\*x 1^3\*x 2^2+4/3\*b^4\*x 1^3\*x 3^2-8\*b^3\*x 1^5\*x 2-8\*b^3\*x 1^5\*x 3-5\*b^3\*x 1^4\*x 2^2-5\*b^3\*x 1^4\*x 3^2+56/9\*b^2\*x 1^6\*x 2+56/9\*b^2\*x 1^6\*x 3+6\*b^2\*x 1^5\*x 2^2+6\*b^2\*x 1^5\*x 3^2-16/9\*b\*x 1^7\*x 2-16/9\*b\*x 1^7\*x 3-7/3\*b\*x 1^6\*x 2^2-7/3\*b\*x 1^6\*x 3^2+32/9\*x 1^7\*x 2\*x 3+14/3\*x 1^6\*x 2^2\*x 3+14/3\*x 1^6\*x 2\*x 3^2+2\*x 1^5\*x 2^2\*x 3^2-8/9\*a^4\*b^2\*x 1^2+4/9\*a^4\*b^2\*x 2^2+4/9\*a^4\*b^2\*x 3^2+4\*a^4\*b\*x 1^3+4\*a^4\*x 1^3\*x 2+4\*a^4\*x 1^3\*x 3-6\*a^4\*x 1^2\*x 2^2-6\*a^4\*x 1^2\*x 3^2+4\*a^4\*x 1\*x 2^3+4\*a^4\*x 1\*x 3^3-2\*a^4\*x 2^3\*x 3-3\*a^4\*x 2^2\*x 3^2-2\*a^4\*x 2\*x 3^3-62/27\*a^3\*b^3\*x 1^2+16/27\*a^3\*b^3\*x 2^2+16/27\*a^3\*b^3\*x 3^2+44/3\*a^3\*b^2\*x 1^3-188/9\*a^3\*b\*x 1^4-4/3\*a^3\*b\*x 2^4-4/3\*a^3\*b\*x 3^4+14/3\*a^3\*x 1^4\*x 2+14/3\*a^3\*x 1^4\*x 3+6\*a^3\*x 1^3\*x 2^2+6\*a^3\*x 1^3\*x 3^2-22/3\*a^3\*x 1^2\*x 2^3-22/3\*a^3\*x 1^2\*x 3^3+4/3\*a^3\*x 1\*x 2^4+4/3\*a^3\*x 1\*x 3^4+4/3\*a^3\*x 2^4\*x 3+8/3\*a^3\*x 2^3\*x 3^2+8/3\*a^3\*x 2^2\*x 3^3+4/3\*a^3\*x 2\*x 3^4-4/3\*a^2\*b^4\*x 1^2+4/27\*a^2\*b^4\*x 3^2+40/3\*a^2\*b^3\*x 1^3-731/27\*a^2\*b^2\*x 1^4-1/3\*a^2\*b^2\*x 2^4-1/3\*a^2\*b^2\*x 3^4+154/9\*a^2\*b\*x 1^5-100/9\*a^2\*x 1^5\*x 2-100/9\*a^2\*x 1^5\*x 3-37/9\*a^2\*x 1^4\*x 2^2-37/9\*a^2\*x 1^4\*x 3^2+8/3\*a^2\*x 1^3\*x 2^3+8/3\*a^2\*x 1^3\*x 3^3-2/9\*a\*b^5\*x 1^2+116/27\*a\*b^4\*x 1^3+4/3\*a\*x 1^5\*x 2^2+4/3\*a\*x 1^5\*x 3^2+2/9\*b^5\*x 1^2\*x 2+2/9\*b^5\*x 1^2\*x 3-16/9\*b^4\*x 1^3\*x 2-16/9\*b^4\*x 1^3\*x 3+a^6-a^2\*x 1^8+2/9\*b^5\*x 1^5-7/9\*b^4\*x 1^6+8/9\*b^3\*x 1^7-1/3\*b^2\*x 1^8-4\*a^3\*



$$\begin{aligned} & \frac{x_1^6 + 8/3 a^2 x_1^7 - 5/9 b^5 x_1^4 + 2 b^4 x_1^5 - 4 a^4 x_1^5}{x_1^4 - a^4 x_1^2 + 2 a^4 x_1^3 + 22/3 a^3 x_1^5 - 7/3 a^2 x_1^6} \\ & \frac{x_1^6 + 4/9 b^5 x_1^3 - 5/3 b^4 x_1^4 + 4 a^4 x_1^3 - 34/9 a^3 x_1^5}{x_1^4 - 1/9 b^5 x_1^2 + 4/9 b^4 x_1^3 - 2/3 a^4 b^2 - 8/9 a^4 x_1^2} \\ & \frac{x_1^2 + 4/9 a^4 x_1^2 - 2 a^2 + 4/9 a^4 x_1^2 - 3 a^2 - 8/27 a^3 b^3 - 1/27 a^2 b^4}{b^4}; \end{aligned}$$

$$\begin{aligned} g_5 := & \frac{2}{3} a^2 b^3 x_1^5 - \frac{7}{3} a^2 b^2 x_1^6 + \frac{8}{3} a^2 b x_1^7 - 9 a^2 x_1^6 x_2^2 - 9 a^2 x_1^6 x_3^2 + 4 a^2 x_1^5 x_2^3 \quad (7.1.5) \\ & + 4 a^2 x_1^5 x_3^3 + \frac{8}{9} a b^4 x_1^5 - \frac{28}{9} a b^3 x_1^6 + \frac{32}{9} a b^2 x_1^7 - \frac{4}{3} a b x_1^8 + \frac{4}{3} a x_1^8 x_2 \\ & + \frac{4}{3} a x_1^8 x_3 + \frac{16}{3} a x_1^7 x_2^2 + \frac{16}{3} a x_1^7 x_3^2 + \frac{10}{9} b^5 x_1^4 x_2 + \frac{10}{9} b^5 x_1^4 x_3 \\ & - \frac{40}{9} b^4 x_1^5 x_2 - \frac{40}{9} b^4 x_1^5 x_3 - \frac{5}{3} b^4 x_1^4 x_2^2 - \frac{5}{3} b^4 x_1^4 x_3^2 + \frac{56}{9} b^3 x_1^6 x_2 \\ & + \frac{56}{9} b^3 x_1^6 x_3 + 6 b^3 x_1^5 x_2^2 + 6 b^3 x_1^5 x_3^2 - \frac{32}{9} b^2 x_1^7 x_2 - \frac{32}{9} b^2 x_1^7 x_3 \\ & - 7 b^2 x_1^6 x_2^2 - 7 b^2 x_1^6 x_3^2 + \frac{2}{3} b x_1^8 x_2 + \frac{2}{3} b x_1^8 x_3 + \frac{8}{3} b x_1^7 x_2^2 + \frac{8}{3} b x_1^7 x_3^2 \\ & - \frac{4}{3} x_1^8 x_2 x_3 - \frac{16}{3} x_1^7 x_2^2 x_3 - \frac{16}{3} x_1^7 x_2 x_3^2 - 7 x_1^6 x_2^2 x_3^2 + \frac{4}{9} a^3 b^3 x_1^3 \\ & - \frac{34}{9} a^3 b^2 x_1^4 + \frac{22}{3} a^3 b x_1^5 + 2 a^3 x_1^5 x_2 + 2 a^3 x_1^5 x_3 - 12 a^3 x_1^4 x_2^2 \\ & - 12 a^3 x_1^4 x_3^2 + 10 a^3 x_1^3 x_2^3 + 10 a^3 x_1^3 x_3^3 - 2 a^3 x_1^2 x_2^4 - 2 a^3 x_1^2 x_3^4 \\ & - 2 a^3 x_2^4 x_3^2 - 4 a^3 x_2^3 x_3^3 - 2 a^3 x_2^2 x_3^4 + \frac{16}{27} a^2 b^4 x_1^3 - \frac{199}{27} a^2 b^3 x_1^4 \\ & + \frac{154}{9} a^2 b^2 x_1^5 - 13 a^2 b x_1^6 + 6 a^2 x_1^6 x_2 + 6 a^2 x_1^6 x_3 + 14 a^2 x_1^5 x_2^2 \\ & + 14 a^2 x_1^5 x_3^2 - \frac{20}{3} a^2 x_1^4 x_2^3 - \frac{20}{3} a^2 x_1^4 x_3^3 + \frac{4}{27} a b^5 x_1^3 - \frac{100}{27} a b^4 x_1^4 \\ & + \frac{98}{9} a b^3 x_1^5 - \frac{98}{9} a b^2 x_1^6 + \frac{32}{9} a b x_1^7 - \frac{32}{9} a x_1^7 x_2 - \frac{32}{9} a x_1^7 x_3 \\ & - \frac{14}{3} a x_1^6 x_2^2 - \frac{14}{3} a x_1^6 x_3^2 - \frac{8}{9} b^5 x_1^3 x_2 - \frac{8}{9} b^5 x_1^3 x_3 + \frac{40}{9} b^4 x_1^4 x_2 \\ & - 3 x_1^8 x_2^2 x_3^2 - 2 a x_1^8 x_2^2 - 2 a x_1^8 x_3^2 - \frac{4}{9} b^5 x_1^5 x_2 - \frac{4}{9} b^5 x_1^5 x_3 \\ & + \frac{14}{9} b^4 x_1^6 x_2 + \frac{14}{9} b^4 x_1^6 x_3 + \frac{2}{3} b^4 x_1^5 x_2^2 + \frac{2}{3} b^4 x_1^5 x_3^2 - \frac{16}{9} b^3 x_1^7 x_2 \\ & - \frac{16}{9} b^3 x_1^7 x_3 - \frac{7}{3} b^3 x_1^6 x_2^2 - \frac{7}{3} b^3 x_1^6 x_3^2 + \frac{2}{3} b^2 x_1^8 x_2 + \frac{2}{3} b^2 x_1^8 x_3 \\ & + \frac{8}{3} b^2 x_1^7 x_2^2 + \frac{8}{3} b^2 x_1^7 x_3^2 - b x_1^8 x_2^2 - b x_1^8 x_3^2 + 2 x_1^8 x_2^2 x_3 + 2 x_1^8 x_2 x_3^2 \end{aligned}$$

$$\begin{aligned}
& + 8x_l^7x_2^2x_3^2 + \frac{10}{9}b^2x_l^4x_2 + \frac{10}{9}b^2x_l^4x_3 - \frac{4}{9}bx_l^5x_2 - \frac{4}{9}bx_l^5x_3 \\
& - \frac{2}{3}a^2b^2x_2^3x_3 - a^2b^2x_2^2x_3^2 - \frac{2}{3}a^2b^2x_2x_3^3 + \frac{230}{9}a^2bx_l^4x_2 \\
& + \frac{230}{9}a^2bx_l^4x_3 + \frac{82}{9}a^2bx_l^3x_2^2 + \frac{82}{9}a^2bx_l^3x_3^2 - 8a^2bx_l^2x_2^3 \\
& - 8a^2bx_l^2x_3^3 + \frac{2}{3}a^2bx_lx_2^4 + \frac{2}{3}a^2bx_lx_3^4 + \frac{2}{3}a^2bx_2^4x_3 + \frac{4}{3}a^2bx_2^3x_3^2 \\
& + \frac{4}{3}a^2bx_2^2x_3^3 + \frac{2}{3}a^2bx_2x_3^4 - \frac{40}{9}a^2x_l^3x_2^2x_3 - \frac{40}{9}a^2x_l^3x_2x_3^2 \\
& + 8a^2x_l^2x_2^3x_3 + 12a^2x_l^2x_2^2x_3^2 + 8a^2x_l^2x_2x_3^3 - \frac{4}{3}a^2x_lx_2^4x_3 \\
& - \frac{8}{3}a^2x_lx_2^3x_3^2 - \frac{8}{3}a^2x_lx_2^2x_3^3 - \frac{4}{3}a^2x_lx_2x_3^4 - \frac{4}{27}ab^5x_lx_2 \\
& - \frac{4}{27}ab^5x_lx_3 + \frac{22}{9}ab^4x_l^2x_2 + \frac{22}{9}ab^4x_l^2x_3 - \frac{2}{9}ab^4x_lx_2^2 - \frac{2}{9}ab^4x_lx_3^2 \\
& - \frac{320}{27}ab^3x_l^3x_2 - \frac{320}{27}ab^3x_l^3x_3 - \frac{4}{3}ab^3x_l^2x_2^2 - \frac{4}{3}ab^3x_l^2x_3^2 \\
& + \frac{2}{3}ab^3x_lx_2^3 + \frac{2}{3}ab^3x_lx_3^3 + \frac{52}{9}ab^2x_l^3x_2^2 + \frac{52}{9}ab^2x_l^3x_3^2 \\
& - 2ab^2x_l^2x_2^3 - 2ab^2x_l^2x_3^3 - \frac{50}{9}abx_l^4x_2^2 - \frac{50}{9}abx_l^4x_3^2 + \frac{4}{3}abx_l^3x_2^3 \\
& + \frac{4}{3}abx_l^3x_3^3 + \frac{104}{9}ax_l^5x_2x_3 + \frac{40}{9}ax_l^4x_2^2x_3 + \frac{40}{9}ax_l^4x_2x_3^2 \\
& - \frac{8}{3}ax_l^3x_2^3x_3 - 4ax_l^3x_2^2x_3^2 - \frac{8}{3}ax_l^3x_2x_3^3 - \frac{8}{9}b^4x_l^2x_2x_3 \\
& + \frac{16}{3}b^3x_l^3x_2x_3 + \frac{2}{3}b^3x_l^2x_2^2x_3 + \frac{2}{3}b^3x_l^2x_2x_3^2 - \frac{100}{9}b^2x_l^4x_2x_3 \\
& - \frac{8}{3}b^2x_l^3x_2^2x_3 - b^3x_l^2x_2^2x_3^2 + \frac{224}{9}b^2x_l^5x_2x_3 + \frac{40}{3}b^2x_l^4x_2^2x_3 \\
& + \frac{40}{3}b^2x_l^4x_2x_3^2 + 4b^2x_l^3x_2^2x_3^2 - \frac{140}{9}bx_l^6x_2x_3 - \frac{40}{3}bx_l^5x_2^2x_3 \\
& - \frac{40}{3}bx_l^5x_2x_3^2 - 5bx_l^4x_2^2x_3^2 - \frac{8}{9}a^4b^2x_lx_2 - \frac{8}{9}a^4b^2x_lx_3 + \frac{4}{9}a^4b^2x_2x_3 \\
& + 2a^4bx_2^2x_3 + 2a^4bx_2x_3^2 - 2a^4x_l^2x_2x_3 + 4a^4x_lx_2^2x_3 + 4a^4x_lx_2x_3^2 \\
& - \frac{44}{27}a^3b^3x_lx_2 - \frac{44}{27}a^3b^3x_lx_3 + \frac{16}{27}a^3b^3x_2x_3 + \frac{38}{9}a^3b^2x_l^2x_2 \\
& + \frac{38}{9}a^3b^2x_l^2x_3 - \frac{22}{9}a^3b^2x_lx_2^2 - \frac{22}{9}a^3b^2x_lx_3^2 + \frac{8}{9}a^3b^2x_2^2x_3
\end{aligned}$$

$$\begin{aligned}
& + \frac{8}{9} a^3 b^2 x_2 x_3^2 - \frac{44}{9} a^3 b x_l^3 x_2 - \frac{44}{9} a^3 b x_l^3 x_3 - \frac{16}{3} a^3 b x_l^2 x_2^2 \\
& - \frac{16}{3} a^3 b x_l^2 x_3^2 + \frac{22}{3} a^3 b x_l x_2^3 + \frac{22}{3} a^3 b x_l x_3^3 - \frac{8}{3} a^3 b x_2^3 x_3 \\
& - 4 a^3 b x_2^2 x_3^2 - \frac{8}{3} a^3 b x_2 x_3^3 - \frac{40}{3} a^3 x_l^3 x_2 x_3 + 2 a^3 x_l^2 x_2^2 x_3 + 2 a^3 x_l^2 x_2 x_3^2 \\
& - \frac{8}{3} a^3 x_l x_2^3 x_3 - 4 a^3 x_l x_2^2 x_3^2 - \frac{8}{3} a^3 x_l x_2 x_3^3 - \frac{8}{9} a^2 b^4 x_l x_2 - \frac{8}{9} a^2 b^4 x_l x_3 \\
& + \frac{4}{27} a^2 b^4 x_2 x_3 + \frac{58}{9} a^2 b^3 x_l^2 x_2 + \frac{58}{9} a^2 b^3 x_l^2 x_3 - \frac{16}{9} a^2 b^3 x_l x_2^2 \\
& - \frac{16}{9} a^2 b^3 x_l x_3^2 - \frac{2}{9} a^2 b^3 x_2^2 x_3 - \frac{2}{9} a^2 b^3 x_2 x_3^2 - \frac{536}{27} a^2 b^2 x_l^3 x_2 \\
& - \frac{536}{27} a^2 b^2 x_l^3 x_3 - \frac{19}{9} a^2 b^2 x_l^2 x_2^2 - \frac{19}{9} a^2 b^2 x_l^2 x_3^2 + 4 a^2 b^2 x_l x_2^3 \\
& + 4 a^2 b^2 x_l x_3^3 + 16 a^2 b x_l^3 x_3^3 - a^2 b x_l^2 x_2^4 - a^2 b x_l^2 x_3^4 - a^2 b x_2^4 x_3^2 \\
& - 2 a^2 b x_2^3 x_3^3 - a^2 b x_2^2 x_3^4 - \frac{16}{3} a^2 x_l^5 x_2 x_3 + \frac{20}{3} a^2 x_l^4 x_2^2 x_3 \\
& + \frac{20}{3} a^2 x_l^4 x_2 x_3^2 - \frac{32}{3} a^2 x_l^3 x_2^3 x_3 - \frac{32}{3} a^2 x_l^3 x_2 x_3^3 + 2 a^2 x_l^2 x_2^4 x_3 \\
& - 8 a^2 x_l^2 x_2^3 x_3^2 - 8 a^2 x_l^2 x_2^2 x_3^3 + 2 a^2 x_l^2 x_2 x_3^4 + 2 a^2 x_l x_2^4 x_3^2 + 4 a^2 x_l x_2^3 x_3^3 \\
& + 2 a^2 x_l x_2^2 x_3^4 + \frac{4}{9} a b^5 x_l^2 x_2 + \frac{4}{9} a b^5 x_l^2 x_3 - \frac{20}{3} a b^4 x_l^3 x_2 - \frac{20}{3} a b^4 x_l^3 x_3 \\
& + \frac{2}{3} a b^4 x_l^2 x_2^2 + \frac{2}{3} a b^4 x_l^2 x_3^2 + \frac{190}{9} a b^3 x_l^4 x_2 + \frac{190}{9} a b^3 x_l^4 x_3 \\
& + \frac{52}{9} a b^3 x_l^3 x_2^2 + \frac{52}{9} a b^3 x_l^3 x_3^2 - 2 a b^3 x_l^2 x_2^3 - 2 a b^3 x_l^2 x_3^3 \\
& - \frac{242}{9} a b^2 x_l^5 x_2 - \frac{242}{9} a b^2 x_l^5 x_3 - \frac{190}{9} a b^2 x_l^4 x_2^2 - \frac{190}{9} a b^2 x_l^4 x_3^2 \\
& + \frac{16}{3} a b^2 x_l^3 x_2^3 + \frac{16}{3} a b^2 x_l^3 x_3^3 + \frac{140}{9} a b x_l^6 x_2 + \frac{140}{9} a b x_l^6 x_3 \\
& + \frac{58}{3} a b x_l^5 x_2^2 + \frac{58}{3} a b x_l^5 x_3^2 - \frac{10}{3} a b x_l^4 x_2^3 - \frac{10}{3} a b x_l^4 x_3^3 \\
& - \frac{56}{9} a x_l^6 x_2 x_3 - \frac{44}{3} a x_l^5 x_2^2 x_3 - \frac{44}{3} a x_l^5 x_2 x_3^2 + \frac{20}{3} a x_l^4 x_2^3 x_3 \\
& + \frac{20}{3} a x_l^4 x_2 x_3^3 + 4 a x_l^3 x_2^3 x_3^2 + 4 a x_l^3 x_2^2 x_3^3 - \frac{4}{9} b^5 x_l^2 x_2 x_3 \\
& + \frac{16}{3} b^4 x_l^3 x_2 x_3 + \frac{2}{3} b^4 x_l^2 x_2^2 x_3 + \frac{2}{3} b^4 x_l^2 x_2 x_3^2 - \frac{160}{9} b^3 x_l^4 x_2 x_3
\end{aligned}$$



$$\begin{aligned}
& -\frac{16}{3} b^3 x_l^3 x_2^2 x_3 - \frac{16}{3} b^3 x_l^3 x_2 x_3^2 + \frac{44}{27} a^3 b^3 x_l^2 x_2 + \frac{44}{27} a^3 b^3 x_l^2 x_3 \\
& - \frac{16}{27} a^3 b^3 x_l x_2^2 - \frac{16}{27} a^3 b^3 x_l x_3^2 - \frac{16}{27} a^3 b^3 x_2^2 x_3 - \frac{16}{27} a^3 b^3 x_2 x_3^2 \\
& - \frac{64}{9} a^3 b^2 x_l^3 x_2 - \frac{64}{9} a^3 b^2 x_l^3 x_3 + \frac{10}{3} a^3 b^2 x_l^2 x_2^2 + \frac{10}{3} a^3 b^2 x_l^2 x_3^2 \\
& + \frac{14}{3} a^3 b x_l^4 x_2 + \frac{14}{3} a^3 b x_l^4 x_3 + 6 a^3 b x_l^3 x_2^2 + 6 a^3 b x_l^3 x_3^2 - \frac{22}{3} a^3 b x_l^2 x_2^3 \\
& - \frac{22}{3} a^3 b x_l^2 x_3^3 + \frac{4}{3} a^3 b x_l x_2^4 + \frac{4}{3} a^3 b x_l x_3^4 + \frac{4}{3} a^3 b x_2^4 x_3 + \frac{8}{3} a^3 b x_2^3 x_3^2 \\
& + \frac{8}{3} a^3 b x_2^2 x_3^3 + \frac{4}{3} a^3 b x_2 x_3^4 + 10 a^3 x_l^3 x_2^2 x_3 + 10 a^3 x_l^3 x_2 x_3^2 - 4 a^3 x_l^2 x_2^3 x_3 \\
& - 14 a^3 x_l^2 x_2^2 x_3^2 - 4 a^3 x_l^2 x_2 x_3^3 + 8 a^3 x_l x_2^3 x_3^2 + 8 a^3 x_l x_2^2 x_3^3 \\
& + \frac{16}{9} a^2 b^4 x_l^2 x_2 + \frac{16}{9} a^2 b^4 x_l^2 x_3 - \frac{8}{27} a^2 b^4 x_l x_2^2 - \frac{8}{27} a^2 b^4 x_l x_3^2 \\
& - \frac{8}{27} a^2 b^4 x_2^2 x_3 - \frac{8}{27} a^2 b^4 x_2 x_3^2 - \frac{356}{27} a^2 b^3 x_l^3 x_2 - \frac{356}{27} a^2 b^3 x_l^3 x_3 \\
& + \frac{28}{9} a^2 b^3 x_l^2 x_2^2 + \frac{28}{9} a^2 b^3 x_l^2 x_3^2 + \frac{230}{9} a^2 b^2 x_l^4 x_2 + \frac{230}{9} a^2 b^2 x_l^4 x_3 \\
& + \frac{82}{9} a^2 b^2 x_l^3 x_2^2 + \frac{82}{9} a^2 b^2 x_l^3 x_3^2 - 8 a^2 b^2 x_l^2 x_2^3 - 8 a^2 b^2 x_l^2 x_3^3 \\
& + \frac{2}{3} a^2 b^2 x_l x_2^4 + \frac{2}{3} a^2 b^2 x_l x_3^4 + \frac{2}{3} a^2 b^2 x_2^4 x_3 + \frac{4}{3} a^2 b^2 x_2^3 x_3^2 \\
& + \frac{4}{3} a^2 b^2 x_2^2 x_3^3 + \frac{2}{3} a^2 b^2 x_2 x_3^4 - \frac{176}{9} a^2 b x_l^5 x_2 - \frac{176}{9} a^2 b x_l^5 x_3 \\
& - \frac{248}{9} a^2 b x_l^4 x_2^2 - \frac{248}{9} a^2 b x_l^4 x_3^2 + 16 a^2 b x_l^3 x_2^3 - \frac{20}{3} a^2 b x_l^4 x_3^3 \\
& + 4 a^2 x_l^5 x_2^2 x_3 + 4 a^2 x_l^5 x_2 x_3^2 - 20 a^2 x_l^4 x_2^2 x_3^2 + 16 a^2 x_l^3 x_2^3 x_3^2 \\
& + 16 a^2 x_l^3 x_2^2 x_3^3 - 3 a^2 x_l^2 x_2^4 x_3^2 - 6 a^2 x_l^2 x_2^3 x_3^3 - 3 a^2 x_l^2 x_2^2 x_3^4 \\
& - \frac{8}{27} a b^5 x_l^3 x_2 - \frac{8}{27} a b^5 x_l^3 x_3 + \frac{140}{27} a b^4 x_l^4 x_2 + \frac{140}{27} a b^4 x_l^4 x_3 \\
& - \frac{4}{9} a b^4 x_l^3 x_2^2 - \frac{4}{9} a b^4 x_l^3 x_3^2 - \frac{44}{3} a b^3 x_l^5 x_2 - \frac{44}{3} a b^3 x_l^5 x_3 \\
& - \frac{50}{9} a b^3 x_l^4 x_2^2 - \frac{50}{9} a b^3 x_l^4 x_3^2 + \frac{4}{3} a b^3 x_l^3 x_2^3 + \frac{4}{3} a b^3 x_l^3 x_3^3 \\
& + \frac{140}{9} a b^2 x_l^6 x_2 + \frac{140}{9} a b^2 x_l^6 x_3 + \frac{58}{3} a b^2 x_l^5 x_2^2 + \frac{58}{3} a b^2 x_l^5 x_3^2 \\
& - \frac{10}{3} a b^2 x_l^4 x_2^3 - \frac{10}{3} a b^2 x_l^4 x_3^3 - \frac{64}{9} a b x_l^7 x_2 - \frac{64}{9} a b x_l^7 x_3
\end{aligned}$$

$$\begin{aligned}
& -\frac{56}{3} a b x_l^6 x_2^2 - \frac{56}{3} a b x_l^6 x_3^2 + 2 a b x_l^5 x_2^3 + 2 a b x_l^5 x_3^3 + \frac{28}{3} a x_l^6 x_2^2 x_3 \\
& + \frac{28}{3} a x_l^6 x_2 x_3^2 - 4 a x_l^5 x_2^3 x_3 + 18 a x_l^5 x_2^2 x_3^2 - 4 a x_l^5 x_2 x_3^3 - 10 a x_l^4 x_2^3 x_3^2 \\
& - 10 a x_l^4 x_2^2 x_3^3 + \frac{16}{9} b^5 x_l^3 x_2 x_3 - \frac{100}{9} b^4 x_l^4 x_2 x_3 - \frac{8}{3} b^4 x_l^3 x_2^2 x_3 \\
& - \frac{8}{3} b^4 x_l^3 x_2 x_3^2 + \frac{224}{9} b^3 x_l^5 x_2 x_3 + \frac{40}{3} b^3 x_l^4 x_2^2 x_3 + \frac{40}{3} b^3 x_l^4 x_2 x_3^2 \\
& + 4 b^3 x_l^3 x_2^2 x_3^2 - \frac{224}{9} b^2 x_l^6 x_2 x_3 - 24 b^2 x_l^5 x_2^2 x_3 - 24 b^2 x_l^5 x_2 x_3^2 \\
& - 15 b^2 x_l^4 x_2^2 x_3^2 + \frac{32}{3} b x_l^7 x_2 x_3 + \frac{56}{3} b x_l^6 x_2^2 x_3 + \frac{56}{3} b x_l^6 x_2 x_3^2 \\
& + 18 b x_l^5 x_2^2 x_3^2 - \frac{16}{3} b^2 x_l^7 x_2^2 x_3 - \frac{16}{3} b^2 x_l^7 x_2 x_3^2 - 7 b^2 x_l^6 x_2^2 x_3^2 \\
& + 2 b x_l^8 x_2^2 x_3 + 2 b x_l^8 x_2 x_3^2 + 8 b x_l^7 x_2^2 x_3^2 - \frac{8}{9} a b^4 x_l^5 x_2 - \frac{8}{9} a b^4 x_l^5 x_3 \\
& + \frac{28}{9} a b^3 x_l^6 x_2 + \frac{28}{9} a b^3 x_l^6 x_3 + \frac{4}{3} a b^3 x_l^5 x_2^2 + \frac{4}{3} a b^3 x_l^5 x_3^2 \\
& - \frac{32}{9} a b^2 x_l^7 x_2 - \frac{32}{9} a b^2 x_l^7 x_3 - \frac{14}{3} a b^2 x_l^6 x_2^2 - \frac{14}{3} a b^2 x_l^6 x_3^2 \\
& + \frac{4}{3} a b x_l^8 x_2 + \frac{4}{3} a b x_l^8 x_3 + \frac{16}{3} a b x_l^7 x_2^2 + \frac{16}{3} a b x_l^7 x_3^2 - 14 a x_l^6 x_2^2 x_3^2 \\
& + 6 a x_l^5 x_2^3 x_3^2 + 6 a x_l^5 x_2^2 x_3^3 - \frac{20}{9} b^5 x_l^4 x_2 x_3 + \frac{88}{9} b^4 x_l^5 x_2 x_3 \\
& + \frac{10}{3} b^4 x_l^4 x_2^2 x_3 + \frac{10}{3} b^4 x_l^4 x_2 x_3^2 - \frac{140}{9} b^3 x_l^6 x_2 x_3 - \frac{40}{3} b^3 x_l^5 x_2^2 x_3 \\
& - \frac{40}{3} b^3 x_l^5 x_2 x_3^2 - 5 b^3 x_l^4 x_2^2 x_3^2 + \frac{32}{3} b^2 x_l^7 x_2 x_3 + \frac{56}{3} b^2 x_l^6 x_2^2 x_3 \\
& + \frac{56}{3} b^2 x_l^6 x_2 x_3^2 + 18 b^2 x_l^5 x_2^2 x_3^2 - \frac{8}{3} b x_l^8 x_2 x_3 - \frac{32}{3} b x_l^7 x_2^2 x_3 \\
& - \frac{32}{3} b x_l^7 x_2 x_3^2 - 21 b x_l^6 x_2^2 x_3^2 - \frac{16}{27} a^2 b^4 x_l^3 x_2 - \frac{16}{27} a^2 b^4 x_l^3 x_3 \\
& + \frac{154}{27} a^2 b^3 x_l^4 x_2 + \frac{154}{27} a^2 b^3 x_l^4 x_3 - \frac{8}{9} a^2 b^3 x_l^3 x_2^2 - \frac{8}{9} a^2 b^3 x_l^3 x_3^2 \\
& - \frac{100}{9} a^2 b^2 x_l^5 x_2 - \frac{100}{9} a^2 b^2 x_l^5 x_3 - \frac{37}{9} a^2 b^2 x_l^4 x_2^2 - \frac{37}{9} a^2 b^2 x_l^4 x_3^2 \\
& + \frac{8}{3} a^2 b^2 x_l^3 x_2^3 + \frac{8}{3} a^2 b^2 x_l^3 x_3^3 + 6 a^2 b x_l^6 x_2 + 6 a^2 b x_l^6 x_3 + 14 a^2 b x_l^5 x_2^2 \\
& + 14 a^2 b x_l^5 x_3^2 - \frac{20}{3} a^2 b x_l^4 x_2^3 + \frac{8}{9} b^5 x_l^5 x_2 x_3 - \frac{28}{9} b^4 x_l^6 x_2 x_3
\end{aligned}$$

$$\begin{aligned}
& -\frac{4}{3} b^4 x_l^5 x_2^2 x_3 - \frac{4}{3} b^4 x_l^5 x_2 x_3^2 + \frac{32}{9} b^3 x_l^7 x_2 x_3 + \frac{14}{3} b^3 x_l^6 x_2^2 x_3 \\
& + \frac{14}{3} b^3 x_l^6 x_2 x_3^2 + 2 b^3 x_l^5 x_2^2 x_3^2 - \frac{4}{3} b^2 x_l^8 x_2 x_3 - \frac{20}{3} a b^2 x_l^3 x_3 \\
& + \frac{140}{27} a b x_l^4 x_2 + \frac{140}{27} a b x_l^4 x_3 - \frac{44}{27} a^3 b x_l x_2 - \frac{44}{27} a^3 b x_l x_3 \\
& + \frac{16}{27} a^3 b x_2 x_3 + \frac{16}{27} a^3 x_l x_2 x_3 - \frac{16}{9} a^2 x_l^2 x_2 x_3 - \frac{8}{27} a b x_l^3 x_2 \\
& - \frac{8}{27} a b x_l^3 x_3 + \frac{16}{27} a x_l^3 x_2 x_3 + \frac{190}{9} a b^2 x_l^4 x_2 + \frac{190}{9} a b^2 x_l^4 x_3 \\
& - \frac{44}{3} a b x_l^5 x_2 - \frac{44}{3} a b x_l^5 x_3 + \frac{58}{9} a^2 b^2 x_l^2 x_2 + \frac{58}{9} a^2 b^2 x_l^2 x_3 \\
& - \frac{356}{27} a^2 b x_l^3 x_2 - \frac{356}{27} a^2 b x_l^3 x_3 + \frac{28}{9} a^2 b x_l^2 x_2^2 + \frac{28}{9} a^2 b x_l^2 x_3^2 \\
& + \frac{160}{27} a^2 x_l^3 x_2 x_3 - \frac{20}{3} a b^2 x_l^3 x_2 - \frac{4}{9} b^3 x_l^2 x_2 x_3 - \frac{7}{3} b^3 x_l^6 + \frac{8}{9} b^2 x_l^7 \\
& + 2 b^3 x_l^5 - \frac{7}{9} b^2 x_l^6 + \frac{2}{3} a^2 x_l^5 - \frac{5}{9} b^3 x_l^4 + \frac{2}{9} b^2 x_l^5 + \frac{4}{9} a^3 x_l^3 \\
& - 6 a^2 b x_l x_2^2 x_3^2 - 4 a^2 b x_l x_2 x_3^3 - \frac{16}{3} a b^3 x_l^2 x_2 x_3 + \frac{14}{9} a b^3 x_l x_2^2 x_3 \\
& + \frac{14}{9} a b^3 x_l x_2 x_3^2 + \frac{592}{27} a b^2 x_l^3 x_2 x_3 - 2 a b^2 x_l^2 x_2^2 x_3 - 2 a b^2 x_l^2 x_2 x_3^2 \\
& - \frac{4}{3} a b^2 x_l x_2^3 x_3 - 2 a b^2 x_l x_2^2 x_3^2 - \frac{4}{3} a b^2 x_l x_2 x_3^3 - \frac{760}{27} a b x_l^4 x_2 x_3 \\
& - 4 a b x_l^3 x_2^2 x_3 - 4 a b x_l^3 x_2 x_3^2 + 4 a b x_l^2 x_2^3 x_3 + 6 a b x_l^2 x_2^2 x_3^2 \\
& + 4 a b x_l^2 x_2 x_3^3 - \frac{56}{9} a^3 b x_l x_2 x_3 - \frac{20}{9} a^2 b^2 x_l x_2 x_3 + \frac{76}{27} a^2 b x_l x_2^2 x_3 \\
& + \frac{76}{27} a^2 b x_l x_2 x_3^2 + \frac{256}{27} a b x_l^3 x_2 x_3 + 14 a^2 b x_l^2 x_2^3 x_3 + 9 a^2 b x_l^2 x_2^2 x_3^2 \\
& + 14 a^2 b x_l^2 x_2 x_3^3 - \frac{8}{3} a^2 b x_l x_2^4 x_3 + \frac{8}{3} a^2 b x_l x_2^3 x_3^2 + \frac{8}{3} a^2 b x_l x_2^2 x_3^3 \\
& - \frac{8}{3} a^2 b x_l x_2 x_3^4 + \frac{8}{27} a b^5 x_l x_2 x_3 - \frac{32}{9} a b^4 x_l^2 x_2 x_3 + \frac{4}{9} a b^4 x_l x_2^2 x_3 \\
& + \frac{4}{9} a b^4 x_l x_2 x_3^2 + \frac{592}{27} a b^3 x_l^3 x_2 x_3 - 2 a b^3 x_l^2 x_2^2 x_3 - 2 a b^3 x_l^2 x_2 x_3^2 \\
& - \frac{4}{3} a b^3 x_l x_2^3 x_3 - 2 a b^3 x_l x_2^2 x_3^2 - \frac{4}{3} a b^3 x_l x_2 x_3^3 - \frac{400}{9} a b^2 x_l^4 x_2 x_3 \\
& - \frac{64}{9} a b^2 x_l^3 x_2^2 x_3 - \frac{64}{9} a b^2 x_l^3 x_2 x_3^2 + 8 a b^2 x_l^2 x_2^3 x_3 + 6 a b^2 x_l^2 x_2^2 x_3^2
\end{aligned}$$

$$\begin{aligned}
& + 8 a b^2 x_1^2 x_2 x_3^3 + 2 a b^2 x_1 x_2^3 x_3^2 + 2 a b^2 x_1 x_2^2 x_3^3 + 32 a b x_1^5 x_2 x_3 \\
& + \frac{70}{3} a b x_1^4 x_2^2 x_3 + \frac{70}{3} a b x_1^4 x_2 x_3^2 - \frac{40}{3} a b x_1^3 x_2^3 x_3 - 4 a b x_1^3 x_2^2 x_3^2 \\
& - \frac{40}{3} a b x_1^3 x_2 x_3^3 - 6 a b x_1^2 x_2^3 x_3^2 - 6 a b x_1^2 x_2^2 x_3^3 - 4 a^4 b x_1 x_2 x_3 \\
& - \frac{56}{9} a^3 b^2 x_1 x_2 x_3 + \frac{104}{9} a^3 b x_1^2 x_2 x_3 + \frac{50}{9} a^3 b x_1 x_2^2 x_3 + \frac{50}{9} a^3 b x_1 x_2 x_3^2 \\
& - \frac{20}{9} a^2 b^3 x_1 x_2 x_3 + \frac{2}{9} a^2 b^2 x_1^2 x_2 x_3 + \frac{76}{9} a^2 b^2 x_1 x_2^2 x_3 + \frac{76}{9} a^2 b^2 x_1 x_2 x_3^2 \\
& + \frac{32}{9} a^2 b x_1^3 x_2 x_3 - \frac{26}{3} a^2 b x_1^2 x_2^2 x_3 - \frac{26}{3} a^2 b x_1^2 x_2 x_3^2 - 4 a^2 b x_1 x_2^3 x_3 \\
& - \frac{4}{3} a b^4 x_1^2 x_2^2 x_3 - \frac{4}{3} a b^4 x_1^2 x_2 x_3^2 - \frac{760}{27} a b^3 x_1^4 x_2 x_3 - 4 a b^3 x_1^3 x_2^2 x_3 \\
& - 4 a b^3 x_1^3 x_2 x_3^2 + 4 a b^3 x_1^2 x_2^3 x_3 + 6 a b^3 x_1^2 x_2^2 x_3^2 + 4 a b^3 x_1^2 x_2 x_3^3 \\
& + 32 a b^2 x_1^5 x_2 x_3 + \frac{70}{3} a b^2 x_1^4 x_2^2 x_3 + \frac{70}{3} a b^2 x_1^4 x_2 x_3^2 - \frac{40}{3} a b^2 x_1^3 x_2^3 x_3 \\
& - 4 a b^2 x_1^3 x_2^2 x_3^2 - \frac{40}{3} a b^2 x_1^3 x_2 x_3^3 - 6 a b^2 x_1^2 x_2^3 x_3^2 - 6 a b^2 x_1^2 x_2^2 x_3^3 \\
& - \frac{112}{9} a b x_1^6 x_2 x_3 - \frac{82}{3} a b x_1^5 x_2^2 x_3 - \frac{82}{3} a b x_1^5 x_2 x_3^2 + \frac{40}{3} a b x_1^4 x_2^3 x_3 \\
& - 20 a b x_1^4 x_2^2 x_3^2 + \frac{40}{3} a b x_1^4 x_2 x_3^3 + 16 a b x_1^3 x_2^3 x_3^2 + 16 a b x_1^3 x_2^2 x_3^3 \\
& + \frac{16}{27} a^3 b^3 x_1 x_2 x_3 + \frac{64}{9} a^3 b^2 x_1^2 x_2 x_3 - \frac{8}{9} a^3 b^2 x_1 x_2^2 x_3 - \frac{8}{9} a^3 b^2 x_1 x_2 x_3^2 \\
& - \frac{40}{3} a^3 b x_1^3 x_2 x_3 + 2 a^3 b x_1^2 x_2^2 x_3 + 2 a^3 b x_1^2 x_2 x_3^2 - \frac{8}{3} a^3 b x_1 x_2^3 x_3 \\
& - 4 a^3 b x_1 x_2^2 x_3^2 - \frac{8}{3} a^3 b x_1 x_2 x_3^3 + \frac{8}{9} a^2 b^4 x_1 x_2 x_3 - \frac{4}{3} a^2 b^3 x_1^2 x_2 x_3 \\
& + \frac{76}{27} a^2 b^3 x_1 x_2^2 x_3 + \frac{76}{27} a^2 b^3 x_1 x_2 x_3^2 + \frac{32}{9} a^2 b^2 x_1^3 x_2 x_3 - \frac{26}{3} a^2 b^2 x_1^2 x_2^2 x_3 \\
& - \frac{26}{3} a^2 b^2 x_1^2 x_2 x_3^2 - 4 a^2 b^2 x_1 x_2^3 x_3 - 6 a^2 b^2 x_1 x_2^2 x_3^2 - 4 a^2 b^2 x_1 x_2 x_3^3 \\
& + \frac{16}{9} a^2 b x_1^3 x_2^2 x_3 + \frac{16}{9} a^2 b x_1^3 x_2 x_3^2 - \frac{8}{3} a b^3 x_1^3 x_2^3 x_3 - 4 a b^3 x_1^3 x_2^2 x_3^2 \\
& - \frac{8}{3} a b^3 x_1^3 x_2 x_3^3 - \frac{56}{9} a b^2 x_1^6 x_2 x_3 - \frac{44}{3} a b^2 x_1^5 x_2^2 x_3 - \frac{44}{3} a b^2 x_1^5 x_2 x_3^2 \\
& + \frac{20}{3} a b^2 x_1^4 x_2^3 x_3 + \frac{20}{3} a b^2 x_1^4 x_2 x_3^3 + 4 a b^2 x_1^3 x_2^3 x_3^2 + 4 a b^2 x_1^3 x_2^2 x_3^3
\end{aligned}$$

$$\begin{aligned}
& + \frac{28}{3} a b x_l^6 x_2^2 x_3 + \frac{28}{3} a b x_l^6 x_2 x_3^2 - 4 a b x_l^5 x_2^3 x_3 + 18 a b x_l^5 x_2^2 x_3^2 \\
& - 4 a b x_l^5 x_2 x_3^3 - 10 a b x_l^4 x_2^3 x_3^2 - 10 a b x_l^4 x_2^2 x_3^3 - \frac{16}{9} a^2 b^4 x_l^2 x_2 x_3 \\
& + \frac{16}{27} a^2 b^4 x_l x_2^2 x_3 + \frac{16}{27} a^2 b^4 x_l x_2 x_3^2 + \frac{160}{27} a^2 b^3 x_l^3 x_2 x_3 \\
& - \frac{32}{9} a^2 b^3 x_l^2 x_2^2 x_3 - \frac{32}{9} a^2 b^3 x_l^2 x_2 x_3^2 - \frac{40}{9} a^2 b^2 x_l^3 x_2^2 x_3 \\
& - \frac{40}{9} a^2 b^2 x_l^3 x_2 x_3^2 + 8 a^2 b^2 x_l^2 x_2^3 x_3 + 12 a^2 b^2 x_l^2 x_2^2 x_3^2 + 8 a^2 b^2 x_l^2 x_2 x_3^3 \\
& - \frac{4}{3} a^2 b^2 x_l x_2^4 x_3 - \frac{8}{3} a^2 b^2 x_l x_2^3 x_3^2 - \frac{8}{3} a^2 b^2 x_l x_2^2 x_3^3 - \frac{4}{3} a^2 b^2 x_l x_2 x_3^4 \\
& - \frac{16}{3} a^2 b x_l^5 x_2 x_3 + \frac{20}{3} a^2 b x_l^4 x_2^2 x_3 + \frac{20}{3} a^2 b x_l^4 x_2 x_3^2 - \frac{32}{3} a^2 b x_l^3 x_2^3 x_3 \\
& - \frac{32}{3} a^2 b x_l^3 x_2 x_3^3 + 2 a^2 b x_l^2 x_2^4 x_3 - 8 a^2 b x_l^2 x_2^3 x_3^2 - 8 a^2 b x_l^2 x_2^2 x_3^3 \\
& + 2 a^2 b x_l^2 x_2 x_3^4 + 2 a^2 b x_l x_2^4 x_3^2 + 4 a^2 b x_l x_2^3 x_3^3 + 2 a^2 b x_l x_2^2 x_3^4 \\
& - \frac{8}{9} a b^5 x_l^2 x_2 x_3 + \frac{256}{27} a b^4 x_l^3 x_2 x_3 + \frac{16}{27} a b^5 x_l^3 x_2 x_3 - \frac{160}{27} a b^4 x_l^4 x_2 x_3 \\
& + \frac{8}{9} a b^4 x_l^3 x_2^2 x_3 + \frac{8}{9} a b^4 x_l^3 x_2 x_3^2 + \frac{104}{9} a b^3 x_l^5 x_2 x_3 + \frac{40}{9} a b^3 x_l^4 x_2^2 x_3 \\
& + \frac{40}{9} a b^3 x_l^4 x_2 x_3^2 - \frac{4}{3} a^2 b x_l^2 x_2 x_3 + \frac{8}{9} a^2 b x_l x_2 x_3 - \frac{32}{9} a b^2 x_l^2 x_2 x_3 \\
& + \frac{4}{9} a b^2 x_l x_2^2 x_3 + \frac{4}{9} a b^2 x_l x_2 x_3^2 - \frac{4}{3} a b x_l^2 x_2^2 x_3 - \frac{4}{3} a b x_l^2 x_2 x_3^2 \\
& + \frac{8}{27} a b^2 x_l x_2 x_3 - \frac{8}{9} a b x_l^2 x_2 x_3 - \frac{320}{27} a b^3 x_l^4 + \frac{98}{9} a b^2 x_l^5 - \frac{28}{9} a b x_l^6 \\
& + \frac{28}{9} a x_l^6 x_2 + \frac{28}{9} a x_l^6 x_3 + \frac{40}{3} a^2 b^2 x_l^3 - \frac{199}{27} a^2 b x_l^4 + \frac{154}{27} a^2 x_l^4 x_2 \\
& + \frac{154}{27} a^2 x_l^4 x_3 - \frac{8}{9} a^2 x_l^3 x_2^2 - \frac{8}{9} a^2 x_l^3 x_3^2 + \frac{116}{27} a b^3 x_l^3 - \frac{100}{27} a b^2 x_l^4 \\
& + \frac{8}{9} a b x_l^5 - \frac{8}{9} a x_l^5 x_2 - \frac{8}{9} a x_l^5 x_3 - \frac{62}{27} a^3 b x_l^2 + \frac{16}{27} a^3 b x_2^2 \\
& + \frac{16}{27} a^3 b x_3^2 + \frac{44}{27} a^3 x_l^2 x_2 + \frac{44}{27} a^3 x_l^2 x_3 - \frac{16}{27} a^3 x_l x_2^2 - \frac{16}{27} a^3 x_l x_3^2 \\
& - \frac{16}{27} a^3 x_2^2 x_3 - \frac{16}{27} a^3 x_2 x_3^2 - \frac{4}{3} a^2 b^2 x_l^2 + \frac{16}{27} a^2 b x_l^3 - \frac{16}{27} a^2 x_l^3 x_2 \\
& - \frac{16}{27} a^2 x_l^3 x_3 - \frac{2}{9} a b^3 x_l^2 + \frac{4}{27} a b^2 x_l^3 - \frac{8}{9} a^2 b^2 x_l x_2 + \frac{16}{9} a^2 b x_l^2 x_2
\end{aligned}$$

$$\begin{aligned}
& -\frac{8}{27} a^2 b x_1 x_2^2 - \frac{4}{27} a b^3 x_1 x_2 + \frac{4}{9} a b^2 x_1^2 x_2 + \frac{16}{9} a^2 b x_1^2 x_3 \\
& -\frac{8}{27} a^2 b x_1 x_3^2 - \frac{4}{27} a b^3 x_1 x_3 + \frac{4}{9} a b^2 x_1^2 x_3 - \frac{8}{9} a^2 b^2 x_1 x_3 - \frac{8}{27} a^2 b x_2^2 x_3 \\
& -\frac{8}{27} a^2 b x_2 x_3^2 + \frac{4}{27} a^2 b^2 x_2 x_3 - \frac{8}{3} b^2 x_1^3 x_2 x_3^2 + \frac{88}{9} b x_1^5 x_2 x_3 \\
& + \frac{10}{3} b x_1^4 x_2^2 x_3 + \frac{10}{3} b x_1^4 x_2 x_3^2 + \frac{20}{9} a^4 b x_1 x_2 + \frac{20}{9} a^4 b x_1 x_3 \\
& -\frac{10}{9} a^4 b x_2 x_3 - 4 a^4 x_1 x_2 x_3 + \frac{38}{27} a^3 b^2 x_1 x_2 + \frac{38}{27} a^3 b^2 x_1 x_3 - \frac{4}{27} a^3 b^2 x_2 x_3 \\
& + \frac{38}{9} a^3 b x_1^2 x_2 + \frac{38}{9} a^3 b x_1^2 x_3 - \frac{22}{9} a^3 b x_1 x_2^2 - \frac{22}{9} a^3 b x_1 x_3^2 \\
& + \frac{8}{9} a^3 b x_2^2 x_3 + \frac{8}{9} a^3 b x_2 x_3^2 + \frac{64}{9} a^3 x_1^2 x_2 x_3 - \frac{8}{9} a^3 x_1 x_2^2 x_3 \\
& -\frac{8}{9} a^3 x_1 x_2 x_3^2 + \frac{2}{27} a^2 b^3 x_2 x_3 - \frac{16}{9} a^2 b^2 x_1 x_2^2 - \frac{16}{9} a^2 b^2 x_1 x_3^2 \\
& -\frac{2}{9} a^2 b^2 x_2^2 x_3 - \frac{2}{9} a^2 b^2 x_2 x_3^2 - \frac{32}{9} a^2 x_1^2 x_2^2 x_3 - \frac{32}{9} a^2 x_1^2 x_2 x_3^2 \\
& -\frac{2}{27} a b^4 x_1 x_2 - \frac{2}{27} a b^4 x_1 x_3 + \frac{22}{9} a b^3 x_1^2 x_2 + \frac{22}{9} a b^3 x_1^2 x_3 \\
& -\frac{2}{9} a b^3 x_1 x_2^2 - \frac{2}{9} a b^3 x_1 x_3^2 + \frac{2}{3} a b^2 x_1^2 x_2^2 + \frac{2}{3} a b^2 x_1^2 x_3^2 - \frac{4}{9} a b x_1^3 x_2^2 \\
& -\frac{4}{9} a b x_1^3 x_3^2 - \frac{160}{27} a x_1^4 x_2 x_3 + \frac{8}{9} a x_1^3 x_2^2 x_3 + \frac{8}{9} a x_1^3 x_2 x_3^2 \\
& + \frac{16}{9} b^2 x_1^3 x_2 x_3 - \frac{20}{9} b x_1^4 x_2 x_3 + \frac{16}{27} a^2 x_1 x_2^2 x_3 + \frac{16}{27} a^2 x_1 x_2 x_3^2 \\
& -\frac{1}{3} b^4 x_1^2 x_2^2 - \frac{1}{3} b^4 x_1^2 x_3^2 + \frac{40}{9} b^3 x_1^4 x_2 + \frac{40}{9} b^3 x_1^4 x_3 + \frac{4}{3} b^3 x_1^3 x_2^2 \\
& + \frac{4}{3} b^3 x_1^3 x_3^2 - \frac{40}{9} b^2 x_1^5 x_2 - \frac{40}{9} b^2 x_1^5 x_3 - \frac{5}{3} b^2 x_1^4 x_2^2 - \frac{5}{3} b^2 x_1^4 x_3^2 \\
& + \frac{14}{9} b x_1^6 x_2 + \frac{14}{9} b x_1^6 x_3 + \frac{2}{3} b x_1^5 x_2^2 + \frac{2}{3} b x_1^5 x_3^2 - \frac{28}{9} x_1^6 x_2 x_3 \\
& -\frac{4}{3} x_1^5 x_2^2 x_3 - \frac{4}{3} x_1^5 x_2 x_3^2 + 2 a^4 b^2 x_1 - \frac{61}{9} a^4 b x_1^2 - \frac{10}{9} a^4 b x_2^2 \\
& -\frac{10}{9} a^4 b x_3^2 + 2 a^4 x_2^2 x_3 + 2 a^4 x_2 x_3^2 + 2 a^3 b^3 x_1 - \frac{322}{27} a^3 b^2 x_1^2 \\
& -\frac{4}{27} a^3 b^2 x_2^2 - \frac{4}{27} a^3 b^2 x_3^2 + \frac{44}{3} a^3 b x_1^3 - \frac{64}{9} a^3 x_1^3 x_2 - \frac{64}{9} a^3 x_1^3 x_3 \\
& + \frac{10}{3} a^3 x_1^2 x_2^2 + \frac{10}{3} a^3 x_1^2 x_3^2 + \frac{2}{3} a^2 b^4 x_1 - \frac{62}{9} a^2 b^3 x_1^2 + \frac{2}{27} a^2 b^3 x_2^2
\end{aligned}$$

$$\begin{aligned}
& + \frac{2}{27} a^2 b^3 x_3^2 + \frac{2}{27} a b^5 x_l - \frac{14}{9} a b^4 x_l^2 + \frac{2}{9} b^4 x_l^2 x_2 + \frac{2}{9} b^4 x_l^2 x_3 \\
& - \frac{8}{9} b^3 x_l^3 x_2 - \frac{8}{9} b^3 x_l^3 x_3 + \frac{8}{9} x_l^5 x_2 x_3 + 2 a^4 b x_l - \frac{8}{9} a^4 x_l x_2 - \frac{8}{9} a^4 x_l x_3 \\
& + \frac{4}{9} a^4 x_2 x_3 + 2 a^3 b^2 x_l + \frac{2}{3} a^2 b^3 x_l + \frac{4}{27} a^2 b^2 x_2^2 + \frac{4}{27} a^2 b^2 x_3^2 \\
& + \frac{2}{27} a b^4 x_l + \frac{40}{9} b^4 x_l^4 x_3 + \frac{4}{3} b^4 x_l^3 x_2^2 + \frac{4}{3} b^4 x_l^3 x_3^2 - 8 b^3 x_l^5 x_2 \\
& - 8 b^3 x_l^5 x_3 - 5 b^3 x_l^4 x_2^2 - 5 b^3 x_l^4 x_3^2 + \frac{56}{9} b^2 x_l^6 x_2 + \frac{56}{9} b^2 x_l^6 x_3 \\
& + 6 b^2 x_l^5 x_2^2 + 6 b^2 x_l^5 x_3^2 - \frac{16}{9} b x_l^7 x_2 - \frac{16}{9} b x_l^7 x_3 - \frac{7}{3} b x_l^6 x_2^2 \\
& - \frac{7}{3} b x_l^6 x_3^2 + \frac{32}{9} x_l^7 x_2 x_3 + \frac{14}{3} x_l^6 x_2^2 x_3 + \frac{14}{3} x_l^6 x_2 x_3^2 + 2 x_l^5 x_2^2 x_3^2 \\
& - \frac{8}{9} a^4 b^2 x_l^2 + \frac{4}{9} a^4 b^2 x_2^2 + \frac{4}{9} a^4 b^2 x_3^2 + 4 a^4 b x_l^3 + 4 a^4 x_l^3 x_2 + 4 a^4 x_l^3 x_3 \\
& - 6 a^4 x_l^2 x_2^2 - 6 a^4 x_l^2 x_3^2 + 4 a^4 x_l x_2^3 + 4 a^4 x_l x_3^3 - 2 a^4 x_2^3 x_3 - 3 a^4 x_2^2 x_3^2 \\
& - 2 a^4 x_2 x_3^3 - \frac{62}{27} a^3 b^3 x_l^2 + \frac{16}{27} a^3 b^3 x_2^2 + \frac{16}{27} a^3 b^3 x_3^2 + \frac{44}{3} a^3 b^2 x_l^3 \\
& - \frac{188}{9} a^3 b x_l^4 - \frac{4}{3} a^3 b x_2^4 - \frac{4}{3} a^3 b x_3^4 + \frac{14}{3} a^3 x_l^4 x_2 + \frac{14}{3} a^3 x_l^4 x_3 \\
& + 6 a^3 x_l^3 x_2^2 + 6 a^3 x_l^3 x_3^2 - \frac{22}{3} a^3 x_l^2 x_2^3 - \frac{22}{3} a^3 x_l^2 x_3^3 + \frac{4}{3} a^3 x_l x_2^4 \\
& + \frac{4}{3} a^3 x_l x_3^4 + \frac{4}{3} a^3 x_2^4 x_3 + \frac{8}{3} a^3 x_2^3 x_3^2 + \frac{8}{3} a^3 x_2^2 x_3^3 + \frac{4}{3} a^3 x_2 x_3^4 \\
& - \frac{4}{3} a^2 b^4 x_l^2 + \frac{4}{27} a^2 b^4 x_2^2 + \frac{4}{27} a^2 b^4 x_3^2 + \frac{40}{3} a^2 b^3 x_l^3 - \frac{731}{27} a^2 b^2 x_l^4 \\
& - \frac{1}{3} a^2 b^2 x_2^4 - \frac{1}{3} a^2 b^2 x_3^4 + \frac{154}{9} a^2 b x_l^5 - \frac{100}{9} a^2 x_l^5 x_2 - \frac{100}{9} a^2 x_l^5 x_3 \\
& - \frac{37}{9} a^2 x_l^4 x_2^2 - \frac{37}{9} a^2 x_l^4 x_3^2 + \frac{8}{3} a^2 x_l^3 x_2^3 + \frac{8}{3} a^2 x_l^3 x_3^3 - \frac{2}{9} a b^5 x_l^2 \\
& + \frac{116}{27} a b^4 x_l^3 + \frac{4}{3} a x_l^5 x_2^2 + \frac{4}{3} a x_l^5 x_3^2 + \frac{2}{9} b^5 x_l^2 x_2 + \frac{2}{9} b^5 x_l^2 x_3 \\
& - \frac{16}{9} b^4 x_l^3 x_2 - \frac{16}{9} b^4 x_l^3 x_3 + a^6 - a^2 x_l^8 + \frac{2}{9} b^5 x_l^5 - \frac{7}{9} b^4 x_l^6 + \frac{8}{9} b^3 x_l^7 \\
& - \frac{1}{3} b^2 x_l^8 - 4 a^3 x_l^6 + \frac{8}{3} a^2 x_l^7 - \frac{5}{9} b^5 x_l^4 + 2 b^4 x_l^5 - 4 a^4 x_l^4 - a^4 x_2^4 - a^4 x_3^4 \\
& + \frac{22}{3} a^3 x_l^5 - \frac{7}{3} a^2 x_l^6 + \frac{4}{9} b^5 x_l^3 - \frac{5}{3} b^4 x_l^4 + 4 a^4 x_l^3 - \frac{34}{9} a^3 x_l^4
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{9} b^5 x_1^2 + \frac{4}{9} b^4 x_1^3 - \frac{2}{3} a^4 b^2 - \frac{8}{9} a^4 x_1^2 + \frac{4}{9} a^4 x_2^2 + \frac{4}{9} a^4 x_3^2 - \frac{8}{27} a^3 b^3 \\
& - \frac{1}{27} a^2 b^4
\end{aligned}$$

After eliminating x4 we have one branch with 4 polynomials.

[g1, x1-x2, a], [g2, a, x1-x3], [g3, g5], [g5]

$$\begin{aligned}
& \text{> indets(g\_1);} \\
& \qquad \qquad \qquad \{a, b, x_1, x_2\} \qquad \qquad \qquad (7.4)
\end{aligned}$$

No x3 in the first polynomial! Bad.

$$\begin{aligned}
& \text{> indets(g\_2);} \\
& \qquad \qquad \qquad \{a, b, x_1, x_3\} \qquad \qquad \qquad (7.5)
\end{aligned}$$

g2 and x1-x3 will disappear from the list in the next level.

$$\begin{aligned}
& \text{> indets(g\_3);} \\
& \qquad \qquad \qquad \{a, b, x_1, x_2, x_3\} \qquad \qquad \qquad (7.6)
\end{aligned}$$

$$\begin{aligned}
& \text{> indets(g\_4);} \\
& \qquad \qquad \qquad \{a, b, x_1, x_2, x_3\} \qquad \qquad \qquad (7.7)
\end{aligned}$$

$$\begin{aligned}
& \text{> indets(g\_5);} \\
& \qquad \qquad \qquad \{a, b, x_1, x_2, x_3\} \qquad \qquad \qquad (7.8)
\end{aligned}$$

These too. So we get the following for the next step after eliminating x3.

One branch with three polynomials.

[g1, x1-x2, a], [g1, x1-x2, a], [g1, x1-x2, a]

We get a new branch adding to the queue which has one polynomial

[g1]

The algorithm is still in the former branch it computes

res(g1, x1-x2, x2) we get g7

$$\begin{aligned}
& \text{> g\_6 := -2/3*x\_1*b+x\_1^2+a+1/3*b-2/3*x\_1;} \\
& \qquad \qquad \qquad g_6 := -\frac{2}{3} x_1 b + x_1^2 + a + \frac{1}{3} b - \frac{2}{3} x_1 \qquad \qquad \qquad (7.9)
\end{aligned}$$

$$\text{> resultant(g\_1, x\_1-x\_2, x\_2);}$$



$$-\frac{2}{3}x_1b + x_1^2 + a + \frac{1}{3}b - \frac{2}{3}x_1 \quad (7.10)$$

As we said.

And it sends down 'a'.

$x_1 - x_2$  and  $g_1$  gives  $g_7$  again. Then it adds a new branch at the end of the queue.

$[x_1 - x_2]$

It sends down  $a$ . Same thing and another new branch at the end of the queue.

$[a]$

Same things happen again. And it finishes this old branch. And goes to the added branches, the first one is

$[g_1]$

This polynomial has  $x_2$ , so we hit line 11 of Algorithm 1 in the paper, this is why we received  $[0]$ .

Using ResChainBranching does not help us to not hit line 11 as we still encounter a branch as  $[g_1]$  in level of  $x_2$ .

End of the file.