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Some Methods Used by Mathematics Teachers in Solving Equations

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ABSTRACT

This study aimed at analyzing and describing Various Methods used by mathematics teacher in solving equations. Type of this study is descriptive by subject of this study comprised 65 mathematics teachers in senior, junior, and primary schools respectively 15, 33, and 17 in numbers. The data were collected from the answer to containing four problems of equation. Data Coding was conducted by two coding personnel to obtain credible data. The data were then analyzed descriptively. It has been found that the teachers have implemented a method for solving equation problems by means of operation on one side of equation and procedural operation. This method has been dominantly used by the teachers to solve to the equation problems. The other method was doing operation on both sides of the equation simultaneously by focusing on similar elements on both sides of the equation.

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1. INTRODUCTION

Problem is not only a word with which students are familiar at school, but it is also frequently found in everyday context. In everyday life, problems can be found in the form of simple personal matters such as the best strategy to cross the street (mostly done without too much of a brain processing) up to the complex ones, such as designing narrow living room. Of course, simple problems such as crossing the street could become not so simple anymore in different occasions. Crossing in different kinds of traffic might be a more complex problem, for instance, because of the difference between crossing roads in Indonesia and in any European countries. This indicates that problems are relative, be it in different place and time or even with different problem solver. Generally, problem is a situation which one needs to solve, that requires resolution, and ways to find the solutions that are often times are not readily known (Polya, 1973; Posamentier & Krulik, 1998). By that, ability and skills in problem solving are very much needed. Problem solving is both mental and physical activity which one performs in order to overcome a problem.

Problem solving in Mathematics is a process of building new mathematical knowledge through the application and adaptation of strategies which are effective to be used in solving problems (Arnold, Heyne & Busser, 2005; Johnson & Schmidt, 2006). The skill to solve problems is essential in Mathematics subject (NCTM, 2012). On the other hand, the main goal of Mathematics learning is to allow students to be able to solve everyday problems. Therefore, it can be said that Mathematics is a tool to train students to solve problem and to build the process of thinking which nurtures the skill to solve non-Mathematical problems. Indeed, there are some talented individuals who possess the inherent skill to solve Mathematical problems. However so, most of us need some practices to develop this particular skill.

In most cases, students seem to consider that problems can only be solved in one single method, especially certain types of problems taught at school. Students often think that Algebraic approach is the only procedure which would "work" in problem solving (Posamentier & Krulik, 1998). Accordingly, teachers need to understand their students' way of thinking in order to develop an environment of problem solving in the class. Teachers' capacity in identifying mathematical key aspects is pivotal in facilitating students' thinking process during problem solving. Developing this skill allows teachers to interpret their students' logic and can result in effective reactive instructional decision making, such as selecting and designing mathematical assignments in problem solving activities (Chamberlin, 2005). Teachers need to know how students can understand mathematical concepts as a way to help them to improve their understanding in Mathematics (Schifter, 2001; Steinberg et.al, 2004).

As a professional educator, the main duties of a teacher are to educate, teach, counsel, guide, train, assess, and evaluate students (the Regulations of Republic of Indonesia No. 14 Article 1:1, 2005). Considering their main duties, teachers' job is not that simple as they have to lead their students to be successful individuals. Teachers design, organize the process of learning, and evaluate in order to change their students. The success of the students in learning at school and in the community relies heavily on teachers (Even & Ball, 2009; Louis, Leithwood, Wahlstrom & Anderson, 2010). Therefore, in learning, teachers need to pay attention to the balance between physical and mental skills of students as well as to implement values that carry out good examples, motivation building, and creativity development of their students during the process of learning (the Decree of Minister of Education and Culture No. 22, 2016). Students' quality rests on the quality of teachers, both in professional and pedagogical senses. What students are doing during the learning process is the reflection of the methods or strategies used and the teachers' mastery of teaching materials. Students' strategy in solving problems is also an illustration of how teachers teach and solve the said problems.

Algebraic problem solving possesses different characters which lie in the application of algorithm to collect figures (arithmetic solution), because the focus of algebra is a relation. Algebraic equation is solved by transforming equation so that the final result from the transformation is a relation expression, such as $y = 7$. Molina, et. al. (2008) divide two types of strategies which students perform in solving arithmetic problems; (1) create calculation to find and compare numeric values of two segments (FC type), or (2) look at the sentence and detect certain characteristics or the relation among elements (LD type). Hejný, Jirotková & Kratochvilová (2006) suggest that the strategy employed in solving the problem is by using procedural and conceptual approaches.

Kieran (1992) identifies several strategies used by students in solving linear problems: 1) using numeral figures (for example: $3 + x = 5$; $5 – 3 = 2$; therefore $x = 2$), 2) counting (for example: $3 + x = 9$, counting from 3 to 9 results in $x = 6$), 3) using cover up method (for example: if replacing 9 in $2x + 9 = 5x$, therefore $2x + ? =5x$, and so "?" must be equal with $3x$, (which is, $3x = 9$), replacing x in $3x = 9$, resulting in $x = 3$), 4) undoing method (working backward or backtracking) (for example: we are looking at an equation $2x + 4 = 18$ with x as the initial, multiply x with 2, add to 4, and the result is 18. We can abort the operation in reverse manner. Subtracting 4 from 18, we find $2x$ (which, $14 = 2x$). We finally divide 14 by 2, resulting in $x = 7$, 5) using trial and error method by substituting certain figure, 6) using transposition method (side alteration, symbol alteration) (for example: in $2x + 4 = 18$, move 4 from the left side to the right and change the symbol, resulting in equation $2x = 18 + (-4)$, and 7) using formal rule (doing similar operation in both sides, or balance method).

Baiduri (2016a) states that most students still use the procedural operation in solving mathematical equations by performing transposition as well as elucidating the elements of the equation in order to acquire the variable values. Baiduri (2016b) further asserts that it is the most common strategy to solve equation by means of computation and transposition (CT) or by paying attention to the numerical relation or the elements of the two sides (relation) (RT).

All students have learned the skill to solve problems by experience or as the products of practicing in Mathematics or Science classes. A different method to solve different problems can facilitate the connection between problems and other knowledge (Silver, Ghoussent, Gosen, Charalambous, & Starwhun, 2005) which become useful in solving problems in the future. Leikin (2007) argues that solving problems by using different approaches facilitates thinking habits (meta-Mathematics) and encourages advanced mathematical thinking. Further, Leikin & Levav-Waynberg (2008) stress that problem solving through different approaches contributes to the students' creativity development and critical thinking. Out of the box solutions in solving problems potentially change the discourse of the class, improving the quality of Mathematics learning and contributing to students' conceptual learning (Silver & Kenney, 1995; Yackel & Cobb, 1996; Boaler, 1998; Stigler & Hiebert, 1999). Boaler (1998) shows that students who learn Mathematics through open activities develop conceptual understanding; while those who follow the traditional approach develop procedural understanding.

Training students about different approaches in problem solving is one of the responsibilities of educators, especially Mathematics teachers. Problem solving, rationalizing, and communicating are processes which must encompass all mathematical instructions and must be modeled by teachers (NCTM, 1991). However, teachers mostly do not realize that there are different approaches in solving problems that are more effective and elegant. Teachers are even rarely aware that they unconsciously show their students that problems can only be solved using procedural approach. Therefore, it is very important to study teachers' method in solving Mathematical problems, especially those of algebraic equations.

2. RESEARCH METHOD

This Study aimed to explaine and analyze methods used by mathematics teachers of Junior high school in solving algebraic equation problems, the research type of this study used descriptive research.

2.1. Subject

The research subjects were 65 Mathematics teachers in secondary schools who volunteered to solve Mathematics problems given; 15 of them were junior high school teachers, 33 senior high school teachers, and 17 vocational high school teachers.

2.2. Instrument

The research instrument was in a form of algebraic equation consisting of four problems. The problems included the elaboration of Mathematics problem proposed by Star & Rittle-Johnson (2008) and Baiduri (2016a and 2016b). The problems were expected to give consistent illustration of the varieties of approaches used by Mathematics teachers in solving algebraic equation problems. In each problem, they were given the chance to solve the problems using different approaches and they were to give explanation of the approach used.

2.3. Data Collection

To describe the solution that the teachers used, task analysis method was used (van Someren et.al, 1994). The tasks performed by the teachers were the visualization or verbalization forms of the problems. Data collection was focused on the written responses/answers by the teachers. The problems must be completed in 10 to 15 minutes of time.

2.4. Data Analysis

Analysis was done by coding the answers of each Mathematics teacher for each problem given, as postulated by Baiduri (2016a and 2016b). The credibility of the data was obtained through coding done by two personnel, where the second coding personnel coded 90% of the data. The agreement between two coders was that 95% for each problem showed similar result. Subsequently, the validated data were analyzed descriptively.

3. RESULTS AND ANALYSIS

Based on the responses written about the problems given, coding process was focused on the methods used by the teachers in solving equation problems. Based on the coding process on the teachers' written responses, the methods used by the teachers were as follows.

3.1. Solving Problem Number 1, Determine The Solution Of This Equation $2(X + 5) = 6$.

Applying method (1), the problem was solved by using multiplication in the left side first (using distributive nature of multiplication against addition), followed by transformation to categorize the similar members in different side, and then performing counting operation by inverse 2 in order to obtain the value of x. As for method (2), the problem was solved in the same method as method (1), but the process was much simpler. The method is illustrated in Figure 1a. In applying method (3), the problem was solved by multiplying with inverse 2 in both sides, followed by categorizing similar members in different side in order to obtain the value of x.

Method (4) used the similar procedures as those of method (3), but using shorter steps as illustrated in Figure 1b. Reflecting on the methods used by the Mathematics teachers in solving Math problem number 1, the first method focused on arithmetic or calculation operation characteristics in one side of the equation. This method has been referred to as breaking method (Baiduri, 2016a) or computation or transposition (Baiduri, 2016b). While on the second method, the method was referred to as simplifying method (Baiduri, 2016a) or relating the two elements of two sides (Baiduri, 2016b) or analyzing expression (Molina & Ambrose, 2008). Looking closely to these, the methods (3 and 4) have been found to be more efficient and flexible (Star & Rittle-Johnson, 2008) compared to the methods (1 and 2) or usually referred to as the best solution. Methods (1) and (2) were used by junior high school teachers (73.33%), senior high school teachers (51.52%), and vocational high school teachers (70.59%). The other teachers used method (3) and (4). Overall, 61.54% of the teachers used method (1) and (2); and 38.46% used method (3) and (4).

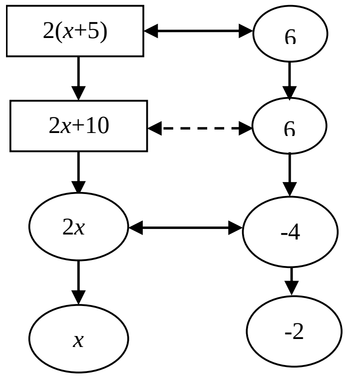

Figure 1(a)

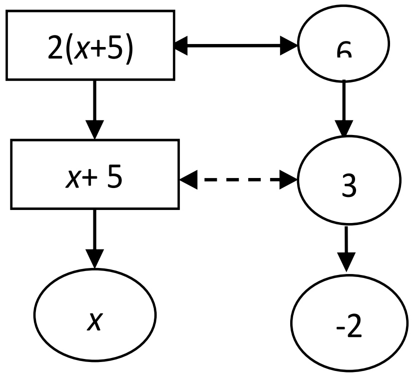

Figure 1(b)

Figure1. a. Solving the equation problems by means of operation in one side, b. Solving the equation problems by means of operation in both sides

Description: : using calculation operation in one or both of the sides
: using transformation in both sides

3.2. Solving Problem Number 2, Find The Result Of This Equation 2(X + 5) + 4(X + 5) = 18

The variations of the solutions done by the teachers are illustrated in Figure 2a to 2d. According to the illustration, for method (1), the problem was solved by doing multiplication in the left side first (using multiplication distributive characteristic against addition), followed by doing calculation operation in similar members of each different side, and then subsequently doing transformation to categorize the similar members of each different side, and then doing addition calculation operation with inverse 30. Finally, in order to obtain the value of x, multiplication inverse 6 was performed in the second segment. This process is illustrated in Figure 2a. Method (2) was done similarly to method (1), but the difference lies in the third step, 2x + 4x +10+20 (method 1) and 6x + 30. Method (2) is illustrated in Figure 2b. Method (3) was done through adding the similar members first, dividing both sides by 6 and subsequently the result was subtracted by 5 to obtain the value of x. This process was mostly done in merely one of the equation sides and is illustrated in Figure 3c. Method (4) was done similarly to method (3), but the calculation process was done in both equation sides, except in the first step. Furthermore, this method was very explicit in a way that the activity was very apparent in the first step of the process. In contrary, the activity on the second process happened inside the cognition of problem solving. This process is displayed in Figure 3d.

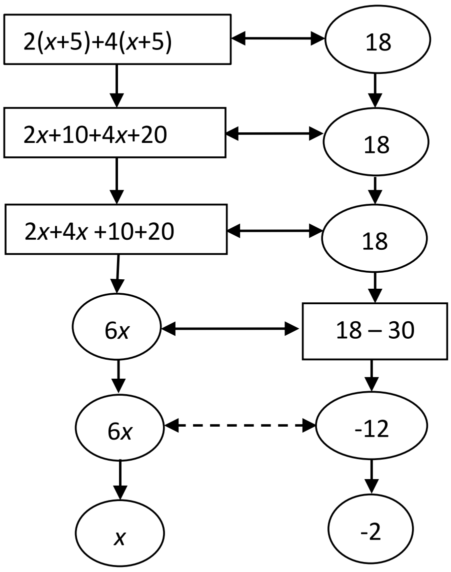

Figure 2(a)

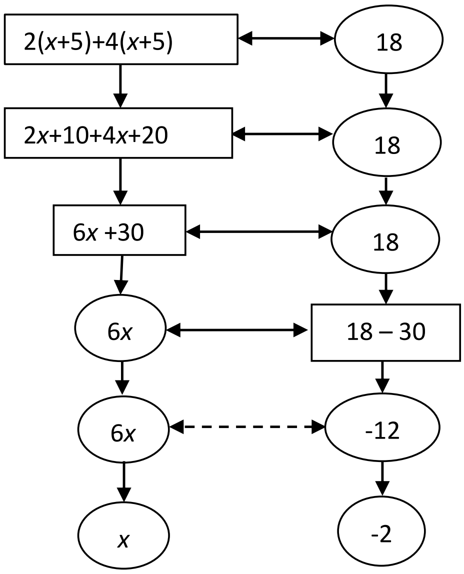

Figure 2(b)

Figure 2a and 2b: Solving the equation problems by means of distributive characteristic and an operation in one side of equation

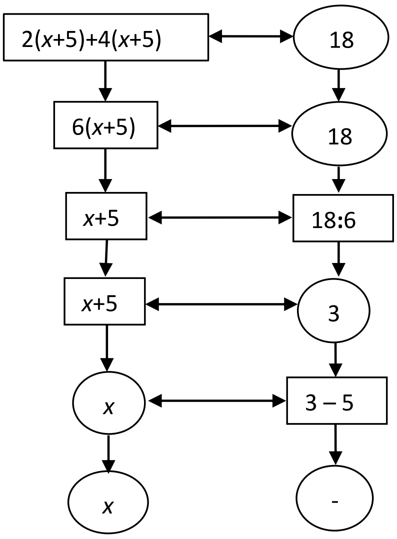

Figure 2(c)

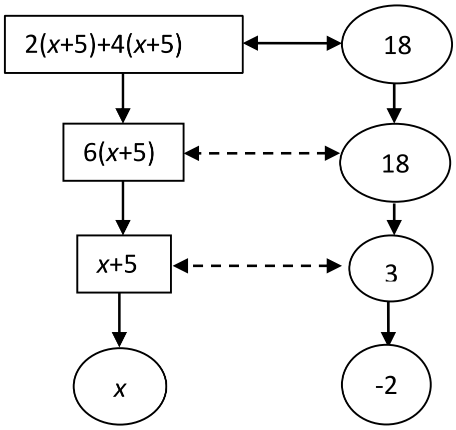

Figure 2(d)

Figure 2c and 2d: Solving the equation problems by means of simplification and an arithmetic operation in both sides of equation

Description: : using calculation operation in one or both of the sides
: using transformation in both sides

The process of problem solving in method (4) has been said to be more efficient or more flexible (Star & Rittle-Johnson, 2008) compared to other methods. The understanding on this matter is classified into understanding relational concept (Skemp, 1976), which is understanding what is being solved and why. Methods (1, 2, and 3), however, emphasized on the procedure in acquiring a solution. These methods are referred to as procedural methods (Hejný, Hejný, Jirotková & Kratochvilová, 2006) or the standard methods (Star & Rittle-Johnson, 2008). Most of the teachers used method (1), (2), and (3), where 73.33%, 45.45%, and 70.59% respectively were representing the percentage of junior high school teachers, senior high school teachers, and vocational high school teachers using the aforementioned methods. Method (4) was only used by minority of junior high and vocational high school teachers, respectively at the percentage of 26.67% and 29.41%. Looking at the percentage, method (4) was mostly used by high school teachers. However, the overall dominating method to solve the problem was the standard or procedural method (58.46%) compared to that of more efficient method (41.54%).

3.3. Solving Problem Number 3, Determine The Result Of This Equation 2(X + 5) + 5x + 9 = 6 +5x + 9

As for method (1), the problem was solved by using multiplication in the left side first (applying the distributive multiplication characteristic against addition), followed by performing calculation operation in the similar members in each side, using transformation to categorize similar members in each different side, and then calculating the multiplication with inverse 2 in the two sides to find the value of x. This process is illustrated in Figure 3a. Applying method (2), the equation was solved by performing multiplication in the left side (applying distributive multiplication characteristics against addition), followed by categorizing similar members in each different side, and then performing multiplication with inverse 2 to find the value of x, as illustrated in Figure 3b. For method (3), the equation was solved by multiplying first, followed by simplifying similar members in both sides, adding with inverse 10 or performing transformation in similar members and then multiplying with inverse 2 to find the value of x. The method is illustrated in Figure 3c. As for method (4), the problem was solved by performing division for similar members in both sides, followed by performing division operation in both sides using nominal 2, and then performing calculation operation of addition inverse (-5) to find the value of x. This process is illustrated in Figure 3d.

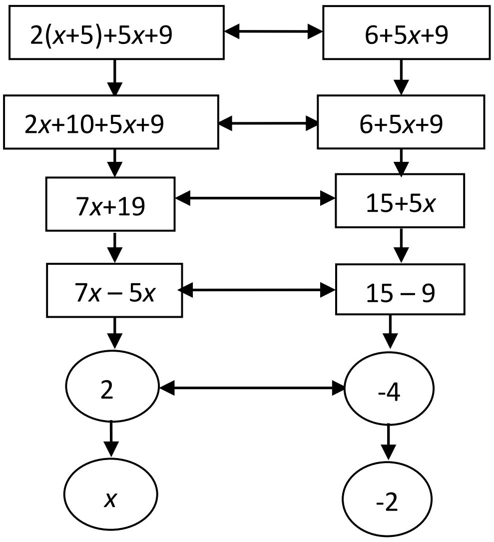

Figure 3(a)

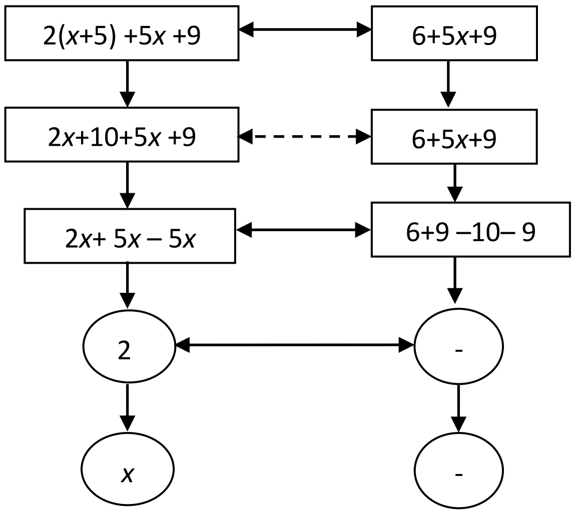

Figure 3(b)

Figure 3a and 3b: Solving the equation problems by means of distributive characteristic and an operation in one side of equation

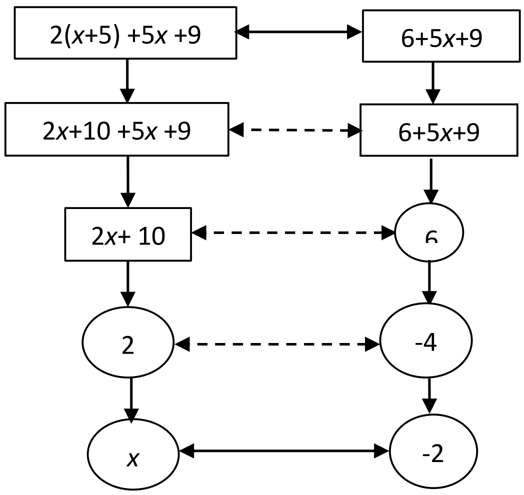

Figure 3(c)

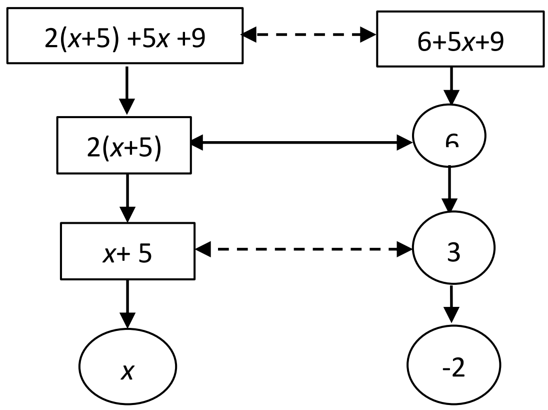

Figure 3(d)

Figure 3c and 3d: Solving the equation problems by concerning on similar members in both sides of equation and an arithmetic operation in both sides of equation

Description: ←——→ : using calculation operation in one or both of the sides
←- - - -→ : using transformation in both sides

Method (1) to (3) are said to be the transposition method (Kieran, 1992); while method (4) focuses on the element of both equation sides and is called as relational approach (Molina & Ambrose, 2008; Molina, Castro & Mason, 2008). Method (1) and (3) were dominantly used by the teachers compared to method (4). Overall, there were 69.23% teachers who solved the problem using method (1) to (3) and only 30.23% of them used method (4). The junior high, senior high, and vocational high school teachers who used method (1) to (3) were respectively 66.67%, 75.76% and 58.82%. The rest of the teachers used method (4). The steps in solving problem using method (1) to (3) were considered too long and focused on one side of the equation. The approach of which was quite different from the approach used in method (4), which was done by performing calculation operation in both sides in each step. This has simplified the problem and therefore the result can be found easier.

3.4. Solving Problem Number 4, Determine The Result Of The Equation $(X+\sqrt{2})2 + 2 = 3(X+\sqrt{2})$

For method (1), the problem was solved by looking at both sides by dividing the value against $(x+\sqrt{2})$. This step disregarded the fact that not all members in the equation contained the form $(x+\sqrt{2})$ and the value of variable x because the divisor cannot be 0. This way, this method cannot be taken into account or is the wrong method. Method (2) was performed by elucidating $(x+\sqrt{2})2$ to gain the value of the square equation in x, $ax^2+bx+c=0, a\neq 0$. Subsequently, they used square formula to obtain the value of x. However, there were some subjects who could not find the right answer using this method. Method (2) was used by 57.14% junior high school teachers, 45.45% senior high school teachers, and 47.06% vocational high school teachers. The total number of teachers solving the problem using this method was 48.44%. Looking closely to it, this method required the skill to multiply two members and to categorize similar members. Besides, this method used very long steps and took much time to finish. This method was very procedural (Hejný, Jirotková & Kratochvilová, 2006; Baiduri 2016b) or could also be called as a standard method (Star & Rittle-Johnson, 2008).

As for method (3), the problem was solved by factoring. The teachers have understood that equation is square equation in a form of $(x+\sqrt{2})$. They have already had in mind that they needed a new variable to substitute $(x+\sqrt{2})$. In applying method (4), the teachers found the similar forms and then assumed the equation in a different variable $(x+\sqrt{2}) = m$. From the assumption, they found the square equation which was simpler m, $m^2+2=3m$ or $m^2-3m+2=0$. The solution for square equation from the transformation was simpler compared to the previous square equation. Therefore, the process to solve this problem has been said to be more flexible (Star and Rittle-Johnson, 2008). The thinking process of the subjects using method (3) and (4) could be seen as relational thinking in problem solving. Method (3) and (4) could be used by the subjects who have better concept understanding (Baiduri, 2014). This method is referred to a conceptual meta-strategy (Hejný, Jirotková & Kratochvilová, 2006) or expression analysis (Molina &Ambrose, 2008). This strategy has used shorter steps and taken relatively short amount of time to finish; and thus it is more flexible compared to method (2) (Star & Rittle-Johnson, 2008). This method was used by the majority of senior high school teachers (54.55%) and vocational high school teachers (52.94%). Overall, this method was more dominantly used by senior high school teachers (51.56%).

The findings of this current study have shown that Mathematics teachers in junior high school and senior high school still focused on using one dominant method in solving Mathematical problems. This is in line with the findings of Bingolbali (2011), Leikin (2007), Ma (1999), and Stigler and Hiebert (1999), showing that teachers do not always solve problem using different approaches, let alone encouraging students to do so. In relation to the implementation of the curriculum, this finding is said to be critical. The standard learning process expects teachers to move from one learning method which emphasizes on single answer towards an answer with multidimensional truths (Decree of Minister of Education and Culture Number 22, 2016) and on encouraging students to show logical, critical, analytical, creative, precise and meticulous, responsible, responsive, and determined characteristics in solving problems (Decree of Minister of Education and Culture Number21, 2016). When teachers' objectives and practices are in consistence, as commonly happened (Kagan, 1992), it can be concluded that teachers would face difficulties in implementing the curriculum in the class.

4. CONCLUSION

Based on the data analysis in this current study, the teachers' methods in solving problems were still dominantly procedural, emphasizing on one operation in one side of the equation. The problem from number one (1) to number three (3) that the teacher did by performing an operation on side of the equation proceeds to grouping similar parts in solving equations. This method was still highly procedural and dominant done by teacher. were As a result, the steps to solve the problem were longer and taking more time to finish. In contrary, if the problem solving focused on the relation between elements of two sides of equation, the steps would be shorter and the time required to solve the problem would also be shorter. A good understanding of the meaning of "equal sign" ("=") as a relation of equation is an important basis for teachers to use a more efficient approach. The problem of number four (4) done by (a) describes the form of rank to obtained a quadraticequation, (b) do factoring, and sunstitutes the variables to obtaied the form of quadratic equations of the standard form with new variables and resolved by the factoring

To further emphasize, the skill to solve mathematical problems is crucial in Mathematics learning. Problem solving is influenced by the type of problems and the steps to solve the problem. Teachers should master different methods or strategies in solving mathematical problems. The information supplemented by teachers in solving problems will be used by students in class, including their strategies and methods. Problem solving by using different approaches will improve the quality of learning Mathematics and students' conceptual understanding.

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REFERENCES

[1]. Anonymous. (2005). "The Regulations of Republic of Indonesia Number 14 Year 2005 about teacher and lecturer". Jakarta: Department of National Education, Indonesia.
[2]. Anonymous, (2016). "Decree of Minister of Education and Culture Number21 Year 2016 about basic and middle education contents standard". Jakarta: Department of Education and Culture, Indonesia.
[3]. Anonymous, (2016). "Decree of Minister of Education and Culture Number 22 Year 2016 about basic and middle education process standard". Jakarta: Department of Education and Culture, Indonesia.
[4]. Arnold,L.M. Heyne, A.L. Busser, A.J. (2005). "Problem Solving: Tools and Techniques For The Park And Recreation Administrator", *Sagamore Publishing*, L.L.C. Champaign, Illinois.
[5]. Baiduri. (2014). "Relational Thinking Profil of Elementary School Students in Based on Mathematics Ability and Gender", *PPS Unesa unpublished dissertation*.
[6]. Baiduri. (2016a). "Mathematics Education Students' Strategies in Solving Equations", *Proceedings of The 7th SEAMS-UGM Conference 2015*, pp. 28 - 33.
[7]. Baiduri. (2016b). "A Typology of Strategies in Solving Algebraic Equations Problems", *Global Journal of Pure and Applied Mathematics.12*(4), pp. 3865-3872
[8]. Bingolbali, E., (2011). "Multiple Solutions to Problems in Mathematics Teaching: Do Teachers Really Value Them?", *Australian Journal of Teacher Education*, 36(1), 18-31.

[9]. Boaler, J. (1998). "Open and closed mathematics: Student experiences and understandings". *Journal for Research in Mathematics Education*, *29* (1), 41-62.

[10]. Chamberlin, M. (2005). "Teachers' discussions of students' thinking: meeting the challenge of attending to students' thinking". *Journal of Mathematics Teacher Education, 8*, 141-170.

[11]. Even, R. and Ball, D.L. (2009). "Setting the stage for the ICMI study on the professional education and development of teachers of mathematics". In Even R.; Ball, D.L. (Eds).*The Professional Education and Development of Teachers of Mathematics.* New York: Springer

[12]. Hejný, M., Jirotková, D. & Kratochvilová, D. (2006). "Early conceptual thinking". In Novotná, J., Moraová, H., Krátká, M. & Stheliková, N. (Eds.), *Proceedings 30th Conferences of the International Group for the Psychology of Mathematics Education,* Vol. 3, pp. 289-296. Prague: PME.

[13]. Johnson, K and Schmidt, A. "The Effects of Teaching Problem Solving Strategies to Low Achieving Students". (2006). *Action Research Projects.* Paper 62. Retrieved from http://digitalcommons.unl.edu/mathmidactionresearch/62

[14]. Kagan, D. (1992). "Implications of research on teacher belief". *Educational Psychologist*, *27*(1), 65-90.

[15]. Kieran, C. (1992). "The learning and teaching of school algebra". In D. Grouws (Ed) *Handbook of Research on Mathematics Teaching and Learning* (pp. 390-419). New York: Macmillan.

[16]. Leikin, R. (2007). "Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks". *Paper presented in the Working Group on Advanced Mathematical Thinking—*CERME-5, Cyprus.

[17]. Leikin, R. & Levav-Waynberg, A. (2008). "Solution Spaces of Multiple-Solution Connecting Tasks as a Mirror of the Development of Mathematics Teachers' Knowledge". *Canadian Journal of Science, Mathematics and Technology Education*, *8*(3), 233 - 251.

[18]. Louis, K.S, ,Leithwood, K, Wahlstrom, K.L, Anderson, S.E, (2010). "Learning from Leadership: Investigating the Links to Improved Student Learning", The University of Minnesota.

[19]. Ma, L. (1999). "Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States". N.J.: Lawrence Erlbaum Asc.

[20]. Molina, M., Castro, E., & Mason, J.(2008)."Elementary school students' approaches to solving true/false number sentences".*PNA* 2 (2), 75 – 86

[21]. Molina, M., and Ambrose, R.(2008)."From an operational to a relational conception of the equal sign: Thirds grades' developing algebraic thinking". *Focus on Learning Problems in Mathematics*,30 (1), 61 – 80.

[22]. National Council of Teachers of Mathematics (NCTM). (2012). "Problem Solving". *In Standards and Focal Points*. Retrieved from http://www.nctm.org/standards/content.aspx?id=26860

[23]. National Council of Teachers of Mathematics (NCTM).(1991). *Professional Standards for Teaching Mathematics*. Reston, VA: Author

[24]. Polya, G. (1973). *How to Solve it.*2nd Ed. Princeton University Press, ISBN 0-691-08097-6

[25]. Posamentier, A.S., Krulik, S. (1998). "Problem-solving strategies for efficient and elegant solutions: A resource for the mathematics teacher". Thousand Oaks; Corwin Press, Inc. California.

[26]. Schifter, D. (2001). "Learning to see the invisible: What skills and knowledge are needed to engage with students' mathematical ideas?", In T. Wood, B.S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 109–134). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

[27]. Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Font Strawhun, B. T. (2005). "Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom".*Journal of Mathematical Behavior*, 24, 287–301.

[28]. Silver, E.A. & Kenney, P.A. (1995). "Sources of Assessment Information for Instructional Guidance in Mathematics". In T.A. Romberg (Ed.) *Reform in School Mathematics and Authentic Assessment* (pp. 38-68), Albany, State University of New York Press.

[29]. Skemp, R.R. (1976). "Relational understanding and Instrumental understanding". *Mathematics Teaching*.

[30]. Star, J.R., and Rittle-Johnson, B. (2008). "Flexibility in problem solving: The case of equation solving, Learning and Instruction", *18*, 565-579

[31]. Steinberg, R., Empson, S. B., & Carpenter, Th. P. (2004). "Inquiry into children's mathematical thinking as a means to teacher change". *Journal of Mathematics Teacher Education, 7*, 237-267.Stigler, J., and Hiebert, J. (1999). *The teaching gap*. New York: The Free Press.

[32]. van Someren, Marteen, W., Barnard, Yvonne. F., Sandberg, Jacobin. A.C., (1994). "The Think Aloud Method. A Practical guide to modeling cognitive processes". London. Academic Press.

[33]. Yackel, E., and Cobb, P. (1996). "Socio-mathematical norms, argumentation, and autonomy in mathematics". *Journal for Research in Mathematics Education*, *27*(4), 458–477.

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