WalkingTime: Dynamic Graph Embedding Using Temporal-Topological Flows

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Embeddings

- Capture meaningful information via intermediate representation for downstream
  - Debates about what other properties are good
- have long history in language processing, information retrivial
  - tf-idf, LSI/ SVD, latent dirlechet processes, etc.

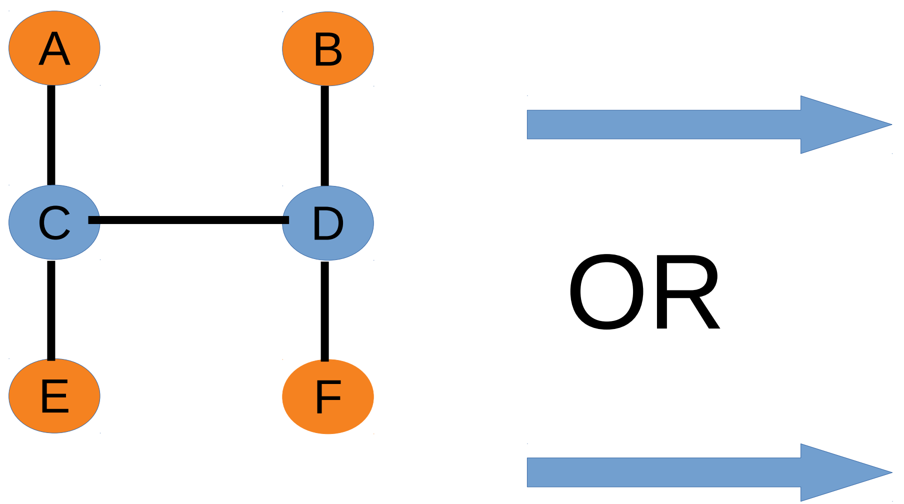
Graph Embeddings

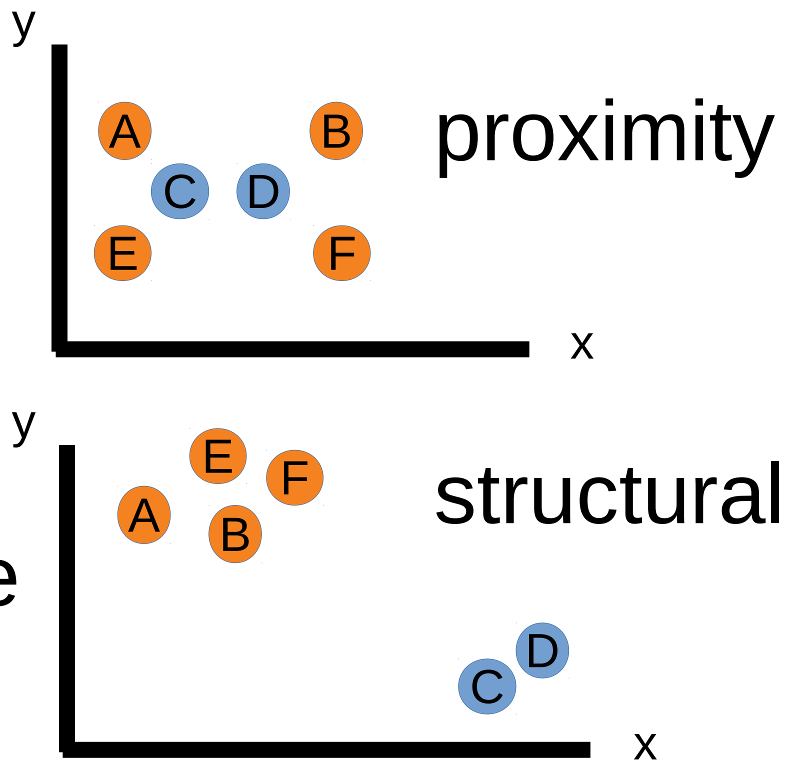
- Have to choose what structures to capture
- Variety of granularities:
  - Whole-graph embeddings
  - Sub-graph embeddings
  - Node embeddings

Focus of this talk

Graph Embeddings

- Emphasis structural similarity or proximity

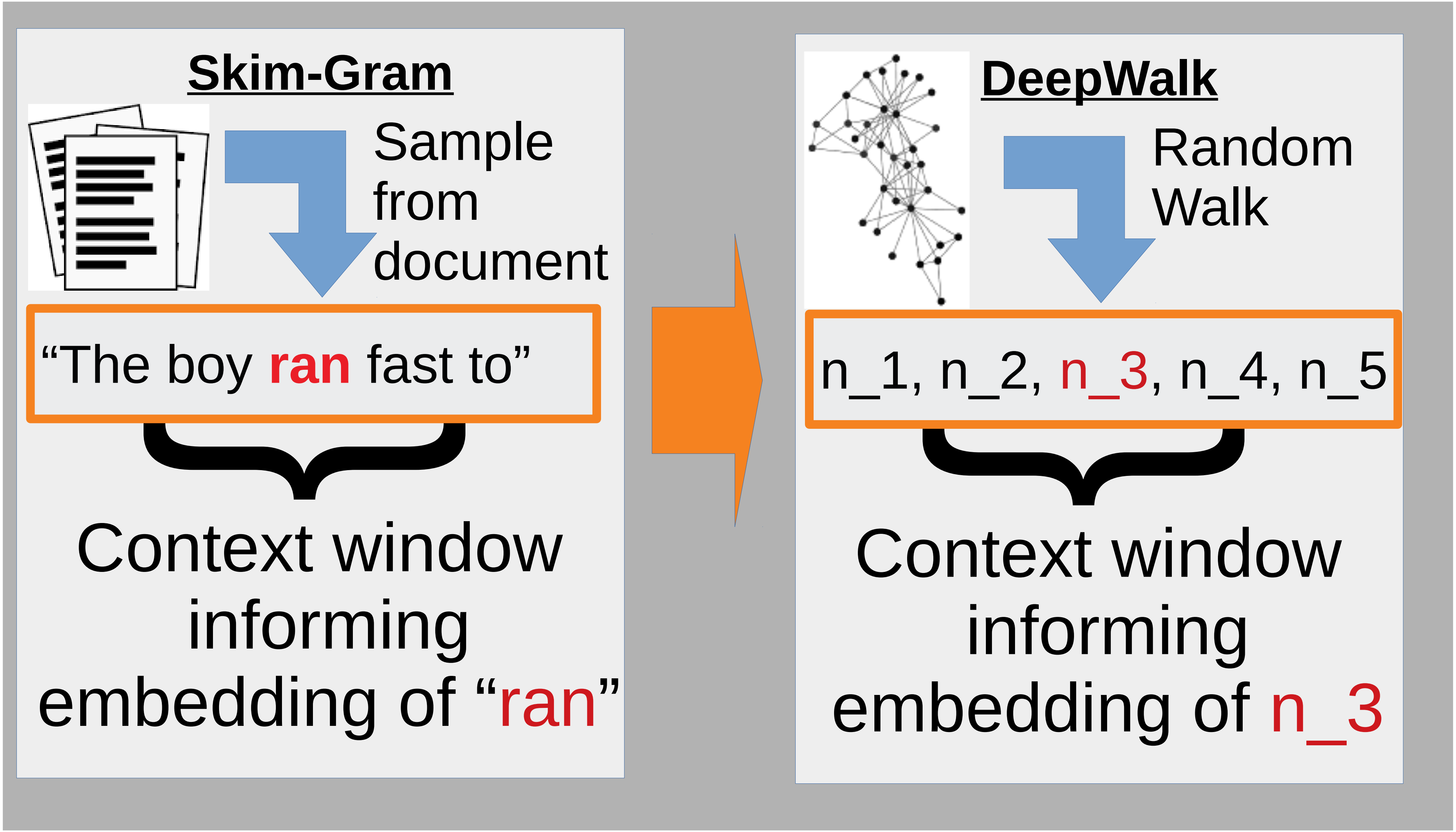




- Can have different distance decays
    - Katz score, A^k

Graph Embedding Techniques

- LLE([11]), Laplacian Eigenmaps ([1])
  - Proximity based, basically matrix factorization
- Autoencoder and convutional neural net approaches
- DeepWalk([10]), node2vec([6])
  - Based on language models, Skip-Gram model ([9]), and rand. walk


Skim-Gram
Sample from document
"The boy ran fast to"
Context window informing embedding of "ran"
DeepWalk
Random Walk
n_1, n_2, n_3, n_4, n_5
Context window informing embedding of n_3

Graph Embedding Techniques

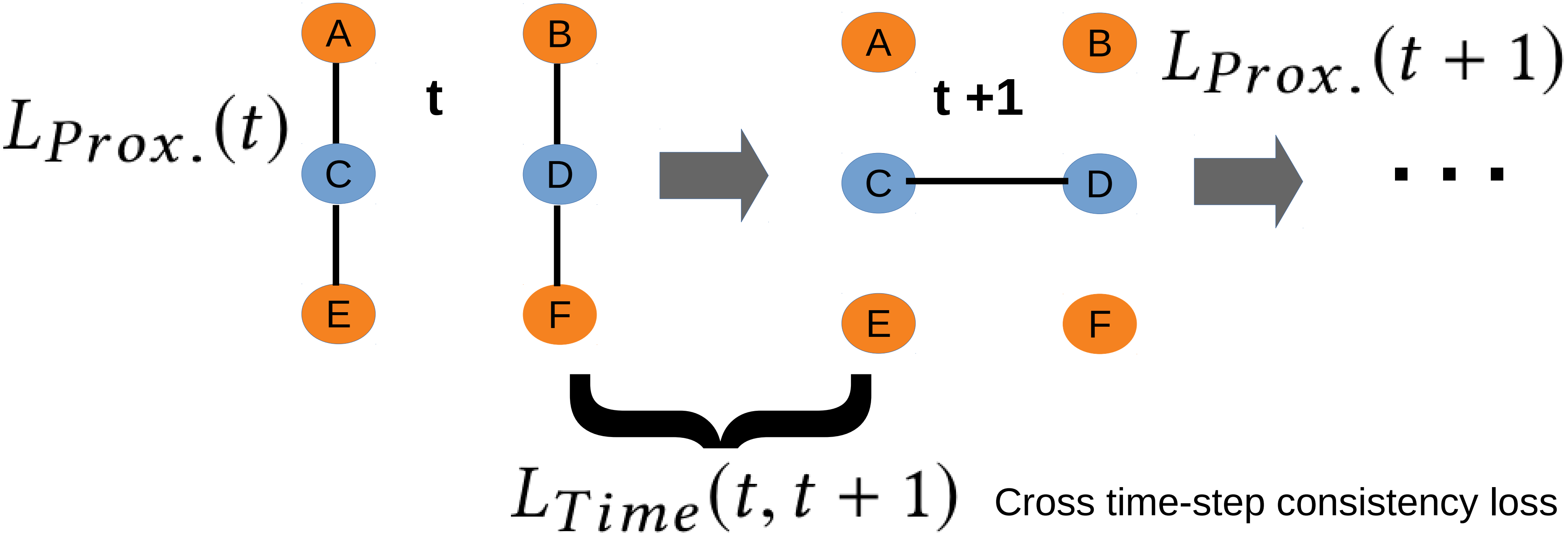
- Many techniques build on node2vec. Ex:
  - Harp ([2])
    - Heirarchy of refinement graphs, repeating embedding to avoid bad local minima
  - Metapath2vec ([4])
    - Bias random walk of node2vec based on edge and node attributes
    - Users provide meta-templates to guide attribute-walks

Temporal Graph Embedding

- Increased attention within last four years
- Motivations from:
  - Disease tracking
  - IoT, autonomous network systems
  - Casuality studies

Temporal Graph Embedding

- Most based on stringing together global snapshots



Temporal Graph Embedding

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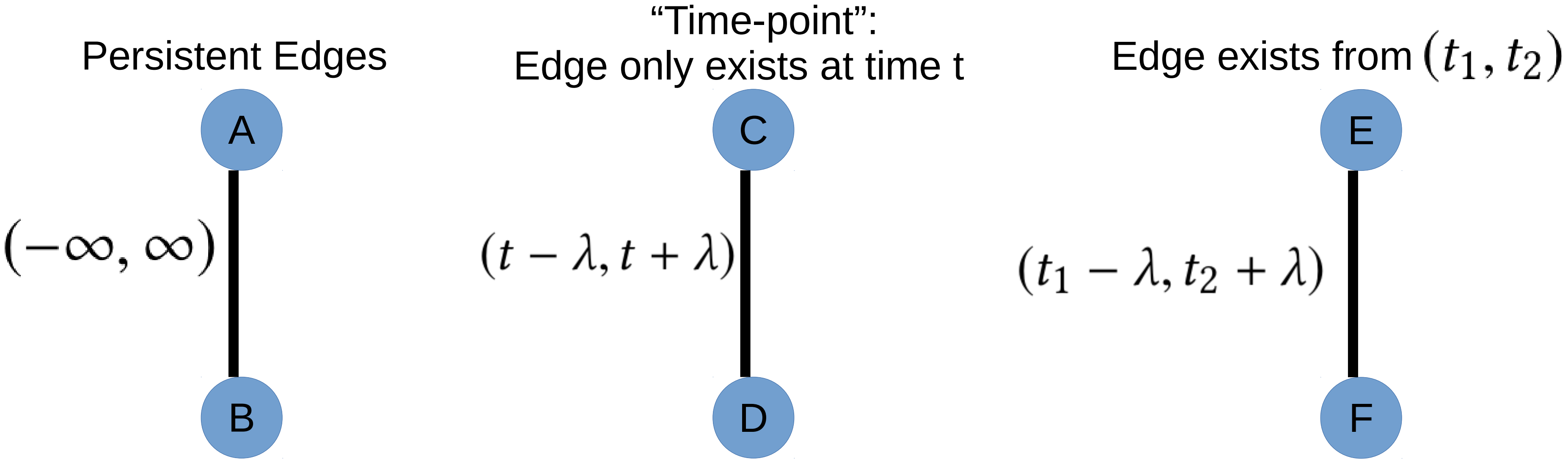
Our Approach: WalkingTime

- Handles time differently:
  - Local
  - Continous
  - Allows forward and backward traversal
- Builds off of node2vec and collection of time-respecting path methods ([7])
  - Technically, handles a multi-graph
  - Maintains set of active times for nodes, only walks to those with overlap

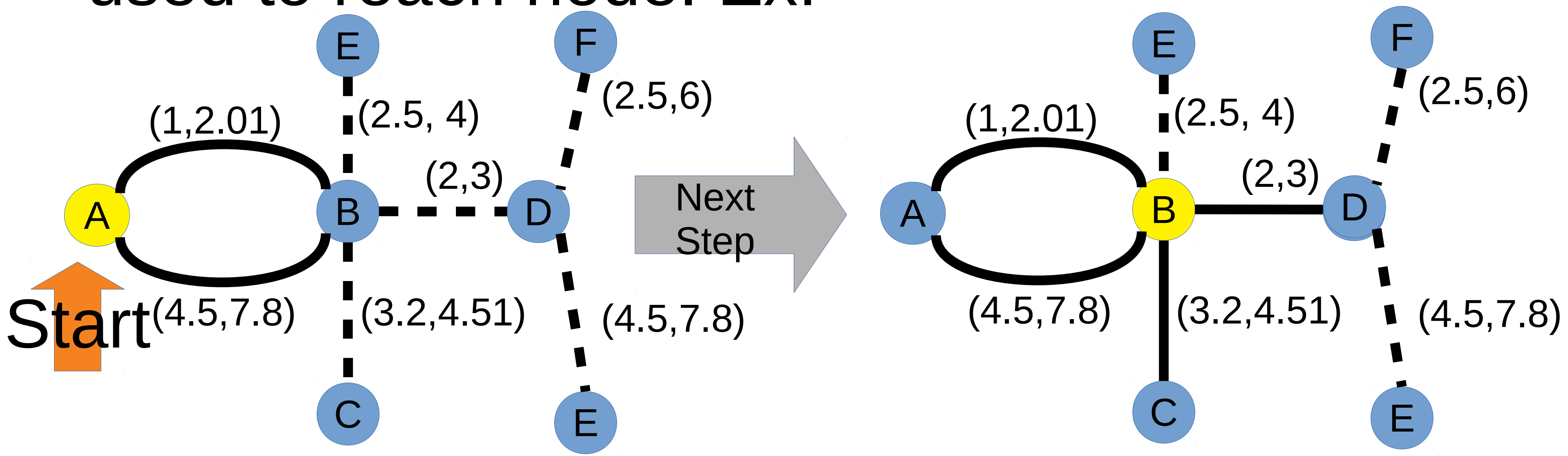
WalkingTime: Pre-processing

- Adds one new parameter compared to node2vec: λ
- Put time intervals on edges



WalkingTime: Random Walk Active Edges

- active edges for each node depends on edges used to reach node. Ex:



Key: **solid line**= "active" edge (can be traversed), **dashed line**="inactive" edge
yellow nodes = node currently on, **blue nodes** = other nodes in the graph

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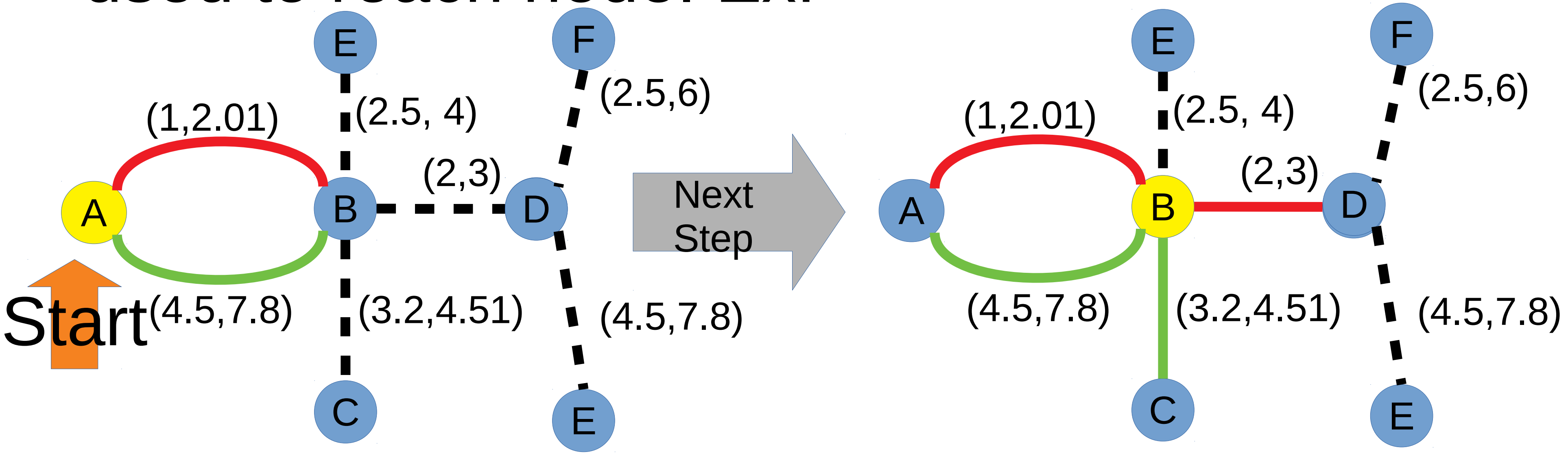

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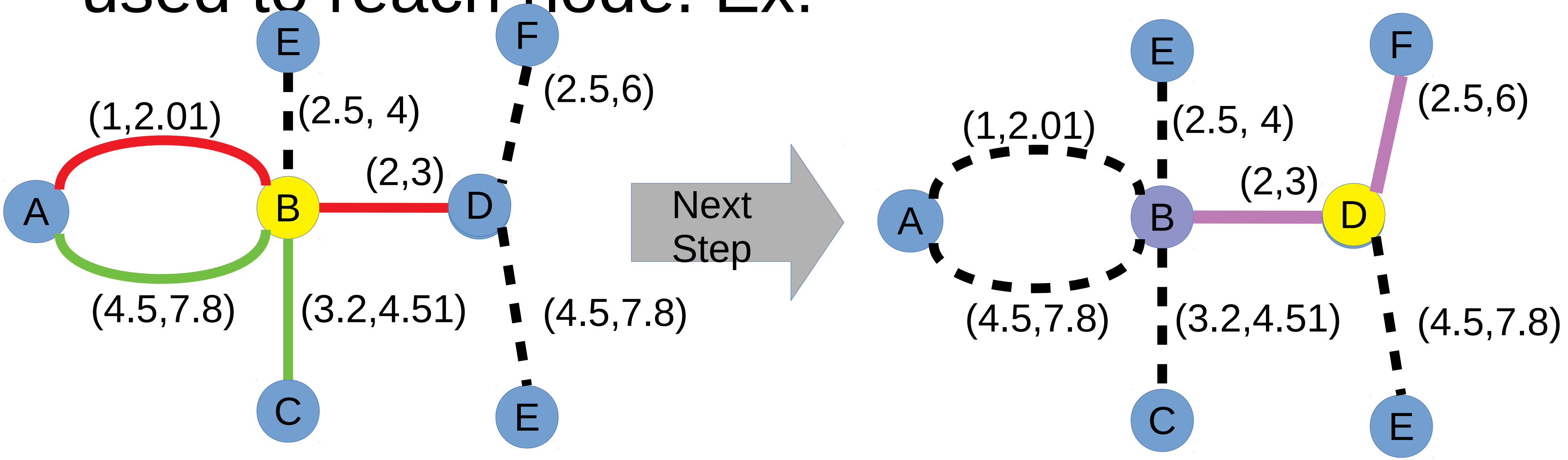

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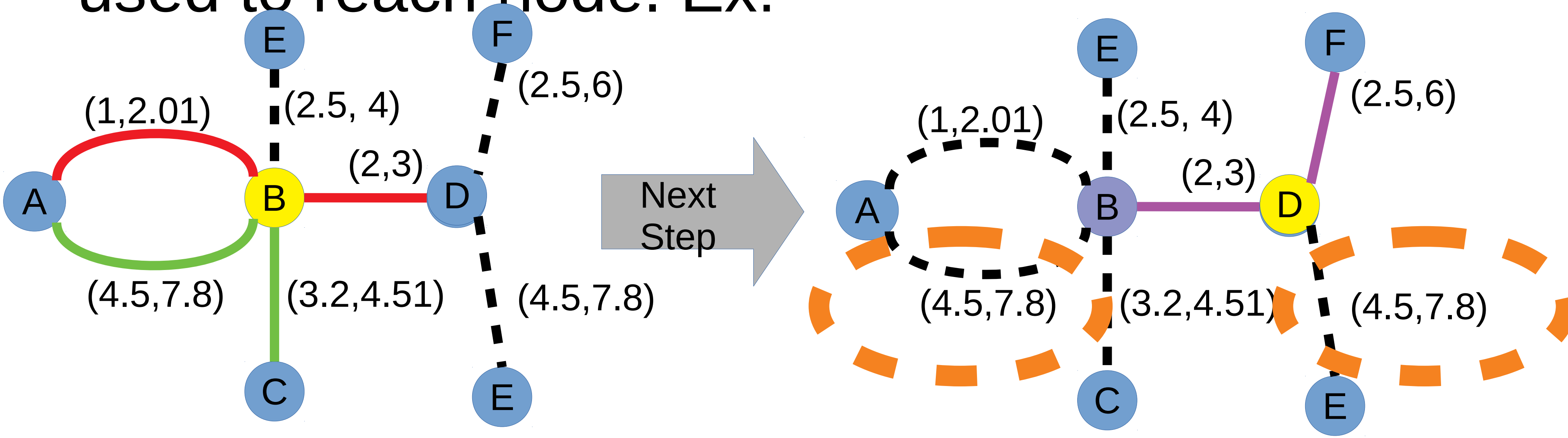

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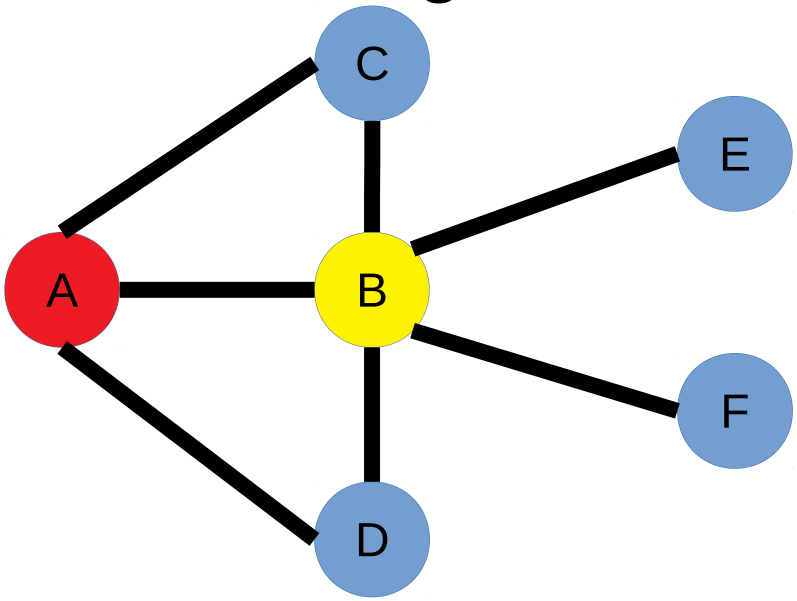
WalkingTime: Random Walk Biased Sampling à la node2vec

- node2vec: p and q parameters influences sampling in same neighborhood

Key:
Yellow node=
Current node

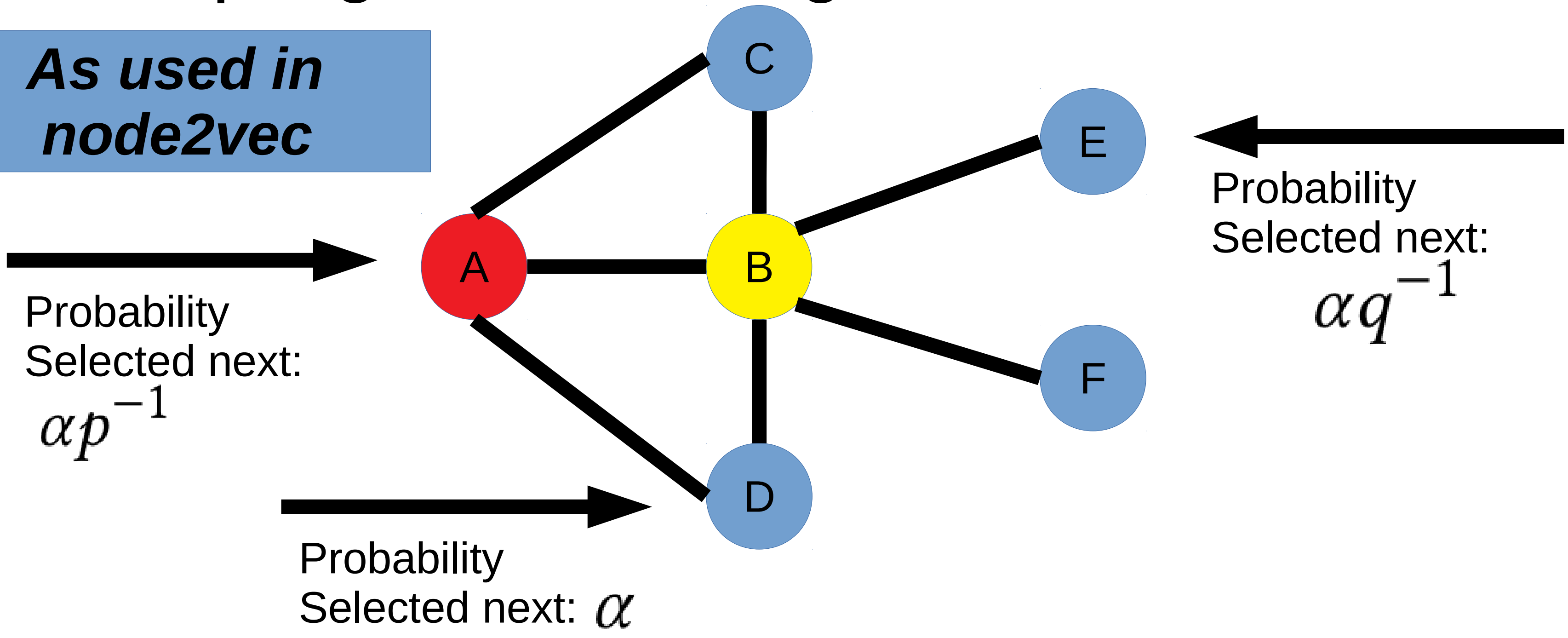
Red node=
Node came from
Last time step



WalkingTime: Random Walk Biased Sampling à la node2vec

- node2vec: p and q parameters influences sampling in same neighborhood

As used in node2vec

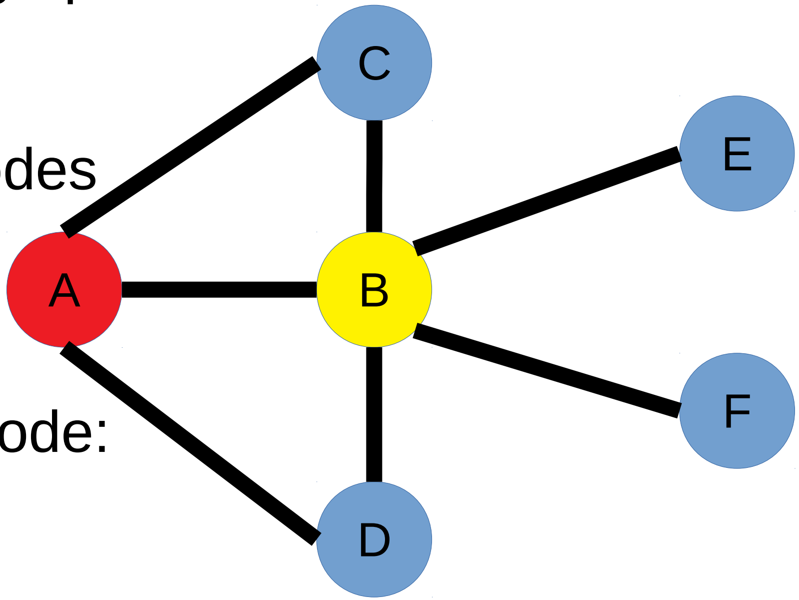


WalkingTime: Random Walk Biased Sampling in Our Method

Added efficiency: reinterprete parameters as rejection sampling probs.

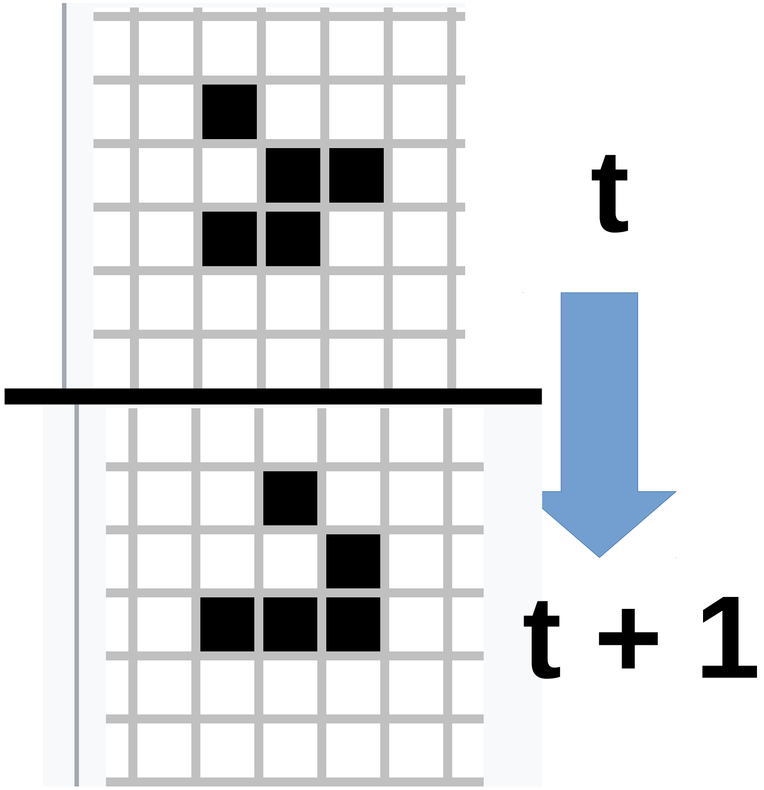
1) Uniform rand. sample a “neighbor in static graph”
   - i.e., edge connects nodes, regardless of if active
2) Find out if there is an active edge linking nodes
   - If not, got to (1)
3) Choose new node with prob. specified by parameters

If sample all nodes and not yet chosen new node: use cached results and alias sampling to do node2vec method



Experiments

- Datasets:
  - Synthetic: Conway's Game of Life ([5])
    - Famous cellular automata
    - One node per grid-cell
    - If two cells share a vertex ***and*** are active within one time-step: form edge



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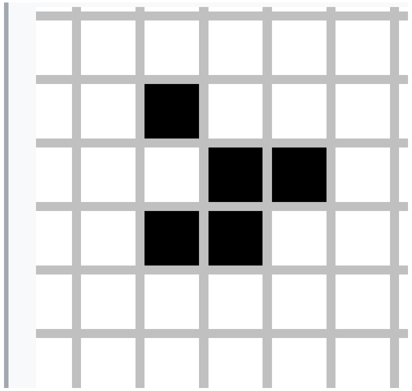
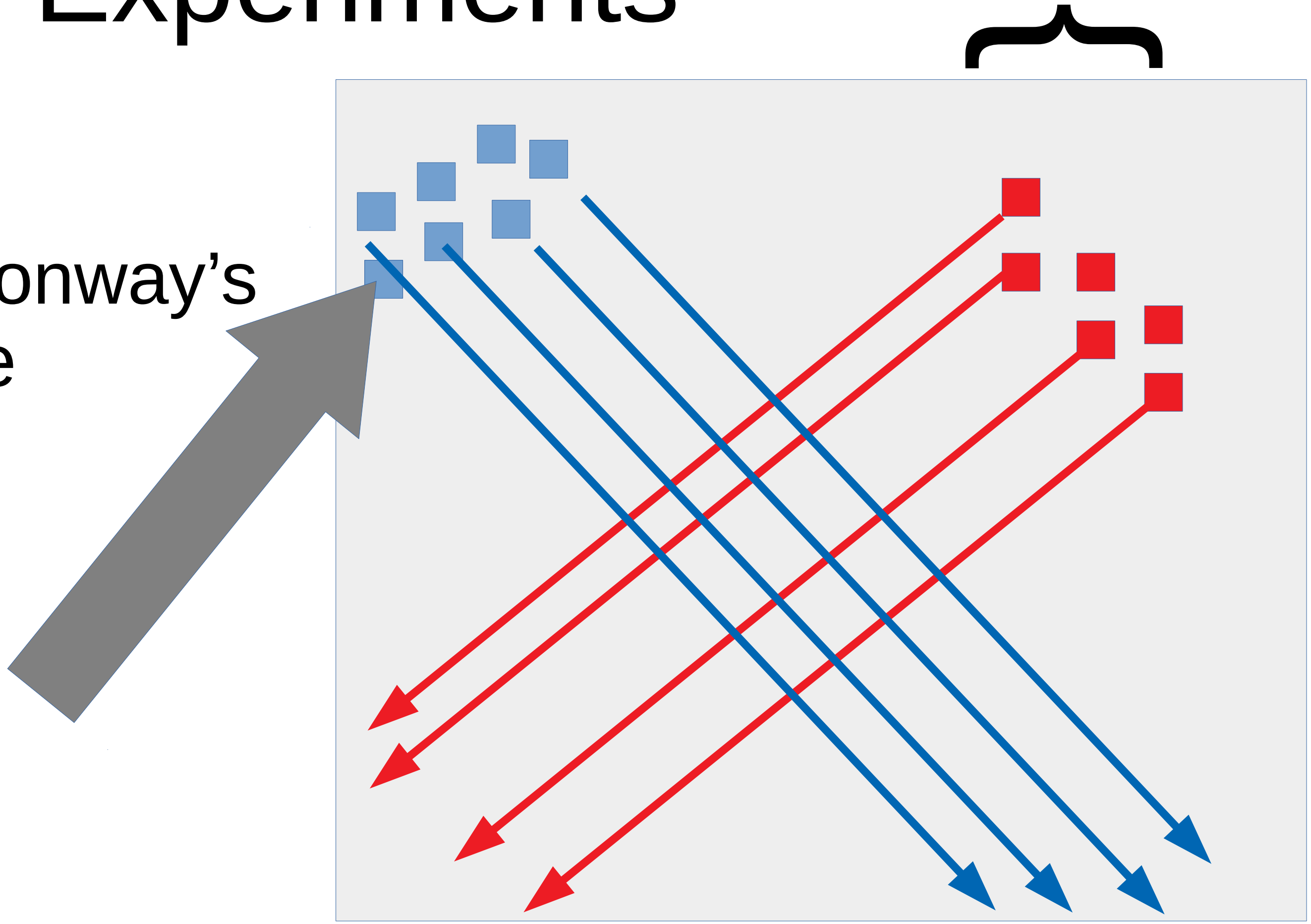
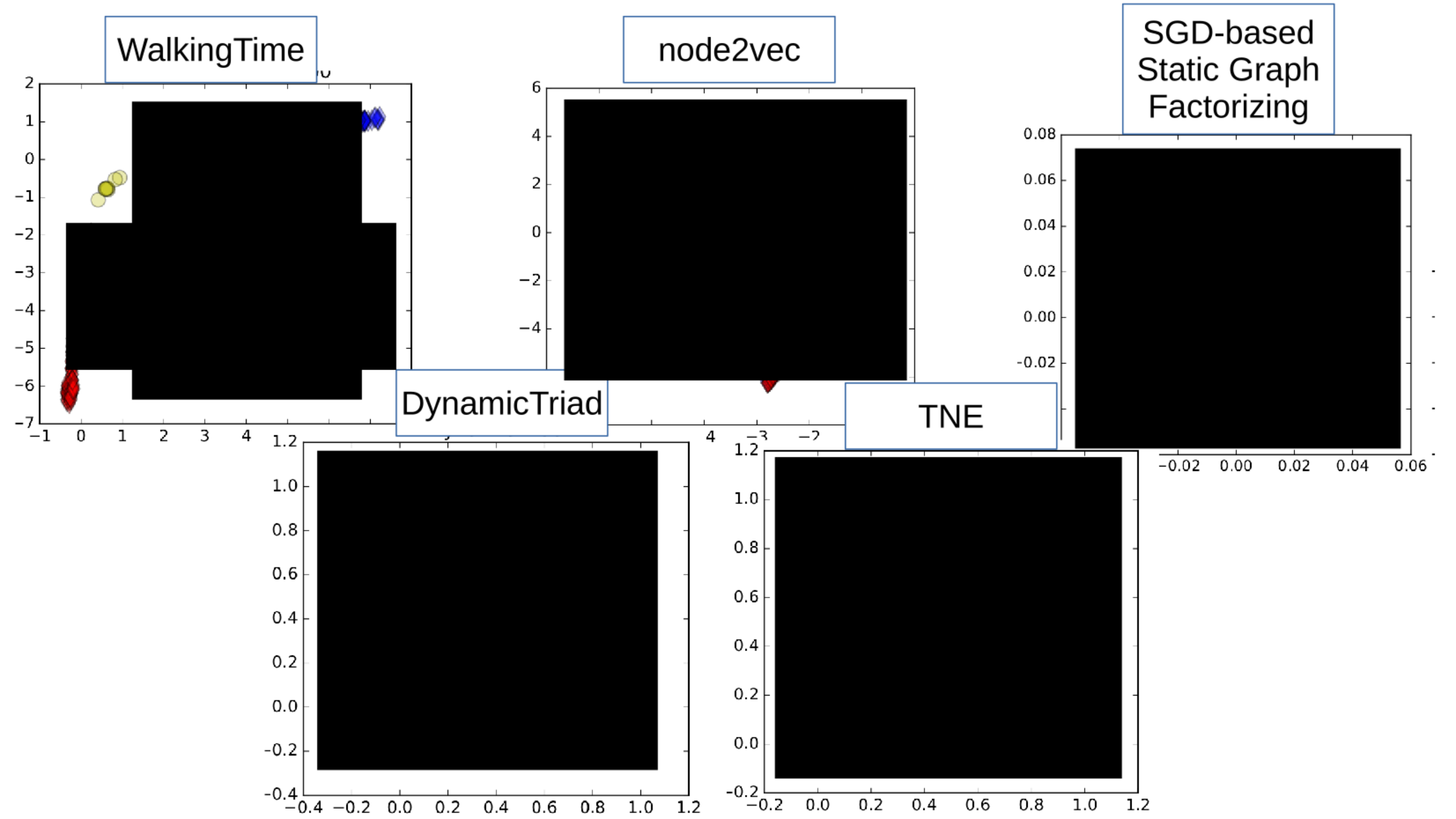


"Glider"

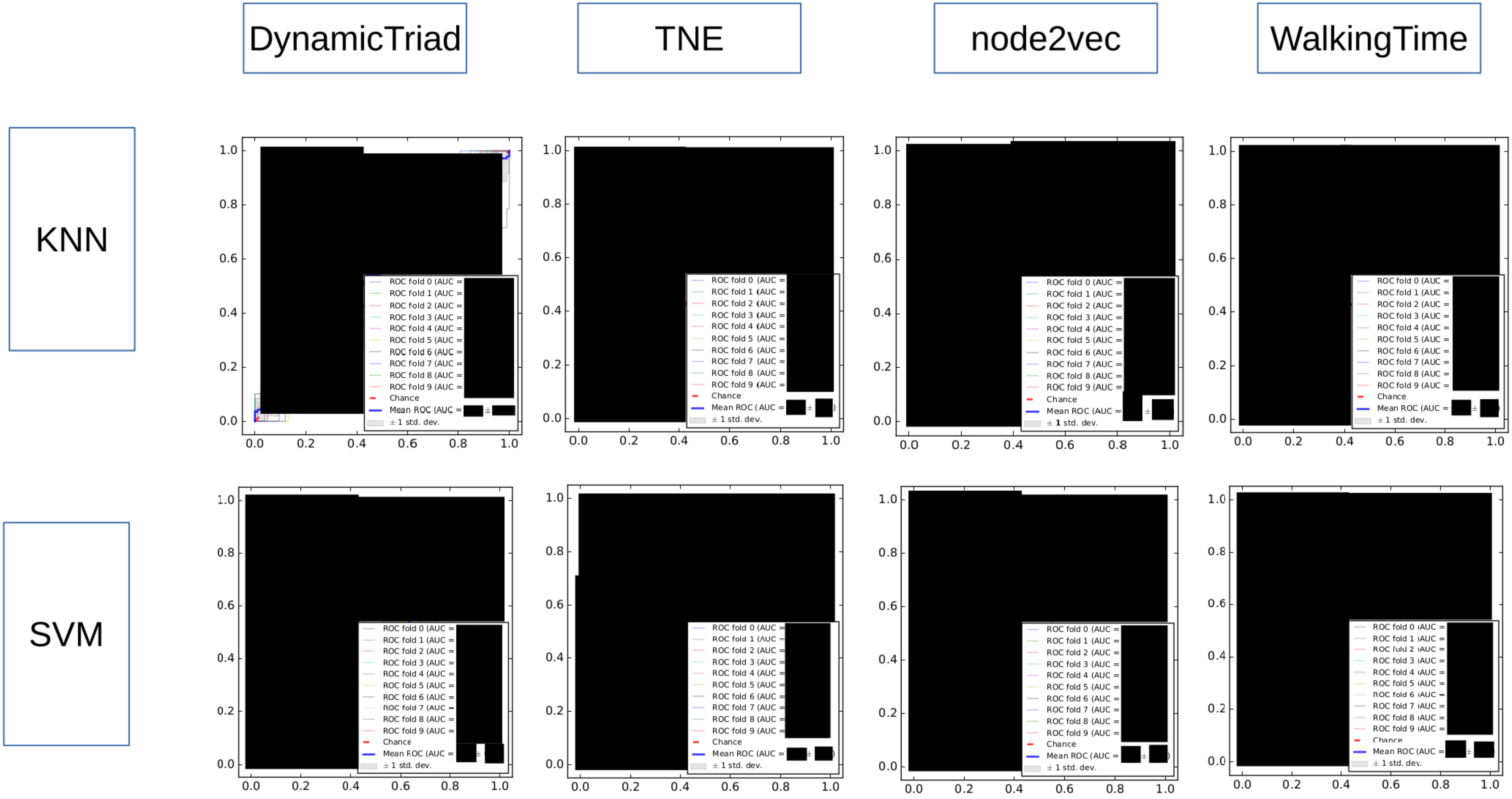


Experiments

- Baseline Algos.:
  - Static graph factorization
  - node2vec
  - TNE ([14])
  - DynamicTriad ([13])
- Evaluation Methods:
  - Node classification
  - 2D Visualization



WalkingTime
2
1
0
−1
−2
−3
−4
−5
−6
−7
−1
0
1
2
3
4
node2vec
6
4
2
0
−2
−4
4
−3
−2
SGD-based Static Graph Factorizing
0.08
0.06
0.04
0.02
0.00
-0.02
−0.02
0.00
0.02
0.04
0.06
DynamicTriad
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
TNE
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2


DynamicTriad
TNE
node2vec
WalkingTime
KNN
SVM
ROC fold 0 (AUC = [redacted]
ROC fold 1 (AUC = [redacted]
ROC fold 2 (AUC = [redacted]
ROC fold 3 (AUC = [redacted]
ROC fold 4 (AUC = [redacted]
ROC fold 5 (AUC = [redacted]
ROC fold 6 (AUC = [redacted]
ROC fold 7 (AUC = [redacted]
ROC fold 8 (AUC = [redacted]
ROC fold 9 (AUC = [redacted]
Chance
Mean ROC (AUC = [redacted] ± [redacted]
± 1 std. dev.

Further Experiments

- Also trying on DBLP ([8,12]) and Higgs-Twitter ([8,3])

Further Experiments

- On-going works:
  - Latent Graph Reconstruction
  - Link Prediction
- Finding datasets that clear and numerous cause-effect relations in lab sciences

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