Final states of decaying 2D turbulence in different geometries with no-slip walls

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Summary. Direct numerical simulations of two-dimensional decaying turbulence in domains of different geometries having no-slip walls are presented. Starting from random initial conditions the flow rapidly exhibits self-organization into coherent vortices. At the same time viscous boundary layers are formed on the walls, become unstable and produce new coherent vortices which are injected into the bulk flow. The computation uses a pseudo-spectral method with volume penalization to model the walls. Each flow is integrated until a quasi-final state is reached.

Two–dimensional turbulence in wall bounded domains has many applications in geophysical flows, *e.g.*, the prediction of coastal currents in oceanography, the transport and mixing of pollutants. Direct numerical simulations of 2D turbulence in circular and square domains can be found, *e.g.* in [2, 3, 5]. The aim of the present paper is to study the influence of the geometry of the domain on the flow dynamics and in particular on the long time behaviour of the flow. We consider different geometries, a circle, a square, a triangle and a torus.

The numerical technique we use here is based on a Fourier pseudo–spectral method with semi-implicit time discretization and adaptive time-stepping [4]. The Navier–Stokes equations are solved in a double periodic square domain of size $L = 2\pi$ using the vorticity–velocity formulation. The bounded domain is thus imbedded in the periodic domain and the no-slip boundary conditions on the wall $\partial\Omega$ are imposed using a volume penalisation method. A mathematical analysis of the method is given in [1], proving its convergence towards the Navier–Stokes equations with no–slip boundary conditions. Details on the code together with its numerical validation can be found in [4]. The governing equations in vorticity-velocity formulation are,

$$\partial_t \omega + \boldsymbol{u} \cdot \nabla \omega - \nu \nabla^2 \omega + \nabla \times \left(\frac{1}{\eta} \chi \boldsymbol{u}\right) = 0$$

where $\boldsymbol{u}$ is the divergence-free velocity field, *i.e.* $\nabla \cdot \boldsymbol{u} = 0$, $\omega = \nabla \times \boldsymbol{u}$ the vorticity, ν the kinematic viscosity and $\chi(\boldsymbol{x})$ a mask function which is 0 inside the fluid, *i.e.* $\boldsymbol{x} \in \Omega$, and 1 inside the solid wall. The penalisation parameter η is chosen to be sufficiently small ($\eta = 10^{-3}$) [4].

Starting with random initial conditions we compute the flow evolution in different geometries for initial Reynolds numbers of about 1000. Figure 1 shows the time evolution of energy, enstrophy and palinstrophy, and figure 2 the vorticity fields at early, intermediate and late times, for different flows in circular, square, triangular and toroidal geometries. More details and discussion of the results can be found in [6].

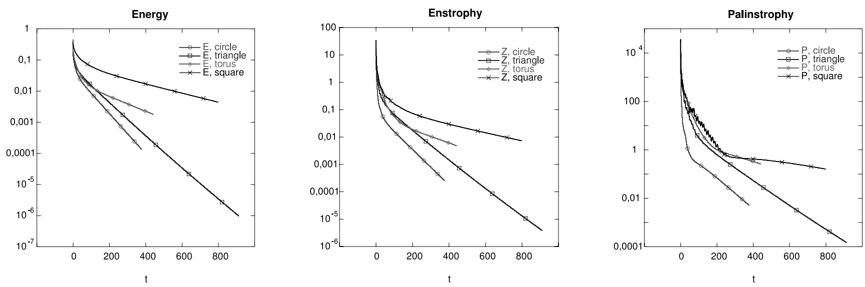

Fig. 1. Two-dimensional decaying turbulence in bounded domains. Time evolution until the final state is reached for energy (left), enstrophy (middle) and palinstrophy (right).

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References

1. P. Angot, C.H. Bruneau and P. Fabrie. *Numer. Math.* **81**, 497–520, 1999.
2. H.J.H. Clercx, A.H. Nielsen, D.J. Torres and E.A. Coutsias. *Eur. J. Mech. B - Fluids* **20**, 557–576, 2001.
3. S. Li, D. Montgomery and B. Jones. *Theor. Comput. Fluid Dyn.* **9**, 167–181, 1997.
4. K. Schneider. *Comput. Fluids*, **34**, 1223–1238, 2005.
5. K. Schneider and M. Farge. *Phys. Rev. Lett.*, **95**, 244502, 2005.
6. K. Schneider and M. Farge. Decaying two–dimensional turbulence in bounded domains. *Preprint, 2007.*

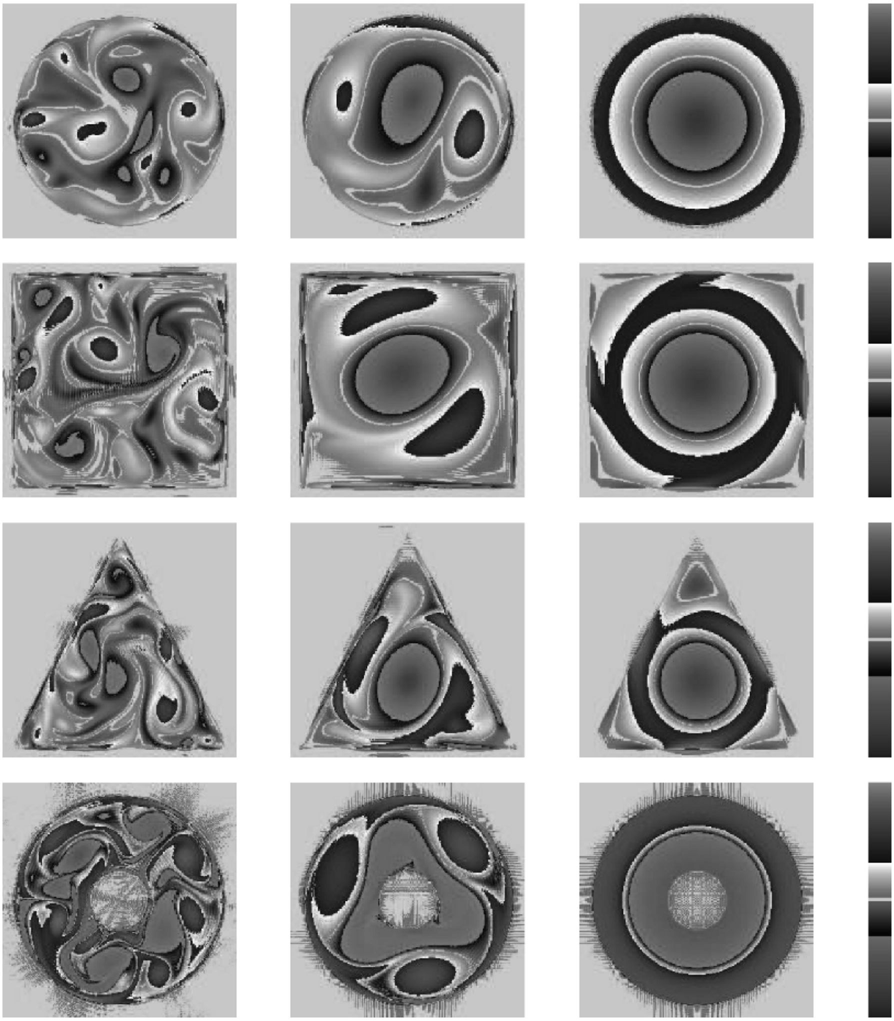

Fig. 2. Two-dimensional decaying turbulence in bounded domains. Vorticity fields at early (left), intermediate (middle) and late times (right). From top to bottom: circular, square, triangular and toroidal domains.