Analysis of a bursting vortex using continuous and orthogonal wavelets

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We study the time evolution of the quasi-periodic bursting of a laboratory produced vortex using orthogonal and continuous wavelets.

1 Laboratory Experiment

The vortex is produced in laminar channel flow. The vortex is both stretched by axial suction and strained by the channel flow and eventually breaks down, resulting in a burst that leads to the production of turbulence. A new vortex is formed after each burst, and the cycle repeats quasi-periodically. The current bursting vortex has been the subject of previous studies of the buildup of the turbulence due to the bursting [1, 2, 3], however previous measurements were not well resolved simultaneously in time and space.

We measure the velocity field in a plane perpendicular to the vortex by particle image velocimetry and calculate the vorticity component perpendicular to the plane, shown in Fig. 1 (a). The current measurements are sufficiently well resolved in time and space to allow us to study the transient buildup.

2 Why Wavelets?

The vortex under study is a quasi-stationary coherent structure which moves in space before bursting. After bursting the evolution of the remaining pieces which have been spread in space is highly nonlinear. As a result the measured signal is inhomogeneous and non-stationary. It is therefore more natural to analyze this flow using a spatially localized set of basis functions rather than a

Fourier basis. Wavelets consist of translations and dilations of a compact function localized in both physical and spectral space. A wavelet basis is a better choice to analyze signals that contain features well localized in physical space and non-stationary in time [4]. Indeed, it has been found in simulation [5] and laboratory experiment [6] that the dynamics of turbulent flows are dominated by the contribution of a relatively small fraction of wavelet coefficients corresponding to the coherent structures.

3 The Orthogonal and Continuous Wavelet Transforms

The orthogonal wavelet transform (OWT) permits a signal to be decomposed into independent contributions, which can be separately reconstructed, possibly after filtering out some coefficients. To insure orthogonality the transform should be performed with discrete values of translation and dilation corresponding to a dyadic grid in wavelet space. A loss of translation invariance results which makes it difficult to read the OWT coefficients. In contrast, the continuous wavelet transform (CWT) permits the dilations and translations to vary continuously, making the coefficients in wavelet space easy to read and interpret [4]. The CWT unfolds signals in space and scale (and possibly direction), allowing one to study how energy is distributed in space and scale by reading the modulus of complex-valued wavelet cofficients. However this also results in a redundancy of the wavelet coefficients and in a correlation between neighboring coefficients which hinders interpretation.

4 Results

We use the OWT to separate the measured vorticity field into a coherent and an incoherent component [shown in Fig. 1 (b) and (c)], following reference [5]. The coherent field retains the dynamical and statistical properties of the total field, such as the evolution of the non-Gaussian PDF and large-scale

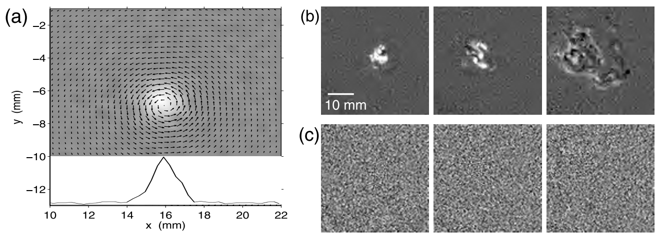

Fig. 1. **(a)** Close-up of a vortex prior to bursting and a 1D cut of the vorticity field along its center. The velocity field is superimposed on the vorticity field. The largest velocity (vorticity) value corresponds to 0.37 m/s (200 s^{-1}). **(b)** Time evolution of the coherent and **(c)** incoherent fields during bursting at 0.33 second intervals

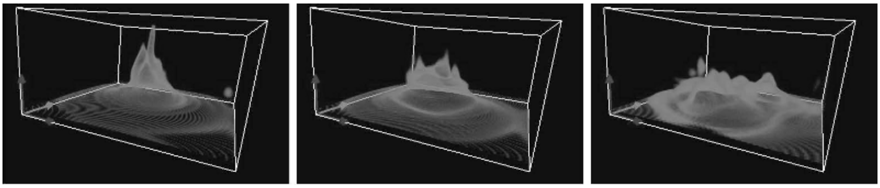

Fig. 2. Isosurfaces of the modulus of the CWT coefficients of three snapshots of the coherent vorticity field during bursting [corresponding to Fig. 1 (b)]. The hortional axes correspond to physical space and vertical axis corresponds to the scale of the transform, with smaller scales at the top

energy spectrum. It is efficiently captured by a small percentage of the large amplitude wavelet coefficients. In contrast, the incoherent field, corresponding to the remaining small amplitude wavelet coefficients, is uncorrelated and featureless with quasi-Gaussian statistics.

We calculate the CWT of the coherent vorticity field, shown in Fig. 2, using a complex-valued Morlet wavelet. The square modulus of the coefficients is thus the local enstrophy density in space and scale. We use the coefficients of the CWT to calculate the evolution of the local intermittency factor, i.e. the deviation from the mean energy spectrum at each location in space [4]. We can thus identify which locations actively contribute to the nonlinear cascade in the inertial range, and which locations are dominated by viscous dissipation.

5 Conclusion

Orthogonal wavelets were used to separate the flow field into a coherent component, capturing the nonlinear dynamics and statistics of the bursting, and an incoherent component void of structure and with quasi-Gaussian statistics. The CWT has allowed us to visualize the wavelet coefficients and track the time evolution of the coherent enstrophy in space and scale. This gives us better insight to interprete the nonlinear cascade of turbulent flows. Each transform has its advantages, thus we recommend that a mixture of the two analyses should be used, each one complementing the other.

References

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