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Prime Conjecture

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Prime Conjecture

Any odd number can be expressed as a sum of two primes minus a third prime, not including the trivial solution $p = p + q - q$.

For example,

$1 = 3 + 5 - 7 = 5 + 7 - 11 = 7 + 11 - 17 = 11 + 13 - 23 = \dots$

$3 = 5 + 11 - 13 = 7 + 19 - 23 = 17 + 23 - 37 = \dots$

$5 = 3 + 13 - 11 = \dots$

$7 = 11 + 13 - 17 = \dots$

$9 = 5 + 7 - 3 = \dots$

$11 = 7 + 17 - 13 = \dots$

a) Is this conjecture equivalent to Coldbach's conjecture? The conjecture is that any odd prime ≥ 9 can be expressed as a sum of three primes. This was solved by Vinogradov in 1937 for any odd number greater then $3^{3^{15}}$.

b) The number of times each odd number can be expressed as a sum of two primes minus a third prime are called prime conjecture numbers. None of them is known!

c)Write a computer program to check this conjecture for as many positive numbers as possible.

There are infinitely many numbers that cannot be expressed as the absolute difference between a cube and a square. These are called bad numbers(!)

For example, F.Smarandache has conjuctured [1] that 5, 6, 7, 10, 13 and 14 are bad numbers. However, 1, 2, 3, 4, 8, 9, 11, 12, and 15 are not as

$$1 = |2^3 - 3^2|,\ 2 = |3^3 - 5^2|,\ 3 = |1^3 - 2^2|,\ 4 = |5^3 - 11^2|,\ 8 = |1^3 - 3^2|,$$

$$9 = |6^3 - 15^2|,\ 11 = |3^3 - 4^2|,\ 12 = |13^3 - 47^2|,\ 15 = |4^3 - 7^2|.$$

a) Write a computer program to determine as many bad numbers as possible. Find an ordered array of a's such that $a = |x^3 - y^2|$, for x and y integers ≥ 1.

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [See also the Arizona State Special Collections, Tempe, AZ., USA].