ON THE STATISTICAL PERFORMANCE OF MUSIC FOR DISTRIBUTED SOURCES

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ABSTRACT

This paper addresses the statistical behaviour of the MUSIC method for DoA estimation, in a scenario where each source signal direct path is disturbed by a clutter spreading in an angular neighborhood around the source DoA. In this scenario, it is well-known that subspace methods performance suffers from an additional clutter subspace, which breaks the orthogonality between the source steering vectors and noise subspace. To perform a statistical analysis of the MUSIC DoA estimates, we consider an asymptotic regime in which both the number of sensors and the sample size tend to infinity at the same rate, and rely on classical random matrix theory results. We establish the consistency of the MUSIC estimates and provide numerical results illustrating their performance in this non standard scenario.

***Index Terms*—** DoA estimation, MUSIC, Distributed Sources, Random Matrix Theory

1. INTRODUCTION

The estimation of the Direction of Arrival (DoA) of source signals using an array of sensors is a fundamental topic in signal processing, having applications in various fields such as radar processing, wireless communications or seismology. Among the high resolution techniques offering a reasonable computation cost, the subspace methods (e.g. MUSIC, ESPRIT) are widely used. The statistical performance of these methods, in terms of Mean Square Error (MSE), asymptotic normality or resolution probability, have been extensively studied (see e.g. [1] and the references therein), usually in the large sample size regime. Moreover, the standard analyses are mostly carried out by considering Line of Sight (LOS) plane waves scenarios, involving a single direct path between each source and the sensor array (usually referred to as *point source model*).

Nevertheless, applications such as wireless communications or ground/sea radar detection may involve additional multipath propagation due to local scattering, leading to non standard models in which a clutter disturbance spread around each source DoA (usually referred to as *distributed source model*).

Let us consider K narrowband and far-field source signals with DoA $\theta_1, \ldots, \theta_K$ impinging on a uniform linear array of M sensors (with each sensor separated by half the wavelength). Assuming rich local scattering around each source DoA, as well as deterministic constant modulus source signals (see e.g. [2], [3]), the received signal can be modeled as a temporally uncorrelated Gaussian M-dimensional time series $(\mathbf{y}_n)$ given by

$$\mathbf{y}_n = \mathbf{R}^{1/2}\mathbf{x}_n, \tag{1}$$

with $\mathbf{x}_n \sim \mathcal{N}_{\mathbb{C}^M}(\mathbf{0}, \mathbf{I})$ and [1]

$$\mathbf{R} = \sum_{k=1}^{K} \left(\frac{\alpha_k \gamma_k}{1+\gamma_k} \frac{\mathbf{a}(\theta_k)\mathbf{a}(\theta_k)^*}{M} + \frac{\alpha_k}{1+\gamma_k}\mathbf{T}_k \right) + \sigma^2 \mathbf{I}, \tag{2}$$

and where

- $\mathbf{a}(\theta) = [1, \mathrm{e}^{-\mathrm{i}\pi\cos(\theta)}, \ldots, \mathrm{e}^{-\mathrm{i}(M-1)\pi\cos(\theta)}]^T$ is the steering vector,
- α_k and γ_k are non negative factors controlling respectively the total energy, and the dispersion of the energy between the direct path and the scatterers, for the k-th source. By convention, setting $\gamma_k = +\infty$ means that the energy of the k-th source is concentrated in the direct path (point source model).
- $\mathbf{T}_k$ is the Toeplitz spatial correlation matrix of the clutter around the k-th source, given by

$$\mathbf{T}_k = \int_0^{\pi} \mathbf{a}(\theta)\mathbf{a}(\theta)^* f_k(\theta)\mathrm{d}\theta, \tag{3}$$

where f_k represents the angular density of the scatterers.

We moreover assume that N samples $\mathbf{y}_1, \ldots, \mathbf{y}_N$ are available.

Several DoA estimation methods or improvements of existing ones have been proposed in this context, usually based on the a priori knowledge or estimation of the clutter statistical parameters, see e.g. [4] [2] [3] [5]. In particular, subspace methods [4] have been modified to take into account the potential increase of the signal subspace dimension, due to the clutter covariances $\mathbf{T}_1, \ldots, \mathbf{T}_K$. In comparison, the performance analysis of classical subspace methods has received much less attention. The statistical behaviour of MUSIC is studied in [6][7], and approximate distribution of the DoA estimates are derived, assuming a clutter with small angular spread around each nominal source DoA. Moreover, most works assume the absence of a direct path between the sources and the sensor array, that is, $\gamma_k = 0$ for all $k = 1, \ldots, K$.

In situations where a direct path between the sources and the array exists and carries a certain amount of the total energy, it is still possible to use classical subspace methods to estimate the nominal DoA. In this case, one major issue is the loss of orthogonality between the source steering vectors $\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)$ and the noise subspace defined as the eigenspace associated with the $M-K$ smallest eigenvalues of $\mathbf{R}$ defined above. Nevertheless, this loss can be mitigated for array equipped with a large number of sensors. In this case, the sample size may not be much larger than the number of

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[1] The role of the normalization factor $\frac{1}{M}$ in the direct path contribution is to keep the SNR bounded, as we will let M goes to infinity for our asymptotic analysis.

sensors, and standard statistical analysis based on asymptotic sample size regime $N \to \infty$, with fixed observations dimension M is not relevant anymore.

In this paper, we consider the same approach as in [8] and resort to the asymptotic regime in which both M and N goes to infinity at the same rate. Based on this, we provide a statistical analysis of the MUSIC DoA estimates, for the distributed source model (1). In particular, we prove that as long as the energy of the direct path of each source (controlled by the coefficients $\alpha_1, \gamma_1, \ldots, \alpha_K, \gamma_K$) is sufficiently large, the MUSIC method still provides consistent DoA estimates. For this purpose, we provide an analysis of the asymptotic behaviour of the K largest eigenvalues of the sample correlation matrix (SCM)

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{y}_n \mathbf{y}_n^*,$$

based on a straightforward extension of the results of [9] (which are valid in particular for a point source model). Most of the proofs in the paper are omitted due to lack of space.

2. SPECTRAL BEHAVIOUR OF THE SCM

For our analysis, we consider the asymptotic regime [2] in which the sample size $N = N(M)$ is a function of M such that $c_M = \frac{M}{N} \to c > 0$ as $M \to \infty$, whereas the source number K is assumed fixed. The source DoA $\theta_1, \ldots, \theta_K$ are considered fixed with respect to M, which models situations where the DoA are widely spaced (see [10] for further details). We assume that θ_k does not belong to the boundary of $\operatorname{supp}(f_k)$, that is, the source DoA may be strictly inside/outside its clutter angular spreading, as well as the non-overlapping conditions [3]

$$\theta_k \notin \operatorname{supp}(f_\ell) \text{ and } \operatorname{supp}(f_k) \cap \operatorname{supp}(f_\ell) = \emptyset,$$

for $k \neq \ell$, and consider $\operatorname{supp}(f_k) \subset (0, \pi)$ for all $k = 1, \ldots, K$. Finally, we assume the technical conditions that f_k is piecewise continuous with no isolated points in its support and that it is Lipschitz continuous [4] when restricted to its support.

For the remainder, we will work with $\mathbb{T} = (-\pi, \pi]$ and rewrite the steering vectors in the Fourier domain as $\mathbf{b}(\varphi_1), \ldots, \mathbf{b}(\varphi_K)$ with

$$\mathbf{b}(\varphi) = [1, \mathrm{e}^{-\mathrm{i}\varphi}, \ldots, \mathrm{e}^{-\mathrm{i}(M-1)\varphi}]^T \text{ and } \varphi_k = \pi \cos(\theta_k).$$

We also rewrite the correlation matrix $\mathbf{R}$ as

$$\mathbf{R} = \frac{1}{M} \mathbf{B}\mathbf{D}\mathbf{B}^* + \mathbf{T} + \sigma^2 \mathbf{I}, \tag{4}$$

where $\mathbf{B} = [\mathbf{b}(\varphi_1), \ldots, \mathbf{b}(\varphi_K)]$, $\mathbf{D} = \operatorname{diag}(d_1, \ldots, d_K)$ with $d_k = \frac{\alpha_k \gamma_k}{1+\gamma_k}$, and where

$$\mathbf{T} = \frac{1}{2\pi} \int_{\mathbb{T}} \mathbf{b}(\varphi) \mathbf{b}(\varphi)^* g(\varphi) \mathrm{d}\varphi,$$

with

$$g(\varphi) = \sum_{k=1}^{K} \frac{2\alpha_k}{1+\gamma_k} \frac{f_k\left(\arccos\left(\frac{\varphi}{\pi}\right)\right)}{\sqrt{1 - \left(\frac{\varphi}{\pi}\right)^2}}.$$

Under the above assumptions, note that $g \in \mathrm{L}^\infty(\mathbb{T})$, and thus $\|\mathbf{T}\|_2 \leq \|g\|_\infty$, where $\|.\|_2$ stands for the spectral norm and $\|.\|_\infty$ the essential sup norm on $\mathrm{L}^\infty(\mathbb{T})$.

Denote by $\lambda_{1,M} \geq \ldots \geq \lambda_{M,M}$ the eigenvalues of the correlation matrix $\mathbf{R}$, and by ν_M the empirical spectral distribution of $\mathbf{R}$ defined as

$$\nu_M = \frac{1}{M} \sum_{m=1}^{M} \delta_{\lambda_{m,M}},$$

where δ_x is the Dirac measure at point x. From Szegö theorem, it is well-known that

$$\nu_M \xrightarrow[M\to\infty]{w} \nu = \tau \circ \left(g + \sigma^2 \mathbb{1}_{\mathbb{T}}\right)^{-1},$$

that is, (ν_M) converges weakly to the image measure of τ by $g + \sigma^2 \mathbb{1}_{\mathbb{T}}$, where τ is the normalized Lebesgue measure on $\mathbb{T}$. Although the global asymptotic behaviour of the spectrum of $\mathbf{R}$ follows the distribution ν, the K largest eigenvalues of $\mathbf{R}$ may split from the support of ν, thus ensuring clear separation between signal and clutter plus noise subspaces, as stated in the following result.

Proposition 1. *If $d_k + g(\varphi_k) > \|g\|_\infty$ for all $k = 1, \ldots, K$, then*

$$\lambda_{k,M} \xrightarrow[M\to\infty]{} \lambda_k = d_k + g(\varphi_k) + \sigma^2,$$

while $\limsup_{M\to\infty} \lambda_{K+1,M} \leq \|g\|_\infty + \sigma^2$.

The proof, which relies on a standard analysis of eigenvalues perturbation and Fourier series computations, is omitted. Note that the condition in the statement of Proposition 1 has a straightforward interpretation, as it requires the energy of the "signal plus clutter" contribution in the directions $\varphi_1, \ldots, \varphi_K$ to be larger than the maximum of the clutter spectral density.

We now examine the conditions under which the K largest eigenvalues of the SCM $\hat{\mathbf{R}}$ also split from the rest. First, we begin by reformulating some well-known results stating that the whole spectrum of $\hat{\mathbf{R}}$ tends to spread following some deterministic distribution. Let us define the empirical spectral distribution of $\hat{\mathbf{R}}$ as the random probability measure

$$\hat{\mu}_M = \frac{1}{M} \sum_{m=1}^{M} \delta_{\hat{\lambda}_{m,M}},$$

where $\hat{\lambda}_{1,M} \geq \ldots \geq \hat{\lambda}_{M,M}$ are the eigenvalues of $\hat{\mathbf{R}}$. Then it is well-known (see [9] and the references therein) that almost surely,

$$\hat{\mu}_M \xrightarrow[M\to\infty]{w} \mu,$$

where μ is a deterministic probability distribution, characterized through its Stieltjes transform $m(z) = \int_{\mathbb{R}} \frac{\mathrm{d}\mu(\lambda)}{\lambda - z}$ satisfying the canonical equation

$$m(z) = \int_{\mathbb{R}} \frac{\mathrm{d}\nu(\lambda)}{\lambda\left(1 - c - czm(z)\right) - z}.$$

[2] Accordingly, we use the subscript M for certain quantities depending on M, when there may be an ambiguity.

[3] The results of this paper could be extended to the case where more than one source DoA belongs to $\operatorname{supp}(f_k)$; nevertheless we stick to the case of "one source per clutter" to lighten the presentation.

[4] Note that this last condition is mainly used to prove the existence of a local minimum at point w_0 for the function ψ defined below in (5).

for all $z \in \mathbb{C} \backslash \operatorname{supp}(\mu)$. Moreover, $\operatorname{supp}(\mu)$ coincides with the disjoint union of compact intervals whose boundary points are given by the local extrema of the function $w \mapsto \psi(w)$ defined as

$$\psi(w) = w \left(1 - c \int_{\mathbb{R}} \frac{\lambda}{\lambda - w} \mathrm{d}\nu(\lambda)\right). \tag{5}$$

In the interval $\left(\|g\|_\infty + \sigma^2, +\infty\right)$, ψ admits in particular a unique local minimum at some point denoted w_0, and $\max\left(\operatorname{supp}(\mu)\right) = \psi(w_0)$.

Following verbatim the steps of [9], we also obtain the following result ensuring the escape of the K largest eigenvalues $\hat{\lambda}_{1,M}, \ldots, \hat{\lambda}_{K,M}$ from the support of μ.

Theorem 1. *Assume that*

$$\lambda_K > w_0. \tag{6}$$

Then, for $k = 1, \ldots, K$, with probability one (w.p.1) as $M \to \infty$, $\hat{\lambda}_{k,M} \to \psi(\lambda_k)$, whereas $\hat{\lambda}_{K+1,M} \to \psi(w_0) < \psi(\lambda_K)$.

The condition in (6) (equivalent to $\psi'(\lambda_K) > 0$) intrisically depends on α_k, γ_k and σ^2 and is roughly satisfied as long as α_k and γ_k are large enough ; it is referred to as *separation condition* (see [8]). Therefore, as long as the separation condition is satisfied, we are able to consistently detect the number of sources in the sense that

$$\hat{K}_M = \max\left\{k : \hat{\lambda}_{k,M} > \psi(w_0) + \epsilon\right\} \xrightarrow[M\to\infty]{a.s.} K,$$

for $0 < \epsilon < \psi(\lambda_K) - \psi(w_0)$, where a.s stands for almost surely.

Remark 1. *Of course, by letting $\gamma_k \to +\infty$ for all k, we have*

$$\psi(w) \to w \frac{w - \sigma^2(1+c)}{w - \sigma^2}, \tag{7}$$

and we retrieve the usual separation condition for point sources [9], that is $\alpha_1, \ldots, \alpha_K > \sigma^2\sqrt{c}$.

3. CONSISTENCY OF THE MUSIC METHOD

In this section, we use the results on the spectrum of the SCM to analyze the statistical behaviour of the MUSIC DoA estimates, in terms of consistency, in the specific doubly asymptotic regime mentioned previously. We assume, to ligthen the presentation, that $\lambda_1 > \ldots > \lambda_K$ (simple eigenvalues case).

Let us define

$$\hat{\eta}_M(\varphi) = \frac{1}{M} \sum_{k=1}^{K} \left|\mathbf{b}(\varphi)^* \hat{\mathbf{u}}_{k,M}\right|^2,$$

the usual MUSIC cost function, where $\hat{\mathbf{u}}_{1,M}, \ldots, \hat{\mathbf{u}}_{K,M}$ are the orthonormalized eigenvectors of $\hat{\mathbf{R}}$ associated with the K largest eigenvalues $\hat{\lambda}_{1,M}, \ldots, \hat{\lambda}_{K,M}$, as well as the related DoA estimates $\hat{\varphi}_{1,M}, \ldots, \hat{\varphi}_{K,M}$ with [5]

$$\hat{\varphi}_{k,M} = \operatorname*{argmax}_{\varphi \in \mathcal{I}_k} \hat{\eta}_M(\varphi),$$

where $\mathcal{I}_1, \ldots, \mathcal{I}_K$ are disjoint compact intervals such that φ_k belongs to the interior of $\mathcal{I}_k$.

The following result gives the asymptotic of $\hat{\eta}_M(\varphi)$. We denote by $\mathbf{u}_{1,M}, \ldots, \mathbf{u}_{K,M}$ the orthonormalized eigenvectors of $\mathbf{R}$ associated with $\lambda_{1,M}, \ldots, \lambda_{K,M}$, and define

$$h(w) = w \frac{\psi'(w)}{\psi(w)}.$$

[5] As it will be clearer below, the argmax is unique w.p.1 for all large M.

Proposition 2. *If the separation condition* (6) *holds, then*

$$\|\hat{\eta}_M - \eta_M\|_\infty \xrightarrow[M\to\infty]{a.s.} 0,$$

where $\eta_M(\varphi) = \frac{1}{M}\sum_{k=1}^{K} h(\lambda_k) \left|\mathbf{b}(\varphi)^* \mathbf{u}_{k,M}\right|^2$.

Proof. For any fixed $\varphi \in \mathbb{T}$, the convergence $\hat{\eta}_M(\varphi) - \eta_M(\varphi) \to 0$ can be obtained as in [8, Th.1], and using the fact that $\nu_M \to \nu$ weakly. The uniformity over all $\varphi \in \mathbb{T}$ can be handled following the arguments as in [11, Th.3.1] □

To study the behaviour of the DoA estimates $\hat{\varphi}_{1,M}, \ldots, \hat{\varphi}_{K,M}$, we thus need to study the local maxima of $\varphi \mapsto \eta_M(\varphi)$ obtained in the previous result. Using standard asymptotic properties of $\mathbf{b}(\varphi)$, we obtain, as $M \to \infty$, that

$$\sup_{\varphi \notin \cup_k \mathcal{I}_k} \eta_M(\varphi) \to 0 \text{ and } \sup_{\varphi \in \mathcal{I}_k} \left|\eta_M(\varphi) - \eta_{k,M}(\varphi)\right| \to 0, \tag{8}$$

where the function $\eta_{k,M}$, defined by

$$\eta_{k,M}(\varphi) = h(\lambda_k) \left|\frac{d_k}{M} \mathbf{b}(\varphi)^* \left(\mathbf{T} + (\sigma^2 - \lambda_k)\mathbf{I}\right)^{-1} \mathbf{b}(\varphi_k)\right|^2,$$

admits a unique local maximum on $\mathcal{I}_k$ at φ_k. This immediately implies that $\hat{\varphi}_{k,M} \to \varphi_k$ a.s. as $M \to \infty$. To precise the rate of convergence, we follow the steps of [10, Th. 5] and obtain that w.p.1,

$$\liminf_{M\to\infty} \hat{\eta}_M\left(\hat{\varphi}_{k,M}\right) \geq h(\lambda_k) > 0. \tag{9}$$

This implies that

$$\limsup_{M\to\infty} M \left|\hat{\varphi}_{k,M} - \varphi_k\right| < \infty \quad \text{w.p.1},$$

otherwise (9) would be contradicted. Finally, if sinc denotes the function satisfying $\operatorname{sinc}(0) = 1$ and $\operatorname{sinc}(x) = \sin(x)/x$ if $x \neq 0$, then

$$\sup_{\delta \in \mathcal{K}} \left|\eta_{k,M}\left(\varphi_k + \frac{\delta}{M}\right) - h(\lambda_k)\operatorname{sinc}\left(\delta/2\right)^2\right| \xrightarrow[M\to\infty]{} 0, \tag{10}$$

for any compact $\mathcal{K}$ of $\mathbb{R}$. This implies $M\left(\hat{\varphi}_{k,M} - \varphi_k\right) \to 0$ w.p.1 as $M \to \infty$, otherwise (9) would be again contradicted. Therefore, the following result holds.

Theorem 2. *If the separation condition* (6) *holds, then*

$$M\left(\hat{\varphi}_{k,M} - \varphi_k\right) \xrightarrow[M\to\infty]{a.s.} 0.$$

Note that this consistency result, referred to as M-consistency, is similar to the one obtained in the point source scenario [10].

Remark 2. *The convergences in* (8) *and* (10) *indicate that the MUSIC cost function asymptotically "removes" the clutter contribution, in the sense that we retrieve its usual behaviour of the point source scenario [10] in/outside neighborhoods of the nominal DoA $\varphi_1, \ldots, \varphi_K$.*

4. NUMERICAL RESULTS FOR UNIFORMLY DISTRIBUTED SCATTERERS

In this section, we provide numerical illustrations of the statistical behaviour of the MUSIC DoA estimates, in the special case where the clutters associated with the K sources are uniformly distributed and centered around the corresponding nominal DoA, that is

$$f_k(\theta) = \frac{1}{\Delta_k} \mathbb{1}_{\left[\theta_k - \frac{\Delta_k}{2}, \theta_k + \frac{\Delta_k}{2}\right]}(\theta).$$

In this case, we have

$$\lambda_k = \frac{\alpha_k}{1+\gamma_k}\left(\gamma_k + \frac{2}{\Delta_k \sin(\theta_k)}\right) + \sigma^2,$$

and the separation condition $\psi'(\lambda_K) > 0$ rewrites more precisely

$$\sum_{k=1}^{K} \int_{\theta_k - \frac{\Delta_k}{2}}^{\theta_k + \frac{\Delta_k}{2}} \left(1 + \frac{\lambda_K}{\frac{2\alpha_k}{(1+\gamma_k)\Delta_k \sin(\theta)} + \sigma^2 - \lambda_K}\right)^2 \sin(\theta) \mathrm{d}\theta$$
$$+ \left(\frac{\sigma^2}{\sigma^2 - \lambda_K}\right)^2 \left(1 - 2\sum_{k=1}^{K} \sin(\theta_k) \sin\left(\frac{\Delta_k}{2}\right)\right) < \frac{2}{c}.$$

We choose $K = 3$ sources with $\theta_1 = 40^\circ$, $\theta_2 = 90^\circ$ and $\theta_3 = 135^\circ$, with clutter spreading $\Delta_1 = \Delta_2 = \Delta_3 = 15^\circ$. The energy factors are set to $\alpha_1 = 2$, $\alpha_2 = 3$, $\alpha_3 = 2.4$ and $\gamma_1 = \gamma_2 = \gamma_3 = 10$.

In Figure 1, we plot the expectation of the periodogram $\varphi \mapsto \frac{1}{M}\|\mathbf{R}^{1/2}\mathbf{b}(\varphi)\|^2$, as well as the asymptotic equivalent $\varphi \mapsto \eta_M(\varphi)$ of the MUSIC cost function obtained in Proposition 2 ($M = 100$, $c = 0.5$ and $\sigma^2 = 0.25$). Both cost functions are renormalized such that their maximum and minimum values are 1 and 0 respectively. As stated in Remark 2, we observe that the MUSIC asymptotic equivalent cost function mitigates the clutter spectral density, which leads in theory to a better "Peak to Sidelobe Ratio" compared to the periodogram.

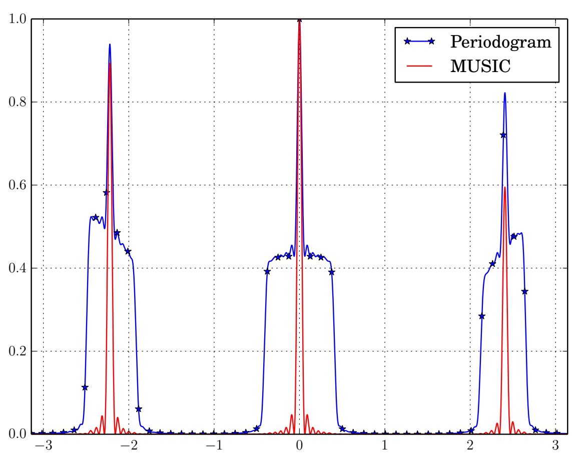

Fig. 1. Periodogram and MUSIC asymptotic cost functions

In Figure 2, we plot the empirical MSE of the MUSIC DoA estimate $\hat{\varphi}_{2,M}$ against a normalized SNR defined as $-10\log(\sigma^2)$, for different values of M (with $N = 2M$). The Cramer-Rao bound for point sources [1] is also plotted (for $M = 150$). We observe a saturation of the MSE at high SNR, due to the loss of orthogonality between the source steering vectors and the noise subspace. The separation condition (6) occurs around SNR=-2dB.

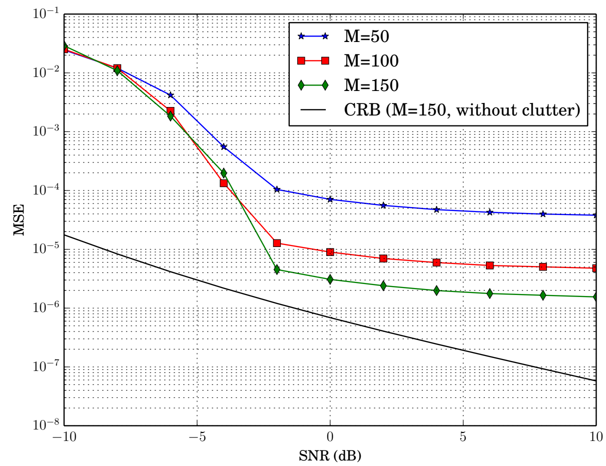

Fig. 2. Empirical MSE of $\hat{\varphi}_{2,M}$ as a function of the SNR

Finally, in Figure 3, we represent the empirical MSE of $\hat{\varphi}_{2,M}$ as a function of M (with $N = 2M$ and SNR = 10 dB). We notice that the saturation of the MSE, observed in Figure 2, can be mitigated if the array is equipped with a large number of sensors, and the order of magnitude is conjectured to be $\mathcal{O}\left(M^{-3}\right)$. Additionally, the estimated DoA are also expected to be asymptotically Gaussian distributed (see e.g. [10] for the case of point sources).

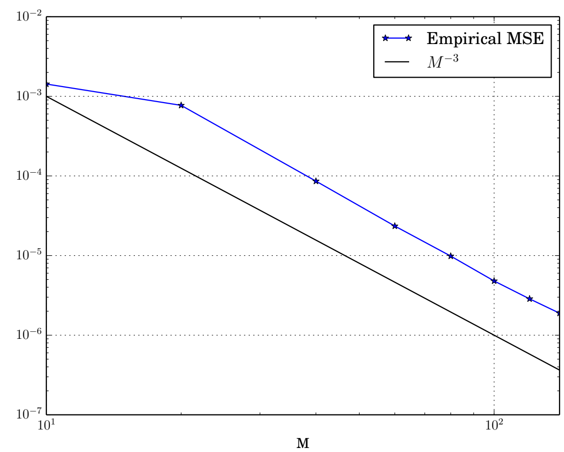

Fig. 3. Empirical MSE of $\hat{\varphi}_{2,M}$ as a function of M

5. CONCLUSION

In this paper, we have addressed a statistical analysis of the MUSIC algorithm for DoA estimation, in a context where source signals LOS component is corrupted by a clutter spreading in a angular sector around the nominal DoA. Considering the asymptotic regime in which the number of sensors and the number of available snapshots tend to infinity at the same rate, we have provided an explicit condition, involving the noise variance as well as the energies of the clutter and LOS components, under which MUSIC provides consistent DoA estimates. Numerical computations of the MSE have been provided as a function of the SNR and the number of antennas, and its theoretical study is currently under investigation.

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