

What could be the Great Debt to Frege? or Gottlobius ab paene omni naevo vindicatus

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IT IS NOW WELL-KNOWN that in late March 1918, not long before finishing a complete draft of “Logisch–philosophische Abhandlung” (1921), Wittgenstein is said to have professed a “great debt of gratitude” to Frege.¹ And, of course, in the “Abhandlung” itself, or the *Tractatus* (1922), as I shall henceforth refer to it, Wittgenstein mentions his indebtedness to “Frege’s magnificent works”.²

Although, starting with G. E. M. Anscombe (1959), many interesting and illuminating studies have appeared on Wittgenstein’s early understanding and criticism of Frege, there remains relatively little consensus on how the criticisms work, or on how thoroughly or deeply Wittgenstein understood Frege.³ I do not see this as a problematic situation, for I do not see that we have the materials for arriving at definitive interpretations. Certainly my aim in this essay is far from such an interpretation. My approach will rather be to re-examine a number of accounts of some of Wittgenstein’s criticisms of Frege, in order to assess the

¹ “einer großen Dankesschuld” (Frege and Wittgenstein 2011, pp. 42–43). It is Frege who said it, in a letter replying to Wittgenstein.

² “den großartigen Werken Freges” (Wittgenstein 1922, pp. 28–29; hereafter cited by remark number). I tinker a bit with the standard translations, relying mostly on a draft of Michael Beaney’s forthcoming translation he is so kind as to allow me to see. I provide the German text in notes.

³ Apart from the works discussed below, some of the most significant studies are Geach (1976), Diamond (1988), Diamond (2010), Floyd (2001b), Ricketts (1996), Goldfarb (2002), Reck (2002a), and Potter (2009).

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extent to which there are resources in Frege's philosophy to meet these criticisms.⁴

How Frege should be understood is itself a matter of controversy. But on this matter I do take a stand, consisting of an interpretation of Frege on judgment, truth, and thought that I sketch in §1. I then turn to an argument, drawn from Thomas Ricketts's "Frege, the *Tractatus*, and the Logocentric Predicament", (1985, hereafter *FTLP*), that there is a tension between Frege's view of logic as a science and a consequence of his conception of logic: we cannot entertain the falsity of a law of logic. Next, I take up an argument extending the tension discerned by Ricketts that is suggested in Jim Conant's "The Search for Logically Alien Thought" (1991, hereafter *SLAT*). This takes Frege's discussion of "logical aliens," a term coined by Ricketts in "Objectivity and Objecthood" (1986b, hereafter *OO*, at p. 86), to show both that the impossibility of aliens is supposed to be the central thesis of Frege's conception of logic, and also that the sentences purporting to express this thesis are in fact nonsense.⁵ Next I take up Kimhi (2018, hereafter *TB*)'s claim that Frege is not able to account for a number of forms of inference involving ascriptions of judgment or belief, and that a rejection along Wittgensteinian lines of Frege's distinction between force and thought is required to account for these inferences. I then proceed to a number of accounts of Wittgenstein's critique of Frege's doctrine that thoughts are senses that denote one of the two objects that are the truth-values, if they have any denotation at all, so that assertoric sentences, which express thoughts, are proper names: Peter Sullivan (1994), Ian Proops (1997), Jean-Philippe Narboux (2009), Cora Diamond (2002), and Ricketts (2002).

As we will see, by and large Frege is not vulnerable to the Wittgensteinian criticisms presented in these interpretations. However, this re-examination, especially of the last group of interpretations, leads to the uncovering of a tension and an insight in Frege's thought that stands in an intriguing relation to the *Tractatus*. Specifically, the insight dovetails with a development in Wittgenstein's conception of propositions between the wartime notebooks (published in Wittgenstein 1979, hereafter *NB*) and the *Tractatus*, and that Tractarian

⁴ To keep this essay at a reasonable length, I will not discuss Wittgenstein's criticisms in 5.02 or in 5.132. For discussion of 5.02 see Diamond (2010). For discussion of 5.132 with respect to Frege, see Ricketts (1985), Proops (2002), and Wischin (2017).

⁵ Recently Conant has distanced himself in various ways from *SLAT* (see esp. Conant 2020, 322–4), but the interest of the arguments in *SLAT* is perhaps independent of anyone's actually propounding or being committed to them.

conception of propositions provides a way of resolving the tension. As I see it, that insight could be, whether or not it in fact is, the great debt to Frege.

A final word on the larger context of this essay. In *Necessity Lost* (2019, hereafter $\square L$), I began a re-tracing of early analytic philosophy from the perspective of the relation between modality and logic, showing how and why Frege and Russell held that “modality ought to be banished from logic” (Russell 1905, p. 520). The Tractarian conception of proposition, as we will see, is founded on a primitive notion of possibility, and so its relation to Frege’s views is part of the story of how Wittgenstein, along with C. I. Lewis, brought modality back into logic.⁶

§1. Frege on Truth, Thought, and Judgment

Over the course of his philosophical career, Frege advanced the following theses about truth and falsity, all but the first and last of which appear after he adopted the sense/reference distinction:

(Acknowledgment)	Judgment is <i>acknowledgment</i> (<i>Anerkennung</i>) of the <i>truth</i> of a content or a thought (See, <i>inter alia</i> , Frege 1879, hereafter <i>L1</i> , p. 3).
(Objecthood)	Truth and falsity are <i>objects</i> , the True and the False, called truth-values (See, <i>inter alia</i> , Frege 1892, hereafter <i>SR</i> , p. 34).
(Referents)	Some but not all thoughts <i>refer to exactly one the two truth-values</i> (See, <i>inter alia</i> , Frege 1893, hereafter <i>GgI</i> , p. X).
(Apparent Thoughts)	Those thoughts which fail to refer are <i>apparent thoughts</i> (<i>Scheingedanken</i>). (See Frege 1897, hereafter <i>L2</i> , p. 130)
(Stepping)	Judgment is taking <i>the step</i> (<i>Schritt</i>) from a thought to a truth-value. (See, <i>inter alia</i> , <i>SR</i> , p. 34)
(Redundancy)	Ascriptions of truth are redundant: “ <i>nothing is added</i> to the thought by my ascribing to it the property of truth” [(Frege 1918, hereafter <i>T</i> , 61). ⁷

⁶ That story is told in *Necessity Regained* (Shieh forthcoming[a]). I take this opportunity to redress a salient omission in $\square L$: many of my ideas about the differences over modality between Frege and Russell on the one hand and Wittgenstein on the other figure already in James Levine (1996). $\square L$ and *Necessity Regained* are in some ways (considerably more long-winded) elaborations of themes from Levine’s essay.

⁷ The English translation here is from (1984, hereafter *CP*, page numbers are to the original publication, indicated in the margins). I will occasionally amend translations, so I will always provide the German text. In this case it is: “dem Gedanken dadurch nichts hinzugefügt zu werden, daß ich ihm die Eigenschaft der Wahrheit beilege.”

(Indefinability)	Truth is <i>not definable</i> (See <i>L2</i> , pp. 128–129; <i>T</i> , p. 60).
(Absoluteness)	The truth or falsity of a thought is <i>not relative</i> to space, time, or circumstance (See <i>L2</i> , p. 135; Frege 1906, hereafter <i>FGII</i> , p. 398)

Frege’s readers have been puzzled by these views. What does (Stepping) mean? Whatever (Stepping) means, how is it related to (Acknowledgment)? Do these doctrines form a consistent package? One seeming inconsistency jumps out: given (Referents), it would seem that truth *is* definable as the property of referring to the True, contradicting (Indefinability). Another well-known seeming inconsistency, pointed out by Michael Dummett, is between (Apparent Thoughts) and (Redundancy): ascription of truth to a thought that doesn’t refer to either truth–value seems to be false, and so not equivalent to that thought.

In $\square L$, chapter 3, I show that (almost all of) these Fregean doctrines on truth *do form a coherent package*. Here I merely sketch how this is so, without going into the details of the interpretive arguments.

Their coherence stems from Frege’s conceptions of thought, judgment, and truth. To begin with, for Frege, a thought *represents* something as the case. The thought that 5 is prime, for example, represents an object, the number 5, as falling under the concept of being a prime number. Second, a judgment is fundamentally the recognition that what a thought represents obtains. To make the judgment that 5 is prime involves (a) recognizing an object as the referent of the proper name “5”, (b) recognizing a concept as the referent of the function name “ ξ is prime”, and (c) recognizing that that object falls under that concept; these recognitions amount to recognizing that the number 5 does indeed fall under the concept of being prime. What is recognized to be the case concerns the referents presented by the senses of “5” and “ ξ is prime”, and these senses compose the thought expressed by the (assertoric) sentence “ ξ is prime”. I call this the *Recognitional Conception of Judgment*.

The nature of truth and falsity is elucidated by reference to the *Recognitional Conception*. Judgment is *primarily* recognizing or knowing what is the case, and truth is involved in judgment only *secondarily*: recognizing the truth of a thought supervenes on, or is grounded in, recognizing the obtaining of what that thought represents. By recognizing that 5 falls under the concept of being prime, one thereby also recognizes the truth of the thought that 5 is prime. Alternatively, recognizing that this thought has the property of truth is, at bottom, just recognizing that 5 has the property of being prime. This supervenience view accounts for (Redundancy). This is not to deny that truth is a property of thoughts, but only to insist that truth is not a *fundamental* property of thoughts.

Truth is that property of thoughts which one recognizes in *virtue of* recognizing the obtaining of what thoughts represent. This contrasts with a view of judgment that Frege is primarily seeking to *reject* in his regress argument(s) for (Indefinability): judgment is fundamentally predicating the property of truth of a thought, or recognizing that that thought has the property of truth. The Recognitional Conception inverts the order of grounding of this view: recognizing what is the case is fundamental, and acknowledging the truth of a thought is explained in terms of that recognition. The availability of this explanation accounts for Frege's continued acceptance of (Acknowledgment): there is no harm in taking judgment to be acknowledgment of the truth of a thought, so long as that acknowledgment is understood as really just recognizing the obtaining of what that thought represents. Note also that the secondariness of truth as a property is what the regress argument for (Indefinability) really establishes.

The notion of recognition is factive: recognizing that something is the case implies that it is the case. It follows that judgment for Frege is factive. But Frege also tacitly operates with a distinction between judgment and judging. Judging is something we do, an activity or practice of cognitive subjects. It is an activity with a constitutive goal: to recognize what thoughts represent as being the case, or as not being the case. That is to say, judgment is constitutive of judging in that no one counts as judging unless they aim at such recognitions. Judging is fallible. One could aim at the recognition in question, and think that one has achieved it, without actually so doing. Judgment, as opposed to judging, is the goal, the achievement of recognition, which is independent of our attempts at recognition and of our thoughts about whether we've succeeded. So one could have judged without having made a judgment; but one could not be judging without aiming to make judgments.

Grounding in judgment as recognition indicates a way to understand (Stepping). The primary function of a thought is to be a step to the acquisition of knowledge, which is to say, to judgment. This is the meaning of Frege's claim that judgment is taking a step from a thought to a truth-value: making a judgment is going beyond a mere representation of what is the case, to recognizing that what is represented obtains. It follows that taking the step to a truth-value is, ultimately, recognizing the obtaining or the failure to obtain of what a thought represents. This account of (Stepping) also points to an elucidation of (Referents). Thoughts are senses, while what they represent involve objects and functions in the realm of reference. By itself this doesn't seem to account for the claim that the True and the False are referents of thoughts. But what are the True and the False? In "Über Sinn und Bedeutung", Frege tells

us that by “truth–value of a sentence” he means “the circumstance that it is true or false,” and that he will call these circumstances “the True” and “the False” (*SR*, p. 34).⁸ Thus the True is what is common to all judgments that are recognitions of the obtaining of what is represented by a thought, the circumstance of the obtaining of what is represented. And, similarly, the False is what is common to all judgments that are recognitions of the failure to obtaining of what is represented by a thought, the circumstance of the non–obtaining of what is represented.

We come now to (Apparent Thoughts). If there is no acquisition of knowledge by means of grasping something which appears to be a thought, then what is grasped fails to fulfil the primary function of thoughts and so is, at best, a radically defective thought, if it is a thought at all. And this leads to an account of (Absoluteness). If the truth or falsity of a thought is relative, then that thought is not either true or false except with respect to a time, a place, or a circumstance. From Frege’s perspective, what this purported thought represents, by itself, without a time, place or circumstance, is not something that one can recognize to obtain, or recognize not to obtain. Hence this purported thought fails to provide what is required for judgment, and is no more than an apparent thought. The truth and falsity of genuine thoughts, thoughts by grasping which we can make judgments and acquire knowledge, are absolute.

§2. The Logical Alien Arguments

I will reconstruct the Wittgensteinian criticism of Frege of *SLAT* as two arguments intended to work in tandem. Since the criticism is intended to establish a difficulty for the expression of Frege’s conception of logic using some sentence like “logical aliens are inconceivable”, I will call these the Logical Alien Arguments *LA* (for short). Exactly what the phrase “logical aliens” is supposed to refer to will be discussed.

§2.1. *LA*₁: the Inconceivability of the Falsity of Logic

The first argument derives from *FTLP*, which is one of the main sources of *SLAT*. It aims for the conclusion that “there is a sense in which we cannot entertain the falsity of logical laws” (*FTLP*, 10). The argument begins from the claim that for every thought, or at least every thought that is part of a science, “the demand for

⁸ Ich verstehe unter dem Wahrheitswerte eines Satzes den Umstand, daß er wahr oder daß er falsch ist. Ich nenne der Kürze halber den einen das Wahre, den andern das Falsche.

judgment is intelligible,” meaning that we can “entertain” its truth, *and* we can “entertain” its falsity. Second, entertaining the falsity of a thought “involves drawing inferences from” its negation,⁹ and drawing inferences “draws on logic” (*FTLP*, 10–11). The argument continues as follows:

This procedure breaks down, if the statement we are considering is refutable on logical grounds. Entertaining the truth or falsity of a statement draws on logic so that we cannot, in this fashion, conceive of the falsity of a logical law. [T]he demand for judgment must be unintelligible in the case of the axioms of the begriffsschrift. (*FTLP*, 10–11)

In this passage it seems the argument moves directly to the conclusion that we cannot entertain the falsity of a logical law. One way to spell out this last move is by assuming that “drawing on logic” means accepting the laws of logic as true. For then, if we assume further that accepting a thought as true implies not entertaining its falsity, then we see how the claim that it is possible to entertain the falsity of a logical law leads to a contradiction. Since Frege evidently holds that logic is a science, this conclusion would indicate a tension if Frege holds that all inference requires accepting the laws of logic.

- 1(a) Frege holds that logic is a science, so that the laws of logic are thoughts that belong to a science. (This is one principal thesis of Frege’s conception of logic.)
- 1(b) Frege also holds that the laws of logic are “the most general laws, prescribing how to think wherever there is thinking at all” (Foreword to *GgI*, XV).¹⁰ (This is the other principal thesis of Frege’s conception of logic.)
- 1(c) This means that accepting the laws of logic is required for making any inference whatsoever.
- 1(d) For every thought p (that is part of a science) we can entertain or conceive of the truth of p and also of the falsity of p (i.e., to face the demand for judgment with respect to p).

⁹ We’ll see below that faithfulness to Frege would require that we speak here of the “opposite” rather than the “negation” of that thought.

¹⁰ The English translation of *Grundgesetze* I use is Frege (2013). The original is “[Die logischen Gesetze] verdienen den Namen „Denkgesetze“ nur dann mit mehr Recht, wenn damit gesagt sein soll, dass sie die allgemeinsten sind, die überall da vorschreiben, wie gedacht werden soll, wo überhaupt gedacht wird.”

- 1(e) To entertain the falsity of p is to draw inferences from the negation of p .
- 1(f) By 1(c), accepting all the laws of logic is required for making any inference.
- 1(g) By 1(a) and 1(d), for each thought L that is a law of logic, we can entertain the falsity of L .
- 1(h) By 1(e) and 1(f), to entertain the falsity of any thought which is a law of logic requires accepting all the laws of logic.
- 1(i) If one accepts a thought, then one does not entertain its falsity.
- 1(j) Hence, by 1(h) and 1(i), for each law of logic L , entertaining L 's falsity requires not entertaining L 's falsity.
- 1(k) Hence, it is impossible to conceive of the falsity of any law of logic.
- 1(l) Hence no law of logic is a thought that is part of any science. This contradicts premise 1(a).

Call this argument LA_1 .

§2.2. Against LA_1 : Judgment vs Judging; Inference vs Inferring

One half of the tension purportedly demonstrated by LA_1 is:

- 1(k) It is impossible to conceive of the falsity of any law of logic.

The premise on which (k) principally rests is:

- 1(c) Accepting the laws of logic is required for drawing any inference whatsoever.

This premise is an interpretation of Frege's remark in the Foreword of *GgI*:

- 1(b) The laws of logic are "the most general laws, prescribing how to think wherever there is thinking at all" (*GgI*, XV).

As stated, 1(c) is a misreading of 1(b). A proper account of the sense in which, for Frege, the laws of logic prescribe all thinking draws on the distinction between

judgment and judging outlined in §1. For Frege, a subset of judgments and judgments consists of inferrings and inferences. Here's how Frege characterizes inferring and its relation to logic in the earlier of the two unpublished manuscripts entitled "Logik":

[T]he grounds which justify the acknowledgment of a truth often reside in other truths which have already been acknowledged. . . .

. . . To judge because one is conscious of other truths as grounds of justification is called inferring. There are laws for this kind of justification, and to set up these laws of correct inference is the goal of logic. (*L1*, p. 3)¹¹

Inferring is something we do, a cognitive activity of judgments on the basis of other judgments. It has a goal: to make judgments on the basis of having made other judgments. The laws of logic set out this goal of inferring. They specify what judgments are based on other judgments. To reach this goal is to infer correctly. It is to make inferences.

So, how do the laws of logic "prescribe how to think wherever there is thinking at all"? No one judges without aiming at making judgments. Once again, inferrings are judgments, those based on other judgments. So no one infers without aiming at making inferences. The laws of logic specify what are inferences. So no one infers without aiming at inferring as specified by the laws of logic.¹²

Inferring is no less fallible than the judging of which it's a sub-species. We make mistakes in deductive reasoning. Could there be beings who reject the laws of logic in thinking? Of course. The supposition that there are beings who *judge* the negation of a law of logic in the course of inferring is not contrary to logic. What is ruled out are beings who make *inferences* involving *judgments* of the negations of the laws of logic. This means that any being who judges the negation of a law of logic is judging *wrongly*; it does not mean that it is not judging or

¹¹ Die Gründe nun, welche die Anerkennung einer Wahrheit rechtfertigen, liegen oft in anderen schon anerkannten Wahrheiten. ...

... Urteilen, indem man sich anderer Wahrheiten als Rechtfertigungsgründen bewusst ist, heisst schliessen. Es gibt Gesetze über diese Art der Rechtfertigung, und diese Gesetze des richtigen Schliessens aufzustellen, ist das Ziel der Logik (Frege 1969, hereafter *NS*, p. 3).

¹² I'd like to note in passing that the judging/judgment and inferring/inference distinctions are analogous to a distinction between exercises of the faculty of understanding and cognitions in Nunez (2019)'s interpretation of Kant's pure general logic. I'm less convinced of than he is that Kant would not allow the possibility of logical aliens.

inferring at all. Frege says pretty much exactly that:

this impossibility, to which we are subject, of rejecting [a logical law] does not prevent us from supposing beings who do reject it; but it does prevent us from supposing that such beings are *right* to do so ... (*GgI*, XVII; emphasis mine).¹³

Thus, once 1(b) is properly understood, we see that Frege doesn't accept 1(c). Inferring, understood as psychological cognitive phenomena, doesn't require accepting, in the sense of judging, the laws of logic. It is only inferences, successes in achieving the constitutive aim of acts of inference, that require judgments of the laws of logic. So, even with the assumption that

1(e) To entertain the falsity of a thought requires drawing inferences from its negation.

Frege's view is in no tension with

1(d) For any thought p that is part of any science, one can conceive of the truth of p and one can conceive of the falsity of p ,

provided that we take the "drawing inferences" in 1(e) to be inferences, not inferences. That is to say, there is no obvious tension between Frege's conception of logic as universally governing inference and his insistence that logic is a science, even if he is committed to holding that a science is a body of thoughts such that, in grasping any of these thoughts, one is faced with the "demand" to judge it or its negation.¹⁴

§2.3 LA_2 and LA_3 : Against Logical Aliens (not Zeros) and Another Tension in Frege

The foregoing account of Frege's conception of inference however, raises a

¹³ diese Unmöglichkeit, die für uns besteht, [ein logisches Gesetz] zu verwerfen, hindert uns zwar nicht, Wesen anzunehmen, die es verwerfen; aber sie hindert uns, anzunehmen, dass jene Wesen darin Recht haben ...

¹⁴ My account of how Frege's "demand for judgment" is consistent with his claim that logic is a science is indebted to May (2018).

question: what grounds the claim that no inference fails to be in accord with the laws of logic? Let's return to Frege's remark in the Foreword of *Grundgesetze* quoted above: "this impossibility, to which we are subject, of rejecting [a logical law] does not prevent us from supposing [*anzunehmen*] beings who do reject it; but it does prevent us from supposing that such beings are *right* to do so". Isn't Frege saying that there's some difficulty, some "impossibility" in even forming the thought that there are beings who infer correctly in accordance to laws of logic different from those governing our inference? Let's be clear about what is supposed to be inconceivable. Not some, shall we say deductively challenged, individuals whose inferences are rife with logical mistakes, as assessed by *the* laws of logic; call them logical zeros. The beings we are supposed to have difficulty conceiving infer correctly, but correctly as assessed by *their* laws of logic, even if wrongly, as assessed by *our* laws of logic. I take such purported beings to be Ricketts's logical aliens. As Frege sees it, a psychological logician PL, who takes logic to be descriptions of how we (human beings, white European phallogocentric racist oppressors, etc.) think, is committed to the possibility or conceivability of logical aliens. But if aliens are impossible or inconceivable, then so are inferences whose correctness is assessable by something other than *the* laws of logic.

LA_2 reconstructs Frege's argument against PL as aimed at a very strong form of impossibility of logical aliens. PL, according to this argument, is committed to accepting a "thesis," expressed by, e.g., the sentence

Logical aliens are possible (L)

The argument aims to show that *L* fails to express any thought at all, and so is *nonsense*. So, when Frege says that it's impossible to suppose that there are aliens, it's because there's no thought there to suppose. And the thesis that no inference fails to accord with the laws of logic is necessary in a correspondingly very strong sense: the denial of that thesis is nonsense.

The argument is that *PL* is caught in a dilemma:

- 2(a) The judgments of the aliens conflict with ours, in which case PL has to rely on a non-psychological notion of incompatibility involving a non-psychological answer to the question "who is right?" (This is horn 1 of the dilemma.)
- 2(b) There is no genuine conflict between the "judgments" of the aliens and our "judgments," but then neither the aliens nor we are really making any

judgments, and so neither they nor we are really thinking. (This is horn 2 of the dilemma.)

- 2(c) The “aliens” of horn 1 are at best *zeros*, who judge wrongly according to the single set of logical laws that govern us and them.
- 2(d) The “aliens” of horn 2 are beings with psychological processes different from ours, not beings whose judgments are governed by logical laws different from those governing our judgments, and so, again, not aliens.
- 2(e) Horns 1 and 2 exhaust the possibilities for the existence of logical aliens.
- 2(f) Hence, we cannot “make any clear sense of the psychologistic logician’s” “thesis” that logical aliens are possible (*SLAT*, p. 83).
- 2(g) It follows that the sentence *L* which purportedly expresses that thesis does not in fact express a thought, and so is nonsense.
- 2(h) Hence logical aliens are impossible.

LA_2 leads to another argument for a tension in Frege. I’ll present this argument, LA_3 , dispensing with the step from the impossibility or inconceivability of logical aliens to the necessity of all inference conforming to the laws of logic, and taking the impossibility to be the fundamental thesis of Frege’s conception of logic. Restoring that step introduces inessential complications. LA_3 aims to establish that this purported “thesis” that logical aliens are inconceivable is itself nonsense:

- 3(a) Frege holds that logical aliens are impossible.
- 3(b) This “conclusion” is based on the claim that sentences like *L* don’t in fact express any thoughts, and so are nonsense.
- 3(c) If *L* is nonsense, then its negation,

Logical aliens are impossible ($\sim L$)

is equally nonsensical.

- 3(d) But this sentence is supposed to express the fundamental thesis of Frege’s conception of logic, i.e., to express a true thought.
- 3(e) Thus, there’s a tension between Frege’s
 - rejection of the purported expression of PL’s supposed “claim,” *L*, as nonsense, and

- taking as expressing the central tenet of his conception of logic.

The form of argument embodied in LA_3 may also be used to build up from LA_1 a purported Fregean conundrum similar to the conclusion of LA_3 :

- 4(a) Steps 1(a)–1(j) of LA_1 is supposed to demonstrate that it is impossible to conceive of the falsity of any law of logic.
- 4(b) They establish this impossibility by showing that the sentence “it is possible to conceive the falsity of any logical law” is nonsense.
- 4(c) But then the negation of this sentence, “it is impossible to conceive the falsity of any logical law” is also nonsense.
- 4(d) So, what was supposed to be a Fregean commitment or insight into the nature of logic turns out to be nonsense.

Call this LA_4 .

§2.4. The LA_2 Fallacy

I now show that LA_2 is fallacious. The argument begins with “[a] priori reflection on the nature of logic” (*SLAT*, p. 83) which yields two or three assumptions,

- the two horns of the dilemma, 2(a) and 2(b),
- perhaps also step 2(e), the claim that the two horns exhaust all the possibilities for what logical aliens could be.

The “a priori reflection” presumably also include the reasoning from these assumptions to steps 2(c)–2(d), which steps together show that none of the supposed alternatives for logical aliens in fact results in logical aliens. The next, decisive step in the conjuring trick is to move to the claim that we can’t “make sense of the psychological logician’s thesis,” and so the words supposedly expressing that “thesis” express no thought at all. This move tacitly rules out another construal of “not being able to make sense” of a claim. For example, one might say that

There are round squares (*R*)

makes no sense, because being square requires having four corners and being round requires having no corners. This characterization, one might think, rests on *a priori* reflection, because its falsity follows logically from the concepts of being round and being square. But in this case we might be less tempted to see *R* as *nonsense*, as opposed to just logically, or analytically, or non-empirically, *false*. Moreover, this view of *R* goes with thinking of it as expressing something logically impossible, namely, a thought whose truth is precluded by logic and the concepts round and square.

Prima facie, the same sort of story may be told about the psychological logician's purported thesis:

Logical aliens are possible (L)

The sense in which we can make no sense of *L* is this:

- The two or three assumptions of LA_2 are products of *a priori* reflection on the nature of logic, spelling out the connections among the concepts of inference, logic, agreement, judgment, and thought.
- Steps 2(c)–2(d) of LA_2 show that the falsity of the thought expressed by PL's sentence *L* follows logically from these concepts.

Moreover, the impossibility of logical aliens consists in the logical (or analytical or non-empirical) falsity of the thought expressed by *L*. PL's "thesis" is not nonsense, it's an *a priori* false thought.¹⁵

Let me be clear about the foregoing. I have not demonstrated that *L* is *not* nonsense. I have, rather, argued that, without an argument showing either why *L* is significantly different from *R*, or that, contrary to appearances, *R* is nonsense, LA_2 fails to establish the nonsensicality of *L*.

¹⁵ The same point holds of Fregean arguments against the possibility of logical aliens that rest on the claim that understanding the contents of judgments requires acknowledging the laws of logic as norms governing inferring. On this line of argument, PL's *L* is not nonsense, but expresses a thought whose truth is ruled out by logic and, say, the conceptual connection between understanding and acknowledgment of logical laws. This is not to say that any of those who suggested or presented such an argument —Joan Weiner (1990), Daniele Mezzadri (2018), Walter Pedriali (2019)— use it as a sub-argument for the conclusion of LA_3 .

And if LA_2 is not a valid argument for the nonsensicality of L , then we have no reason to accept premise 2 of LA_3 . Hence we have no reason to take LA_3 to be a sound argument, and so no reason to think that it indicates any tension in Frege's view of logic.

A similar fallacy afflicts LA_4 . According to 4(b), 1(a)–1(j) of LA_1 show that a sentence like

It is possible to conceive of the falsity of a law of logic (L)

is nonsense. But do they? It seems to me not. Rather, by the lights of this argument, L' expresses a thought which logically contradicts three other thoughts:

- 1(e) To conceive of the falsity of a thought involves drawing inferences from the negation of an expression of that thought.
- 1(f) To draw any inference requires accepting all the laws of logic as true.
- 1(i) Accepting a thought as true implies not entertaining its falsity.

Thus, we have no reason to take LA_4 to demonstrate any tension in Frege's view of logic.

§2.5. Concepts vs Objects?

One manoeuvre that might be attractive to proponents of the LA arguments at this point is to appeal to Frege's troubles with the distinction between concept and object. The idea is to claim that

Logical aliens are possible (L)

and

It is possible to conceive of the falsity of a law of logic (L)

are nonsense because they are like

No concept is an object (C)

which Frege cannot avoid taking to be nonsense.

But, to the extent that a case may be made for C to be nonsense, that case doesn't obviously apply to logical aliens or to entertaining the falsity of logical laws.

Here's an account of the line of thinking about concept and object in which Frege is caught. Frege starts by apparently advancing a thesis about a difference in logical functioning between two types of expressions, which underlies an apparent ontological categorial difference. But if this thesis is true, then it follows that the words Frege uses to express that thesis fail to express it, and indeed it's not clear that any words, functioning logically "in accordance with" that "thesis" would express it. This makes it hard to see just what thought is expressed by the words that Frege uses to advance the "doctrine" of the absolute distinction between concept and object.

But no case has been presented that either LA_1 or LA_2 has the same self-reflexive structure of the concept/object distinction problem for Frege.

In LA_1 , what would have to come out as not expressing any sense is the negation of L' :

It is impossible to conceive of the falsity of a law of logic ($\sim L$)

The argument yields this "impossibility" from three "theses," 1(e), 1(f) and 1(i) that are products of *a priori* reflection. The sensicality of none of these sentences, so far as I can see, is impugned by the (logical) impossibility of entertaining the falsity of a logical law. Until an argument is given for why these sentences are nonsense, based on $\sim L'$, we have no reason to think that either L' or $\sim L'$ is analogous to C . As opposed to taking them to express thoughts contradicted by or following from, respectively, the thoughts expressed by 1(e), 1(f) and 1(i).

In the case of LA_2 , what would have to come out as nonsense is the negation of L ,

Logical aliens are impossible ($\sim L$)

The horn of the dilemma that does the heavy lifting is 2(b): if there are no non-psychological standards determining whether the purported aliens and we are right, then neither the aliens nor we are thinking. Does this turn out to be nonsense once we come to the "conclusion" that logical aliens are impossible?

Again, it seems to me not. And again, until we have an argument for how the nonsensicality of 2(b) “follows” from the “impossibility” of aliens apparently expressed by $\sim L'$, there’s no reason to think that the impossibility of aliens is analogous to the concept/object distinction. As opposed to being a thought that follows logically from the thought expressed by 2(b).

§2.6 Frege on Aliens and Logical Laws

I’ve left two questions hanging. Is Frege in fact committed to the logical impossibility of logical aliens? And if not, what would be his ground for holding that all inference conforms to the laws of logic? These are difficult questions, to which I venture two suggestions.

First, let’s return one more time to the Foreword of *Grundgesetze* where Frege says that it is “impossible” for us to suppose that logical aliens are *right* to reject logical laws. Frege begins the paragraph in which this remark occurs by saying that in the case of basic laws of logic, laws not reducible to other laws, logic can provide no answer “to the question, why and with what right we acknowledge a logical law to be true” (*GgI*, XVII).¹⁶ What follows then are non-logical answers to this question. Frege states that he “neither want[s] to dispute nor to endorse” one such answer: “when we judge we cannot reject [a logical] law, but have to acknowledge it if we do not want to lead our thinking into confusion and in the end abandon judgment altogether” (*GgI*, XVII).¹⁷ This answer Frege then characterizes as an impossibility for us to reject such basic laws of logic. It is at this point that Frege brings in a second impossibility, the one on which we have been focusing, for us to take the aliens to be *right* to reject basic logical laws. Thus, Frege’s words here do not foreclose reading the second impossibility as also presenting “a ground of ... our taking to be true” rather than “a ground of being true” (*GgI*, XVII).¹⁸ That is to say, both “impossibilities” are subjective, psychological features of us. This idea is reinforced by how Frege continues after presenting the second impossibility. He gives a slightly different characterization of the second impossibility: it is the impossibility of “doubting whether it is we or [the aliens] who are right” (*GgI*, XVII).¹⁹ Then he says, “At least this holds *of*

¹⁶ Die Frage nun, warum und mit welchem Rechte wir ein logisches Gesetz als wahr anerkennen, kann die Logik nur dadurch beantworten, dass sie es auf andere logische Gesetze zurückführt.

¹⁷ wenn wir urtheilen, können wir [ein logisches] Gesetz ... nicht verwerfen, wir müssen es anerkennen, wenn wir nicht unser Denken in Verwirrung bringen und zuletzt auf jedes Urtheil verzichten wollen.

¹⁸ Nicht ein Grund des Wahrseins wird angegeben, sondern unseres Fürwahrhaltens.

¹⁹ daran zu zweifeln, ob wir oder jene Recht haben.

myself (*GgI*, XVII; emphasis mine).²⁰

It has struck many that the way of understanding Frege just outlined can't be right, because of what he says next:

If others dare in the same breath to both acknowledge a law and doubt it, then that seems to me to be an attempt to jump out of one's own skin against which I can only urgently warn. Whoever has once acknowledged a law of being true has thereby also acknowledged a law that prescribes what ought to be judged, wherever, whenever and by whomsoever it may be judged (*GgI*, XVII).²¹

Is Frege not here insisting that the psychological logician's position is incoherent, because it involves "acknowledging and doubting a law of logic at the same time"? The incoherence is not a logical contradiction, but it's something close, namely something contrary to our conception of logical inference or logical law. But an astute psychological logician has the resources to resist this charge of incoherence. She would say that she's not acknowledging and doubting logical laws at the same time, since acknowledgment and doubt are both psychologically relative: given our psychological makeup, it is right for us to acknowledge and wrong for us to doubt, but given the aliens' psychological makeup, it is right for them to doubt and wrong for them to acknowledge. Where's the incoherence?

This is *not to deny* that

2(b) If there is no genuine conflict between the "judgments" of the aliens and our "judgments," then neither the aliens nor we are really making any judgments, and so neither they nor we are really thinking.

is getting at something in Frege's thinking. What it's getting at underlies what Frege says later:

If everyone designated something different by the name 'Moon', namely one of his ideas, much like he voices his pain with the exclamation 'ouch!', then of course a psychological

²⁰ Wenigstens gilt das von mir.

²¹ Wenn Andere es wagen, in einem Athem ein Gesetz anzuerkennen und es zu bezweifeln, so erscheint mir das als ein Versuch, aus der eignen Haut zu fahren, vor dem ich nur dringend warnen kann. Wer einmal ein Gesetz des Wahrseins anerkannt hat, der hat damit auch ein Gesetz anerkannt, das vorschreibt, wie geurtheilt werden soll, wo immer, wann immer und von wem immer geurtheilt werden mag.

viewpoint would be justified; but a dispute concerning the properties of the moon would be pointless: one could perfectly well assert of his moon the opposite of what another with the same right says of his (*GgI*, XIX).²²

Following Ricketts's *OO*, I take the argument to rest on the suggestion that it is unclear how the apparent disagreement between us and logical aliens is, at bottom, any different from, e.g., differential responses to stimuli exhibited by two groups of animals that have been trained differently. Thus psychological logic is incompatible with there being such things as agreement or disagreement in our assertions and judgments, as we intuitively conceive of these actions. I want to emphasize, however, that this consequence does not provide a conclusive refutation of psychologism, since Frege has not shown that there are indeed such things as agreement and disagreement, as we conceive of them. Frege has not shown that logical aliens are impossible, nor does he aim to do so. All he tries to do is to display how distant a consistently pursued psychologism would end up from the way in which we conceive of our intellectual lives.²³

§3. Thinking, Being, and the Content/Force Distinction

I turn now to a recent criticism of Frege advanced in *TB*. It is claimed to be inspired by Wittgenstein, but not based on any of Wittgenstein's texts.

§3.1. Syllogisms

Kimhi claims that Frege runs into trouble with certain "syllogisms of thinking and being" (*TB*, p. 10); for example:

A.

1. A judges p
2. p
3. A truly judges p

²² Wenn jeder mit dem Namen „Mond“ etwas Anderes bezeichnete, nämlich eine seiner Vorstellungen, etwa so, wie er mit dem Ausrufe „au!“ seinen Schmerz äusserte, so wäre freilich die psychologische Betrachtungsweise gerechtfertigt; aber ein Streit über die Eigenschaften des Mondes wäre gegenstandslos: der Eine könnte von seinem Monde ganz gut das Gegentheil von dem behaupten, was der Andere mit demselben Rechte von seinem sagte.

²³ See Shieh (2002) for more details of this reading.

B.

1. A judges p
2. not- p
3. A falsely judges p

C.

1. A judges not- p
2. not- p
3. A truly judges not- p

D.

1. A judges not- p
2. p
3. \therefore A falsely judges not- p (TB, pp. 122–123).

Kimhi claims that the “philosophical task” is to show how these syllogisms may be “revealed together, in one glance, as valid and complete —that is, as requiring no extra premises” (TB, p. 122). Frege, it is said, cannot carry out this task, cannot “capture the logical unity of the above syllogisms,” because

they all involve the occurrence of the same expression, “ p ”, both inside and outside the context of indirect discourse. The former is an intensional context, and thus, according to Frege, one in which the expression occurs with a different signification. Hence, from a Fregean point of view, the premises of these syllogisms are logically unconnected. Yet to recognize these syllogisms as self-evident, one must recognize that the conclusion reveals the logical unity of the premises (TB, p. 123).

Kimhi’s project in this is to provide an anti-Fregean account of the validity and completeness of these syllogisms, one which involves rejecting Frege’s function/argument conception of “propositional complexity” as well as Frege’s distinction between force and content or thought.

It is hard to see how exactly Kimhi’s criticism is supposed to work, since there is a straightforward Fregean account of the validity of all of these “syllogisms”. This account depends on answers to the question, what do “judges truly” and

“judges falsely” mean? An obvious Fregean answer is: A judges truly just in case the thought A acknowledges as true is true, and A judges falsely just in case the thought A acknowledges as true is false. Frege’s theory of oblique (*ungerade*) contexts is that the sense expressed by (an assertoric statement) p when p occurs in

A judges p

is not the thought G expressed by p in syllogisms A and D, but a sense which presents G ; similarly in

A judges not- p

“not- p ” expresses a sense that denotes the thought G' expressed by the second premises of B and C. In a number of texts Frege appears to hold that G' is composed of G and a sense presenting the truth function denoted by “not”, so that G' is true just in case G is false (As we will see below, in §5, there are other texts in which Frege appears to be committed to a different view). Let’s suppose further that, for Frege a statement that ascribes a judging to a thinker is true just in case that thinker acknowledges as true the thought denoted by the complement clause of that statement.

Now we show that syllogism A is valid as follows. Suppose premise A1 is true, then the thinker who is the referent of “A” acknowledges G as true. Suppose A2 is true. Since A2 expresses G , this is the supposition that G is true. It follows that the conditions for A3 to be true are satisfied, as desired. The validity of syllogism C is demonstrated in a parallel way, with G' in place of G .

Now suppose that the premises of syllogism B are true. B1 is true just in case the referent of “A” acknowledges G as true. B2 is true just in case G' is true, just in case G is false. Hence the condition for the conclusion B3 to be true is satisfied, and B is valid, as desired. The validity of syllogism D is established by a parallel argument in which G and G' are substituted for one another.

Given the demonstrations just presented, Frege’s theory may be seen to provide an unproblematic account of the logical connection between the premises of each syllogism: the referent of the complement clause in each ascription of judging that is the first premise is a thought that either is, or is the opposite of, the thought expressed by the second premise.

So far as I can see, the only thing Kimhi says that might be taken as addressing the foregoing Fregean account of A–D is in a footnote:

The Fregean’s deflationary construal of the conclusions as conjunctions of the premises offers only the illusion of an account, since it assumes that the complex signified by “*p*” in intensional contexts has the same truth–value as the assertion “*p*” (TB, p. 123 n. 8).

This is no less difficult to understand than the claim that Frege cannot account for the validity of A–D. Perhaps Kimhi’s objection is that the Fregean explanations of the A–D rely on an assumption in addition to the two premises of each of these “syllogisms,” namely, “the complex signified by ‘*p*’ in intensional contexts has the same truth–value as the assertion ‘*p*,’” and so do not show the syllogisms to be “complete”. But what, on the Fregean account, is “the complex signified by ‘*p*’ in intensional contexts”? Surely it is the denotation of the complement clause in “propositional attitude” ascriptions, in the case of A–D, ascriptions of judging. That denotation, according to the Fregean theory, *is* the thought expressed by an assertion ‘*p*’. It would then be very surprising indeed if the complex and the assertion fail to have the same truth–value. That is to say, the identity in question is intrinsic to the Fregean account, not an additional *ad hoc* assumption that has to be added to the syllogisms.

Since Frege has no trouble with the “syllogisms of thinking and being,” it’s unclear what rationale there is for seeking an alternative to Frege’s function/argument conception of propositional complexity, or for rejecting Frege’s force/content distinction.

§3.2. Display

Kimhi’s rejection of Frege’s force/content distinction is based on contesting an argument for that distinction formulated by Geach grounded on an observation: “a proposition may occur in discourse now asserted, now unasserted, and yet be recognizably the same proposition” (Geach 1965, p. 37). In particular, a statement *p* as it occurs in its negation “not–*p*” does not assert that *p*, nor does it assert that *p* when it occurs as the antecedent of a conditional “if *p* then *q*”. From this observation, which Kimhi labels “Frege’s observation,” Geach infers, on Frege’s behalf, that asserting force does not belong to the thought expressed by *p*, since, otherwise, it would have asserting force as it occurs as component senses of thoughts expressed negations and conditionals in which *p* occurs.

Against this inference Kimhi argues that there is a different way of accounting

for Frege's observation. Exactly how the account works is far from clear, but it appears to depend on a

distinction between a display and a self-identifying display We say that a sample displays a repeatable. For example, Pantone #15-5519 displays turquoise. But not every sample has to be an instance of the repeatable. We can display swimming without actually swimming (e.g., by performing certain gestures on dry land). In this case the gestures display an act of a certain kind by means of its characteristic appearance without being an instance of the act. In contrast, a color sample is a self-identifying display, insofar as it is an instance of the very color it displays (*TB* p. 41).

This distinction in turn funds a

distinction between displaying an assertion and being an assertion. The display is of a repeatable: a concrete occurrence of a propositional sign is a display of a repeatable act. This display can be either an assertion (a self-identifying display) or a gesture (a mere display) (*TB*, p. 51).

The account of Frege's observation seems then to be this. An assertion is a self-identifying display of a propositional sign. If this propositional sign is a negation "not- p " or a conditional "if p then q ," then the component propositional sign p is a mere display of an assertion, not an assertion. In contrast, a "concrete occurrence of" p by itself would be an assertion. Thus, instead of accounting for Frege's observation as (Geach's) Frege does, by asserted and unasserted occurrences of a proposition, Kimhi accounts for it by occurrences of a proposition in which it is an assertion as against occurrences in which it merely displays an assertion.

It's not clear to me, however, that Kimhi's account is a rejection of the content/force distinction. As it occurs in "not- p ," p is supposed not to be an assertion, but to be a mere display of one. But isn't p still a proposition(al) symbol in that occurrence? Does it not follow that that merely assertion-displaying occurrence of p is a proposition that is *not an assertion*. Moreover, does Kimhi not insist that p in this occurrence is the very same proposition as p in an assertion-being occurrence? How could it be, then, that a proposition is inseparable from assertoric force?

Perhaps the idea is that a proposition somehow essentially involves assertoric force, but this assertoric force which is woven into the proposition's being is merely potential. In some occurrences it is actualized, in others not. But this position seems a mere verbal variation on a content/force distinction: content is

proposition–cum–potential assertion, the distinct forces to which this content may be joined are the actualization and the mere displaying of that content.

Perhaps Kimhi takes himself to have rejected the content/force distinction because a proposition in merely assertion–displaying occurrences is not pure content, but has some other force than assertoric. That is, proposition never occurs without some force. But what force could it be? Something like suppositional or neutral force? If so, then it seems again little more than a verbal variant of Frege’s (as opposed to Geach’s) content/force distinction. I take the intuitive basis of Frege’s content/force distinction to be expressed in this passage

[I]s it not a great result when the scientist after much hesitation and laborious researches can finally say “My conjecture is true” (*T*, p. 61)?²⁴

The key here is the very thought grasped earlier in forming the conjecture is later acknowledged as true. One could, if one wishes, re–describe this as: previously the scientist grasped a thought with neutral force, now she judges the same thought with asserting force. I doubt Frege would find this reformulation objectionable. No more objectionable than: previously she was involved in a merely assertion–displaying occurrence of a proposition; now she was involved in a self–identifying occurrence of that same proposition.²⁵

§3.3. Frege on Thought vs Force

The most fundamental reason for taking Frege to be impervious to Kimhi’s criticisms is that Kimhi misunderstands Frege’s conception of thought. As noted above, I take Frege to be committed to the recognitional conception of judgment.

²⁴ ist es nicht ein großer Erfolg, wenn nach langem Schwanken und mühsamen Untersuchungen der Forscher schließlich sagen kann ‘was ich vermutet habe, ist wahr’?

²⁵ Kimhi’s notion of merely assertion–displaying occurrences of propositions is akin in certain ways to Hanks (2007, 2015) who holds that although a proposition is essentially assertoric, that force can be “cancelled” in embedded contexts. Here the question is, of course, how a proposition whose assertoric force has been cancelled differs from a proposition with no force. Jean–Philippe Narboux’s (2021) review of Kimhi brought to my attention another construal of Frege’s observation akin to Kimhi’s, in Anscombe (2015a). Anscombe’s account involves a “basic notion of assertedness, *assertedness in a context* — which may be the proposition itself, in which context it is always an assertion— and an absolute notion of assertion ... to be defined in terms of assertion in, and completeness of, a context” (ibid., p. 169). But then does it not follow that a proposition *p* which is asserted in a complete context which is *p* may not be asserted in another complete context, and so, according to the theory, not asserted?

Talk of taking the step to a truth–value has to be cashed out in the more fundamental term of recognizing what is the case in the realm of reference, and sense plays an ineliminable role in this, because this “what is the case” concerns what is presented by the senses that make up the thought judged. Moreover, for Frege there is no conception of thought that isn’t something that puts a thinker in a position to make a judgment. In Wittgensteinian terms, a thought for Frege essentially presents how things are in the world, though of course Frege himself would be suspicious of the invocation of essence here.

The thought/force distinction is not optional for Frege, for two reasons. The first I have already mentioned. First, Frege conceives of scientific advance (which for him includes mathematical advance) as often precipitated by conjecturing something to be the case, such that the advance is describable as, e.g., “I see now that my conjecture is true”. Second, judging may be recognizing that what a thought represents is *not* the case, and in such cases of judging the scientist may express her recognition as “I see now that my conjecture is false”. A Fregean thought, like a Wittgensteinian proposition, says how things are in the realm of reference (Wittgenstein: reality); but, in judging, a thinker who has grasped a thought or understood a proposition may come to recognize that things are *not* as the proposition or thought represent them to be. How a thought represents things to be is independent of the agreement or disagreement of that representation with reality, and it’s on that independence that Frege insists in drawing the distinction between thought and force.

§4. Thoughts as Senses of Names of Truth–Values

A strand of Wittgenstein’s criticism of Frege appears very early, in the “Notes on Logic” dictated in the fall of 1913:

Frege said “propositions are names”; Russell said “propositions correspond to complexes”. Both are false; and especially false is the statement “propositions are names of complexes”. (Wittgenstein 1913, hereafter *NL*, 1–4; 97)^{26, 27}

Frege’s conception, as we saw in §1 above, is that assertoric sentences express thoughts, refer to one of the two truth–values, and belong to the same logical

²⁶ References to “Notes on Logic” are given in the form “x–y”, where “x” is “S” if the citation is to the “Summary,” and is the numeral “n” if the citation is to the nth Manuscript, followed by a semi–colon and page number(s) in *NB*.

²⁷ Contrast May (2018, p. 114) who suggests that Wittgenstein takes propositions to refer to facts.

category as proper names. Many readers of the *Tractatus* take this passage from *NL* to be based on problems that Wittgenstein discerns in the consequence of Frege’s conception that thoughts are senses that present an object that is a truth-value. In this section I discuss five accounts of what these problems are, and show that none of the five, as they stand, constitutes an objection that should trouble Frege.

§4.1. Functions whose Values are Truth-Values alongside other Objects

The first Wittgensteinian criticism of Frege I will discuss is based on Peter Sullivan’s “The Sense of ‘A Name of a Truth-Value’” (1994). If the truth-values are objects, then nothing bars definitions of functions such as

$$f(x) \begin{cases} \text{the True,} & \text{if } x = 1 \pmod 3 \\ \text{the False,} & \text{if } x = 2 \pmod 3 \\ x^x & \text{if } x = 0 \pmod 3 \\ \dot{\varepsilon}(\varepsilon = \varepsilon) & \text{if } x \text{ is not a} \\ & \text{number} \end{cases}$$

Does $f(235 + 3917)$ express a thought? It looks as if we have to do a calculation first, to figure out if this expression refers to a truth-value, before we can know if it expresses a thought.

But the issue for functions such as f arises because it’s not a concept; it doesn’t map all objects to only one of two truth-values. So, suppose we changed the last two clauses of f to:

$$f(x) \begin{cases} \text{the True,} & \text{if } x = 1 \pmod 3 \\ \text{the False,} & \text{if } x = 2 \pmod 3 \\ \text{the False} & \text{if } x = 0 \pmod 3 \\ \text{the False} & \text{if } x \text{ is not a number} \end{cases}$$

Then $f(235 + 3917)$ is merely a different way of expressing the thought that $235 + 3917 = 1 \pmod 3$.

On my interpretation, Frege takes concepts to be, or to be representable by, truth-valued functions *not merely* because the predicative nature of concepts may be captured by characteristic functions, in which case *any* two objects would do. The reason for taking the values of these characteristic functions to be the truth-values is that the falling of an object under a concept is what is represented by thoughts expressed by logically simple predications. Thus the recognition of an object falling under a concept is nothing other than the judgment that it does so, i.e., of the acknowledgment of that thought as true, and of taking the step from that thought to the True. A parallel point holds of the failure of an object to fall under a concept: recognition of that underlies stepping to the False. Thus, taking concepts to be (represented by) truth-valued functions reflects not merely the predicative nature of concepts but also the role concepts play in judgment.

§4.2. A Black Spot

The next two accounts take the problem Wittgenstein discerns as the point of one of the weirder *Tractatus* remarks about truth and falsity. This occurs already in “Notes on Logic,” and so is one of Wittgenstein’s earliest ideas about logic that made it into the *Tractatus*:

4.063

A picture to explain the concept of truth: black spot on white paper; the form of the spot can be described by stating for each point of the surface whether it is white or black. The fact that a point is black corresponds to a positive fact, that a point is white (not black) to a negative fact. If I designate a point of the surface (a Fregean truth-value), then this corresponds to the supposition that is put up for judgement, etc. etc.

To be able to say that a point is black or white, however, I must first know when a point is called black and when white; to be able to say that “*p*” is true (or false), I must have determined under what circumstances I call “*p*” true, and I thereby determine the sense of the proposition.

The point at which the analogy breaks down is now this: we can indicate a point on the paper without even knowing what white and black are; but nothing at all corresponds to a proposition without sense, since it does not designate a thing (truth-value) whose properties are called, say, “false” or “true”; the verb of a proposition is not “is true” or “is false” —as Frege

supposed— but that which is “true” must already contain the verb.²⁸

Proops’s “The Early Wittgenstein on Logical Assertion” (1997) provides a clear account of the analogy and its breakdown. To begin with, a Fregean thought is what Wittgenstein calls the sense of proposition. On the side of the spot, “we can single out a point without knowing what black and white are, for we can make use of the coordinates of the plane” (Proops 1997, p. 131). There are two parts to this claim about the spot, with two analogous counterparty claims about thoughts and truth-values. First, a point may be identified by its coordinates. The analogous claim is that a truth-value is referred to by means of a thought. Second, the coordinates identification of a point is entirely independent of the colour of that point. The analogous claim is that the reference to a truth-value by a thought is entirely independent of the concepts of truth and falsity. Proops reads Wittgenstein as claiming that the analogy breaks down at the second claim. The identification of a truth-value as the reference of a thought would take the form of a definite description: the truth-value of such-and-such a thought. This requires provision of a specification of that thought, which is the specification of the sense of a proposition. But, by 4.024, the sense of a proposition is the condition for it to be true. Hence the specification of a thought involves the concept of truth, and the second part of the analogy fails.

What exactly is the criticism, though? Assuming that the criticism is meant to have bite against Frege, it appears to have to be this. Frege accepts that the thought expressed by an assertoric sentence is its truth-conditions. But his conception of thoughts as referring to the truth-value objects debars him from

²⁸ Ein Bild zur Erklärung des Wahrheitsbegriffes: Schwarzer Fleck auf weißem Papier; die Form des Fleckes kann man beschreiben, indem man für jeden Punkt der Fläche angibt, ob er weiß oder schwarz ist. Der Tatsache, daß ein Punkt schwarz ist, entspricht eine positive —, der, daß ein Punkt weiß (nicht schwarz) ist, eine negative Tatsache. Bezeichne ich einen Punkt der Fläche (einen Frege’schen Wahrheitswert), so entspricht dies der Annahme, die zur Beurteilung aufgestellt wird, etc. etc.

Um aber sagen zu können, ein Punkt sei schwarz oder weiß, muß ich vorerst wissen, wann man einen Punkt schwarz und wann man ihn weiß nennt; um sagen zu können: »*p*« ist wahr (oder falsch), muß ich bestimmt haben, unter welchen Umständen ich »*p*« wahr nenne, und damit bestimme ich den Sinn des Satzes.

Der Punkt an dem das Gleichnis hinkt ist nun der: Wir können auf einen Punkt des Papiers zeigen, auch ohne zu wissen, was weiß und schwarz ist; einem Satz ohne Sinn aber entspricht gar nichts, denn er bezeichnet kein Ding (Wahrheitswert) dessen Eigenschaften etwa »falsch« oder »wahr« heißen; das Verbum eines Satzes ist nicht »ist wahr« oder »ist falsch« —wie Frege glaubte—, sondern das, was »wahr« ist muß das Verbum schon enthalten.

taking thoughts to be truth–conditions.

Narboux’s “Négation et totalité dans le *Tractatus* de Wittgenstein” (2009) advances a slightly different account of the failure of the analogy. Understanding a proposition requires knowing its truth–conditions. But, either in addition to or as an elaboration of the notion of truth–conditions, Narboux invokes the ideas of a proposition being “posed–as–true,” and of how reality is to be compared with that proposition:

it is not possible to understand a proposition that would not be posed–as–true Now, according to Frege’s conception, not only is it possible to understand a thought without knowing whether it is true or false, but it is also possible to understand it without knowing how reality is to be compared to it, that is to say, without knowing what its possible truth and falsity consist in. Above all, according to Frege’s conception, a thought could present a situation (“the circumstance that p ”), thus symbolizing, without already being posed–as–true. (2009, p. 174).²⁹

Again, for this to constitute an internal critique of Frege, it seems to have to comprise two claims. First, Frege is committed to taking assertoric sentences or the thoughts they express to be “posed–as–true,” and to taking grasp of thought to require knowing how reality is to be compared with it. Second, Frege’s conception of thought precludes that thought as being “posed–as–true”.

That Frege holds thoughts to be or to involve truth–conditions is well–attested by *Grundgesetze* §32:

not only a reference but also a sense belongs to all names correctly formed from our signs. Every such name of a truth–value *expresses* a sense, a *thought*. For owing to our stipulations, it is determined under which conditions it refers to the True. The sense of this name, the *thought*, is: that these conditions are fulfilled (*GgI*, p. 50; emphases Frege’s).³⁰

²⁹ il n’est pas possible ... de comprendre une proposition qui ne serait pas posée–comme–vraie Or, d’après la conception de Frege, non seulement il est possible de comprendre une pensée sans savoir si elle est vraie ou fausse, mais encore il est possible de la comprendre sans savoir comment la réalité doit lui être comparée, c’est–à–dire sans savoir en quoi consistent sa vérité et sa fausseté éventuelles. Surtout, à suivre la conception de Frege, une pensée pourrait présenter une situation (« la circonstance que p »), donc symboliser, sans être encore posée–comme–vraie.

³⁰ nicht nur eine Bedeutung, sondern auch ein Sinn kommt allen rechtmässig aus unsern Zeichen gebildeten Namen zu. Jeder solche Name eines Wahrheitswerthes drückt einen Sinn, einen Gedanken aus. Durch unsere Festsetzungen ist nämlich bestimmt, unter welchen Bedingungen er das Wahre bedeute. Der Sinn dieses Namens, der Gedanke ist der, dass diese Bedingungen erfüllt sind.

Frege states here that his stipulations governing the signs of the *Grundgesetze* Begriffsschrift fixes the conditions under which a Begriffsschrift name of a truth-value refers to the True, i.e., the conditions under which that name expresses a true thought. Most importantly, Frege characterizes that thought as that the truth-conditions are *fulfilled*. The fulfillment of these conditions clearly implies that the thought *is true*. But, these conditions do not involve that thought. I mean that if a Begriffsschrift name N of a truth-value expresses the thought T that the condition C for N to be true are fulfilled, C is not that T is true, nor does C have any sub-condition that T is true. One might rephrase these last points as: no thought states of itself that it is true, but every thought is that a certain condition is fulfilled, a condition in which the truth of that thought consists.

It thus seems *prima facie* reasonable to take it that on Frege's *Grundgesetze* §32 conception, every thought is, in Narboux terminology, "posed-as-true". This also means that, in grasping such a thought, one would then know what would have to be the case in reality for the thought to be true —the conditions would have to be fulfilled, and what would have to be the case in reality for the thought to be false— the failure of the conditions to be fulfilled. It seems reasonable to rephrase this as: by grasping a thought one knows how it is to be compared to reality.

The question, then, for the readings of Proops and Narboux is this. Why should we think that Frege is also committed to characterizations of thought that are in tension with his view of thoughts as truth-conditions, and as posed-as-true so that grasp of a thought furnishes knowledge of how it is to be compared to reality?

§ 4.3 Reversibility and Duality

Something like an answer to this question may be found in Diamond's "Truth before Tarski" (2002). She ascribes to Wittgenstein a criterion that has to be satisfied by any expression that functions logically as a proposition that expresses thought. The bases of the criterion are in 4.0621

... However, it is important that the signs " p " and " $\sim p$ " can say the same. For it shows that nothing in reality corresponds to the sign " \sim ".

...

The propositions " p " and " $\sim p$ " have opposite sense, but one and the same reality

corresponds to them.³¹

and in Geach (1982)'s attempt to make more precise how it is that " p " and " $\sim p$ " have the opposite sense and yet can say the same, in terms of a logic of duality. For " p " and " $\sim p$ " to have opposite senses is for a sentence p in a given language L and its negation in L to be "contradictory opposites" of one another. " p " and " $\sim p$ " can say the same because, for every language L there exists a dual language L_D , with the same vocabulary and syntax, such that each sentence of L_D "equiform" with a sentence of L says the contradictory opposite of that sentence of L . In terms of this logic of duality one may formulate the logical distinction between propositions and names. To begin with, Geach provides as a compositional translation from sentences of L to their duals in L_D that *preserves what is said*. In such a translation, sentences of L are mapped to their negations, negations to the sub-sentence negated, conjunctions to disjunctions, etc. But proper names and function names are mapped to themselves; names are self-dual. The logical criterion of being a proposition expressing a thought is thus non-self-duality.

The problem for Frege then is that he takes assertoric sentences that express thoughts to have a function–argument structure. In particular, simple predications, such as "Theaetetus flies" are taken to have the logical structure of complex functional names such as "the present Queen of England", which is the completion of a function name, "the present Queen of ξ ", with the proper name "England". Since both proper and function names are self-dual, it follows that assertoric sentences as Frege conceives of them are also self-dual, and so fail the logical criterion of expressing a thought.

In order to account for the remark that "the verb of a proposition is not 'is true' or 'is false'—as Frege supposed," Diamond claims that the "main verb ... in simple predications ... will be dual to its negation," and so, for Wittgenstein,

A verb is a verb, logically speaking, only if it is not self-dual, and no sentence is a sentence, logically speaking, unless it has a verb that is genuinely a verb in this sense (2002, p. 264).

Diamond then sees Wittgenstein as taking the horizontal sign of the

³¹ Daß aber die Zeichen " p " and " $\sim p$ " das gleiche sagen können, ist wichtig. Denn es zeigt, daß dem Zeichen " \sim " in der Wirklichkeit nichts entspricht.

...

Die Sätze " p " and " $\sim p$ " haben entgegengesetzten Sinn, aber es entspricht ihnen eine und dieselbe Wirklichkeit.

Grundgesetze to function logically as a verb, since the horizontal may be understood as expressing “ ‘is one and the same as the True’ or something like that, in asserted propositions”, so Frege “allows for an opposite verb ‘is not the True’ (or ‘is the False’, as Wittgenstein has it) in asserted propositions” (ibid., pp. 264–265). Since, as we saw, Wittgenstein thinks that assertoric sentences on Frege’s conception lack verbs, Wittgenstein takes Frege to use the horizontal together with the negation stroke in an attempt to restore the expression of thoughts at least to the sentences of Begriffsschrift.

As it stands, this line of argument presents an internal criticism of Frege only if Frege is committed to non-self-duality as a necessary condition of expression of thoughts. Diamond in fact appears to be ambivalent on whether Frege is so committed, and thus whether Wittgenstein’s argument would be compelling for Frege. I’ll now show that, even on the assumption that Frege is committed to non-self-duality as a criterion of thought expression, he would not be moved by this argument.

The key is to focus on the nature and role of the two primitive logical objects the True and the False. Concepts are (represented by) truth-valued functions. As discussed in §1, this means that talk of a concept mapping an object to the True supervenes on that object falling under that concept.

Now let’s look again at *Grundgesetze* §32: Frege claims that, given his stipulations, each well-formed name of the Begriffsschrift is determined under which conditions it refers to the True. The sense of this name, the *thought*, is: that these conditions are fulfilled. It is determined, for instance, that the name “*Fa*” refers to the True just in case the value of the function denoted by “ $F\xi$ ” for the argument denoted by “*a*” is the True. The thought expressed by “*Fa*” then is that these conditions are fulfilled, i.e., that the value of the function denoted by “ $F\xi$ ” for the argument denoted by “*a*” is the True. But this condition supervenes on the condition that the object denoted by “*a*” falls under the concept denoted by “ $F\xi$ ”. This condition is what, fundamentally, is expressed by “*Fa*”; it is the thought expressed by “*Fa*”. Could “*Fa*” express the “contradictory opposite” of this thought? *Prima facie* the answer is yes, by a simple “reversal” of the stipulation for “*Fa*” to refer to the True: it does so just in case the value of the function denoted by “ $F\xi$ ” for the argument denoted by “*a*” is the False. This condition supervenes on the condition that the object denoted by “*a*” does not fall under the concept denoted by “ $F\xi$ ”. Thus, even if it’s not clear whether, for Frege, which if any component of the name “*Fa*” counts as a “verb,” there is a *prima facie* case that the logic of duality may be made to apply to “*Fa*”.

§4.4. Opposition of Thoughts as Opposition of Objects

In “Wittgenstein against Frege and Russell” (2002), Ricketts advances an account of a Wittgensteinian objection to Frege closely related to Diamond’s. Ricketts sees Frege as holding a conception of “a sentence and its negation” as expressing “opposed contents or thoughts” (ibid., p. 244). Here’s one example, from the likely very early unpublished manuscript “Logik”:

Before we judge, we often question. ... We grasp the content of a truth before we acknowledge it as true, but not merely this but rather also the opposite; with the question we waver between opposites [*Gegensätzen*]. ... This opposition or contradiction [*Gegensatz oder Widerstreit*] is so understood that we automatically reject one side as false when we acknowledge the other as true, and conversely. The rejection of the one and the acknowledgment of the other are one and the same (*L*₁, pp. 7–8).³²

This view of the “opposition” between a sentence and its negation is an acknowledgment of “sense in something like the way Wittgenstein understands this notion” (Ricketts 2002, p. 244). But once Frege comes to take sentences to be proper names of the truth-values, he moves “the opposition between a thought and its negation from the realm of *Sinn* to the realm of *Bedeutung*” (ibid., pp. 244–245). Taking sentences to be proper names of truth-values means that assertions of Begriffsschrift sentences are conceived as “putting forward” the objects designated by different names as identical to the True. In particular, assertions of a sentence and its negation are the putting forward of two objects as the True. Given Frege’s stipulation of the function expressed by the negation stroke in *Grundgesetze* §6, if Δ is a name of the True, then “ $\neg\Delta$ ” names the False, and if Δ names the False, then “ $\neg\Delta$ ” names the True, and Frege describes the negation-stroke “ $\neg\xi$ ” as “a [function] sign by means of which every truth-value is transformed into its *opposite*” (IGgI §§ 6, 10; emphasis mine).³³ That is to say, Frege characterizes the True and the False as “opposite” truth-values.

The problem, for Ricketts’s Wittgenstein, is that although there is a sense in which objects “can oppose each other, as when one chair faces another” (Ricketts

³² Ehe wir urteilen, fragen wir oft. Wir erfassen den Inhalt der Wahrheit, ehe wir ihn als wahr anerkennen, aber nicht bloss diesen, sondern auch den entgegengesetzten; denn bei der Frage schwanken wir zwischen Gegensätzen. Dieser Gegensatz oder Widerstreit ist so zu verstehen, dass wir das eine Glied von selbst als falsch verwerfen, wenn wir das andere als wahr anerkennen und umgekehrt. Die Verwerfung des einen und die Anerkennung des anderen sind dasselbe (*NS*, 8).

³³ ein [Functionsz]eichen ... durch welches jeder Wahrheitswerth in den entgegengesetzten verwandelt wird.

2002, p. 244), it is not in this sense that a sentence and its negation express opposite thoughts. What it is for a thought to oppose its negation is for that thought to disagree with or to contradict its negation (*idem*).

The force of this criticism evidently depends on what it is for a thought to disagree or contradict another thought. I now sketch an account of how Frege's conception of a thought and its negation as denoting opposed truth-values does in fact rest on the disagreement or contradiction between those thoughts, on one interpretation of disagreement or contradiction.

The account begins from the account of Frege's conception of truth and falsity of §1. A thought's denoting the True supervenes on the obtaining of what that thought represents as being the case in the realm of reference. A thought's denoting the False supervenes on the failure to obtain of what that thought represents. On this account, what the "opposition" of the truth-values ultimately comes to, is an opposition between the obtaining of what is represented by a thought and non-obtaining of what a thought represents. In these terms, the "transformation" of the True into the False effected by attaching the negation-stroke to a sentence is at bottom a "transformation" of the obtaining of what a thought represents into its non-obtaining.

But now, one plausible way of characterizing how the negation of a statement *S* contradicts or disagrees with *S* is that the negation says that what *S* says is the case is not in fact the case, does not in fact obtain. But then, it is unclear that Frege's talk of opposed truth-values fails to capture the contradiction or disagreement between a thought and its negation.

The foregoing is no more than a *prima facie* case for Frege's being able to meet the Wittgensteinian criticism elaborated by Ricketts. In fact, as we will see in §5 next, the case begins to unravel when one considers what thought is expressed by a negated sentence and what that thought represents. The key insight leading to this unravelling is indicated by Ricketts's focus on Frege's conception of negation and opposite thoughts.

§5. Negation, Opposite Thought, and Falsity

In this section I speculate on the deepest difference between Frege and early Wittgenstein.³⁴

The difference arises from what I see as tensions in Frege's thinking among his conceptions of judgment, of falsity and the "opposite" of a thought, and of

³⁴ In many moments of these speculations I am very much indebted to Narboux (2009).

negation as a truth–function.

Wittgenstein, I will suggest, saw a way to resolve these tensions on the basis of a primitive notion of possibility that undergirds the conception in the *Tractatus* of propositions as pictures and of the sense of propositions.

§5.1. The Composition of Thoughts and Negation

There is a longstanding debate over whether Frege holds that thoughts are constituted from senses, as opposed to having components that result from analyses imposed on a fundamentally undivided whole.³⁵ I do not take sides on this debate. But I suggest that Wittgenstein’s understanding of Frege include ascribing to Frege the view that thoughts are constituted from senses. Here are two expressions of this view in Frege’s writings. The first is from *Grundgesetze* §32, a work Wittgenstein most likely had read:

the simple or complex names of which the name of a truth–value consists contribute to expressing the thought, and this contribution of the individual name is its *sense*. If a name is part of the name of a truth–value, then the *sense* of the former name is *part of the thought* [*Sinn jenes Namens Theil des Gedankens ... ist*] expressed by the latter (*GgI*, p. 51; emphasis Frege’s).³⁶

The second is from “Gedankengefüge,” the last essay Frege published, and so not something Wittgenstein could have read before writing the *Tractatus*. It brings up three additional points about the composition of thoughts. First, as in *Grundgesetze*, Frege claims that parts of thoughts correspond to parts of sentences expressing those thoughts:

[W]e [can] distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought

³⁵ For the former view see Bell (1987); for the latter see Levine (2002); Dummett (1981, chapter 15) offers a reconciliation of both sides. For a fuller bibliography of all the sides to this controversy see Kemmerling (2011), who champions the analytic imposition interpretation.

³⁶ Die einfachen oder selbst schon zusammengesetzten Namen nun, aus denen der Name eines Wahrheitswerthes besteht, tragen dazu bei, den Gedanken auszudrücken, und dieser Beitrag des einzelnen ist sein Sinn. Wenn ein Name Theil des Namens eines Wahrheitswerthes ist, so ist der Sinn jenes Namens Theil des Gedankens, den dieser ausdrückt.

(Frege 1923, hereafter *CT*, p. 36).^{37,38}

Second, Frege notes that “we really talk figuratively when we transfer the relation of whole and part to thoughts” (*CT*, p. 36).³⁹ Finally, Frege claims that a phrase of negation expresses a part of thoughts:

In my article “Negation” I considered the case where a thought seems to be composed of a part needing completion (the unsaturated part, as one may call it) which corresponds in language to the negating word, and a thought (*CT*, pp. 36–37).⁴⁰

Is there a non-figurative account of what it is for senses to be parts of a thought? I take such an account to be that a thought is composed of a set of senses just in case there is a rule-governed determination of the truth or falsity of that thought in terms of the referents presented by those senses. Sticking again to simple predications like “Theaetetus flies”, the thought it expresses is constituted from the senses of the name “Theaetetus” and the sense of the predicate “ξ flies” because the statement is true just in case the object denoted by the sense of the name falls under the concept denoted by the sense of the predicate. Finally, it seems clear that here Frege takes a phrase of negation to express a sense that (a) is a component of thoughts expressed by use of these phrase and (b) it is an unsaturated component. The unsaturatedness of the sense of negation indicates that it is, and denotes a function. Presumably Frege has in mind the function stipulated in *Grundgesetze* §6, mapping each of the truth-values to the other, and all other objects to the True. The view that a phrase of negation expresses a sense that is part of the thoughts expressed by sentences in which that phrase occurs and denotes the negation truth-function I’ll call the “orthodox” Fregean position on negation.

³⁷ [W]ir in dem Gedanken [] Teile unterscheiden [können], denen Satzteile entsprächen, so daß der Aufbau des Satzes als Bild gelten könnte des Aufbaues des Gedankens.

³⁸ Frege seems to take the claim that parts of thoughts correspond to parts of sentences to explain how “[w]ith a few syllables, [language] expresses an incalculable number of thoughts” (“die Sprache ... mit wenigen Silben unübersehbar viele Gedanken ausdrückt”) (*CT*, p. 36). The phenomenon to be explain is perhaps akin to what is sometimes called the productivity of language in contemporary linguistics (see Chomsky 1968).

³⁹ sprechen wir eigentlich in einem Gleichnisse, wenn wir das Verhältnis von Ganzem und Teil auf den Gedanken übertragen.

⁴⁰ In meinem Aufsätze “Die Verneinung” habe ich den Fall betrachtet, daß ein Gedanke zusammengesetzt erscheint aus einem ergänzungsbedürftigen oder, wie man auch sagen kann, ungesättigten Teile, dem sprachlich das Verneinungswort entspricht, und einem Gedanken.

§5.2. Opposite Thoughts and the Function of Negation

Throughout his writings, Frege claims that to each thought there correspond an opposite thought. In §4 we saw an example, from the first “Logik” manuscript. The same claim appears in the last but one paper Frege published, “Die Verneinung”:

[F]or every thought there is a contradictory {footnote: We could also say “an opposite”.} thought; we declare a thought as false by acknowledging the truth of its contradictory (Frege 1918, hereafter *Neg*; p. 154).⁴¹

What is an opposite thought? Evidently it has two features:

- When we ponder a (yes–no) question we grasp a thought without judging it, and at the same time we also grasp the opposite thought.
- Acknowledging a thought as true *is the same as* rejecting the opposite thought as false, rejecting that thought is the same act as acknowledging its opposite.

These features don’t quite answer my question. What I would like to know is the composition of the opposite of a thought: suppose that a thought T is composed of such–and–such senses, of what senses is the opposite of T , T' , composed?

It might seem that there’s a fairly simple answer to this question, because there’s evidence that Frege thinks that signs of negation are used to express opposite thoughts. For example, here’s a compact presentation of Frege’s argument that there is no need to adopt a sign to indicate rejection or denial in addition to an (assertion) sign which indicates assertion:

Each thought is set against an opposite, such that rejecting one of them coincides with acknowledging the other. Judging is the choice between opposite thoughts. The acknowledgment of one of them and the rejection of the other is one act. So there is no need of a special sign for the rejection of a thought, but rather only one for negation without

⁴¹ Zu jedem Gedanken gehört demnach ein ihm widersprechender {Fußnote: Man könnte auch sagen »ein entgegengesetzter«.} Gedanke derart, daß ein Gedanke dadurch als falsch erklärt wird, daß der ihm widersprechende als wahr anerkannt wird.

asserting force. (Frege 1979, hereafter *PW*, p. 198)⁴²

Here Frege does not claim that there is no act of rejecting thoughts distinct from acknowledging, i.e., judging them (Although in “Die Verneinung,” he does argue against the need for positing rejection). Here Frege’s reasoning is that, because rejecting a thought is one and the same act as judging the opposite thought, one could dispense with a sign for rejection, if there is a sign for expressing the opposite thought. And there is, it’s the negation sign. So, given the orthodox Fregean view of negation, the components of T' are just those of T with the addition of the sense of a negation sign.

But Frege also gives us reason to doubt this account of the composition of opposite thoughts. Consider an argument Frege gives for double negation cancellation:

If the first thought is the opposite of the second, then the second is the opposite of the first. To declare false the thought that Peter did not come to Rome is to assert that Peter came to Rome. We could declare it false by inserting a second “not” and saying “Peter did (not) not come to Rome” or “It is not true that Peter did not come to Rome”. And from this it follows that double negation cancels. *The opposite of the opposite is the original* (*L₂*, p. 149; emphases mine).⁴³

Frege’s argument for double–negation cancellation here is *not* based on the claim that a doubly–negated sentence has the same truth–value as that sentence (though clearly what he says is consistent with that claim). The fundamental premise of his argument is rather that the opposite of the opposite of a thought is *that very thought*. Thus, if a sentence S expresses T , not– S expresses T' and not–not– S expresses the opposite of T' , but that’s just T . This is why double–negation cancels.

It’s not clear that this view is consistent with the orthodox Fregean account of

⁴² Jedem Gedanken steht ein entgegengesetzter gegenüber, sodass die Verwerfung des einen mit der Anerkennung des anderen zusammenfällt. Das Urteilen ist die Wahl zwischen entgegengesetzten Gedanken. Die Anerkennung des einen und die Verwerfung des andern ist eine Tat. Man braucht also für die Verwerfung kein besonderes Zeichen, sondern nur für die Verneinung ohne behauptende Kraft. (*NS*, p. 214).

⁴³ Wenn der erste Gedanke dem zweiten entgegengesetzt ist, so ist auch der zweite dem ersten entgegengesetzt. Indem man den Gedanken, dass Petrus nicht nach Rom kam, für falsch erklärt, behauptet man, dass Petrus nach Rom kam. Man könnte das für falsch erklären mit einem zweiten eingeschobenen „nicht“ bewirken: „Petrus kam [nicht] nicht nach Rom“ oder „Es ist nicht wahr, dass Petrus nicht nach Rom kam“. Und es ergibt sich so, dass die doppelte Verneinung sich aufhebt. Das Entgegengesetzte des Entgegengesetzten ist das Ursprüngliche (*NS*, 161).

negation. The negation sign, on this view, denotes a function, and so expresses a sense that presents this function. As we have seen, Frege takes the sense of the negation sign to be a component of the thoughts expressed by sentences in which it occurs. But then, *prima facie*, the thought expressed by the negation of a sentence would have to differ from that expressed by the sentence. How, then, could the thought expressed by the double negation of a sentence be the same as the thought expressed by that sentence? If the sense of “not” is an addition to T to yield its opposite T' , then surely to get to the thought expressed by not–not– S a second dose of the sense of “not” would have to be added to T' . How then could we get just plain old T as a result?

Now, of course, Frege holds that there are expressions whose sense do not make a contribution to the senses of expressions in which they occur. The truth predicate is such an expression. So maybe double–negation has the same kind of logically anomalous features as the truth predicate. But this idea, I now try to show, doesn’t restore the “orthodox” Fregean account of negation but rather points towards the *Tractatus*.

What exactly Frege’s views of the truth predicate are has been subject to much dispute; I will consider only a late note:⁴⁴

So the word “true” seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought. And although this attempt miscarries, or rather through the very fact that it miscarries, it indicates what is characteristic of logic. ... “[T]rue” only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word “true” at all but in the asserting force with which a sentence is uttered (*PW*, p. 252).⁴⁵

Suppose that double negation is like the truth predicate. What does this passage suggest about how double negation works? Frege claims that the predication of truth seems, *per impossibile*, to make asserting force a contribution to thought. But what is asserting force? Here’s a suggestion. Asserting force is the way in

⁴⁴ See Ricketts (1986a) and Taschek (2008) for further discussion of this passage. In $\square L$, chapter 3, I advance an account of Frege’s view of the truth–predicate, as well as provide a partial survey of some extant positions. But I do not discuss this text. I believe the story I’ll tell below is consistent with my $\square L$, chapter 3 story.

⁴⁵ So scheint das Wort »wahr« das Unmögliche möglich zu machen, nämlich das, was der behauptenden Kraft entspricht, als Beitrag zum Gedanken erscheinen zu lassen. ... »[W]ahr« eigentlich nur einen missglückten Versuch macht, auf die Logik hinzuweisen, indem das, worauf es eigentlich dabei ankommt, gar nicht in dem Worte »wahr« liegt, sondern in der behauptenden Kraft, mit der der Satz ausgesprochen wird (*NS*, p. 272).

which the uttering of a sentence effects an assertion, i.e., expresses a judgment.

Here then is an analogous claim about double negation. Doubly negating a sentence seems, *per impossibile*, to make the way in which a certain judgment is expressed into a contribution to the thought expressed. What judgment? Obviously that which the sentence itself, uttered with asserting force, would express. That is to say, double negation merely provides a way of expressing a judgment that could also be expressed using the unnegated sentence, it does not affect the thought judged.

We can explain how this alternative method of expression works in terms of Frege's view of the opposition of thoughts and its connection to their acknowledgment and rejection. On this view, the negation sign *does* affect the thought expressed by its occurrence, but its effect on a sentence is to express the opposite thought, and, most importantly, the judgment of this opposite thought is the rejection of the original thought.

I take this to suggest that there are two equivalent ways of conceiving what negation expresses:

1. "changing" the thought to be judged to the opposite thought.
2. "switching" what one is doing with that thought, from acknowledging to rejecting, or *vice versa*.

There are then two ways of explaining why double negation cancels:

1. The opposite of the opposite of a thought is that thought itself.
2. Since acknowledging and rejecting are the only cognitive acts, "switching" twice results in the original act.

But now, what is "changing" a thought to its opposite? Construing this "change" as adding the sense of negation to it is in some tension with Frege's views of opposite thoughts. In particular, it's not clear how, on this model of "changing" to an opposite thought, the opposite of the opposite of a thought could be that thought itself.

So, a Fregean who wants to hold on to this last doctrine of Frege's on opposite thoughts would seem to have some difficulties accounting for the composition of opposite thoughts. Such a Fregean might come to hypothesize that conceiving of negation as "changing" to the opposite thought is best taken to be metaphorical.

Its “cash value” is the second way of conceiving of negation: “switching” from acknowledging to rejecting, or *vice versa*.

It’s not clear, however, that this view is fully satisfactory, from a Fregean perspective. The reason lies in an insight buried in Frege’s idea that rejecting a thought as false is acknowledging the opposite thought as true. Let’s consider this idea in light of Frege’s conception of judgment as recognition of what is the case in the realm of reference. In §1 I characterized rejecting a thought as false or taking the step from that thought to the False as, at bottom, recognizing that what that thought represents fails to obtain. But such a failure to obtain is *also what is the case* concerning the referents of parts of that thought. If, for example, the thought is what is expressed by a simple predication “*Fa*”, then rejecting that thought as false is recognizing what is the case concerning the referents of “*a*” and of “ ξ is *F*”, namely, the object that is the referent of “*a*” does not fall under the concept that is the referent of “ ξ is *F*”. This recognition, then, is a judgment, and what is recognized is what is represented by some thought, specifically the opposite of the thought rejected as false.

So the idea of negation as a switch from acknowledgment to rejection or *vice versa* of a single thought doesn’t capture fully the role that opposite thoughts play in Frege’s conception of judgment.⁴⁶

But then it seems we are back at the problem of the composition of opposite thoughts for Frege. Our last reflections brings out another aspect of the problem. How could the *failure to obtain* of what is represented by one thought *be precisely* the *obtaining* of what another thought represents?

We can elaborate the difficulty for Frege in two closely connected ways.

At one level, Frege’s conception of the truth–value determination of a thought is functional. Let’s set aside complication arising from truth–functional and quantificational complexity, and consider again a simple predication “*Fa*”, expressing a thought *G*. The truth or falsity of *G* is the value of the function that is the referent of “ ξ is *F*” for argument that is the referent of “*a*”. If what *G* represents doesn’t obtain, then the value is the False. No function has more than one value for a single argument. Now, if the thought opposite to *G*, *G'* is composed of the same senses that compose *G*, then the truth–value of *G'* is also the False. In that case what *G'* represents also fails to obtain, contrary to what

⁴⁶ Dummett makes a related point as follows: “there is no such thing as a game in which the object is to try to lose. Rather than say that two people are playing chess, but that they have both agreed to try to lose, we shall say that they are trying to win, but that what they are playing is not chess but a variant of it” (1973, p. 320). For further discussion see Narboux (2009); I do not see, however, that this line of argument is in *Neg*, as Narboux claims.

Frege's notion of opposite thought requires of G' . It follows that G' must be composed of different senses than composes G . But it's not clear that G' can be composed of different senses. Recognizing the non-obtaining of what G represents is recognizing that the referent of " a " does not fall under the referent of " ξ is F ", and this failing of the referent of " a " to fall under the referent of " ξ is F " is that which *is the case* concerning these *very same* referents that G' is supposed to represent. Thus the senses that compose G' would have to present the very same referents as are presented by the senses that compose G .

Now let's reconsider the situation from a more fundamental level than the modelling of concepts by truth-valued functions. At that level, G is true or false depending on whether the object that is the referent of " a " falls or does not fall under the concept that is the referent of " ξ is F ". What it is for what G represents not to obtain is for the referent of ' a ' not to fall under the the referent of " ξ is F ". Now, if G' is composed of the same senses as compose G , then G' would also be true or false depending on whether the referent of " a " falls or does not fall under the referent of " ξ is F ". It follows that if what G represents does not obtain, then neither does what G' represents, again contrary to G' 's being opposite to G . So, again, G' must be composed of different senses than composes G . But it also appears that G and G' cannot be composed of different senses, by the same line of argument presented in the last paragraph. If they are composed of different senses, then it's unclear how the recognition of the non-obtaining of what G represents concerning the referents of its component senses can be, at the same time, recognition of what is the case concerning these very referents.

The upshot of these reflections is this. Frege operates with a model of what a thought represents that is the falling of an object under a concept. The obtaining of what is represented then is that object falling under the concept. The difficulty just rehearsed shows that, in order to be consistent with his insight that rejecting as false is the very same thing as acknowledging the opposite as true, this model has to be given up. An object's *not* falling under a concept has to be on equal footing with that object's falling under that concept as what a thought can represent as holding in the realm of reference. And these two are also on equal footing as what may be the case in the realm of reference. Truth remains univocal, as what is represented being the case. But the thoughts that are mutually opposite representations differ neither in their component senses, nor, therefore, in the referents that the thought may be taken to be about.

This difficulty for Frege, I suggest, posed a task for Wittgenstein: how to conceive of thought so that thoughts that do *not* differ in the referents of their component senses could nevertheless be mutual opposites, in conformity with

the Fregean insight that the failure to obtain of what a thought represents is something that is the case, and is the obtaining of the opposite thought.

§5.3. A Path to the *Tractatus*

Wittgenstein completes the task in the *Tractatus*. In this sub-section, I briefly sketch the philosophical path that leads to that completion, consisting of the Tractarian conception of proposition.⁴⁷

The path begins with Wittgenstein's criticisms of Russell's multiple-relation theories of judgment and understanding, theories in which Russell abandoned the view he held together with Moore that judgment or belief, together with understanding, consist of a subject standing in a cognitive relation to a proposition, a complex entity composed of entities in the world.

The "Notes on Logic" include a theory of proposition and judgment intended to escape the criticisms Wittgenstein directed at Russell's multiple-relation theories. Wittgenstein *reinstates propositions as elements of judgment*. Propositions are *facts* that represent facts in the world. (Not the whole truth, as we'll see.) A fact is an *aspect* of a collection or a composite. Facts have *forms*, something that they have in common with other facts.

For example, consider the English sentence:

Desdemona loves Othello (1)

The sentence is not itself a proposition. Rather, a proposition involves some one among many facts about the sequence of Latin alphabet letters and blank spaces that is (1). One such fact is:

The word "Desdemona" occurs to the left of the word "loves", and
the word "Othello" occurs to the right of "loves" (2)

This fact consists of two syntactical entities, the words "Desdemona" and "Othello", standing in a complex spatial relation, which we may roughly indicate, in Fregean fashion, as

⁴⁷ I give fuller accounts in Shieh (forthcoming[a],[b]).

ξ occurs to the left of the word “loves”, and ζ occurs to the right of “loves”

The *form* of this fact is

Some entity stands in some relation to another entity

Wittgenstein indicates this form with

xRy

This form is something fact (2) about sentence (1) shares with indefinitely many other facts, for instance, taking, as Russell does, *Othello* to be history rather than fiction, the fact consisting of two people, Desdemona and Othello, standing in the relation we indicate as

ξ loves ζ

Fact (2) can be used to represent, or, in Wittgenstein’s terminology in *NL*, symbolize, any fact with this form. The symbolization requires *conventionally adopted* or *stipulated* (*NL*, “convention,” S-15; 94 n 2, 3-14; 102; for stipulation see “laying down”, 4-8; 104) *rules* which fix

- What entities words ‘Desdemona and ‘Othello’ stand for, the “meanings” of these words (*NL*, 2-17; 99, 3-15; 102, 4-8; 104).
- What fact about these meanings are “of like sense” with the syntactic fact (2), i.e., what fact about these meanings makes fact (2) true (*NL*, 4-8; 104).
- What fact about these meanings are “of opposite sense” with fact (2), i.e., makes fact (2) false (*NL*, 4-8; 104).

For example, one could stipulate

- “Desdemona” stands for Aristotle
- “Othello” stands for Plato

- The fact that Aristotle is younger than Plato makes (2) true
- The fact that Aristotle is not younger than Plato makes (2) false.

Call these stipulations (*). These *rules* specify *how* a propositional fact is *compared with* facts, or as Wittgenstein puts it, “A proposition is a standard to which facts behave” (*NL*, S-18; 95; see also 4-8; 104). Propositions are *not only* facts, they are facts *together with* rules of comparison. The proposition that consists of fact (2) together with stipulations (*) we’ll call *p*.

Note that what is stipulated to make (2) false is the fact that Aristotle is *not* younger than Plato. This is an instance of what Wittgenstein calls a “negative fact” (*NL*, 1–7,8; 97, 3–1; 99). Negative facts are also involved in Wittgenstein’s account of negation in *NL*. This account foreshadows 4.0621(1) and (3):

[I]t is important that we *can* mean the same by “*q*” as by “not-*q*”, for it shows that neither to the symbol “not” nor to the manner of its combination with “*q*” does a characteristic of the denotation of “*q*” correspond (*NL* 1–9; 97).

If one interchanged the positive fact stipulated to make fact (2) true with the negative fact stipulated to make (2) false, the result is a different set of stipulation for (2), call them (*'). The proposition that consists of fact (2) together with stipulations (*') is the negation of *p*.

The *NL* theory is coherent provided that the notion of negative fact is. From the first of Wittgenstein’s wartime notebooks we see that he had by late 1914 come to realize that he did not have a coherent account of negative fact, and thus also no coherent accounts of falsity and of negation. This is what Wittgenstein called the “truth–problem (*Wahrheits–Problem*)” (*NB*, 24 Sept 1914).

Together with a standard set of stipulated rules, a fact about the sentence

Catiline denounced Cicero (3)

is false if the negative fact that Lucius Sergius Catalina did not denounce Marcus Tullius Cicero obtains in the world. But, if (3) is *false*, it *does not* describe any aspect of the world. Doesn’t this mean that there is *no fact* that *corresponds to* (3)?

Could we hold that a negative fact is just the absence of a fact? Then it is the *absence of any fact* that corresponds to (3) that makes it false? Well, then

wouldn't it *also* be the absence of any fact corresponding to

Iago loves Cassio

that makes *this* propositional fact false? The absence of a fact is not a feature of the world, not something obtaining in the world. Then it's *not clear how*

1. the absence of any fact corresponding to "Catiline denounced Cicero"
2. the absence of any fact corresponding to "Iago loves Cassio"

are *different*. But it seems these absences *have to be different*, because

- absence 1 doesn't make "Iago loves Cassio" false,
- absence 2 doesn't make "Catiline denounced Cicero" false.

Wittgenstein's truth–problem is a version of an ancient problem of falsity as posed, e.g., by Plato: how is it possible to "say, speak, or think *that which is not ... correctly ... ?*" (*Sophista*, 238c).⁴⁸

I will here mention only one moment of Wittgenstein's struggles with the Truth–Problem, *NB* entries on the 30 and 31 of October, 1914. He starts on the 30th with

Could we say: In " $\sim\phi(x)$ " " $\phi(x)$ " presents [*stellt vor*] how things are *not* [*wie es sich nicht verhält*]?

Even in a picture we could represent [*darstellen*] a negative fact by representing what is *not* the case.

If, however, we admit these methods of representation [*Darstellungsmethoden*], then what is really characteristic of the relation of *representing*?

....

There are different ways of giving a representation [*Darstellungsweisen*], even by means of a picture, and what represents is not merely the sign or picture but also the method of representation [*Methode der Darstellung*]. *What is common to all representation is that*

⁴⁸ See Narboux (2009) for an illuminating discussion of the relations between Plato's discussion of not-being and falsity in Plato: *Sophista* and Wittgenstein's concerns leading to the *Tractatus*.

*they can be right or wrong, true or false (NB, p. 21)*⁴⁹

Here representing what is the case and representing what is not the case are distinct methods or ways of representing “Darstellungsweisen” or “Darstellungsmethoden”. The conception is that we have a picture, say



(4)

and we use that picture to represent in two ways: (a) to represent, e.g., two people fencing, or (b) to represent these two people not fencing. The picture (or sign) by itself does not yet represent; only by adjoining a way of representing (as is the case or as is not the case) is there a representation, is there something that can be true or false.

On the 31st Wittgenstein adds

The way of representing [*Darstellungsweise*] determines how reality has to be compared with the picture (NB, 22).⁵⁰

It may be that this is an echo of how forms symbolize in *NL*. If a picture represents in the “is the case” way, then a positive fact is of like sense with it, and the negative fact matched with that positive fact is of opposite sense with it. If that represents in the “is not the case” way, then the negative fact is of like sense, the matched positive fact of opposite sense.

Now, recall from our earlier discussion Frege’s idea that the opposite of an opposite thought is that thought itself. I said that a Fregean philosopher who wants to hold on to this idea would have reason to be suspicious of the orthodox

⁴⁹ Könnte man sagen: in “ $\sim\phi(x)$ ” stellt “ $\phi(x)$ ” vor, wie es sich nicht verhält?

Man könnte auch auf einem Bild eine negative Tatsache darstellen, indem man darstellt, was nicht der Fall ist.

Wenn wir aber diese Darstellungsmethoden einräumen, was ist dann eigentlich charakteristisch für die Beziehung des Darstellens?

...

Es gibt eben verschiedene Darstellungsweisen, auch durch das Bild, und das Darstellende ist nicht nur das Zeichen oder Bild, sondern auch die Methode der Darstellung. Aller Darstellung ist gemeinsam, daß sie stimmen oder nicht stimmen, wahr oder falsch sein kann.

⁵⁰ Die Darstellungsweise bestimmt, wie die Wirklichkeit mit dem Bild verglichen werden muß.

Fregean view of negation. Such a Fregean might come to conceive of negation, not as “changing” to the opposite thought, but as “switching” from acknowledging to rejecting, or *vice versa*, of a single thought. Wittgenstein at the end of October 1914 was that Fregean, taking negation to work by “switching” between acknowledging that reality is how a thought represents it as being, and rejecting that thought as representing how reality is.

But, “Darstellungsweise” disappears from both *Proto-Tractatus* (Wittgenstein 1971) and *Tractatus*.⁵¹ Why?

I propose that the reason is the Fregean insight that the failure to obtain of what a thought represents is something that is the case, and is the obtaining of the opposite thought. I see this Fregean insight in

4.062

Can't we make ourselves understood with false propositions as we have done up to now with true ones? So long as we know that they are meant to be false. No! For a proposition is true if things are as we say they are by using it [*wahr ist ein Satz, wenn es sich so verhält, wie wir es durch ihn sagen*]; and if we mean $\sim p$ by “*p*”, and things are as we mean it, then “*p*” in the new conception is true and not false.⁵²

To use a proposition is to say how things are, *wie es sich verhält*, which is, in the terms of my reading of Frege, what is the case. And saying how things are is one: there are no affirmative or negative ways of saying how things are, as Wittgenstein thought in *NB*. These former “affirmative or negative ways,” in the Tractarian conception become, *as it were, folded into the “how things are”* that a proposition is used to say.

This Fregean insight is necessary but not sufficient for Wittgenstein to reach the *Tractatus* from *NB*. In late 1914 Wittgenstein was still struggling to substantiate his claim in *NB* that “we *can* mean the same by ‘*q*’ as by ‘not-*q*’” (*NL*, 1–9; 97), in face of the loss of confidence in negative facts occasioned by the Truth–Problem. How did he do it? How did he get from this remark in *NL* to 4.062, with only a change in sentence letter? I want to note that the Truth–

⁵¹ I am grateful to Josh Eissenthal and Tom Ricketts for bringing this disappearance to my attention, thereby leading me to realize that Wittgenstein, by the *Tractatus*, was no longer the Fregean of the previous paragraph, possibly because Frege showed him he should not be.

⁵² Kann man sich nicht mit falschen Sätzen, wie bisher mit wahren, verständigen? Solange man nur weiß, daß sie falsch gemeint sind. Nein! Denn, wahr ist ein Satz, wenn es sich so verhält, wie wir es durch ihn sagen; und wenn wir mit »*p*« $\sim p$ meinen, und es sich so verhält, wie wir es meinen, so ist »*p*« in der neuen Auffassung wahr und nicht falsch.

Problem is not far from the dilemma we left Frege with: how could opposite thoughts be opposite to one another, and yet differ neither in their component senses, nor, therefore, in the referents that the thoughts may be taken to be about? This is just about the question how a single picture, (4), can be used to say that how things are is *not* two people fencing, *or* to say that how things are *is* those two people fencing? When Wittgenstein had negative facts at his disposal, he could answer: by switching the positive and negative facts in the truth- and falsity-making stipulations for (4). But now negative facts are gone, or at least in peril.

What Wittgenstein needed to solve the Truth-Problem he might have glimpsed only at the eleventh hour. Two remarks absent from *Proto-Tractatus*, together with an addition to a remark of *Proto-Tractatus*, are among the last items to make it into the *Tractatus*.

2.033

Form is the *possibility of structure*. (Emphasis mine)⁵³

2.15

That the elements of the picture stand to one another in a determinate way [*bestimmter Art und Weise*] presents [*stellt vor*] things as so standing to one another.

This connection of the elements of a picture is called its structure, and the possibility of this structure its form of depiction [*Form der Abbildung*].⁵⁴

2.151

The form of depiction is the possibility that the things stand to one another as do the elements of the picture.⁵⁵

The coming into being of the *Tractatus* involved the adoption of a distinction between structure and form. The significance of this distinction is, among other things, a transformation of the meaning of a number of formulations from *NL* and *NB*:

⁵³ Die Form ist die Möglichkeit der Struktur.

⁵⁴ 2.15(1) is *Proto-Tractatus* 2.151: »Daß sich die Elemente des Bildes in bestimmter Art und Weise zu einander verhalten stellt vor, daß sich die Sachen so zu einander verhalten«. 2.15(2) incorporates the structure/form distinction absent from *Proto-Tractatus* 2.15101: »Dieser Zusammenhang der Elemente des Bildes heißt [Bildes heiße seine Struktur und ihre Möglichkeit] seine Form der Abbildung«. 2.15(2) additions in square brackets.

⁵⁵ Die Form der Abbildung ist die Möglichkeit, daß sich die Dinge so zu einander verhalten, wie die Elemente des Bildes.

- Propositions are *facts*.
- Propositions are *pictures* of facts.
- A proposition pictures a fact by having a *form* that the fact also has.
- The truth and falsity of propositions result from a *comparison* of propositional facts with what they picture, namely, facts in the world.

In the *Tractatus*, the meanings of these formulations are inflected with modality, in virtue of the notion of form's incorporation of a *primitive conception of possibility*.

A key effect of this transformation is the notion of state-of-things (*Sachverhalt*):

2.031

In a state-of-things (*Sachverhalt*) objects stand to one another in a determinate way.

2.032

The way in which objects hang together in a state-of-things is the *structure* of the state-of-things.⁵⁶

In view of 2.033, we see that a *state-of-things* is the *realization* or the *obtaining* of a *possibility*. A state-of-things “*obtains*” (*besteht*) just in case a *possibility of things* standing to one another in a determinate way *is realized*. One could put it in a slightly different way: a fact that may be a picture is an embodiment of a possibility, suggesting that pictorial possibility is like an expression of a face.⁵⁷

Picturing in the *Tractatus* is fundamentally modal. A picture is a fact in virtue of its elements standing to each other in a determinate way. This *obtaining state* of pictorial elements is a *realization of a possibility* for those pictorial elements. It is a realization of a *form*.

Suppose that the *possibility* of structure *realized by a picturing fact* is *also a possibility for the things* correlated with pictorial elements to be connected in a

⁵⁶ 2.031 Im Sachverhalt verhalten sich die Gegenstände in bestimmter Art und Weise zueinander.
2.032 Die Art und Weise, wie die Gegenstände im Sachverhalt zusammenhängen, ist die Struktur des Sachverhaltes.

⁵⁷ So that discernment of pictorial possibility is like “our recognition of facial expressions” (Diamond 1968, p. 243).

state-of-things. Then the picturing fact can *represent* a *possible* obtaining of a *state-of-things* involving the objects correlated with the pictorial elements.

A propositional picture is *true* if the *possibility it presents* as obtaining is *realized by the things* whose representatives are the elements of the picture. The *falsity* of a proposition consists of the *non-realization by things* of the possibility presented by that proposition. Distinct *unrealized possibilities individuate* distinct *falsehoods*. The Truth-Problem is blocked by incorporating modality into the nature of propositions. This incorporation reinstates negative facts: they are unrealized possibilities.⁵⁸

Now we have also a resolution of Frege's dilemma over opposite thoughts. If mutually opposite thoughts differ neither in their component senses, nor in the referents they are about, how do they differ? From his Tractarian modal perspective, Wittgenstein can now answer: the difference in mutually opposite thoughts consists in the difference between representing the realization and representing the non-realization of a possibility for their common referents.

On the story I'm now bringing to a close, what Wittgenstein could have inherited from Frege that constitutes the great debt he could have owed to Frege, is Frege's insight into the nature of the relation between opposite thoughts. That relation, which Wittgenstein would perhaps characterize as internal, ties the absence from reality of what would answer to a thought to a presence of what would answer to its opposite. Wittgenstein's repayment of that debt would then be to dispel the conundrum Frege's insight posed for Frege, through describing modality at the centre of the nature of thought.

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⁵⁸ The claim that possibility is central to propositional representation in the *Tractatus* is not news. See, for instance, Glock (2006), Zalabardo (2015). I differ from these commentators over some details of how possibility figures in Tractarian picturing, and, unlike them, I trace the philosophical motivation of the Tractarian modal conception of proposition to the Truth-Problem of *NB*.

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What could be the Great Debt to Frege? or *Gottlobius ab paene omni naevo vindicatus*

In this paper I examine a number of interpretations of Wittgenstein’s criticisms of Frege’s conception of logic. One is based on Frege’s rejection of psychologism and alleges that this rejection engenders a tension that is resolved in the *Tractatus*. Another is based on the claim that there are patterns of inference involving what are now known as propositional attitude ascriptions that Frege’s conception of logic is allegedly not equipped to handle. Five others invoke difficulties supposed to arise from Frege’s view of thoughts as senses of names of the two truth-values. I show that none of these interpretations present a compelling criticism of Frege. I then suggest that Wittgenstein inherited from Frege an insight underlying Frege’s claim that every thought has an “opposite,” such that to reject the thought as false is to affirm the opposite as true. Frege’s insight, however, is in tension with Frege’s conception of thoughts as representing the falling of objects under concepts and his conception of negation as (expressing) a truth-function. The resolution of this tension in the *Tractatus* involves (a) its *Grundgedanke* that neither negation nor any other “logical constant” is a representative in our picturing of the world, and (b) a fundamentally modal conception of the picturing of propositions as requiring a primitive conception of possibility

Keywords: Logical Laws · Inference · Propositional Attitudes · Negation · Normativity · Representation · Truth and Falsity

¿Cuál podría ser la gran deuda a Frege? o *Gottlobius ab paene omni naevo vindicatus*

En este trabajo examino algunas interpretaciones de la crítica que Wittgenstein hace a la concepción de lógica de Frege. Una se basa en el rechazo de Frege del psicologismo y alega que este rechazo engendra una tensión

que es resuelta en el *Tractatus*. Otra más se basa en la afirmación de que hay patrones de inferencias involucrando lo que se conoce como adscripciones de actitudes proposicionales, alegando que la concepción de la lógica de Frege no es capaz de manejarlas. Cinco otras apelan a dificultades que supuestamente surgen a raíz del punto de vista de Frege de los pensamientos como sentidos de los nombres dos valores de verdad. Yo demuestro que ninguna de estas interpretaciones presenta una crítica contundente de Frege. Sugiero, entonces, que Wittgenstein heredó de Frege una idea que es el fundamento de la afirmación de Frege, de que cada pensamiento tiene un «opuesto», de manera que rechazar el pensamiento como falso es afirmar que lo opuesto sea verdadero. Hay cierta tensión, sin embargo, entre esta idea fundamental de Frege y la concepción de los pensamientos como la representación del caer de objetos bajo conceptos, y su concepción de la negación como (expresando) una función veritativa. Resolver esta tensión en el *Tractatus* involucra (a) su *Grundgedanke* de que ni la negación ni ninguna otra «constante lógica» sea un elemento representativo en nuestro retratar del mundo, y (b) una concepción fundamentalmente modal del retratar de las proposiciones, la cual requiere una concepción primitiva de la posibilidad.

Palabras Clave: Leyes lógicas · Inferencia · Actitudes proposicionales · Negación · Normatividad · Representación · Verdad y falsedad.

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