Draft version December 13, 2016
Preprint typeset using LaTeX style AASTeX6 v. 1.0

# MODELING THE SPATIAL DISTRIBUTION OF FRAGMENTS FORMED FROM TIDALLY DISRUPTED STARS

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## ABSTRACT

Roughly once every $10^4$ years, a star passes close enough to the supermassive black hole Sgr A* at the center of the Milky Way to be pulled apart by the black holes tidal forces. The star is then spaghettified into a long stream of mass, with approximately one half being bound to Sgr A* and the other half unbound. Hydrodynamical simulations of this process have revealed that within this stream, the local self-gravity dominates the tidal field of Sgr A*. This residual self-gravity allows for planetary-mass fragments to form along the stream that are then shot out into the galaxy at velocities determined by a spread of binding energies. We develop a Monte Carlo code in Python that models and plots the evolving position of these fragments for a variety of initial conditions that are likely realized in nature. This code utilizes an n-body integrator based in `Mathematica` to differentially solve for the position, velocity, and acceleration of each fragment at every time step. From the produced data we determine the probability distribution of bound and unbound fragments, along with a possible fraction of fragments end up within a 8 kpc shell around the galactic center. This enables the calculation of the distance at which the nearest fragment to our sun could potentially lie, which turns out to be approximately 200 parsecs.

*Keywords:* black hole physics – gravitational – hydrodynamics – n-body integration

## 1. INTRODUCTION

Stars orbiting the supermassive black hole at the center of our galaxy have the potential to pass too close and be disrupted by the black hole's overwhelming gravitational force. This tidal disruption event (TDE) is dependent on the star passing within a distance known as the tidal radius, determined by the mass of the black hole ($M_{BH}$) and the stellar mass and radius ($M_\star$, $R_\star$),

$$r_t \simeq (R_\star/R_\odot) \cdot \left(\frac{M_{BH}/10^6 M_\odot}{M_\star/M_\odot}\right)^{\frac{1}{3}} \quad (1)$$

at which the black hole's gravity overpowers the star's self-gravity (Rees 1988). A full or partially disrupted star is characterized by the impact parameter $\beta \equiv \frac{r_t}{r_p}$ (where $r_p$ is the distance to the pericenter), and results in a stream of debris whose volume increases linearly with time.

Hydrodynamical simulations of this process have demonstrated that the local self-gravity of the tidal stream dominates the tidal gravity of the hole (Coughlin & Nixon 2015). This results in the formation of fragments along the stream that are then launched out into the galaxy with a range of binding energies. This fragmentation process poses various questions regarding the evolution of these objects. In this paper, we seek to answer: (1) What is the final spatial distribution of the fragments produced from TDEs originating at the galactic center, and (2) how near is the closest fragment to our sun? Some fragments may move fast enough to escape the galaxy entirely, with the other extreme being fragments that remain closely bound to Sgr A*. There may also be fragments with large elliptical orbits about the galaxy, allowing them to travel far enough to be deposited near our Sun.

Our simulation of these fragments' motion consists of an initialization package written in Python and an N-body integrator based in `Mathematica`. In Section 2, we present an analytic analysis of the environmental assumptions underlying our simulation, and in Section 3 describe in more detail the construction and steps taken with our Python/`Mathematica` code. Section 4 presents the results of our simulation and the analysis. We conclude in Section 5 with a discussion of our results' implications and additional questions to be posed regarding fragment observability and the existence of fragments

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produced by TDEs in nearby galaxies.

## 2. FRAGMENTATION

For the purposes of our simulation, we work under the assumption that fragments begin to form once the disrupted stream cools to temperatures low enough for hydrogen recombination, $T_f = 5 \times 10^3$ K. The initial temperature $T_i$ at the core of the star can be calculated with the stellar mass and radius, using the microscopic ideal gas law $P = n_i k_b T_i$ where $n_i = \frac{\rho_\star}{\mu m_p}$ is the initial number density calculated from stellar density $\rho_\star$, the mean molecular weight of hydrogen $\mu = 0.5$, and the mass of a proton $m_p$:

$$\begin{aligned} T_i &= \frac{P}{n_i k_b} \\ &= \left[\frac{1}{4\pi R_\star^2}\left(\frac{GM_\star^2}{2R_\star}\right)\right]\left(\frac{\left(\frac{4}{3}\pi R_\star^3\right)\mu m_p}{M_\star k_b}\right) \end{aligned}$$

Due to the dynamics of the stream's expansion, the change in volume is proportional to $T^{\frac{3}{2}}$. Thus we calculate an expansion factor $\alpha = \left(\frac{T_f}{T_i}\right)^{\frac{3}{2}}$ to determine the final gas density when fragmentation occurs, given a value $\beta$ for our tidal disruption event

$$n_f = \frac{0.5\ n_i\ \mathcal{C}(\beta)}{\alpha} \tag{2}$$

where

$$\mathcal{C}(\beta) \equiv \begin{cases} \exp\left[\frac{3.1647 - 6.3777\beta + 3.1797\beta^2}{1 - 3.4137\beta + 2.4616\beta^2}\right],\ 0.5 \leq \beta \leq 0.9 \\ 1,\ \beta > 0.9 \end{cases}$$

describes the fraction of stellar mass removed by the disruption (Guillochon & Ramirez-Ruiz 2015). The mass of each fragment is simply the Jean's mass:

$$M_{frag} = \frac{\pi}{6}\frac{c_s^3}{G^{3/2}\rho^{1/2}} \tag{3}$$

$$= 1.74 \times 10^{-8} M_J\ c_s^3\ n_f^{-\frac{1}{2}} \tag{4}$$

where $c_s = \sqrt{\frac{\gamma k_b T_f}{\mu m_p}}$ is the sound speed, with $\gamma = \frac{5}{3}$ for a gas-pressure dominated polytropic fluid, and $\rho = \rho_*/\alpha$.

The number of fragments can then be calculated by dividing the fraction of stellar mass removed in the disruption ($M = M_\star \cdot \mathcal{C}(\beta)$ given a specific $\beta$ value) by the mass of the fragment as determined in Equations (3-4):

$$N_{frag} = \frac{0.5\ M_\star\ \mathcal{C}(\beta)}{M_{frag}} \tag{5}$$

Given the initial position and velocity vectors for a fragment (the determination process of this is described in Section 3), we solve for the evolution of a fragment's position $\vec{r}(t)$ through the second order differential force equation $m_{frag} \cdot \vec{r}''(t) = \sum_{i=1}^{N} F_i$ at each time step $0 \leq t \leq t_{max}$. The forces that a fragment experiences include the gravitational attraction of Sgr A* and forces derived from the gravitational potential of the Milky Way, as described in Kenyon et al. (2014):

$$F_i = (F_{BH})_i + (F_b)_i + (F_c)_i + (F_d)_i + (F_h)_i \tag{6}$$

where $i = 1, 2, 3$ indicates the $x$, $y$, and $z$ component of the force, and

$$(F_{BH})_i = \frac{-GM_{BH}\vec{r}_i}{r^3} \tag{7}$$

$$(F_b)_i = \frac{-GM_b\vec{r}_i}{r^2(r_b + r)} \tag{8}$$

$$(F_c)_i = \frac{-2GM_{BH}\vec{r}_i}{\max(r_c, r) \cdot r^2} \tag{9}$$

$$(F_d)_i = \frac{-GM_d\vec{r}_i}{(x^2 + y^2 + [a_d + z^2 + b_d^2]^2)^{1.5}} \tag{10}$$

$$(F_h)_i = -GM_h\vec{r}_i \cdot \left(\frac{\ln(1 + \frac{r}{r_h})}{r^3} - \frac{1}{r^2(r + r_h)}\right) \tag{11}$$

are the forces due to Sgr A*, the galaxy bulge, cluster, disk, and halo (respectively). The necessary parameters are set as defined in Kenyon et. al (2014): for the bulge, disk, and halo, $M_b = 3.76 \times 10^9 M_\odot$, $M_d = 6 \times 10^{10} M_\odot$, and $M_h = 10^{12} M_\odot$. The radius of the halo and bulge are $r_h = 20$ kpc and $r_b = 0.1$ kpc. The parameters $a_d = 2.75$ kpc and $b_d = 0.3$ kpc are set such that the disk potential matches a circular velocity of 235 km s$^{-1}$ at the position of the sun.

## 3. METHODS

Our simulation is first initialized through a package written in Python, which inputs the user-driven variables of number of stars disrupted. The main "TDESim" initializing class defined within the package randomly draws the necessary parameters that define the star being disrupted (e.g. stellar mass, radius, tidal radius), and the disruption itself ($\beta$, the number of fragments produced, the specific binding energy spread). It then calculates, using these parameters, an initial position and velocity vector for each fragment . These values are written into a JSON file that is uploaded into `Mathematica`, and used as starting positions for a integrator written within a `Mathematica` notebook. This integrator outputs as solutions for each fragment an interpolation function describing the evolution of $x$, $y$, and $z$ positions over the integrated time.

### 3.1. *Python Initialization*

The mass of Sgr A* is initialized as $M_{BH} = 4 \times 10^6 M_\odot$. For each star, the stellar mass is randomly drawn over the interval $[0.1 M_\odot, 100 M_\odot]$ using an inverted cumulative distribution function derived from Salpeter's initial mass function. The stellar radius is calculated through a mass-radius power law $R_\star \propto M_\star^{0.8}$, with the tidal radius determined from $M_{BH}$ and the stellar mass and radius.

To calculate the number of fragments produced in a given simulated TDE, $\beta$ is drawn over the interval $[0.5, 2.5]$ from the appropriate distribution. The mass of the fragment is then calculated as a function of $\beta$ and used to determine the number of fragments given the stellar mass.

We then set the position vector $\vec{r_\star}$ of the star using random sphere point picking at a distance $r_p$ away from the galactic center. $r_p$ signifies the periapse of the star's orbit and is determined by the relation $\beta = \frac{r_t}{r_p}$. To determine the position vectors of fragments produced in each simulated TDE, we take the same directional vector $\hat{r_\star}$ of the disrupted star, multiply this unit vector by the magnitude of the stellar radius, and partition it evenly based on the number of fragments being produced.

The direction of the each fragments' velocity vector is randomly determined on a plane perpendicular to $\vec{r_\star}$ using an rotation angle $\phi$ randomly drawn from a uniform distribution over the interval $[0, 2\pi)$. The magnitude of each velocity is determined by the spread of binding energy of the fragments. This binding energy spread is calculated using our drawn $\beta$ to interpolate a proper energy distribution associated with this value. Given the number of fragments, the binding energy of each fragment is calculated by partitioning the interval [0,1] into $N_{frag}$ points and evaluating our interpolated energy distribution at each point. This binding energy is then multiplied by an energy scale, and the scaled energy is used to calculate total velocity of the fragment,

$$v_{frag} = v_\infty + v_h = \sqrt{2E} + \frac{2GM_{BH}}{r_t} \tag{12}$$

To transfer all this data describing the initial position and velocity vectors of each fragment produced in each simulated TDE, the initialization code writes it into a JSON file. The JSON file is formatted as a nested list, with the elements being dictionaries associated with each star. Within each dictionary are the keys `x`, `y`, `z`, `vx`, `vy`, `vz`; the values associated with these keys are lists containing the x/y/z/vx/vy/vz values of each fragment produced in the indexed star's disruption.

### 3.2. *Mathematica-Based N-body Integrator*

`Mathematica` possesses a powerful capacity for integration and is unparalleled by open-source but numerically limited n-body integrators based in Python. Namely, it can calculate arbitrary order solutions which allow for accurate resolution on a range of scales, i.e. solutions that model both short-range distances when the force of the black hole overpowers all other contributions, and long-range distances when the forces of the disk and halo dominate. For this reason, we were compelled to utilize this program as our main machinery in integrating, as our simulation consists of thousands of particles experiencing such complicated forces. We define necessary constants in cgs units (e.g. 1 $M_\odot$, 1 year, 1 parsec) and the forces that each particle is experiencing due to the black hole, the galactic bulge, cluster, disk, and halo. The JSON data file is then imported, with the number of stars extracted from the length of the data file, and each $N_{frag}$ varying based on the index of the star.

The integrator uses the `Do` iteration within `Mathematica` to iterate over each index $1 \leq i \leq N_{stars}$. The number of fragments is determined from the $i^{th}$ element of our data set (which is a dictionary containing the initial values associated with each fragment), and serves as the range of our second iteration. The initial position and velocity components are then extracted utilizing this indexed dictionary, and are extracted for all fragments by iterating over $1 \leq j \leq N_{frag}$.

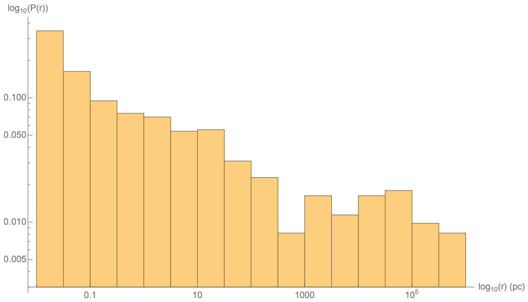
Once the inital values for each fragment are extracted, the integrator uses `NDSolve` to find a solution for the fragment's $F = ma$ equations with each time-step. `NDSolve` is parameterized by a maximum integration time ($10^{10}$ years), a maximum number of steps ($10^4$), and an "Event Method" which stops the integrator if a negative radial velocity is calculated. This is to avoid the repeated calculation of a bounded fragment's orbit. Thus, we calculate a fragment's position up to the time at which the fragment is the farthest away from the galactic center.

Our solutions $x(t)$, $y(t)$, and $z(t)$ are in the form interpolated functions, with varying domains of $t$. These solutions are organized within a nested list, in which a solution is indexed by the star and the fragment. Similarly, the maximum time for each fragments interpolated solution is stored within a nested list. Remark that for unbounded fragments, $t_{max} = 10^{10}$ years, whereas for bounded fragments, $t_{max} < 10^{10}$ years. We use this distinction to separate the unbounded from the bounded fragments into nested lists objects. Data is visualized via `Histogram` and `Plot` methods.

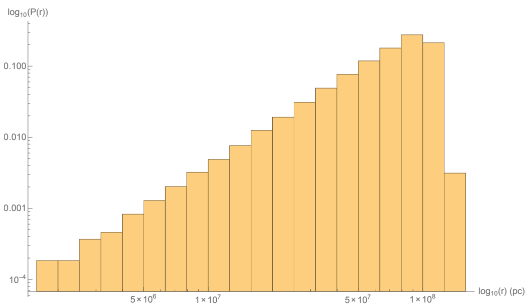
## 4. RESULTS

A set of 50 tidal disruptions were simulated, with the number of fragments per disruption determined from the randomly drawn stellar mass and beta through the method described in Section 2. Bound and unbound fragments were separated and plotted in a combined histogram and two separate histograms. For the bound fragments, the histogram binned the length of the fragments' apoapsis, while the histogram of unbound fragments binned the distance of the fragment from the galactic center at time $t_{\rm max} = 10^{10}$ years.

There were a total of 613 fragments characterized as bound and 10,860 characterized as unbound. Approximately 50.9% of the bound fragments had an apoap-




(a) Histogram of bound fragments




(b) Histogram of unbound fragments

**Figure 1**: Histogram plots of bound and unbound fragments. The $\log_{10}$ of $r$ defines each bin. For the bound fragments, $r$ signifies the fragments apoapsis. For the unbound fragments, $r$ signifies the final distance the fragment has travelled. Each bin $r$ has a height equal to $\log_{10}(P(r))$, where $P(r) = \frac{\text{Count}(r)}{N}$ with $N$ representing the total number of bound [unbound] fragments.

sis within 0.1pc. Furthermore, approximately 88.9% of bound fragments had an apoapsis less than or equal to 100 pc. Thus, the vast majority of bound fragments end up being very closely bound to the black hole. A sizable drop in fragment count occurred past the $10^3$ parsecs, with only 38 total fragments possessing an apoapsis within the range $10^3 - 10^4$ pc (1-10 kpc). However, restricting to the $10^3$ pc - $10^6$ pc range, we find a small bump in fragment count: 3.425% of fragments having an apoapsis in the range $10^4 - 10^5$ pc, while 2.773% and 1.794% lie within the range $10^3 - 10^4$ pc and $10^5 - 10^6$ pc respectively.

For the unbound fragments, the probability count of each bin increases in a practically logarithmic fashion. Thus, the majority of the fragments (77.514%) end up at a distance $10^7 - 10^8$ pc from the galactic center. The maximum distance traveled by an unbound fragment is $1.334 \times 10^8$ pc, with 21.63% of unbound fragments travelling a distance within the range $10^8 - 10^9$ pc from the galactic center. Only 93 unbound fragments (0.856 %) traveled a distance less than $10^7$ pc from the galactic center.

To estimate a minimal distance at which a fragment might be deposited near our sun, we must first have a sense of how many bound fragments have been produced by tidal disruption throughout Hubble time, $N_{\text{tot}}$. We can visualize a certain fraction $F$ of these bound fragments having an apoapsis $r_f$ in the range 7 kpc to 9 kpc, i.e. a fraction of bound fragments being deposited within a spherical shell of $\delta = 1$ kpc at a distance 8 kpc from the galactic center. If we position ourselves at the sun and denote a "surveying radius" $r$ in parsecs, we can calculate the amount of fragments deposited within our surveying sphere by taking the number of fragments within our 1 kpc thick spherical shell, $N_f = N_{\text{tot}} \cdot F$, and multiplying it by the ratio of the volume of our surveying sphere and the volume of the spherical shell:

$$
\begin{aligned}
f(r) &= N_f \frac{\frac{4}{3}\pi r^3}{V_\delta} \\
&= (N_{\text{tot}} \cdot F) \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(9000)^3 - \frac{4}{3}\pi(7000)^3} \\
&= (N_{\text{tot}} \cdot F) \frac{r^3}{3.86 \times 10^{11}}
\end{aligned}
$$

Thus, by setting $f(r) = 1$ in the above equation, we can solve it for the minimal distance $r$:

$$1 = (N_{\text{tot}} \cdot F) \frac{r^3}{3.86 \times 10^{11}}$$

From our data regarding bound fragments, we can calculate the approximate fraction bound of fragments produced by a tidal disruption that are deposited with a spherical shell of thickness $\delta = 10^3$ pc at a distance $8 \times 10^3$ pc from the galactic center:

$$
\begin{aligned}
F &= \frac{\#\text{ of bound fragments, } 7000 < r_f < 9000}{\#\text{ of bound fragments}} \\
&= 3.26 \times 10^{-3}
\end{aligned}
$$

The data also provides an approximate number of bound fragments produced per tidal disruption event,

$$
\begin{aligned}
n_{\text{bound}} &= \frac{\#\text{ of bound fragments in simulation}}{\#\text{ of stars disrupted in simulation}} \\
&= 12.26
\end{aligned}
$$

Assuming that tidal disruption events occur at a rate $\Gamma = 10^{-4}\ \text{yr}^{-1}$, we surmise the total number of bound fragments produced from every tidal disruption that occurs in $10^{10}$ years:

$$N_{\text{tot}} = n_{\text{bound}}\, \Gamma\, t_{\text{max}} = 1.226 \times 10^7$$

The distance in parsecs of the nearest fragment to our sun is the solution to the equation

$$
\begin{aligned}
1 &= (N_{\text{tot}} \cdot F) \frac{r^3}{3.86 \times 10^{11}} \\
&= \frac{4 \times 10^4}{3.86 \times 10^{11}} r^3 \\
&= (1.04 \times 10^{-7})\, r
\end{aligned}
$$

We find a solution to the above with $r = 212.9$ parsecs.

## 5. DISCUSSION

### 5.1. *Observability*

Given the existence of fragments produced in tidal disruption events, along with their potential proximity to our solar system, our work motivates questions regarding their observability. Namely, is it possible to observe one of these fragments considering their material composition and evolution and our own limited observational instruments? To answer this question requires additional insights into the process of cooling that each fragment inevitable undergoes as it is ejected out of the galactic center.

The fragments produced in a tidal disruption event can be broadly thought of as Jupiter-sized masses comprised pure hydrogen and helium, with the existence of other elements depending on where exactly in the stream the fragment forms. It has no core and thus no source of internal energy, and will be hottest right at the onset of its collapse. We propose that given their probable material composition and cold temperature, these fragments would look somewhat like extremely cold brown dwarfs. Additional insight into the evolution of the material composition of tidally disrupted stellar fragments could be gained from utilizing stellar evolution simulation codes, such as MESA, to simulate this process more in depth.

For the purpose of this discussion, we can roughly describe the cooling curve of the fragment, with the luminosity proportional to the mass of the fragment $M$ and $\frac{1}{t}$ (Marleau & Cumming 2014):

$$L = 7.85 \times 10^{-6} L_{\odot} \frac{(M/3\ \mathrm{M_J})^{2.641}}{(t/10\ \mathrm{Myr})^{1.297}}$$

From this, we derive the luminosity-time dependency for a 1.5 Jupiter-mass fragment:

$$L = 7.85 \times 10^{-6} L_{\odot} \frac{(1.5/3)^{2.641}}{(t/10\ \mathrm{Myr})^{1.297}}$$
$$= \frac{1.26 \times 10^{-6}}{(t/10\ \mathrm{Myr})^{1.297}} L_{\odot}$$

An obvious result is that the older the fragment is, the less luminous it will be. Considering our oldest fragments, which will have existed no longer than Hubble time ($10^4$ Myr), the luminosity of such objects would be around $1.62 \times 10^{-10}$ $L_{\odot}$, which corresponds to an absolute magnitude of 29.25. At 212.9 parsecs, an object with this magnitude would have an apparent magnitude of 35.88. This apparent magnitude is on the order of the the faintest objects observed optically by the European Extremely Large Telescope, meaning that with our most sensitive telescopes, a stellar fragment could possibly be detected optically. More certainly would such objects be detectable in the infrared spectrum, having cooled down significantly (1,000 - 10,000 K) as it traveled through the galaxy. An additional technique that could be used to identify these objects is microlensing, with the fragment serving as the lens to some bright background source (e.g. a star). The characteristic angle for gravitational lensing scales as $\theta_E \sim \sqrt{M_{frag}}$, and given that most fragments move at speeds on the order of 1000 km s$^{-1}$, a microlensing event can be transient enough for the apparent change in the source's brightness to be detected.

### 5.2. *Intergalactic TDE Fragments*

Our simulations have demonstrated that a majority of fragments produced by tidal disruptions in our galaxy are unbound to the black hole and traveling at relativistic speeds - the fastest of these being shot out at velocities on the order of $1 \times 10^7$ m s$^{-1}$! Fragments that are bound are more likely to possess tightly bound orbits around Sgr A*, with only 0.326 % of fragments ever traveling within a spherical shell of thickness $10^3$ pc at a distance $8 \times 10^3$ pc from the galactic center. It is the frequency of tidal disruption events in our galaxy that allow for large enough fragment production, such that fragments are adequately deposited in this shell and we can derive the nearest fragment to be 212.9 parsecs from our sun.

However, the sheer magnitude of unbound hypervelocity fragments produced by TDEs in our galaxy, all of which definitively escape our galaxy, pose the question: to what extent is the Milky Way populated by stellar fragments produced by TDEs originating in other galaxies? Consider the Virgo cluster, a cluster of roughly 2000 galaxies lying approximately $2 \times 10^7$ parsecs away from the Milky Way. As our data demonstrated, we could possibly conjecture that over 70% of the unbound fragments produced by tidal disruption events originating from galaxies within the virgo cluster travel the necessary distance to reach our own galaxy. Singular galaxies that lie a similar distance from our own include NGC 1300 ($1.87 \times 10^7$ pc) and the Tadpole galaxy ($1.29 \times 10^8$ pc).

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