Burn and coast method timing optimization for BYU Supermileage vehicle

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ABSTRACT

The Mechanical Engineering Department at Brigham Young University sends a team each year to compete internationally with a Supermileage vehicle of their own design and build. The object of participation with their vehicle is to achieve the maximum fuel efficiency possible. High fuel efficiency is achieved by minimizing energy losses and developing optimal driving methods specific to the vehicle and the race track. This research details procedures and results of optimization applied to the "burn and coast" method of driving that BYU implements in competition. Total fuel consumption was the objective of the optimization. An objective function was derived from an energy balance defining the inputs and losses of the Supermileage vehicle. Optimization constraints were drawn from competition rules dictating average velocity limits. A constrained gradient based optimization was performed in Matlab using the fmincon function. The design variables used were the start and end positions of the engine burns along the competition track. Results were calculated by the optimizer for one, two and three burns per lap, and several sets of initial conditions were implemented for each case thereof. Results from the optimization limited total position of engine burn to ~16 m regardless of the number of burns or the initial conditions. Based on the given track and the results of the optimization it is suggested that the vehicle run the engine twice for each lap; once at the beginning of the lap and again shortly after the fourth turn.

NOMENCLATURE

V – Velocity (m/s)

x – Position on the track (m)

A – Frontal area of vehicle (m^2)

m – Mass of vehicle and driver (kg)

C_d – Drag Coefficient

μ – Coefficient of Friction (tire to road)

η – Engine Efficiency

$\dot{m}_f$ – mass flow rate of fuel (kg/s)

ρ – Air density (kg/m^3)

R – Turning Radius (m)

g – Gravity (m/s^2)

h – Height (m)

LHV – Lower Heating Value of fuel (kJ/kg)

δ_e – Engine Boolean value (1 for engine on, 0 for engine off

δ_c – Cornering Boolean value

P – Engine Ignition Penalty (kg)

INTRODUCTION

Each year a team of senior students from the Brigham Young University Mechanical Engineering Program designs and builds a Supermileage Vehicle to compete in an international, intercollegiate competition. The competition this year is the Shell Eco-marathon, and is held amongst participating universities to demonstrate the best solution for fuel efficiency of a single occupant vehicle powered by an internal-combustion engine (SAE International, 2016). Historically, the team strives to focus their efforts on building an efficient engine for their vehicle and minimize energy losses in their car due to friction and drag (see Figure 1 for a picture of the BYU vehicle).

Figure 1: The 2014 BYU Supermileage Vehicle. It achieved 1300 mpg to win the 2014 SAE Supermileage competition.

As part of their classic race-day strategy for maximizing fuel economy BYU implements a “burn and coast” racing technique. The burn and coast method involves selectively running the engine for short periods of time to minimize fuel consumption and maintain a competition-specified average velocity. In general the method has proven successful in obtaining improved fuel economy.

Although the burn and coast method of driving the vehicle has proved more successful than other methods of driving the vehicle in terms of fuel efficiency, it is this aspect of the team's strategy that is the subject of this research. To this point in the team's competing history, BYU has utilized, but not optimized the timing of the burn and coast method for fuel economy. Thus, given data describing the physical characteristics of this year's vehicle and the competition track, the goal of this research was to apply optimization techniques to the timing of running the given engine on the specified track in order to minimize fuel consumption

Competition rules drove several constraints on this optimization problem. Most notably, as per the Shell Eco-Marathon rules, the average velocity of the vehicle over the course of an entire run may not exceed 25 mph (11.176 m/s), nor may it be less than 15 mph (6.7056 m/s) (SAE Supermileage Rules and Important Documents, 2016). The track itself is ~ 1 km in length and a full run consists of 10 laps of the track.

FORMULATION OF OBJECTIVE

The objective for the optimization of the burn and coast method was derived from an energy balance of the Supermileage Vehicle as derived by Dr. Jerry Bowman. The energy balance (1) was first derived in the time domain. It states that the change of the kinetic and potential energy of the vehicle with respect to time are equal to the energy output by the engine minus energy lost to friction at the tires, air drag, and cornering. The time-domain energy balance was simplified to a relationship that demonstrates the equivalence to the vehicle's change of velocity with respect to time (Bowman, 2016).

$$\frac{d}{dt}\left[\frac{1}{2}mV^2 + mgh\right] = \delta_e\left(\eta\dot{m}_f LHV - P\right) - \mu mgV - \frac{1}{2}\rho V^3 C_d A - \frac{\delta_c 0.95V^{3.9}}{R^{1.4}} \tag{1}$$

$$mV\frac{dV}{dt} + mg\frac{dh}{dt} = \delta_e\left(\eta\dot{m}_f LHV - P\right) - \mu mgV - \frac{1}{2}\rho V^3 C_d A - \frac{\delta_c 0.95V^{3.9}}{R^{1.4}}$$

$$\frac{dV}{dt} = \frac{\delta_e\left(\eta\dot{m}_f LHV - P\right)}{mV} - \mu g - \frac{\rho V^2 C_d A}{2m} - \frac{\delta_c 0.95V^{3.9}}{mR^{1.4}} - \frac{g}{V}\frac{dh}{dt} \tag{2}$$

Due to the nature of the race and the information available to us, it was more useful to minimize the fuel consumption in the space domain instead of the time domain. This is because the design variables we have chosen to use are discrete positions (m) along the track. Therefore, using several substitutions for dV/dt, dh/dt and dx/dt, we were able to solve equation 2 in the space domain (3).

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt}$$

$$\frac{dh}{dt} = \frac{dh}{dx}\frac{dx}{dt}$$

$$\frac{dx}{dt} = V$$

$$mV^2\frac{dV}{dx} + mgV\frac{dh}{dx} = \delta_e\left(\eta\dot{m}_f LHV - P\right) - \mu mgV - \frac{1}{2}\rho V^3 C_d A - \frac{\delta_c 0.95V^{1.9}}{R^{1.4}}$$

$$\frac{dV}{dx} = \frac{\delta_e\left(\eta\dot{m}_f LHV - P\right)}{mV^2} - \frac{\mu g}{V} - \frac{\rho V C_d A}{2m} - \frac{\delta_c 0.95V^{1.9}}{mR^{1.4}} - \frac{g}{V}\frac{dh}{dx} \tag{3}$$

Now that we have the derivative of velocity with respect to position on the track, that formula is used to calculate the velocity at each respective velocity on the track ($V_i=V_{i-1}+dV_i/dt$). Given the velocities at every position on the track, an average velocity can then be calculated.

Ultimately, the objective (f) that we minimize is the total mass of fuel consumed by the engine (m_f). The total fuel consumption is measured by summing the discrete amounts of fuel burned at each interval for which the engine is on. Also, an ignition penalty of .7 g of fuel is applied for each time the engine is run as per dynamometer testing of the 2016 supermileage engine. The function space is a result multiplying the $\dot{m}_f$ by the corresponding burn time to get the fuel burned at each position and then summing the values of fuel burned at every position on the track.

Simplifications to the model were made in order to develop a clean, generalized representation of the vehicle. Some of the assumptions made include: 1) the turning radius at all corners is constant for each corner and relatively large (10 m), 2) the engine ignition penalty is always the same, 3) the engine experiences consistent performance (i.e. the efficiency is constant), and 4) there are no effects from wind or weather on the vehicle.

OPTIMIZATION APPROACH

We performed the optimization of this objective using a constrained gradient based method. Matlab's fmincon() function was implemented to perform the optimization (MathWorks, Inc., 2016). Critical to the performance of the optimizer was the proper application of constraints to the function space. Constraints on the average velocity can now be set corresponding to competition rules:

$$6.7056\frac{m}{s} < V_{avg} < 11.176\frac{m}{s}$$

Other constraints are set such that the position of the start of an engine burn must be less than the end burn position:

$$x_i < x_{i+1}$$

The energy balance equation provided to us by Dr. Bowman is a nonlinear differential equation. Our numerical approximation to the equation is undefined at x= 0 and loses physical meaning if the velocity is negative. To prevent this from happening we attempted to constrain the velocity at each point along the track. Unfortunately our discretization of the solution created 10,000 constraints for one lap around the track. Attempts to optimize the full course, 10 laps, created 100,000 constraints. We thought an active set method would be deal with this large number of constraints but even it was unable to operate under these conditions (Nocedal, 2006). A decision was made to hard code a fail-safe to prevent the velocity from ever going below zero. Optimized solutions that triggered the fail safes were discarded and we input initial variables that would avoid these discontinuities.

When supplying gradients to the optimizer two methods were tested, namely the finite difference method and complex step method. The difficulty in calculating these gradients lied in the nature of the input variables. Our inputs were used as switches for the burning fuel and turning fuel off. We feared that a small perturbation of the variables would not provide accurate gradients using the finite difference method. To check the gradient we moved to the complex method. However, because the imaginary perturbation was used in the switch and not directly on the function space no gradient was found. To work around this a new approach was needed.

To calculate the needed time we used the average value theorem at each interval of burning and coasting to find the average velocity at burn and coast intervals (Stewart, 2008). This could in turn be used to find the optimal fuel usage while allowing for imaginary perturbations to take effect.

$$V_{avg} = \frac{P_1\%}{x_2 - x_1}\int_{x_1}^{x_2} V(x)dx + \frac{P_2\%}{x_3 - x_2}\int_{x_2}^{x_3} V(x)dx + \ldots$$

The results converged to similar results while optimizing with 2 input variables but for more variables the complex method gave nonsensical answers. This implies errors in implementing the mean value theorem to define the objective function. For simplicity and accuracy we elected to move forward with the finite difference method.

In addition, many possible solutions fall outside constraint values. In order to move the optimization search towards feasible solutions, the optimization algorithm employs a penalty method (Chandrupatla., 1999). Each time the constraint is violated, a penalty of one gram of fuel is given to the objective function. This penalty for constraint violation assisted the optimization process in obtaining an optimal performance strategy.

Difficulties arise in the optimization when solving for more than 2 variables. In several cases the first order optimality is on the order or 10^{-2} instead of 10^{-6}. These may very well be due to the fact that our discretization of the energy balance equation is of order 10^{-1} limiting our resolution. To combat this problem we compared the diagnostics of the resulting optimization to determine by eye a reasonable optimal point that satisfied the constraints. To improve our understanding of the optimum we compare the achieved results from testing different number of burns for one lap around the track. In addition we compare our optimization of one lap to an optimization of the entire race (see Results).

RESULTS

Before reasonable results could be reached for any case, the function space was analyzed for 1 burn in order to get an initial understanding of where to expect to find minimums in the results. A low-resolution example of the function space for one burn can be seen in Figure 2. Within the function space it is obvious that the optimizer will trend towards the front-left face of the valley. The dark floor towards

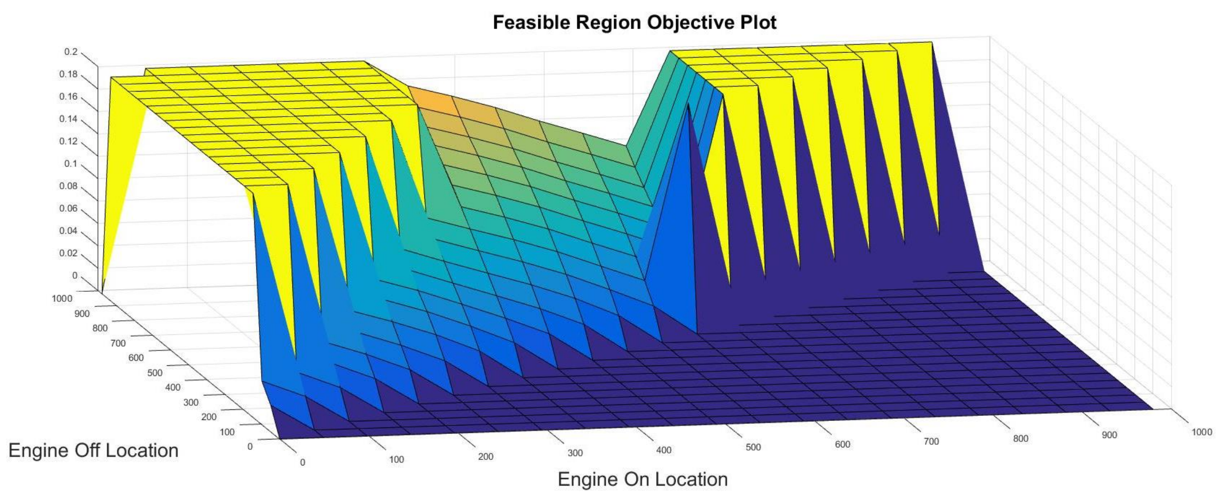

Figure 2: Preliminary contour plot of function space (fuel consumption) for one burn

the right of the plot represents the portion of the space where the beginning burn location is greater than the ending location and thus is infeasible. The yellow plateaus represent areas that have violated the constraints.

As mentioned previously, optimization was then performed for a single lap at one, two and three burns. For each burn case, four sets of initial points were tested in attempt to lower the risk of missing optimal results due to getting stuck in local minima.

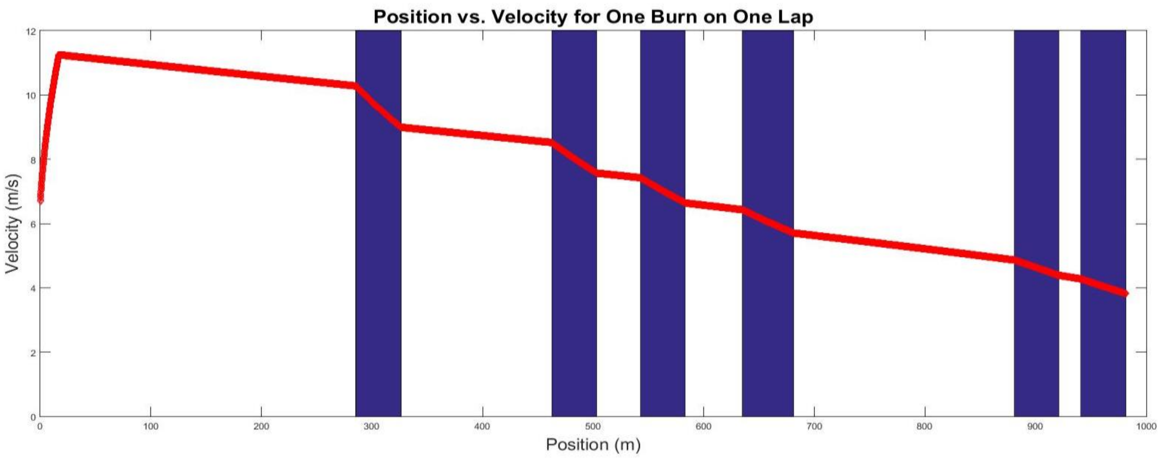

Figure 3: Plot of optimization results for Velocity vs. Position with one burn on a single lap of the course. The Blue lines indicate turns on the course.

The first optimizations performed were for one lap and one burn at various initial points. All of them demonstrated accordance with Figure 2 as the (x_1,x_2) location of the single burn resulted in a relatively short burn towards the beginning of the lap. For example, in Figure 3, it is obvious that the location of the burn is at the very beginning of the lap, lasting only 15 or 20 meters.

In order to validate the single-lap analysis approach, an entire run was put together of 10 laps of the track and an optimization of 10 burn positions was run. The result is seen in Figure 4:

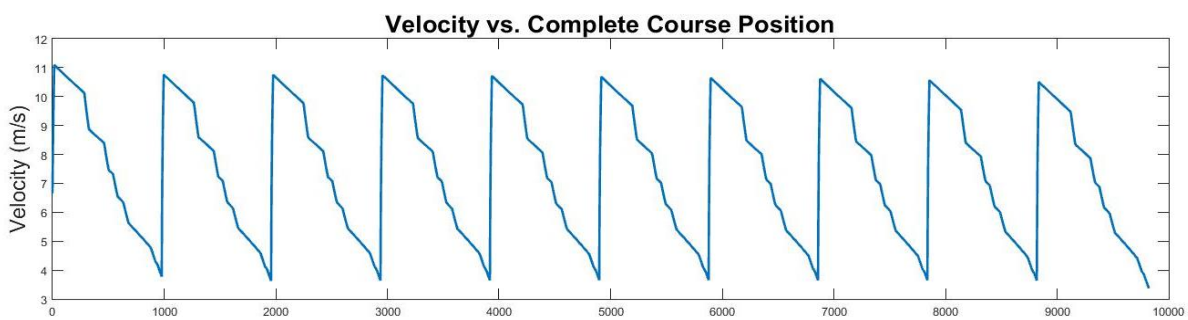

Figure 4: Optimization results for 10 burns over complete course. Position vs. Velocity is plotted, and indicates a single burn per lap towards the beginning of each lap.

The results of the 10 burn, full track optimization served to demonstrate that a single-lap analysis would be effective. This conclusion was drawn from the fact that the burn position converged at the beginning of every lap and demonstrated distinct repeatability. Therefore, further optimization proceeded in a single-lap analysis fashion. Single laps were given initial velocities near the minimum acceptable average velocity (~6.7 m/s) and continued to be optimized for one, two and three burns per/lap at 4 different sets of initial burn positions.

A rough analysis of the various retrieved velocity plots indicated that though elevation change over the length of the course does play some role in the energy loss and gain of the vehicle, cornering is a more significant loss and tended to impact the optimization much more. When looking at equation 3, this is a logical result. The cornering term in equation 3 includes a velocity term of power 1.9, which results in cornering being the greatest power loss of the given model.

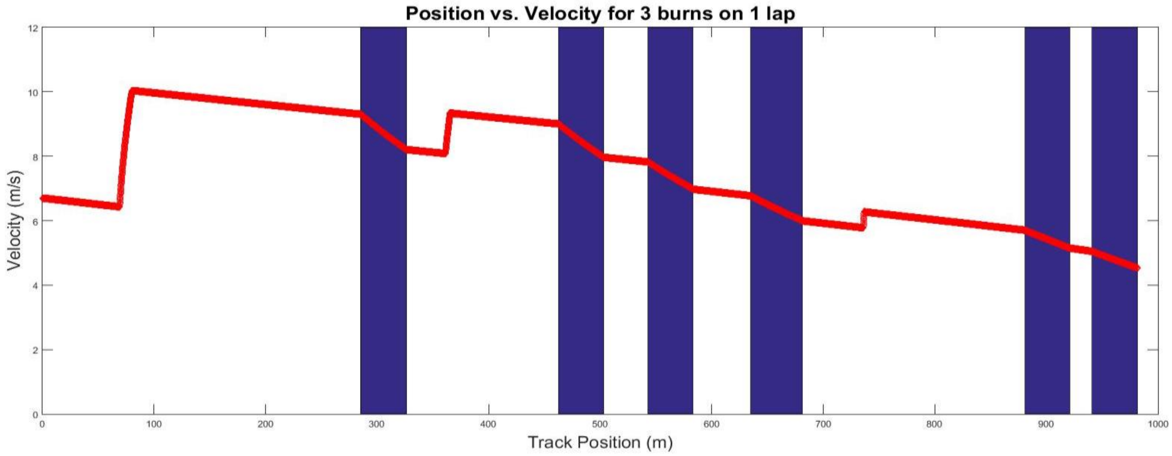

Figure 5: Plot of an optimization results for Velocity vs. Position with three burns on one lap. The Blue lines indicate turns on the course. It seems significant that engine burns come after corners where there is significant space before the succeeding corner.

Table 1: Results of one, two and three burn optimizations of various initial points.

	One-Burn Lap				
	Case #1	Case #2	Case #3	Case #4	Case #5
Initial Conditions	0, 200	200, 400	400, 600	600, 800	800, 980
Optimal Run Positions (m)	**0, 16.9**	**291.5, 331.7**	**291.0, 331.3**	**292.3, 332.5**	**310.2, 326.5**
Objective Function Value	5.88	11.18	11.18	11.16	5.52
Fuel Consumed (g)	**9.41**	**17.89**	**17.89**	**17.86**	**8.83**
Predicted fuel efficiency (MPG)	1321.27	694.51	694.51	695.81	1406.68
Average Velociy (m/s)	7.03	7.86	7.86	7.87	6.73
	Two-Burn Lap				
Initial Conditions	0, 400, 500, 980	0, 300, 301, 500	0, 50, 690, 740	300, 500, 700, 900	
Optimal Run Positions (m)	**356.5, 371.3 758.4, 759.9**	**0, 10.2731, 215.2099, 219.4550**	**24.9, 34.7, 715.6, 717.5**	**0.7, 11.7, 747.7, 749.0**	
Objective Function Value	5.52169	4.29628	4.02	4.95	
Fuel Consumed (g)	**8.83**	**6.87**	**6.44**	**7.92**	
Predicted fuel efficiency (MPG)	1406.64	1807.84	1929.75	1569.09	
Average Velociy (m/s)	6.7339	6.7408	6.7166	6.7169	
	Three-Burn Lap				
Initial Conditions	0, 300, 300, 600, 600, 900	0, 100, 100, 200, 200, 300	0, 300, 400, 500, 800, 900	300, 500, 600, 700, 800, 900	
Optimal Run Positions (m)	**242.4, 251.0, 553.66, 557.03,**	**104.090, 119.453, 175.73, 175.84,**	**216.01, 228.37, 433.12, 437.37,**	**401.61, 411.89, 632.35, 639.26,**	
Objective Function Value	5.15	4.7105	5.313	6.09	
Fuel Consumed (g)	**8.24**	**7.54**	**8.5**	**9.74**	
Predicted fuel efficiency (MPG)	1508.16	1648.87	1461.89	1275.37	
Average Velociy (m/s)	6.7245	6.1494	6.732	6.701	

Ultimately, the optimal result found was a two burn per lap run as seen in case 3 of the two-burn lap section of Table 1. Overall optimal results tend towards a burn at the beginning of the run and after the fourth lap. Total burn distance is about 15 meters, and the average velocity hovers around the minimum constraint.

DISCUSSION

There was a lot of perceivably valuable knowledge brought out through this optimization study. One of the most significant portions of information drawn therefrom relates to the length of burn. In most cases the total length of track over which the engine was turned on was between 14 and 20 meters. This was true for one, two or even three burn positions, irrespective of the initial conditions. We believe that the engine is not currently modeled very accurately. We hypothesize that a more accurately modeled engine term would demonstrate a need for a longer overall burn in order to meet the constraining average velocities, however it appears that the same trend of consistent burn-distance would hold across all burn cases and their respective initial conditions for the given design variables.

Another very important observation from the results was that initial conditions (starting points for the design variables) had a great deal of bearing on the final result. We suppose that this is due largely to shape of the function space. Scaling of the function objective as well as the derivative thereof was applied to the algorithm. Generally, scaling aided in the reaching a better optimum. However as seen in **Error! Reference source not found.**, the initial conditions can result in very different burn locations albeit providing similar objective values. In **Error! Reference source not found.** the pink trail shows an

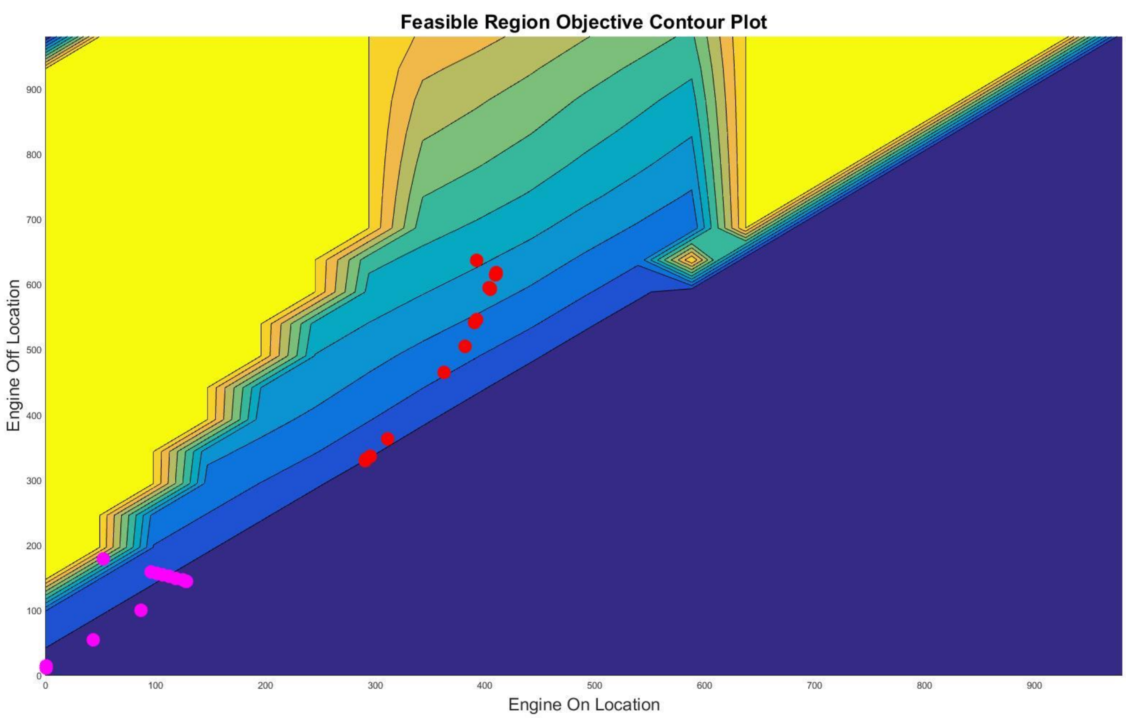

Figure 6: Contour plot of one burn for one lap. Overlaid are the optimization history of two initial conditions. The plot demonstrates the importance of picking initial conditions wisely.

optimization history originating at (0,200) whereas the red history starts at (400,600). Although both result in similar fuel consumption values, this may have interesting implications about the model used in the optimization and the assumptions made. It is possible that assuming an initial velocity of 6.7 m/s isn't appropriate. Perhaps the single lap analysis is good for laps 2-9 but not as good for the first and last lap.

Added constraints dictating initial and final velocities for the first and last laps respectively may be necessary in order to truly model a full run.

After examination of the results it is apparent that the highest fuel efficiencies that the team can achieve given the reported physical characteristics of the current vehicle (according to this model) will be through a two-burn lap. Not only is this indicated by the max fuel efficiency being found in a 2-burn lap (see TABLE 1), but the 3 burn lap results also all resulted in only two effective burns as one of them was forced to a near-zero burn time.

Results for most cases also seemed to indicate, though not exclusively, that burns should be positioned before corners so as to avoid high velocities in the corners.

CONCLUSION

From our results, we conclude that the best approach to the Supermileage Vehicle burn and coast method is to burn about 20 meters at the beginning of the lap and after turn four. Using this strategy, and under ideal conditions, the Supermileage Vehicle could achieve 1,930 miles per gallon. According to our model, the velocity of the car is most adversely affected by the turning radius of the four modeled energy losses. The effects of cornering drag can best be addressed by approaching the series of three turns with a minimal velocity, while still maintaining an average velocity of 15mph (6.7056 m/s). This last plot illustrates the resulting velocity for a lap with the engine running from positions 24.9 to 34.7 meters and 715.6 to 717.5 meters.

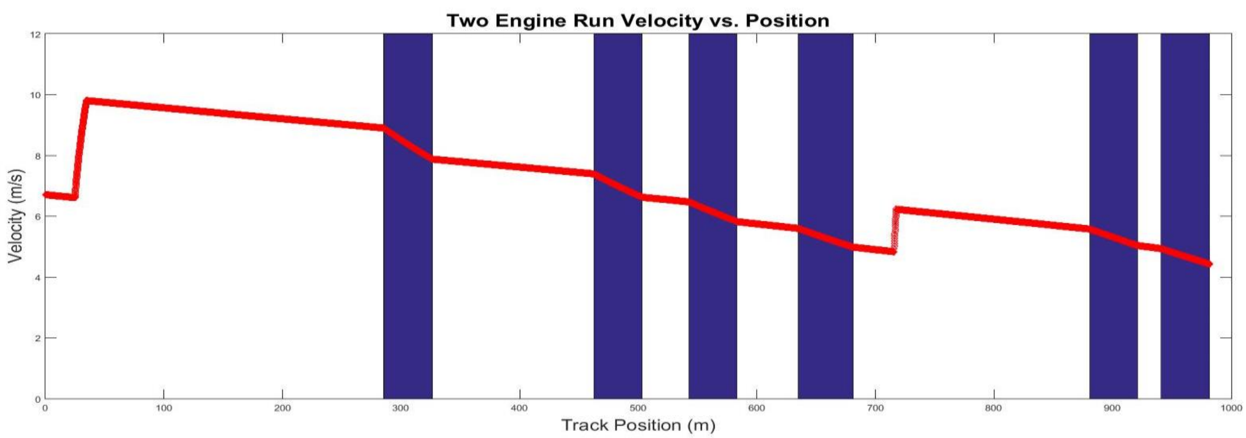

Figure 7: Optimal Strategy for Lap

The average velocity for this strategy over a lap is 6.72 m/s, or 15.1 mph, meeting the required velocity constraints. Admittedly, there remains many factors not consider in this calculation, human error notably. A person's ability to correctly time the turning on and off of the engine involves a fair amount of unmeasurably uncertainty. Furthermore, the error associated with turning is a rough approximation. As a final recommendation to the Supermileage Team, we suggest improving the model of engine efficiency and power output. The current model lacks the ability to accurately model change in velocity at values greater than one. By addressing these issues, and following this proposed strategy, the Supermileage Team will be able to improve their performance at this year's competition.

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