Properties of Massive Stars on the Main Sequence

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- Review simple physics of massive stars on the H Burning Main Sequence
- Using MESA, each student will get to know one star of a fixed mass, M
- We will use all your data to check for trends as a function of M
- Low Metallicity, NO rotation or mass loss

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Stars 101!

- Hydrostatic balance demands a certain temperature in the interior.
- That hot core leads to energy loss to the exterior at a rate fixed by the physics of the heat transfer (i.e. radiative diffusion and/or convection)
- Until some other energy source can be tapped (e.g. nuclear fusion), that energy loss leads to gravitational contraction to smaller radii at a timescale set by the heat loss.

Hydrostatic Balance and Gas Pressure

Since sound waves travel around a young star over days to weeks, there is plenty of time for hydrostatic balance to apply:

$$\rightarrow \frac{dP}{dr} = -\rho g, \text{ where } g(r) = \frac{Gm(r)}{r^2}$$

We now cheat, and assume that at a typical place in the star, m=total mass M, r=radius R, then hydrostatic balance gives:

$$\frac{P}{R} \sim \rho \frac{GM}{R^2}, \text{combine with ideal gas } P \approx \frac{\rho k_B T}{m_p}$$

Where m_p=proton mass (everything is ionized). This yields the relation between the central temperature, T_c, and the stellar mass and radius as well as the central (or average) density

$$k_{\mathrm{B}} T_c \approx \frac{GM\mu m_p}{R} \qquad T_c \propto M^{2/3} \rho^{1/3}$$

Hydrostatic Balance Outcomes

$$k_{\mathrm{B}} T_c \approx \frac{G M \mu m_p}{R}$$

As R shrinks (i.e. as the star contracts from the large cloud it started in), the core temperature rises! This is the same as what happens to a particle in orbit, as it loses energy (radiates!), it moves in (radius shrinks!), and moves faster (higher temperature!).

Prior to prevalence of any nuclear energy source, the loss of energy leads to a slow contraction of the star. If the Sun were powered this way, it would change its radius on a time

$$t_{\mathrm{Kelvin}} \approx \frac{G M^2 / R}{L} \approx 10^7 \mathrm{yr}$$

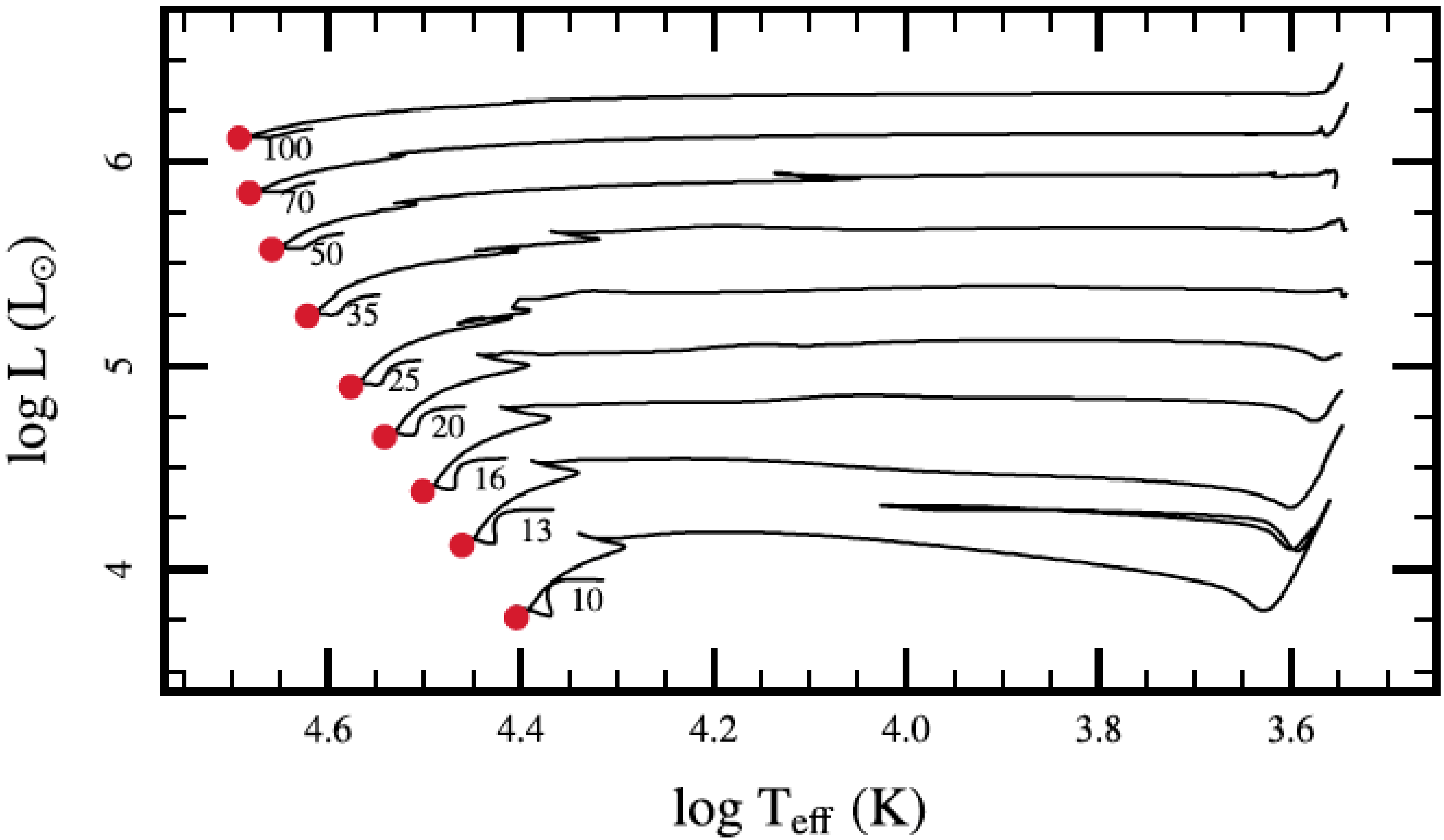
Stellar Luminosity Prediction

The luminosity is determined by heat transport, since the core is hot, and the surface is cold (VACUUM!). Unlike in your house, the heat is transported by diffusion of photons, which leads to a flux equation:

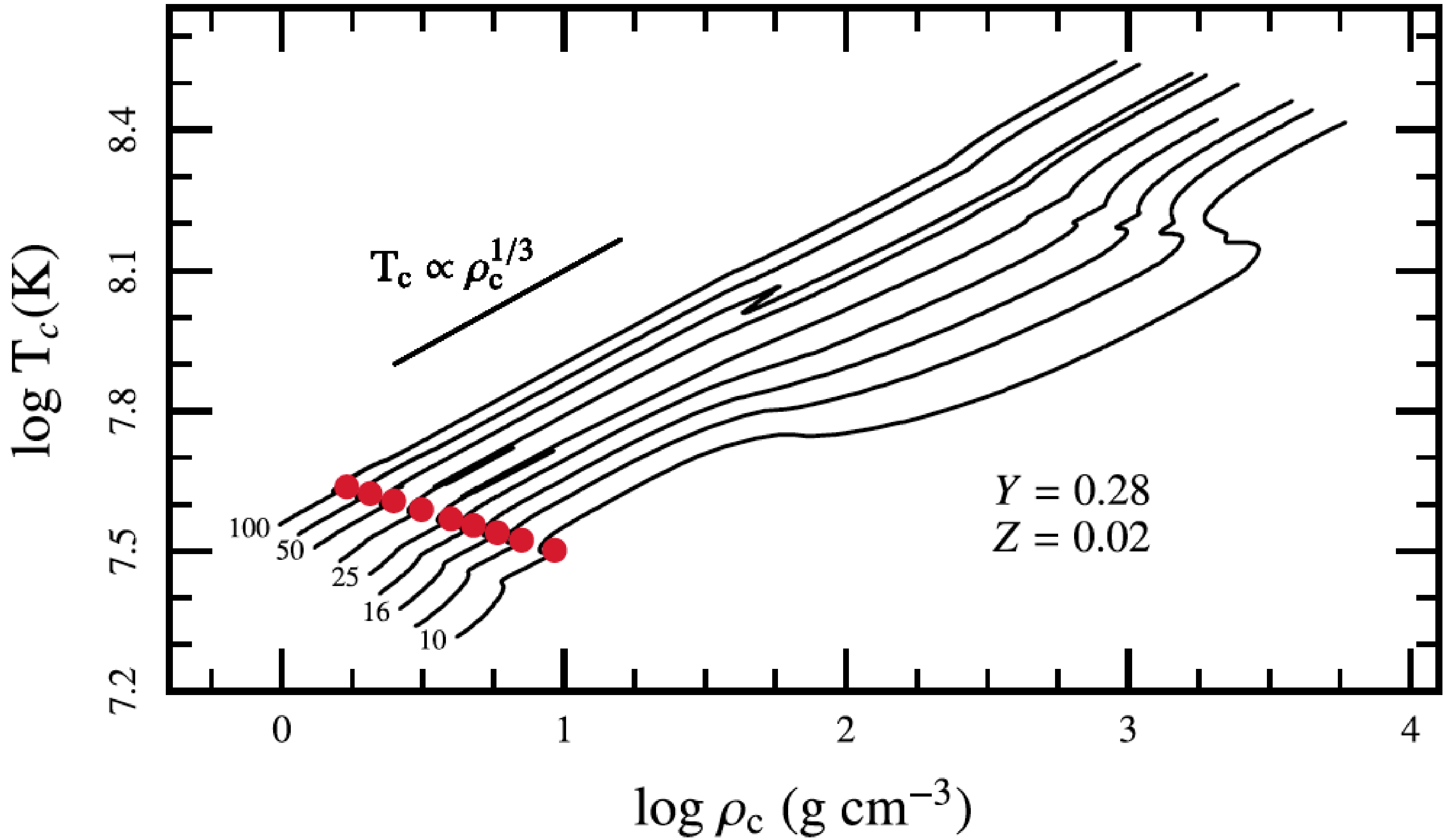
$$F = -\frac{c}{3\kappa\rho}\frac{daT^4}{dr} \sim \frac{acT_c^4}{\kappa\rho R} \to L \sim R^2 F \propto M^3$$

Where the last step assumed that the opacity, Kappa, is constant and we used the HB relations from the previous page assuming only ideal gas pressure.

Note the nearly constant L as R changes and a strong Mass Dependence



Red Points are the ZAMS

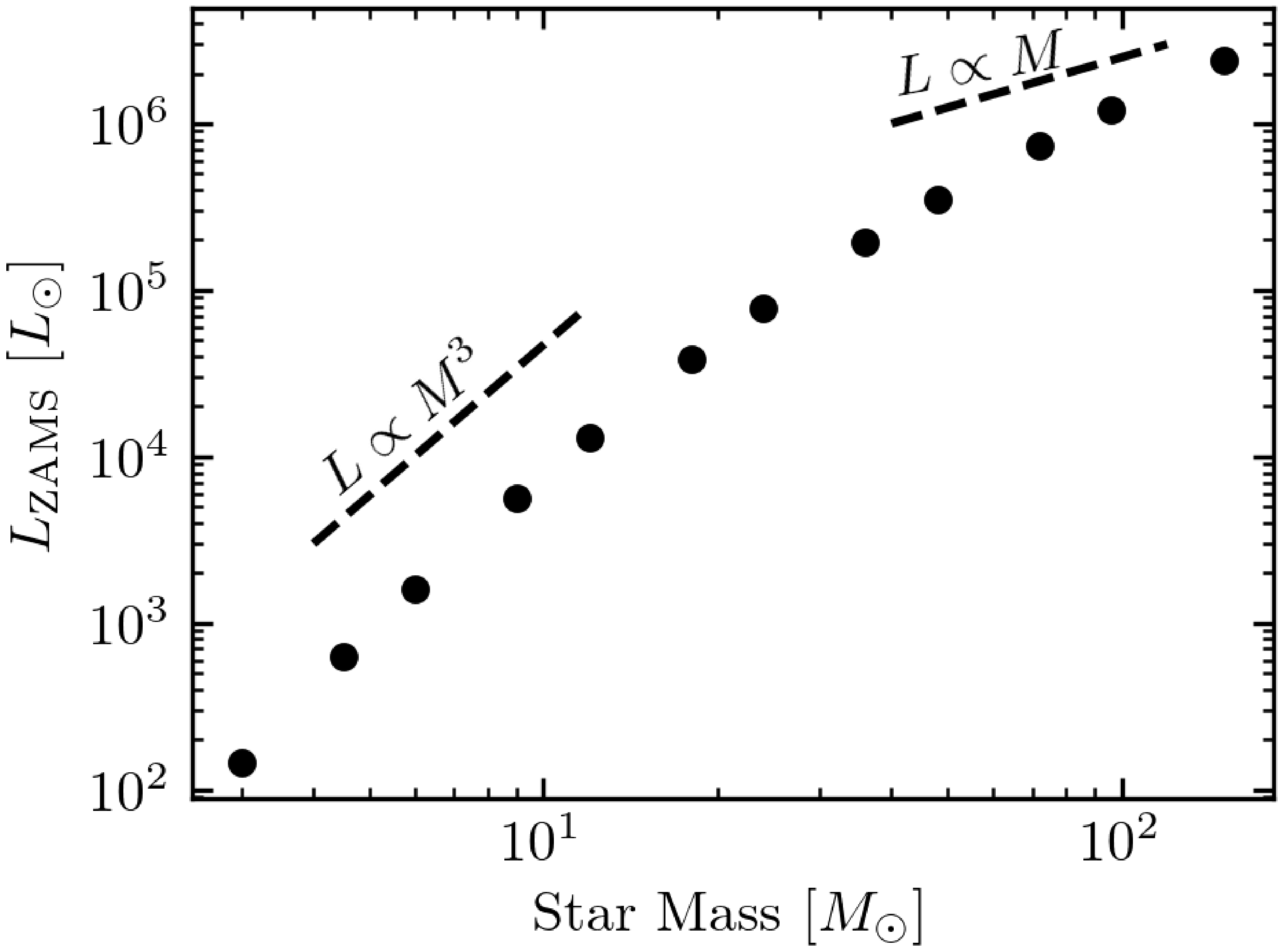


The CNO cycle is operative and very T dependent, allowing for $L=L_{nuc}$ at nearly a constant core temperature over a large range in mass! We will say nothing more about CNO today.

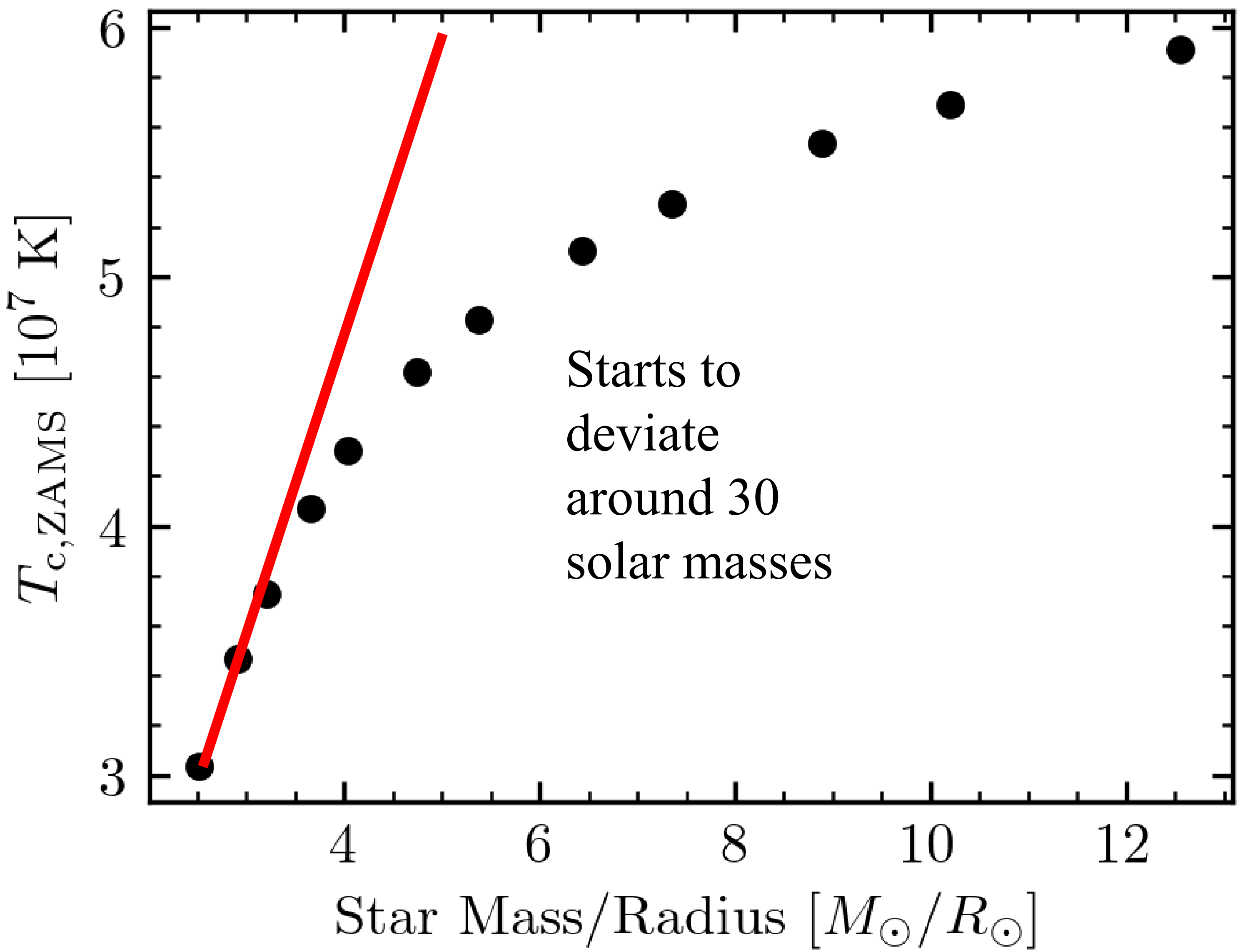
Mini Lab One

- Every Student chooses a mass from the table in the range of 3 to 100 solar masses.
- We choose a low metallicity ($Z=10^{-4}$) so that the opacity is mostly just electron scattering.
- You will run to the Zero-Age Main Sequence, defined as when $L_{nuc}=0.99\ L$
- Follow the instructions in the write-up and record the data in the spreadsheet.
- Stare at the T-rho profile and talk with people at your table about how your star differs from others.

Luminosity vs. Mass



Core Temperature vs. M/R



Radiation Pressure Prevalence

A naïve calculation that assumes gas pressure and then checks the ratio of radiation pressure to gas pressure predicts

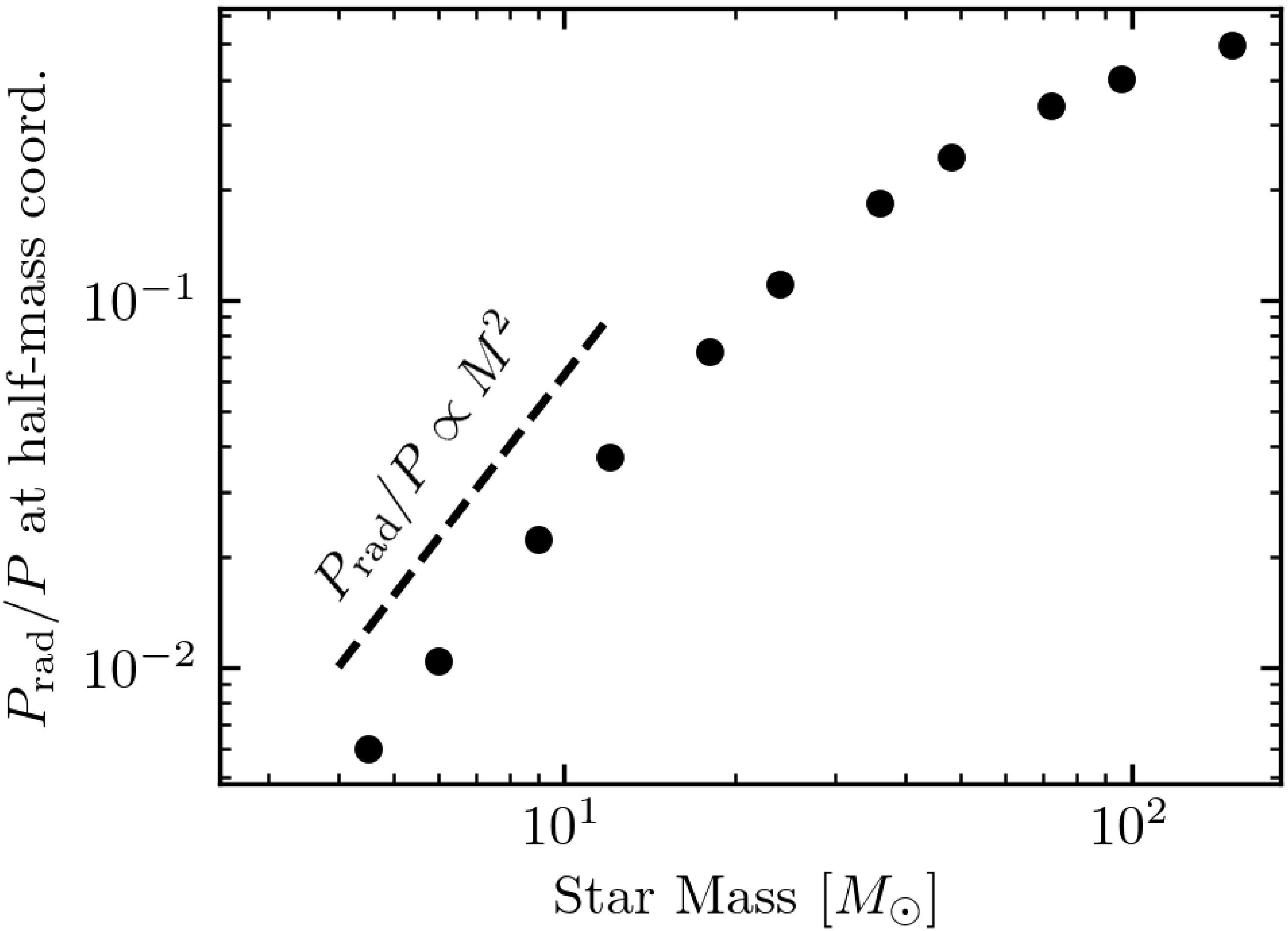
$$\frac{P_{\rm rad}}{P_{\rm gas}} \propto \frac{T^3}{\rho} \approx \left(\frac{M}{M_c}\right)^2$$

Where the physical mass scale is set by fundamental constants:

$$M_c \approx m_p \left(\frac{\hbar c}{G m_p^2}\right)^{3/2} \sim M_\odot$$

A nice example of ``deriving'' the solar mass scale in terms of fundamental constants !!

Uh-Oh! Radiation pressure is becoming important as Mass Increases



50% of Pressure is from radiation at 100 solar masses!!

Radiation Pressure only?

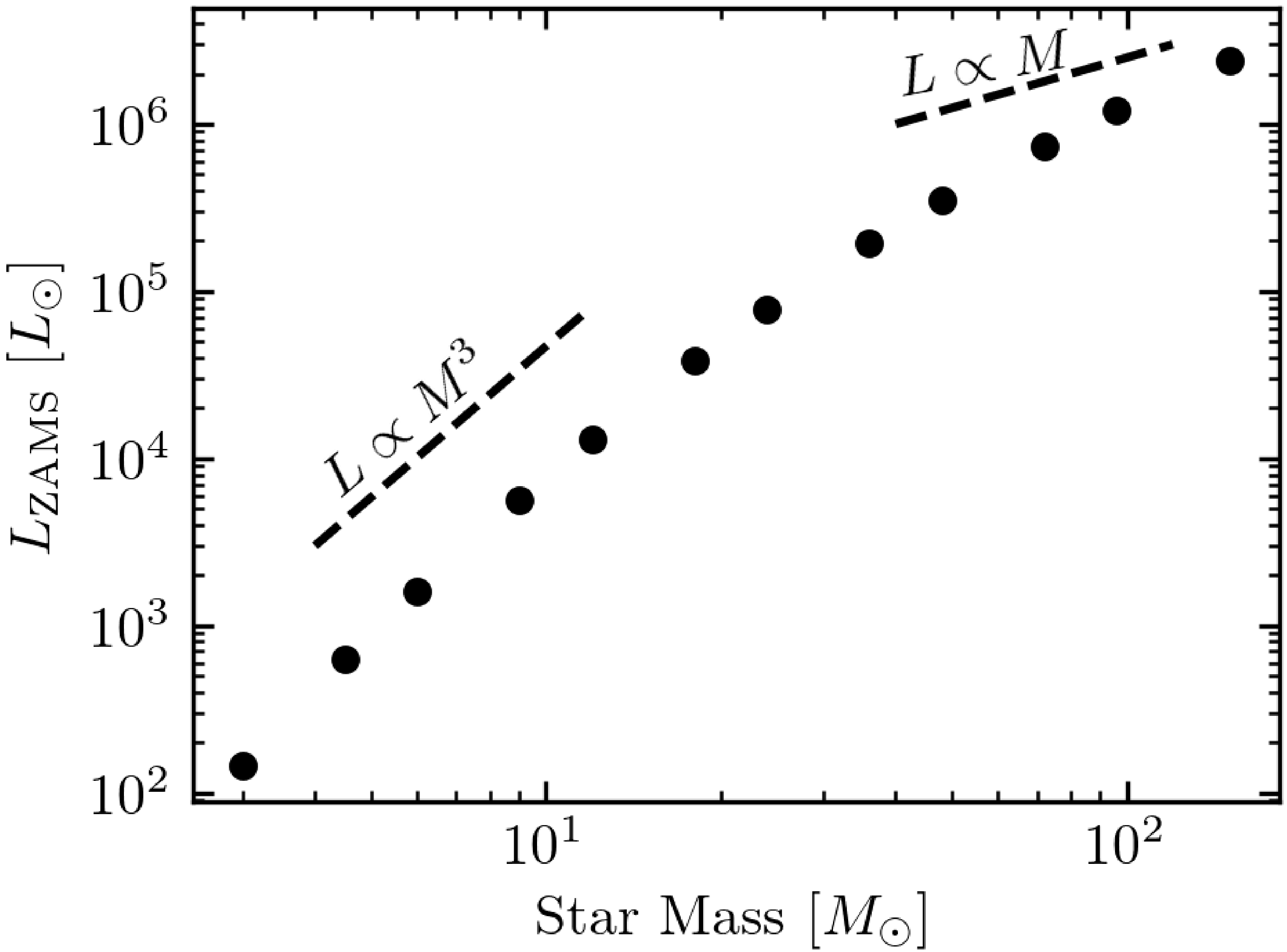
Let's check the extreme limit where radiation pressure dominates. Hydrostatic balance then implies

$$P_c \approx \frac{GM^2}{R^4} \approx aT_c^4$$

Which can be placed back in the radiation diffusion equation to find that the luminosity scale changes to:

$$L \to L_{\mathrm{Edd}} \equiv \frac{4\pi c G M}{\kappa} \approx 3 \times 10^4 L_\odot \left(\frac{M}{M_\odot}\right)$$

Luminosity vs. Mass



Radiative Heat Transfer in Stars

- Let's start with a reminder of stellar structure

$$L = 4\pi r^2 F = 4\pi r^2 \left[-\frac{c}{3\kappa\rho} \frac{daT^4}{dr} \right]$$

$$\frac{dP}{dr} = -\rho \frac{Gm(r)}{r^2}$$

- Combine nicely to yield:

$$\frac{dP_{\rm rad}}{dP} = \frac{L(r)\kappa(r)}{4\pi G m(r) c} = \frac{L(r)}{L_{\rm Edd}(r)}$$

Eddington "Standard Model"

- IF we set L(r)=Lm(r)/M and assume opacity is constant, then we obtain:

$$P_{\mathrm{rad}}(r) \approx \frac{L}{L_{\mathrm{Edd}}} \int_r^R dP \approx \frac{L}{L_{\mathrm{Edd}}} P(r)$$

- So that gas pressure dominates for stars with

$$L \ll L_{\mathrm{Edd}} = \frac{4\pi c G M}{\kappa}$$

- And there is an easy polytropic relation to write down when gas pressure dominates:

$$T(r)^4 \propto \rho(r) T(r) \rightarrow T(r)^3 \propto \rho(r) \rightarrow P(r) \propto \rho(r)^{4/3}$$

Mini Lab Two

- You will now stare at the profile of your model on the ZAMS to see how it looks relative to the Eddington Standard Model

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Minilab 2: Explore the model Profile in Detail!

1 Using pgstar to gain physical insight

Often it is useful to compare `MESA` to analytic expectations. This is often done in post-processing, but can also be done real-time using `pgstar`!

We want to check if our stars are well-described by a polytrope, $P \propto \rho^{\gamma}$ where γ is the polytropic index. To do this, we want to stare at 2 pgstar plots: the T-rho profile that comes on by default, and a profile plot of P versus ρ.

In the `&pgstar` section of `inlist_pgstar`, set

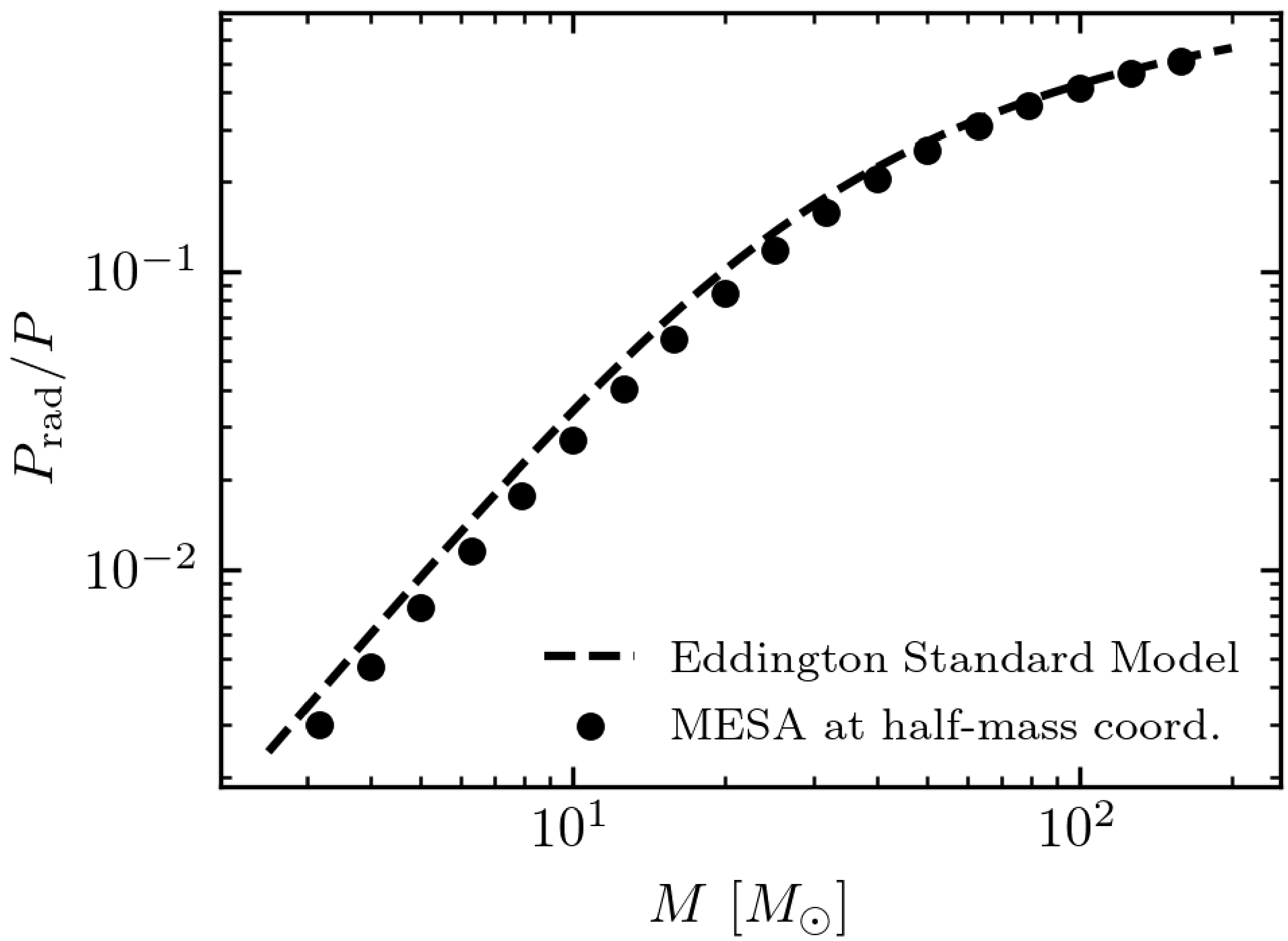
```
Profile_Panels1_win_flag = .true.
```

By default the Profile panels x-axis is `'mass'` and the y-axis of the first row is `'logT'`. In your `inlist_pgstar`, change that by adding

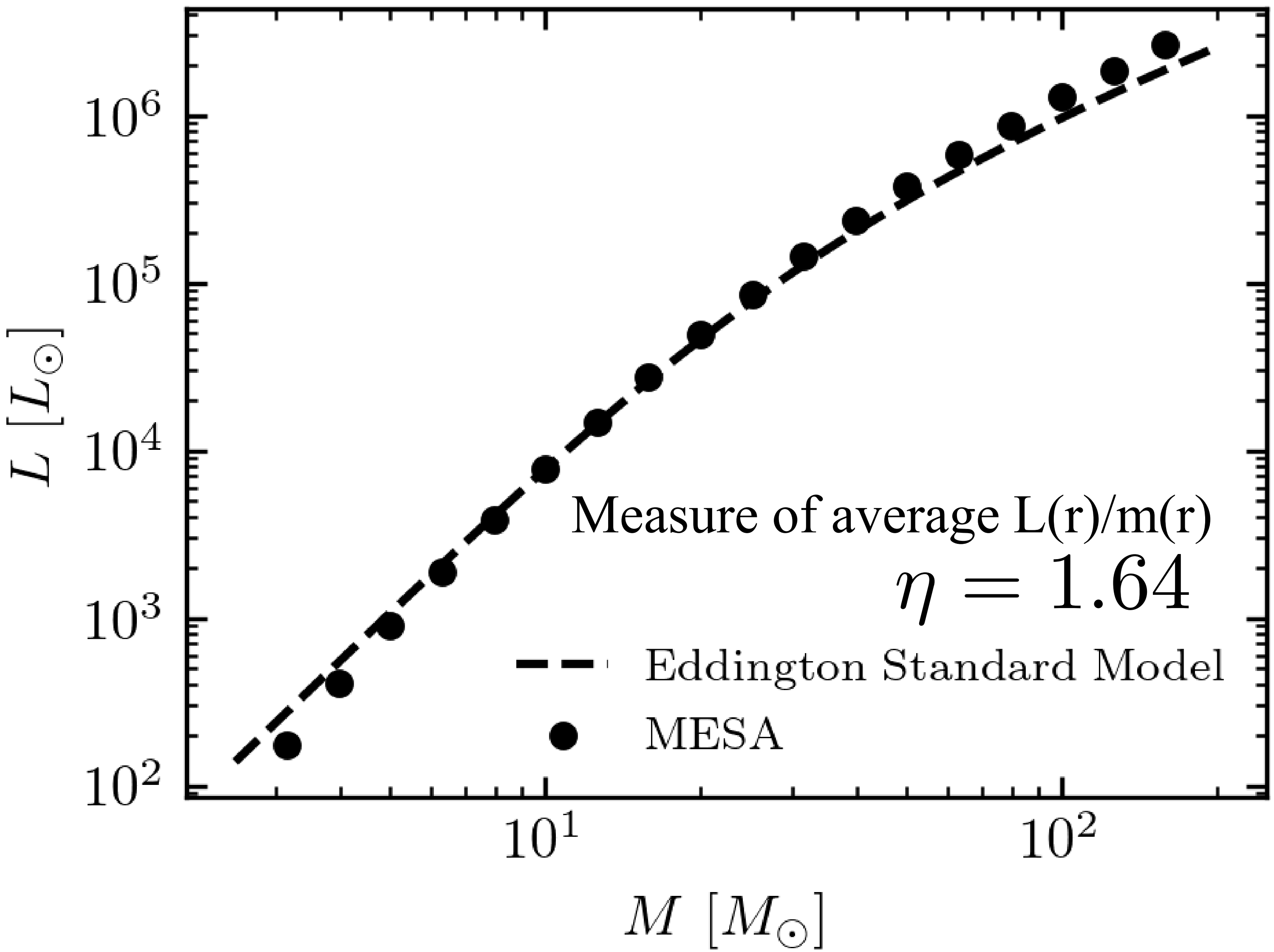
Polytrope Predictions

- These polytropes, when true, allow for the derivation of a number of relations. I'll spare you the algebra, and simply show the outcomes.
- For example, the central density is 54 times the average density. Please check!!!

Eddington's Prediction for Radiation Pressure



Eddington's Prediction for Luminosity



Evolution on the Main Sequence

- For our H and He mixtures, the mean molecular weight is defined:

$$P = \frac{\rho k_{\mathrm{B}} T}{\mu m_p}, \quad \frac{1}{\mu} \equiv 2X + \frac{3Y}{4}$$

- This changes as the H burns to He, giving a contrast:

$$\mu(ZAMS) \approx 0.6, \quad \mu(X = 0.3) \approx 0.9$$

Mini Lab Three

- Now run your star to when H has reached 30% by mass in the core, and let's see what happens.

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Minilab 3: Evolution along the Main Sequence

1 Reporting Quantities near the TAMS

We now want to watch evolution along the Main Sequence. Previously, you saved a ZAMS model of your selected mass. Time to put that model to use!

Using the same `inlist_project`, `inlist_pgstar`, `history_columns.list`, and `run_star_extras`, modify the `&star_job` section of your `inlist_project` to tell the model not to start on the pre-main sequence, but rather to load in your saved model:

Some Changes, Some Not

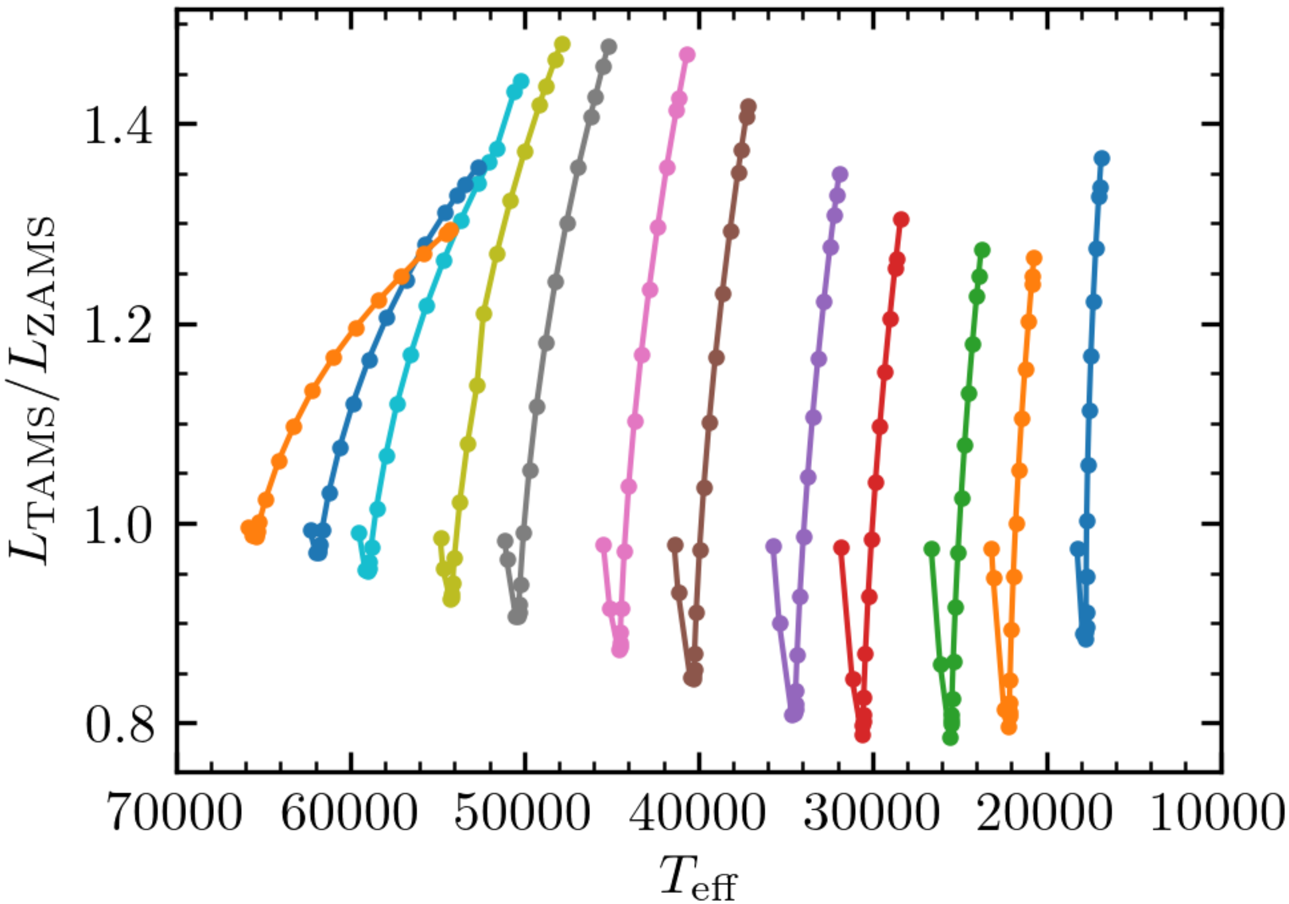
- The core temperature did not change much, that's because the slight luminosity increase can be easily matched by the CNO cycle burning by only slightly increasing T_c
- Luminosity does change, as does radius.
- In our simplest, naïve limit calculated earlier, we predict a Luminosity that scales as

$$L \propto \frac{M^3 \mu^4}{1+X}$$

- Implying a factor of 7 times brighter. . And, if we built a pure He star, 40 times brighter!!

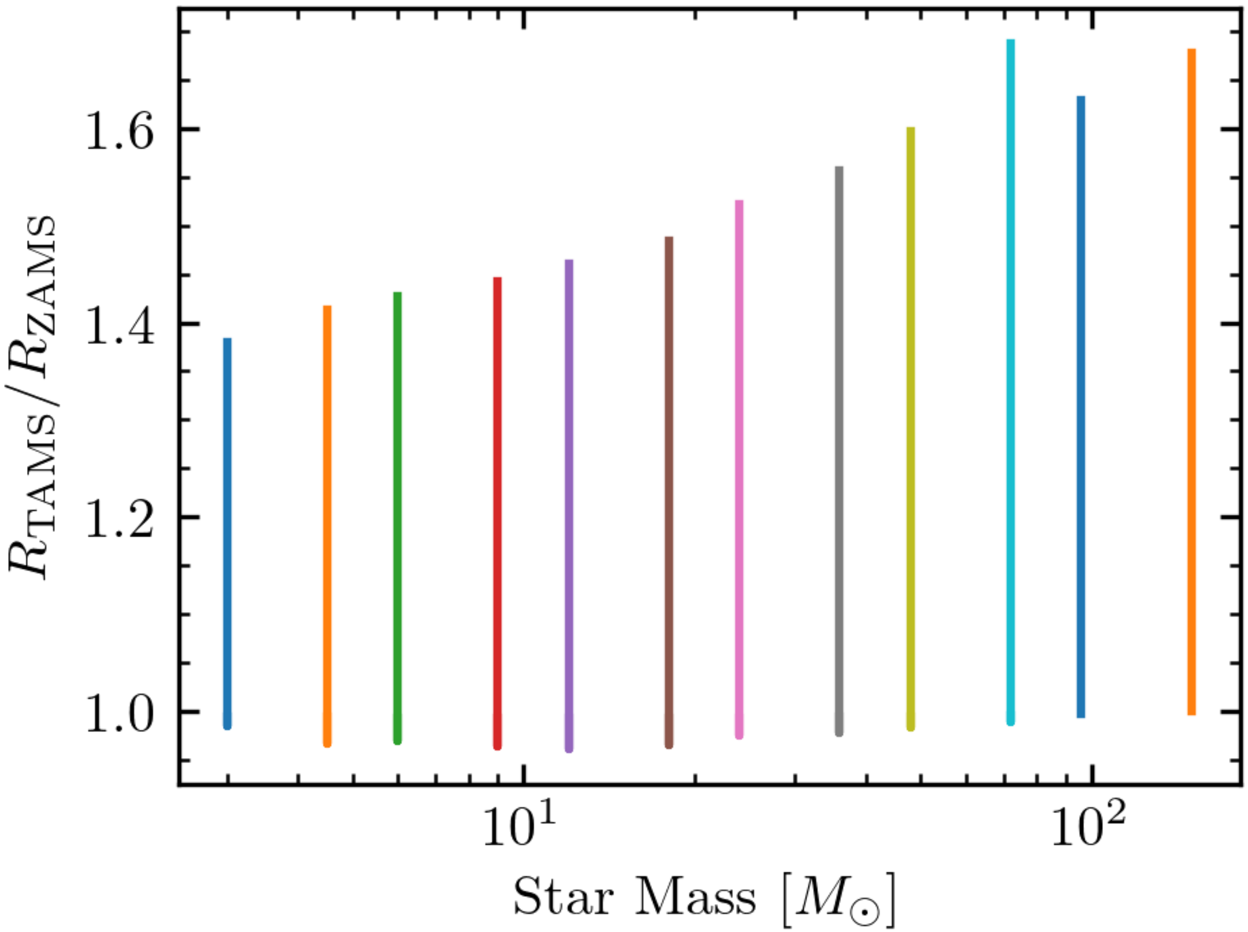
Luminosity Change

- Though it does brighten, nothing like that seen. This is due to the convective core breaking the simple expectations. . .



Radius Expansion

$$k_B T_c \approx \frac{G M \mu m_p}{R}$$



- If the core temperature stays the same, then the radius should increase by the ratio of mu's or 0.9/0.6=1.5!!
- Pretty close!

What I did not tell you about!

- All of these stars have convective cores. That ‘breaks’ some of the relations a bit, especially when a sizeable fraction of the star is convective.
- Low metals was for a reason! As opacity increases with metals, many of these stars can get into a Super-Eddington challenge that is a forefront problem in stellar evolution. Ask Will Schultz!

All Done for Today

- Thanks to Jared and Sunny for a great lab, and for making it all possible!
- Hope you all had a chance to cut your teeth on MESA while learning some fun bits about massive stars.