SIGNALS OF OPPORTUNITY IN MOBILE RADIO POSITIONING

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ABSTRACT

In this paper we address the benefit of observing a signal of opportunity for mobile terminal positioning. In an exemplary environment we arrange three mobile terminals in an equilateral triangle. These mobile terminals determine their position relatively to each other by transmitting and receiving mobile radio signals. In addition they observe a signal of opportunity which is emitted from a transmitter. In our case this signal is a single carrier. We evaluate the performance improvement for positioning of the mobile terminals when observing such a signal of opportunity. The evaluation is based on the calculation of the Cramér-Rao lower bound. For an exemplary environment with three mobile terminals and one signal of opportunity a performance gain of up to $\approx$ 38.8 % can be expected theoretically.

***Index Terms*—** cooperative positioning, signals of opportunity, Cramér-Rao lower bound, Fisher information

1. INTRODUCTION

The availability of position information at wireless mobile terminals has become a key feature in recent years to drive location and context aware services. Providing position information anywhere with sufficient accuracy is a challenging task. Global satellite navigation systems like the US Global Positioning System (GPS) or the European satellite navigation system Galileo provide accurate position information in suitable environments [1]. However, in critical environments like indoors such systems provide a poor performance due to weak signals, multipath or non-line-of-sight signal propagation. Therefore, it is desirable to exploit hybrid or multi-sensor positioning approaches which can complement each other in different situations [2, 3]. An obvious alternative is to use a wireless communications network for positioning [4] since they provide good coverage with broadband signals like the primary and secondary synchronization signals (PSS, SSS) or even the positioning reference signal (PRS) in the 3GPP-LTE[1] standard [5]. Even external systems like digital terrestrial video broadcasting are capable of improving positioning performance as long as their signals can be received by the mobile device [6]. Generalizing this approach of observing signals for positioning purposes leads to the concept of cognitive positioning or positioning with *signals of opportunity (SoO)* [7]. For positioning the transmitter (source) of such a SoO can be considered as a landmark.

In this paper we aim to evaluate the benefit of a SoO for mobile radio positioning in an exemplary scenario. Here, mobile terminals (MTs) in a 2-dimensional area cooperatively locate each other and exploit an additional SoO observation for improving positioning performance. The MTs can be considered as an array of antennas for receiving a SoO, which is a single carrier in our case, Different to the concepts of antenna arrays [8] or MIMO radar [9], we are not primarily interested in the localization of the SoO source. Instead, we aim to improve the positioning performance of the MTs. For evaluating the achievable performance improvement, we derive and calculate the Cramér-Rao lower bound for this kind of scenarios.

2. PREREQUISITES

2.1. Environment

We consider a 2D environment as shown in Fig. 1. Mobile transceivers (MT) transmit signals $s_p(t)$. By observing these signals at adjacent MTs the spatial distance (the range) between the MTs can be determined. In general we assume that the MTs are not synchronized, i.e., they do not have a common time base. For this reason we include the unknown offset of the MTs' time bases T_p to a global time base as a further coordinate. So we generally have to estimate both spatial and temporal parameters. Note we do not consider spatially fixed mobile transmitters like base stations in a mobile communications system, which inherently could serve as a global spatial and temporal reference. Instead we are going to define a local coordinate system later on by fixing some of the MTs' parameters. We further assume that there is a transmitter (TX), emitting a signal $s_0(t)$. The MTs exploit this *signal of opportunity (SoO)* to improve their positioning performance. The estimation of the unknown spatial and temporal parameters can be performed centrally by providing signal observations or sufficient statistics about these parameters to a central unit, which calculates the parameter estimates jointly. Another approach is to estimate the unknown parameters decentrally and

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[1] 3rd Generation Partnership Project - Long Term Evolution

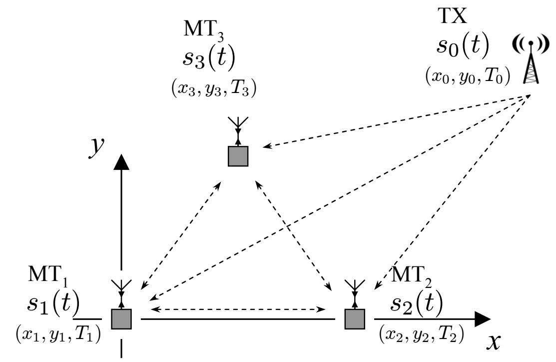


Fig. 1. Three mobile radio transceivers (MT) in an uniform circular array arrangement. The MTs cooperatively and relatively locate each other. For proving their positioning performance the MTs observe a *signal of opportunity*, transmitted from a source TX.

iteratively at each MT based on signal observations and estimates provided by adjacent MTs. In both cases, cooperation between the MTs is required for exchanging observed or already processed data.

2.2. Signal Model

2.2.1. Generic Description

With complex valued baseband signals $s_p(t)$, transmitted at the transceivers MT_p ($p = 1, 2, \dots$) or the SoO source TX ($p = 0$), we obtain the time signal

$$r_{q,p}(t) = \underbrace{s_p\left(t - \frac{\tilde{d}_{q,p}}{c_0}\right)}_{\stackrel{\text{def}}{=} s_{q,p}(t)} + n_{q,p}(t) \tag{1}$$

at transceiver MT_q. We usually sample these signals at time instances $t = k\,T$. The term $n_{q,p}(k\,T) = n_{q,p}(k)$ is complex valued additive white Gaussian noise (AWGN) with zero mean and variance

$$\mathrm{E}\left\{\left|n_{q,p}(k)\right|^2\right\} = \sigma^2_{q,p}. \tag{2}$$

As shown in Eq. (1) signal s_p transmitted at MT_p arrives at receiver MT_q with a delay which corresponds to a *pseudo range*

$$\tilde{d}_{q,p} = \underbrace{\sqrt{(x_q - x_p)^2 + (y_q - y_p)^2}}_{\substack{\stackrel{\text{def}}{=} d_{q,p} \\ \text{Euclidean TX-RX distance}}} + \underbrace{c_0\,(T_q - T_p)}_{\substack{\text{bias due to} \\ \text{time base offsets}}}, \tag{3}$$

where c_0 is the speed of light. The pseudo range consists of the Euclidean transmitter-receiver distance and a bias term which is caused by different time base offsets T_p and T_q at the transmitter and the receiver of the signal. With the signal model in Eq. (1) we assume that signals observed from different transmitters do not interfere each other, meaning that we perfectly can separate these signals at a receiver. This can be achieved by an appropriate multiplex of these signals in time, frequency or code domain. The design of the signals $s_p(t)$, $p = 1, 2, \dots$ transmitted from transceivers MT_p is not within the scope of this paper. Instead we are going to use a real valued pseudo range signal model, which will follow from the derivation of the Fisher information later on in Sec. 3.1.

2.2.2. The Signal of Opportunity

For the SoO, transmitted from TX, we consider a real valued waveform $s_0(t) = \sqrt{2P_{\text{TX}}}\cos(2\pi f_{\text{S}}\,t)$ with a frequency f_{S} and TX power P_{TX}. This signal arrives at the MTs with different delays which correspond to the pseudo range defined in Eq. (3). At transceiver MT_q we obtain the bandpass signal

$$\check{r}_{q,0}(t) = \sqrt{P_q}\,\cos\left(2\pi f_{\text{S}}\left(t - \frac{\tilde{d}_{q,0}}{c_0}\right)\right) + n_{q,0}(t) \tag{4}$$

with a real valued AWGN process having an autocorrelation function of $\frac{N_0}{2}\delta(\tau)$. The received power P_q is subject to a signal propagation loss between the SoO source and transceiver MT_q, and therefore, dependent on the distance between them. For our investigations, however, we do not exploit this dependency for the estimation of the distance between the MT and the SoS transmitter. Since we are interested in the phases of these observed signals we describe them in the complex baseband and get

$$r_{q,0} = \sqrt{P_q}\,\exp\left(\mathrm{j}\,2\pi f_{\text{S}}\left(-\frac{\tilde{d}_{q,0}}{c_0}\right)\right) + n_{q,0} \tag{5}$$

at transceiver MT_q. Similar to Eq. (1) we define

$$s_{q,0}(t) = \sqrt{P_q}\,\exp\left(\mathrm{j}\,2\pi f_{\text{S}}\,t\right). \tag{6}$$

The term $n_{q,0}$ is a complex valued AWGN sample with zero mean and variance $\sigma^2_{q,0}$. We observe such a constant phasor by observing one period with duration $T_{\text{S}} = \frac{1}{f_{\text{S}}}$ of the waveform in Eq. (4).

3. FISHER INFORMATION FOR SIGNAL PARAMETER ESTIMATION IN COMPLEX GAUSSIAN NOISE

The variance of unbiased signal parameter estimates is bounded by the Cramér-Rao lower bound (CRLB), which is obtained from inverting the corresponding Fisher information matrix (FIM). The FIM components at row u and column v for signal parameter estimation in complex Gaussian noise are [10, Ch. 15.7]

$$\begin{aligned}[\mathbf{F}(\boldsymbol{\alpha})]_{u,v} = &\operatorname{tr}\left[\mathbf{C}_{\mathbf{r}}^{-1}(\boldsymbol{\alpha})\,\frac{\partial\,\mathbf{C}_{\mathbf{r}}(\boldsymbol{\alpha})}{\partial\alpha_u}\,\mathbf{C}_{\mathbf{r}}^{-1}(\boldsymbol{\alpha})\,\frac{\partial\,\mathbf{C}_{\mathbf{r}}(\boldsymbol{\alpha})}{\partial\alpha_v}\right] \\ &+ 2\,\mathrm{Re}\left[\frac{\partial\,\mathbf{s}^{\mathrm{H}}(\boldsymbol{\alpha})}{\partial\alpha_u}\,\mathbf{C}_{\mathbf{r}}^{-1}(\boldsymbol{\alpha})\,\frac{\partial\,\mathbf{s}(\boldsymbol{\alpha})}{\partial\alpha_v}\right].\end{aligned} \tag{7}$$

The subscript $(\cdot)^{\mathrm{H}}$ denotes the Hermitian of a vector or matrix. For the equation above, we collect all the received signal samples $r_{q,p}(k\,T)$, indexed by p, q and k, in a vector $\mathbf{r}$

and their respective mean values $s_p\left(k\,T - \frac{\tilde{d}_{q,p}}{c_0}\right)$ in vector $\mathbf{s}$. With that, $\mathbf{C_r}(\boldsymbol{\alpha})$ denotes the covariance matrix of $\mathbf{r}$. Vector $\boldsymbol{\alpha}$ is composed of the unknown parameters which we wish to estimate. For our exemplary scenario these parameters are the spatial and temporal coordinates of the MTs, i.e., $\boldsymbol{\alpha} = [x_2, T_2, x_3, y_3, T_3]^{\mathrm{T}}$. Note the parameters x_1, y_1, y_2 and T_1 are fixed (known) in order to define a spatial and temporal coordinate system.

3.1. Uncorrelated Gaussian Noise

In our case the covariance matrix does not depend on the parameter vector $\boldsymbol{\alpha}$. The first term in Eq. (7) (the *trace* term) vanishes for this reason. Further, we consider uncorrelated Gaussian noise, which results in a diagonal covariance matrix $\mathbf{C_r}$ for the observed signal samples. With these assumptions, Eq. (7) modifies to

$$[\mathbf{F}(\boldsymbol{\alpha})]_{u,v} = 2\,\mathrm{Re}\left[\sum_{q,p} \frac{1}{\sigma_{q,p}^2} \sum_k \frac{\partial\, s_p^*\left(k\,T - \frac{\tilde{d}_{q,p}}{c_0}\right)}{\partial \alpha_u} \frac{\partial\, s_p\left(k\,T - \frac{\tilde{d}_{q,p}}{c_0}\right)}{\partial \alpha_v}\right] \quad (8)$$

with complex AWGN variances $\sigma_{q,p}^2$ according to Eq. (2). For the calculation of the FIM according to Eq. (8) we need the derivatives of the observed signal with respect to the unknown parameters $\boldsymbol{\alpha}$, which we wish to estimate. Note that our signal defined in Eq. (1) depends on the parameters $\boldsymbol{\alpha}$ solely through the pseudo range introduced in Eq. (3). With this structure we get

$$\frac{\partial\, s_p\left(k\,T - \frac{\tilde{d}_{q,p}}{c_0}\right)}{\partial \alpha_u} = -\frac{1}{c_0}\,\dot{s}_{q,p}(k\,T)\,\frac{\partial\,\tilde{d}_{q,p}}{\partial \alpha_u} \quad (9)$$

by applying the chain rule. For convenience we denote $\frac{\partial}{\partial t}\, s(t) = \dot{s}(t)$. We insert (9) into (8) and obtain

$$[\mathbf{F}(\boldsymbol{\alpha})]_{u,v} = \sum_{q,p} \frac{\partial\,\tilde{d}_{q,p}}{\partial \alpha_u} \underbrace{\left(\frac{2}{\sigma_{q,p}^2\,c_0^2} \sum_k \left|\dot{s}_{q,p}(k\,T)\right|^2\right)}_{:=\tilde{\sigma}_{q,p}^{-2}} \frac{\partial\,\tilde{d}_{q,p}}{\partial \alpha_v}. \quad (10)$$

With the gradient for M unknown parameters $\nabla_{\boldsymbol{\alpha}} = \left[\frac{\partial}{\partial\alpha_1}, \frac{\partial}{\partial\alpha_2}, \dots \frac{\partial}{\partial\alpha_M}\right]^{\mathrm{T}}$ we can express the FIM as

$$\mathbf{F}(\boldsymbol{\alpha}) = \sum_{q,p} \frac{1}{\tilde{\sigma}_{q,p}^2}\,\nabla_{\boldsymbol{\alpha}}\tilde{d}_{q,p}\,\nabla_{\boldsymbol{\alpha}}^{\mathrm{T}}\tilde{d}_{q,p} \quad (11)$$

For L pseudo range observations, indexed by p and q, we collect the gradient vectors $\nabla_{\boldsymbol{\alpha}}\tilde{d}_{q,p}$ in an $M\times L$ matrix $\mathbf{G}$, which is called the *geometry matrix*. If we arrange the corresponding pseudo range variances $\tilde{\sigma}_{q,p}^2$ in an $L\times L$ diagonal matrix $\boldsymbol{\Sigma}$ we can rewrite Eq. (11) in matrix form as $\mathbf{F}(\boldsymbol{\alpha}) = \mathbf{G}\,\boldsymbol{\Sigma}^{-1}\,\mathbf{G}^{\mathrm{T}}$ and get the corresponding Cramér-Rao lower bound as

$$\mathbf{CRLB}(\boldsymbol{\alpha}) = \left(\mathbf{G}\,\boldsymbol{\Sigma}^{-1}\,\mathbf{G}^{\mathrm{T}}\right)^{-1}. \quad (12)$$

For the special case where the diagonal elements in $\boldsymbol{\Sigma}$ are equal, i.e., $\boldsymbol{\Sigma} = \sigma^2\mathbf{I}$, Eq. (12) simplifies to

$$\mathbf{CRLB}(\boldsymbol{\alpha}) = \sigma^2\left(\mathbf{G}\,\mathbf{G}^{\mathrm{T}}\right)^{-1}. \quad (13)$$

The diagonal elements of matrix $\mathbf{CRLB}(\boldsymbol{\alpha})$ provide the lower bound on the variance of an unbiased estimator for the corresponding parameters. Matrix $\left(\mathbf{G}\,\mathbf{G}^{\mathrm{T}}\right)$ is only dependent on the geometrical constellation of the MTs and TX to each other.

3.2. The Geometry Matrix

For calculation of the geometry matrix we have to derive the pseudo ranges $\tilde{d}_{q,p}$ with respect to the unknown parameters according to Eq. (11). For the derivatives with respect to the spatial parameters we get

$$\frac{\partial}{\partial x_q}\,\tilde{d}_{q,p} = \frac{x_q - x_p}{d_{q,p}} \qquad \frac{\partial}{\partial y_q}\,\tilde{d}_{q,p} = \frac{y_q - y_p}{d_{q,p}} \quad (14a)$$

$$\frac{\partial}{\partial x_p}\,\tilde{d}_{q,p} = \frac{x_p - x_q}{d_{q,p}} \qquad \frac{\partial}{\partial y_p}\,\tilde{d}_{q,p} = \frac{y_p - y_q}{d_{q,p}} \quad (14b)$$

with the true Euclidean distance $d_{q,p} = d_{p,q} = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2}$ between transmitter p and receiver q. The derivatives with respect to the time base offsets are

$$\frac{\partial}{\partial T_q}\,\tilde{d}_{q,p} = c_0 \qquad \frac{\partial}{\partial T_p}\,\tilde{d}_{q,p} = -c_0 \quad (15)$$

3.3. Pseudo Range Signal Model

The result of Eq. (10) corresponds to a real valued Gaussian signal model for the pseudo range observations

$$\breve{d}_{q,p} = \tilde{d}_{q,p} + \tilde{n}_{q,p}, \quad (16)$$

where $\tilde{n}_{q,p}$ is real AWGN with zero mean and variance

$$\mathrm{E}\left\{\tilde{n}_{q,p}^2\right\} = \tilde{\sigma}_{q,p}^2 = \left(\frac{2}{\sigma_{q,p}^2\,c_0^2} \sum_k \left|\dot{s}_{q,p}(k\,T)\right|^2\right)^{-1}. \quad (17)$$

We apply this model directly to the signals transmitted from the mobile transceivers MT_p, meaning that a signal design yields a variance of $\tilde{\sigma}_{q,p}^2$ for pseudo range estimation between the MTs.

For the SoO we have one complex valued sample $r_{q,0}$ according to Eq. (5). Inserting this into Eq. (17) yields

$$\mathrm{E}\left\{\tilde{n}_{q,0}^2\right\} = \tilde{\sigma}_{q,0}^2 = \frac{c_0^2\,\sigma_{q,0}^2}{8\pi^2\,f_{\mathrm{S}}^2\,P_q} = \frac{\lambda_{\mathrm{S}}^2}{8\pi^2}\,\mathrm{SNR}_q^{-1} \quad (18)$$

with wavelength $\lambda_{\mathrm{S}} = \frac{c_0}{f_{\mathrm{S}}}$ and the signal-to-noise ratio $\mathrm{SNR}_q = \frac{P_q}{\sigma_{q,0}^2}$ at transceiver MT_q. With TX power P_{TX} we can expect a receiver power level of

$$P_q = P_{\mathrm{TX}}\,G_{\mathrm{TX}}G_{\mathrm{RX}}\left(\frac{4\pi\,d_{q,0}}{\lambda_{\mathrm{S}}}\right)^{-2} \quad (19)$$

for free space signal propagation [11]. G_{TX} and G_{RX} denote the antenna gains at the TX and RX side respectively. With the Boltzmann constant $k_{\mathrm{B}} = 1.381 \times 10^{-23}\,\mathrm{Ws/K}$ and a noise temperature $T = 300\,\mathrm{K}$ we get a noise power of $\sigma_{q,0}^2 = k_{\mathrm{B}}\, T\, B_{\mathrm{S}}$ with bandwidth $B_{\mathrm{S}} = \frac{1}{T_{\mathrm{S}}}$ which is the inverse of the SoO period duration. Inserting this together with Eq. (19) into Eq. (18) yields

$$\tilde{\sigma}_{q,0} = d_{q,0} \sqrt{\frac{2\, k_{\mathrm{B}}\, T B_{\mathrm{S}}}{P_{\mathrm{TX}}\, G_{\mathrm{TX}} G_{\mathrm{RX}}}} \tag{20}$$

for the pseudo range SoO model.

4. RESULTS

We refer to Fig. 1 and define the origin of a local 2D coordinate system at the position of MT_1. Further, we choose the time base of MT_1 as our local temporal reference. MT_2 is located on the positive x-axis. The position of MT_3 is chosen such that the three MTs form an equilateral triangle with distance d_{MT}, i.e., the MTs form an uniform circular array with three elements as shown in Fig. 1. Thus, we set

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = d_{\mathrm{MT}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = d_{\mathrm{MT}} \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \tag{21}$$

as ground truth positions of the MTs. With the definition of the spatial and temporal reference above we can fix the estimates $\hat{x}_1 = 0$, $\hat{y}_1 = 0$, $\hat{T}_1 = 0$ and $\hat{y}_2 = 0$. Subsequently, we evaluate the position accuracy of MT_3.

4.1. Reference

As reference scenario we consider the arrangement of the MTs as described above without the SoO source TX. Thus, the unknown parameters to estimate are $\boldsymbol{\alpha} = [x_2, T_2, x_3, y_3, T_3]^{\mathrm{T}}$, where at the end we are interested in the estimation performance for the position coordinates x_3, y_3 of MT_3. With 5 unknowns and 6 possible pseudo range observations $\tilde{d}_{2,1}, \ldots, \tilde{d}_{2,3}$ the 5×6 geometry matrix for this reference scenario is

$$\mathbf{G} = \left(\nabla_{\alpha} \tilde{d}_{2,1},\ \nabla_{\alpha} \tilde{d}_{3,1},\ \nabla_{\alpha} \tilde{d}_{1,2},\ \nabla_{\alpha} \tilde{d}_{3,2},\ \nabla_{\alpha} \tilde{d}_{1,3},\ \nabla_{\alpha} \tilde{d}_{2,3}, \right). \tag{22}$$

For the signals transmitted from the MTs we use the pseudo range signal model according to Eqs. (16) and (17) with equal variance $\tilde{\sigma}_{q,p}^2 = \sigma_{\mathrm{MT}}^2$. Thus, we calculate the Cramér-Rao lower bound according to Eq. (13). The square root of the sum of the 3rd and 4th diagonal element of that matrix yields the Cramér-Rao lower bound for the position root mean square error (RMSE) of MT_3 as

$$\begin{aligned} \mathrm{RMSE}_{\mathrm{MT}_3} &= \sqrt{\mathrm{RMSE}_{\hat{x}_3}^2 + \mathrm{RMSE}_{\hat{y}_3}^2} \\ &\geq \sigma_{\mathrm{MT}} \sqrt{\left[(\mathbf{G}\,\mathbf{G}^{\mathrm{T}})^{-1} \right]_{3,3} + \left[(\mathbf{G}\,\mathbf{G}^{\mathrm{T}})^{-1} \right]_{4,4}} \\ &= \sigma_{\mathrm{MT}} \sqrt{1.0607^2 + 0.6124^2} \\ &= 1.225 \times \sigma_{\mathrm{MT}} = \sqrt{\mathrm{CRLB}_{\mathrm{ref}}}. \end{aligned} \tag{23}$$

Note the result above is only dependent on the pseudo range noise variance σ_{MT}^2 and the geometry of the MTs. It is invariant with respect to a scale factor, e.g., d_{MT} in Eq. (21). We use this result as a reference in order to assess the positioning performance improvement when introducing a SoO source.

4.2. Positioning using a Signal of Opportunity

As a next step we introduce an additional SoO source TX. We assume that we know the coordinates of TX but not its local time base offset T_{S}. Thus, the new vector of unknown parameters is $\boldsymbol{\alpha} = [x_2, T_2, x_3, y_3, T_3, T_0]^{\mathrm{T}}$ and extended by T_0 compared to our reference scenario. We calculate the CRLB, in particular its square root $\sqrt{\mathrm{CRLB}_{\mathrm{MT}_3}}$, according to Eq. (12), where we focus on the position parameters of MT_3 similar to Eq. (23). The diagonal matrix $\boldsymbol{\Sigma}$ in this case consists of pseudo range variances $\tilde{\sigma}_{q,p}^2 = \sigma_{\mathrm{MT}}^2$ for the observation of signals transmitted from MTs and $\tilde{\sigma}_{q,0}^2 = \sigma_{\mathrm{S}}^2$ for the SoO. Fig. 2(a) shows the achievable gain

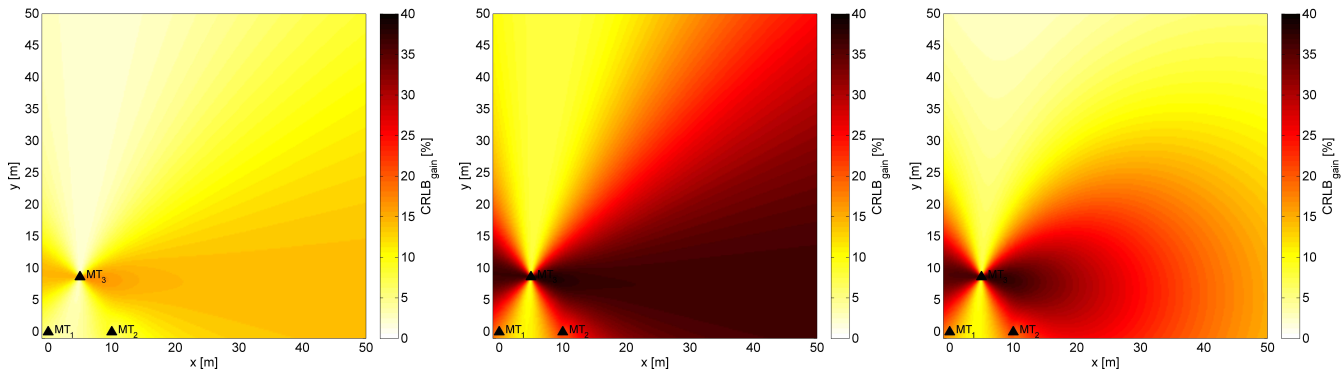
$$\mathrm{CRLB}_{\mathrm{gain}} = 1 - \sqrt{\frac{\mathrm{CRLB}_{\mathrm{MT}_3}}{\mathrm{CRLB}_{\mathrm{ref}}}} \tag{24}$$

in percent for $\sigma_{\mathrm{MT}} = 1\,\mathrm{m}$ versus the location of the SoO source TX in a 2D area. The pseudo range variance for SoO observations is $\sigma_{\mathrm{S}} = 1\,\mathrm{m}$ and independent of the distance between TX and MTs for the moment. It should be mentioned that the achievable gain for this example depends only on the ratio between the pseudo range variances σ_{MT} and σ_{S}. If both variances are equal, the achievable gain in positioning accuracy is $\approx 17.2\,\%$. This gain vanishes as $\sigma_{\mathrm{S}} \gg \sigma_{\mathrm{MT}}$.

To assess the maximum achievable gain for this example we let $\sigma_{\mathrm{S}} \ll \sigma_{\mathrm{MT}}$, i.e., $\sigma_{\mathrm{S}} \to 0$. Fig. 2(b) shows the corresponding result. We observe a maximum achievable gain of $\approx 38.8\,\%$. The gain is higher if TX is located horizontally to MT_3. This can be explained from the constellation of the MTs and TX itself. MT_1 and MT_2 are located south to MT_3, and therefore, provide higher accuracy in vertical direction for positioning of MT_3. Thus an additional source can improve the positioning performance of MT_3 compared to the reference scenario if it is located such that it improves in particular the horizontal positioning performance of MT_3.

We now consider the case where σ_{S} depends on the distance between TX and the MTs due to signal propagation loss. As already mentioned, the MTs form a uniform circular array with element distance $d_{\mathrm{MT}} = 10\,\mathrm{m}$. We choose this distance such that it is half the wavelength of the observed SoO carrier. Thus the period duration of our SoO is $T_{\mathrm{S}} = \frac{\lambda_{\mathrm{S}}}{c_0} = \frac{2\, d_{\mathrm{MT}}}{c_0} = 66.7\,\mathrm{ns}$, which corresponds to a carrier frequency of $B_{\mathrm{S}} = 15\,\mathrm{MHz}$. We choose the parameters in Eq. (20) such that $\tilde{\sigma}_{q,0} = 1\,\mathrm{m}$ at a reference distance of 50 m. Assuming free space signal propagation and isotropic antenna gains of $G_{\mathrm{TX}} = G_{\mathrm{RX}} = 1$, this is already achieved for a TX power of

$$P_{\mathrm{TX}} = d_{q,0}^2\, \frac{2\, k_{\mathrm{B}}\, T\, B_{\mathrm{S}}}{\tilde{\sigma}_{q,0}^2\, G_{\mathrm{TX}} G_{\mathrm{RX}}} = -65\,\mathrm{dBm}. \tag{25}$$



(a) $\sigma_{\mathrm{MT}} = 1\,\mathrm{m}$, $\sigma_{\mathrm{S}} = 1\,\mathrm{m}$

(b) $\sigma_{\mathrm{MT}} = 1\,\mathrm{m}$, $\sigma_{\mathrm{S}} \to 0$

(c) $\sigma_{\mathrm{MT}} = 1\,\mathrm{m}$, $\sigma_{\mathrm{S}} = 1\,\mathrm{m}$ at a distance of 50 m, the SoO is subject to free space loss

Fig. 2. Gain in positioning accuracy in percent for MT_3, shown in different colors, versus all positions of the SoO source TX in a 2D area.

Fig. 2(c) shows the corresponding results, where we can observe that the gain achieves the asymptotic gain (c.f. Fig. 2(b)) as the SoO source TX gets closer to the MTs with respect to the reference distance (50 m). As the MT-TX distances increase, $\sigma_{\mathrm{S}} \gg \sigma_{\mathrm{MT}}$ and the achievable gain vanishes. The maximum gain for this example is $\approx 38.0\,\%$.

5. CONCLUSIONS

In this paper we have exemplarily evaluated the benefit of a signal of opportunity for mobile terminal positioning. For this case we have derived the appropriate form of the Cramér-Rao lower bound. On that basis we have calculated the performance gains which can be expected theoretically. In the considered scenario three MTs are arranged in a equilateral triangle and observe a single carrier, transmitted from a SoO source with known position. By observing this signal of opportunity the Cramér-Rao lower bound predicts a performance gain of up to $\approx 38.8\,\%$ for this example. Of further interest are the evaluation of the performance gains with an increasing number of observed SoOs as well as relaxing the assumption of known SoO source positions.

6. REFERENCES

[1] P. Misra and P. Enge, *Global Positioning System — Signals, Measurements and Performance*, 2nd ed. Ganga-Jamuna Press, 2005, ISBN 0-9709544-1-7.

[2] P. M. F. Dovis and D. Orgiazzi, "Assessment study of navigation and communication terminal-based hybrid positioning," *Proceedings of ION/GNSS*, May 2002.

[3] C. Mensing, S. Sand, and A. Dammann, "Hybrid data fusion and tracking for positioning with GNSS and 3GPP-LTE," *International Journal of Navigation and Observation*, vol. 2010, Aug. 2010, doi:10.1155/2010/812945.

[4] F. Gustafsson and F. Gunnarsson, "Mobile positioning using wireless networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 41–53, Jul. 2005.

[5] *LTE; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical channels and modulation (3GPP TS 36.211 version 10.1.0 Release 10)*, ETSI, Apr. 2011, ETSI TS 136 211 V10.1.0.

[6] M. Rabinowitz and J. J. Spilker, Jr., "A new positioning system using television synchronization signals," *IEEE Transactions on Broadcasting*, vol. 51, no. 1, pp. 51–61, Mar. 2005.

[7] R. K. Martin, J. S. Velotta, and J. F. Raquet, "Bandwidth efficient cooperative TDOA computation for multicarrier signals of opportunity," *IEEE Transactions on Signal Processing*, vol. 57, no. 6, pp. 2311–2322, Jun. 2009.

[8] P. Ioannides and C. Balanis, "Uniform circular arrays for smart antennas," *IEEE Antennas and Propagation Magazine*, vol. 47, no. 4, pp. 192–206, Aug. 2005.

[9] H. Godrich, A. Haimovich, and R. Blum, "Cramér-Rao bound on target localization estimation in MIMO radar systems," in *Information Sciences and Systems, 2008. CISS 2008. 42nd Annual Conference on*, Mar. 2008, pp. 134–139.

[10] S. M. Kay, *Fundamentals of Statistical Processing — Estimation Theory*. Prentice Hall, 1993, ISBN 0-13-345711-7.

[11] H. Friis, "A note on a simple transmission formula," *Proceedings of the IRE*, vol. 34, no. 5, pp. 254–256, may 1946.