Deciphering the Collatz Conjecture Through Recursion

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Abstract

More than 80 years has passed since the Collatz conjecture has been proposed, but since then there has been no concrete proof as to why it is stuck in the 4 to 2 to 1 loop endlessly. There had been various attempts to find the reason as to why this infinite loop occurs, but there hasn't been any widely recognized proof for the Collatz conjecture.

This paper details a different approach to explain why this infinite loop occurs and at the same time explain why the Collatz conjecture is similar to a computer source code. Section 2, 3, 4, and 5 details the process used to determine the purpose of 2n and n + 1 functions inside an iterative loop, while section 6 and 7 reveals the logical reasoning behind how the 2n + 2 algebraic expression was obtained based on 3n + 1 or 2n + n + 1.

Section 8 provides proof for 2n + 2 as the hidden algebraic expression of the Collatz conjecture, while both sections 9 and 10 provides proof that it is also possible to get stuck in the -4 to -2 to -1 loop endlessly when n is any negative odd integer by simply replacing addition with subtraction. Sections 11 provides the key reason why the Collatz conjecture is getting stuck in an endless loop of 4 to 2 to 1.

1. Introduction

Collatz conjecture states:

- a. Let n be any positive odd integer greater than 0
- b. If n is odd, use 3n + 1
- c. If *n* is even, use $\frac{n}{2}$
- d. Repeat the loop until 1 is reached

2. Recursion and Iterations

In computer science, recursion and iteration are two related methods for solving problems that depends on the solution of a previous computation from the same lines of code.

Recursion is a recursive function that calls itself repeatedly if a specific condition is met and will only stop once the predefined condition for it to stop is met.

Iteration simply means executing the same lines of code over and over until the predefined condition for it to stop is met.

The very definition of recursion and iteration perfectly describes the Collatz conjecture in its entirety. Therefore, it is now possible to infer that the Collatz conjecture is just like any other computer source code, which can be further examined to determine what the purposes are of each function inside the iterative loop. Representing the Collatz conjecture as a computer algorithm using the python language. See the link for the python code in Appendix A.

Iterative function	Recursive function
filename: 3n _plus_ 1_whileloop.py	filename: 3n _plus_ 1_recursion.py
# Using a while loop for the	<pre># Creating a function called</pre>
Collatz conjecture	Collatz conjecture
<pre># created by Glenn Patrick King</pre>	<pre># created by Glenn Patrick</pre>
Ang 10/18/2021	King Ang 10/18/2021
# n = 1 or any positive odd	def Collatz_Conjecture(n,
integer > 0	previous_n=0):
n = 1	<pre>if previous_n == 2 or n == 2:</pre>
$\begin{array}{llllllllllllllllllllllllllllllllllll$	2. print("1")
	print("end recursive
while $n > 0$ and previous $n != 2$:	programming")
previous_n = n	else:
	previous_n = n
if previous_n == 2:	
print("1")	if n % 2 == 1:
<pre>print("end loop")</pre>	n = (3 * n) + 1
break	elif n % 2 == 0:
elif n % 2 == 1:	n = n / 2
n = (3 * n) + 1	
elif n % 2 == 0:	<pre>print(int(n))</pre>
n = n / 2	Collatz Conjecture(n,
<pre>print(int(n))</pre>	previous n)
P++110 (+110 (11))	

<pre># n = 1 or any positive odd</pre>
integer > 0
n = 1
Collatz Conjecture(n)

3. Rewriting the 3n + 1 algebraic expression

In mathematics, an algebraic expression is an expression composed of variables, constants, and algebraic operations. Furthermore, algebraic expressions can be rewritten more than one way without affecting the original algebraic expression. Rewriting 3n + 1:

3n+1= 2n+n+1

Rewriting the original algebraic expression is the key to not only better understand what the function of each part of the algebraic expression is for, but also in deciphering the hidden form of the 3n + 1 algebraic expression inside an iterative loop.

4. Revealing the functions of 2n and n+1

In computer programming, when there are multiple functions in the source code of a program, the coder or programmer must look at and study each function to fully comprehend what it is for or what it exactly does. The same logic applies in this instance to fully prove that the 3n + 1 algebraic expression was designed from the very beginning to create an infinite loop that goes from 4 to 2 to 1 then back to 4 loops.

By using the rewritten algebraic expression, the possibility of evaluating each part of the algebraic expression for its true purpose has finally been opened up. For this proof, the rewritten algebraic expression will be separated into 2n and n + 1.

Starting with 2*n*:

The purpose of 2n is to always output an even integer that is always evenly divisible by 2.

Proof: Properties of even and odd numbers.

The sum of two odd numbers is always an even number.

• Rewriting 2*n* as:

n + n

For *n* + 1:

Its purpose is to always output an even integer that is always evenly divisible by 2.

Proof: Properties of even and odd numbers

The sum of two odd numbers is always an even number.

• Let n be any positive odd integer

n + 1

This proves that both 2n and n + 1 is designed to not only always output an even integer but ensure that the integer is always evenly divisible by 2. Since it has already been proven what 2n and n + 1 does, it is now time to decipher the code hidden inside 3n + 1.

5. Deciphering the Collatz Conjecture's code

Although using the recursive function is far more ideal, for the purpose of keeping things simple, the while loop will be used throughout the proof instead. Both 2n and n + 1 will be put inside an iterative loop separately in order to shed light as to the true purpose of each function.

a. Starting with 2n:

This experiment will be using the exact same conditions as the Collatz conjecture, but with a slight variation where 2n will be used instead of 3n + 1. Representing the Collatz conjecture as a computer algorithm using the python language. See the link for the python code in Appendix A. filename: 2n_whileloop.py

<pre># n = 1 or any positive odd integer > 0</pre>	Example:
# created by Glenn Patrick King Ang 10/18/2021	n = 19
n = 19 previous_n = 0	if odd: 2n n = 2(19) n = 38
while n > 0 and previous_n != 2:	if even: $\frac{n}{2}$
previous_n = n	$n = \frac{38}{2}$
if previous_n == 2:	2

print("1")	<i>n</i> = 19
<pre>print("end loop")</pre>	
break	The loop is forever stuck in the 19 to 38
elif n % 2 == 1:	then back to 19 loops.
n = (2 * n)	
elif n % 2 == 0:	
n = n / 2	
<pre>print(int(n))</pre>	

By looping the code with 2n as the algebraic expression, it is now possible to infer that for any positive odd integer, the output will always get stuck in a loop. The loop consists mainly of multiplying and dividing n by 2, which will then revert n back to its original value. Therefore, the only purpose of 2n is to convert the positive odd integer into a positive even integer that will always be evenly divisible by 2.

Logically speaking, the true purpose of 2n when combined with n + 1 is to ensure that the iterative loop will never stop as long as a positive odd integer exists inside the iterative loop.

b. Starting with n+1:

This experiment will be using the exact same conditions as the Collatz conjecture, but with a slight variation where n + 1 will be used instead of 3n + 1. Representing the Collatz conjecture as a computer algorithm using the python language. See the link for the python code in Appendix A. filename: n_plus_1_whileloop.py

<pre># n = 1 or any positive odd integer > 0</pre>	Example:
<pre># created by Glenn Patrick King Ang 10/18/2021</pre>	n = 1
n = 1	<i>if odd</i> : <i>n</i> + 1
previous $n = 0$	n = 1 + 1
while $n > 0$ and previous_n != 2:	n = 2
previous_n = n	n
if previous_n == 2:	if even: $\frac{\pi}{2}$
print("1")	2
print("end loop")	$n=\frac{1}{2}$
break	$n = \overline{1}$
elif n % 2 == 1:	
n = n + 1	

Executing the iterative loop with more examples:

Example:		
n = 7	n = 23	n = 85
<i>if odd</i> : $n = n + 1$ n = 7 + 1 n = 8	if odd: $n = n + 1$ n = 23 + 1 n = 24	if odd: n = n + 1 n = 85 + 1 n = 86
$if even: n = \frac{n}{2}$ $n = \frac{8}{2}$ $n = 4$	$if even: n = \frac{n}{2}$ $n = \frac{24}{2}$ $n = 12$	if even: $n = \frac{n}{2}$ $n = \frac{86}{2}$ n = 43
Repeat the loop.	Repeat the loop.	Repeat the loop.
Output: $n = \frac{4}{2} = 2$ $n = \frac{2}{2} = 1$ $n = 1 + 1 = 2$ The loop is forever stuck in the 2 to 1 then back to 2 loops	Output: $n = \frac{12}{2} = 6$ $n = \frac{6}{2} = 3$ $n = 3 + 1 = 4$ $n = \frac{4}{2} = 2$ $n = \frac{2}{2} = 1$ $n = 1 + 1 = 2$ The loop is forever stuck in the 2 to 1 then back to 2 loops	Output: n = 43 + 1 = 44 $n = \frac{44}{2} = 22$ $n = \frac{22}{2} = 11$ n = 11 + 1 = 12 $n = \frac{12}{2} = 6$ $n = \frac{6}{2} = 3$ n = 3 + 1 = 4 $n = \frac{4}{2} = 2$ $n = \frac{2}{2} = 1$ n = 1 + 1 = 2 The loop is forever stuck in the 2 to 1 then back to 2 loops

By looping the code with n + 1 as the algebraic expression, it is now safe to infer that for any positive odd integer, the output will always be forever stuck in the 2 to 1 then back to 2 loops. Therefore, the only purpose of n + 1 is to keep adding 1 to any positive odd integer in order to eventually convert the positive odd integer into a positive even integer that will always be evenly divisible by 2.

Logically speaking, the true purpose of n + 1 is to ensure that the iterative loop will never stop until n becomes 2. By simply using 1 as n inside the iterative loop for n + 1, the output is always going to be 2 because n + 1 is designed to eventually convert any positive odd integer inside the iterative loop into 2.

Therefore, both 2n and n + 1 was designed from the very beginning not only to ensure that the iterative loop will never stop until all positive odd integers are exhausted, but also for n + 1 to eventually convert any positive odd integer into 2 inside the iterative loop.

6. The hidden form of the 3n + 1 algebraic expression

After proving the functions of the 2n and n + 1 inside the iterative loop, it is now possible to make a confident inference that will reveal the algebraic expression's hidden form.

Based on observations:

- The main purpose of 2*n* is to turn any positive odd integer into an even integer that is always evenly divisible by 2. This ensures that the iterative loop will never break until all positive odd integers are exhausted inside the iterative loop.
- Moreover, n + 1 ensures that any positive odd integer will eventually be reduced to 2 at some point inside the iterative loop. In essence, n + 1 is nothing but a function with the end goal of converting any positive odd integer inside the iterative loop into 2.

Therefore, it is safe to confidently infer that n + 1 is basically just 2.

Now, the hidden form of the 3n + 1 algebraic expression is finally revealed. By substituting n + 1 with 2, the hidden form of the 3n + 1 or 2n + n + 1 algebraic expression is:

$$2n + 2$$

7. Proving the logic of inference using both iterative loop and recursive function for 2n + n + 1

The 2n + 2 algebraic expression provides a clearer picture as to how the Collatz conjecture truly works. In essence, 2n mainly functions as a multiplier to ensure that the positive odd integer will always be evenly divisible by 2, whereas n+1's sole objective is to eventually convert any positive odd integer into 2. In order to prove the logic of the inference, a python code was constructed using both iterative loop and recursive function for 2n+n+1 or more commonly known as 3n+1. Representing the Collatz conjecture as a computer algorithm using the python language.

See the link for the python code in Appendix A. filename: 2n_plus_n_plus_1_recursion.py

```
\# n = 1 or any positive odd integer > 0
                                                              3n + 1 = 2n + n + 1
# created by Glenn Patrick King Ang 10/18/2021
                                                              Using a while loop for
n = 1
                                                              2n + n + 1
previous n = 0
                                                              Replacing n + 1 with a
                                                              recursive function
def n plus 1(n recursion, previous n recursion=0):
                                                              called n plus 1
    if previous n recursion == 2 or n recursion ==
2:
                                                              This python code for
         return n recursion
                                                              the Collatz conjecture
    else:
                                                              provides a conclusive
         previous n recursion = n recursion
                                                              proof that n + 1 is
         if n recursion % 2 == 1:
                                                              basically just 2.
              n recursion = n recursion + 1
         elif \overline{n} recursion \% \overline{2} == 0:
              n recursion = n recursion / 2
                                                              Therefore, this also
                                                              proves that the 2n + 2
         return n plus 1(n recursion,
                                                              algebraic expression
previous n recursion)
                                                              for the Collatz
                                                              conjecture is its hidden
while n > 0 and previous n != 2:
                                                              form.
    previous n = n
    if previous n == 2:
         print("end while loop for 2n + recursion at
1")
         break
    elif n % 2 == 1:
         n = (2 * n) + n plus 1(n)
    elif n % 2 == 0:
         n = n / 2
    print(f"While loop for 2n + recursion (n+1):
{int(n)}")
```

After obtaining the hidden form of the Collatz conjecture's 3n + 1 algebraic expression, it is now possible to prove that all positive odd integer will always end up in the 4 to 2 to 1 loop using 2n + 2.

8. All positive odd integers will loop 4 to 2 to 1 endlessly using 2n+2

Using the same conditions as the Collatz conjecture:

- a. Let *n* be any positive odd integer greater than 0
- b. If n is odd, use 2n + 2
- c. If *n* is even, use $\frac{n}{2}$
- d. Repeat the loop until 1 is reached

Representing the Collatz conjecture as a computer algorithm using python language. See the link for the python code in Appendix A. filename: 2n_plus_2_whileloop.py

```
# n = 1 or any positive odd
                                            n = 7
integer > 0
# created by Glenn Patrick King
                                            if odd: n = 2n + 2
Ang 10/18/2021
                                            n = 2(7) + 2
                                            n = 16
n = 7
previous n = 0
                                           if even: n = \frac{n}{2}
while n > 0 and previous n != 2:
                                           n = \frac{16}{2}
     previous n = n
                                            n = 8
     if previous n == 2:
                                            Repeat the loop.
          print("1")
          print("end loop")
                                            Output:
          break
                                           n = \frac{8}{2} = 4
     elif n % 2 == 1:
         n = (2 * n) + 2
                                           n = \frac{\overline{4}}{2} = 2
     elif n % 2 == 0:
          n = n / 2
                                            n = \frac{2}{2} = 1
     print(int(n))
                                            n = 2(1) + 2 = 4
                                            The loop is forever stuck in the 4 to 2 to 1
                                            then back to 4 loops
```

9. Further Proof: All negative odd integers will loop -4 to -2 to -1 endlessly using 3n-1

It is now possible to prove that the Collatz conjecture also works with all negative odd integers using 3n + 1 as long as some changes are made. Since it has already been proven that the only function of n + 1 is to output 2, it is now safe to infer that to get the -4 to -2 to -1 then back to -4 loops, a simple switch from addition to subtraction will suffice to make the 3n + 1 work for all negative odd integers. Therefore, the new algebraic expression is:

3n - 1

Using the same conditions as the Collatz conjecture:

- a. Let *n* be any negative odd integer less than 0
- b. If n is odd, use 3n 1
- c. If *n* is even, use $\frac{n}{2}$
- d. Repeat the loop until -1 is reached

Representing the Collatz conjecture as a computer algorithm using python language. See the link for the python code in Appendix A. filename: 3n_minus_1_whileloop.py

```
\# n = -1 or any negative odd
                                           Example:
integer < 0
# created by Glenn Patrick King
                                           n = -1
Ang 10/18/2021
                                           if odd: 3n - 1
n = -1
                                           n = 3(-1) - 1 = -3 - 1
previous n = 0
                                           n = -4
while n < 0 and previous n != -2:
                                           if even: \frac{n}{2}
    previous n = n
                                           n = \frac{-4}{2} = -2
    if previous n == -2:
         print("-1")
                                           n = \frac{-2}{2} = -1
         print("end loop")
         break
                                           n = 3(-1) - 1 = -4
    elif n % 2 == 1:
         n = (3 * n) - 1
                                           The loop is forever stuck in the -4 to -2
    elif n % 2 == 0:
                                           to -1 then back to -4 loops. More
         n = n / 2
                                           examples are provided in Appendix B
    print(int(n))
```

10. Further proof: 2n + 2 will also adhere to the same changes made on the 3n + 1 for all negative odd integers

Since it has already been proven that 3n + 1 works on all negative odd integer by simply replacing addition with subtraction, it is now possible to infer that its hidden form will also adhere to the same changes as well. To get the -4 to -2 to -1 then back to -4 loops with 2n + 2, replacing addition with subtraction will give a new algebraic expression:

2n - 2

Using the same conditions as the Collatz conjecture:

- a. Let *n* be any negative odd integer less than 0
- b. If n is odd, use 2n 2
- c. If *n* is even, use $\frac{n}{2}$
- d. Repeat the loop until you reach -1

Representing the Collatz conjecture into a computer algorithm using python language. See the link for the python code in Appendix A. filename: 2n_minus_2_whileloop.py

```
\# n = -1 or any negative odd
                                          Example:
integer < 0
# created by Glenn Patrick King
                                          n = -1
Ang 10/18/2021
                                          if odd: 2n - 2
n = -1
                                          n = 2(-1) - 2 = -2 - 2
previous n = 0
                                          n = -4
while n < 0 and previous n != -2:
                                         if even: \frac{n}{2}
    previous n = n
                                         n = \frac{-4}{2} = -2
    if previous n == -2:
                                         n = \frac{-2}{2} = -1
         print("-1")
         print("end loop")
         break
                                          n = 2(-1) - 2 = -4
    elif n % 2 == 1:
         n = (2 * n) - 2
    elif n % 2 == 0:
                                          The loop is forever stuck in the -4 to -2
         n = n / 2
                                          to -1 then back to -4 loops.
    print(int(n))
```

11. Key reason why the Collatz conjecture is always getting stuck in an endless loop of 4 to 2 to 1

The only reason why all positive odd integers are always getting stuck in the 4 to 2 to 1 loop endlessly is because of 2n in 2n + n + 1. The 2n ensures that any positive odd integer will always be at least greater than or equal to 2 in order for it to be evenly divisible by 2. This can be proven by simply removing 2n from 2n + n + 1, the iterative loop will always get stuck in the 2 to 1 then back to 2 loops for any positive odd integer.

Moreover, it can also be observed that 3n + 1 is basically almost the same as n + 1, where 3 multiplied by any positive odd integer will still be a positive odd integer. 3n ensures that any positive odd integer multiplied by 3 will always result in a positive odd integer greater than 2. Thus, adding 1 to the result of 3n will always output a positive even integer greater than 2 that will then be evenly divided by 2 until it reaches 1 to begin the iterative loop again.

Therefore, the Collatz conjecture is going to loop endlessly for any positive integer because the 3n + 1 algebraic expression used in the iterative loop was designed not only to reduce any positive odd integer to 2, but to also make sure that once the positive even integer 2 is further reduced to 1, the next positive even number created is 4.

12. Conclusion

Perceiving the Collatz conjecture as a computer code revealed the hidden function deep inside the seemingly simple 3n + 1 algebraic expression. The beauty and complexity of this problem lies on the need to discern the patterns that emerges from evaluating the intent or purpose of the programmer and or designer for each function inside the iterative loop.

From all the proof that has been given, it is now possible to confirm that:

- a. All positive odd integer will loop 4 to 2 to 1 then back to 4 endlessly given that the equation is 3n + 1 or its hidden form 2n + 2.
- b. All negative odd integer will loop -4 to -2 to -1 then back to -4 endlessly given that the equation is 3n 1 or its hidden form 2n 2.
- c. The Collatz conjecture is looping endlessly because the 3n + 1 algebraic expression used in the iterative loop was designed not only to reduce any positive odd integer to 2, but also to make sure that once the positive even integer 2 is further reduced to 1, the next positive even number created is 4.

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Appendix A:

Link to the GitHub repository containing the python codes used in this research paper:

https://github.com/07231985/collatzconjecture

Appendix B:

More examples:

- a. n = -7
- b. n = -23
- c. c = -85

2 p = -7	h n - 22	c n85
a. n = -7	b. n = -23	c. n = -85
<i>if odd</i> : $n = 3n - 1$	if odd: $n = 3n - 1$	if $odd: n = 3n - 1$
n = 3(-7) - 1	n = 3(-23) - 1	n = 3(-85) - 1
n = -21 - 1	n = -70	n = -256
n = -22		
n	if even: $n = \frac{n}{2}$	<i>if even</i> : $n = \frac{n}{2}$
if even: $n = \frac{n}{2}$	-70	
2	$n = \frac{-70}{2}$	$n = \frac{-256}{2}$
$n = \frac{-22}{2}$	n = -35	n = -128
n = -11		
Depend the lease	Repeat the loop.	Repeat the loop.
Repeat the loop.	Output	Output
Output:	Output: n = 3(-35) - 1 = -106	Output: -128
n = 3(-11) - 1 = -34	-106	$n = \frac{-128}{2} = -64$
$n = \frac{-34}{2} = -17$	$n = \frac{-106}{2} = -53$	
	n = 3(-53) - 1 = -160	$n = \frac{-64}{2} = -32$ $n = \frac{-32}{2} = -16$
n = 3(-17) - 1 = -52	$n = \frac{-160}{2} = -80$	$n = \frac{-32}{2} = -16$
$n = \frac{-52}{2} = -26$	<u> </u>	-16
$n = \frac{-26}{2} = -13$	$n = \frac{-80}{2} = -40$	$n = \frac{1}{2} = -8$
2	$n = \frac{-40}{2} = -20$	$n = \frac{-16}{2} = -8$ $n = \frac{-8}{2} = -4$
n = 3(-13) - 1 = -40	$n = \frac{2}{20}$	
$n = \frac{-40}{2} = -20$	$n = \frac{-20}{2} = -10$	$n = \frac{-4}{2} = -2$
$n = \frac{-20}{2} = -10$	$n = \frac{-10}{2} = -5$	$n = \frac{-2}{2} = -1$
	L	$n = \frac{2}{2}$ n = 3(-1) - 1 = -4
$n = \frac{-10}{2} = -5$	n = 3(-5) - 1 = -16 -16	n = 3(-1) - 14
n = 3(-5) - 1 = -16	$n = \frac{-16}{2} = -8$	The loop is forever stuck in
$n = \frac{-16}{2} = -8$	$n = \frac{-8}{-4} = -4$	the -4 to -2 to -1 then
$\begin{bmatrix} n - 2 \\ 2 \\ 0 \end{bmatrix} = 0$	$\begin{bmatrix} 2 \\ -4 \end{bmatrix}$	back to -4 loops
$n = \frac{-8}{2} = -4$	$n = \frac{\frac{-8}{2}}{\frac{-4}{2}} = -4$ $n = \frac{\frac{-4}{2}}{\frac{-2}{2}} = -2$ $n = \frac{\frac{-2}{2}}{\frac{-2}{2}} = -1$	
$n = \frac{-4}{2} = -2$ $n = \frac{-2}{2} = -1$	$n = \frac{-2}{2} = -1$	
$\int \frac{-2}{-2} - 1$	n = 3(-1) - 1 = -4	
n = 3(-1) - 1 = -4	The loop is forever stuck in	
The loop is forever stuck in	the -4 to -2 to -1 then back to -4 loops	
the -4 to -2 to -1 then	uack to -4 100ps	
back to -4 loops		
L !	1	1