



An Algebraic Approach to Neutrosophic Euclidean Geometry

¹Mohammad Abobala,²Ahmad Hatip

¹Faculty of Science, Department of Mathematics Tishreen University, Lattakia Syria

¹e-mail: mohammadabobala777@gmail.com

²Department of Mathematics, Gaziantep University, Gaziantep, Turkey

²e-mail: Kollnaar5@gmail.com

Abstract: The objective of this paper is to present a general definition of Neutrosophic Euclidean geometry. The mechanism of comparison between neutrosophic numbers is introduced, as well as the absolute value is extended to the neutrosophical case. In addition, the concept of the neutrosophic plane with n neutrosophic dimensions is obtained. Also, Euclidean geometric concepts are extended neutrosophically such as neutrosophic distance, neutrosophic midpoint, neutrosophic vectors, neutrosophic circles, and lines. A connection between neutrosophic geometrical concepts and classical Euclidean geometry is described and established.

Keywords: Neutrosophic Plane, Neutrosophic Absolute Value, Neutrosophic Distance, Neutrosophic Midpoint, Neutrosophic Vectors, Neutrosophic Circles, Neutrosophic Lines.

1. Introduction

Neutrosophic logic is a generalization of intuitionistic fuzzy logic by adding an indeterminacy I with property $I=I^2$. Neutrosophic set concept has wide applications in different areas of science, such as decision making [7,20], health care [8,21], machine learning [9], artificial intelligence [10], soft computing [22], industry [23], and statistics [11].

On the other hand, neutrosophic sets played an interesting role in pure mathematics such as topology and analysis [12,13], spaces [1,2], and algebraic structures [3,4,5,6].

Neutrosophic spaces theory began with Agboola et.al [14], where they studied neutrosophic vector spaces and their properties. Recently, many studies have been carried out on these spaces, where AH-subspaces and homomorphisms were presented [15]. In [16,17,18,19], Hatip et. al studied neutrosophic modules (a generalized form of neutrosophic spaces) with their substructures such as homomorphisms and AH-submodules.

Euclidean geometry with two dimensions was built over points (a, b) taken from the space $R \times R$, Where constants are taken from the field of real numbers R . To extend the classical geometric system neutrosophically, we shall build an algebraic map, which we call (AH-isometry), between the Cartesian product $R(I) \times R(I)$, and the classical space $R^2 \times R^2$.

In the classical geometry, multiplying by scalars is depending on taking these scalars from the field R , in neutrosophic geometry, multiplying is depending on neutrosophic real numbers from the neutrosophic field $R(I)$.

We show that the AH-isometry (**Abobala-Hatip** isometry) map will preserve addition and distances, in other words, it allows us to study neutrosophic vectors, points, shapes by going back to their corresponding classical concepts. This work is considered as a first step in the study of neutrosophic geometry, it has many benefits in the progression of neutrosophic studies and establishing neutrosophic functional analysis in the future since we can define neutrosophic geometrical shapes and study their relation with classical geometrical shapes.

To achieve our goals, we shall define an order relation between neutrosophic numbers as well as neutrosophic absolute values. Thus, we can build the concept of neutrosophic distances and norms in an easy way.

This work clarifies a charming connection between neutrosophic algebra and geometry.

Motivation

We regard that we don't have a geometrical system based on neutrosophic spaces, thus our motivation is to close this important research gap by defining the basic theoretical concepts of a new geometrical system based on neutrosophic numbers and spaces.

Preliminaries

Definition 2.1: [32] Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $\{x, (\mu A(x), \delta A(x), \gamma A(x)): x \in X\}$, where $\mu A(x)$, $\delta A(x)$ and $\gamma A(x)$ represent the degree of membership, the degree of indeterminacy, and the degree of non-membership respectively of each element $x \in X$ to the set A .

Definition 2.1: [31] Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such as $0 \cdot I = 0$ and $I^2 = I$ which results in that $I^n = I$ for all positive integers n .

Definition 2.2: [30] Let $(G, *)$ be any group, the neutrosophic group is generated by I and G under $*$ denoted by $N(G) = \{ \langle G \cup I \rangle, * \}$.

Definition 2.3: [30] Let R be any ring. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operations of R .

Definition 2.4: [33] Let K be a field, the neutrosophic field generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 2.4: [29] Let $(M, +, \cdot)$ be any R -module over a neutrosophic ring $R(I)$. The triple $(M(I), +, \cdot)$ is called a strong neutrosophic R -module over a neutrosophic ring $R(I)$ generated by M and I .

3. Main concepts and discussion

In the beginning, we will define the basic concepts in neutrosophic Euclidean geometry and then we will study their relations with classical geometry.

Definition 3.1:

Let $R(I) = \{a + bI; a, b \in R\}$ be the real neutrosophic field, we say that $a + bI \leq c + dI$ if and only if $a \leq c$ and $a + b \leq c + d$.

Theorem 3.2:

The relation defined in Definition 3.1 is a partial order relation (reflexive, anti-symmetric, and transitive).

Proof:

Let $x = a + bI, y = c + dI, z = m + nI \in R(I)$, we have

$x \leq x$ that is because $a \leq a$ and $a + b \leq a + b$.

Now, suppose that $x \leq y$ and $y \leq x$, then $a \leq c, a + b \leq c + d, c \leq a, c + d \leq a + b$, hence $a = c, a + b = c + d$, which means that $d = b$ and $x = y$.

Assume that $x \leq y$ and $y \leq z$, hence $a \leq c, a + b \leq c + d, c \leq m, c + d \leq m + n$, this implies that $a \leq m, a + b \leq m + n$, so $x \leq z$. Thus \leq is a partial order relation on $R(I)$.

Remark 3.3:

According to Theorem 3.2, we can define positive neutrosophic real numbers as follows:

$a + bI \geq 0 = 0 + 0.I$ implies that $a \geq 0, a + b \geq 0$.

Absolute value on $R(I)$ can be defined as follows:

$|a + bI| = |a| + I[|a + b| - |a|]$, we can see that $|a + bI| \geq 0$.

We can compute the square root of a neutrosophic positive real number as follows:

$\sqrt{a + bI} = \sqrt{a} + I[\sqrt{a + b} - \sqrt{a}]$, it is clear that $(\sqrt{a} + I[\sqrt{a + b} - \sqrt{a}])^2 = a + bI$ and $\sqrt{a + bI} \geq 0$.

Example 3.4:

- $x = 2 - I$ is a neutrosophic positive real number, since $2 \geq 0$ and $(2 - 1) = 1 \geq 0$.
- $2 + I \geq 2$, that is because $2 \geq 2$ and $(2 + 1) = 3 \geq (2 + 0) = 2$.
- $|1 + 3I| = |1| + I[|1 + 3| - |1|] = 1 + 3I$.
- $|-3 + 2I| = |-3| + I[|-3 + 2| - |-3|] = 3 - 2I$ notice $0 + 0I \geq -3 + 2I$
- $\sqrt{9 + 4I} = \sqrt{9} + I[\sqrt{13} - \sqrt{9}] = 3 + (\sqrt{13} - 3)I$.

Definition 3.5:

We define the neutrosophic plane with n neutrosophic dimensions (N-dimensions) as follows:

$$R(I) \times R(I) \times R(I) \times \underbrace{\dots}_{n\text{-times}} \times R(I)$$

Example 3.6:

$R(I) = \{a + bI; a, b \in R\}$ is a neutrosophic plane with one N-dimension.

$R(I)^2 = \{(a + bI, c + dI); a, b, c, d \in R\}$ is a neutrosophic plane with two N-dimensions.

In the following, we will focus on the two N-dimensional neutrosophic planes.

Definition 3.7:

Let $A(a + bI, c + dI), B(x + yI, z + tI)$ be two neutrosophic points from $R(I)^2$, we define:

$\overrightarrow{AB} = ([x + yI] - [a + bI], [z + tI] - [c + dI])$, is called a neutrosophic vector with two N-dimensions.

Definition 3.8:

Let $\vec{u} = (a + bI, c + dI)$ be a neutrosophic vector, we define its norm as follows:

$$\|\vec{u}\| = \sqrt{(a + bI)^2 + (c + dI)^2} = \sqrt{a^2 + c^2 + I[(a + b)^2 + (c + d)^2 - a^2 - c^2]}.$$

It is easy to see that $\|\vec{u}\| \geq 0$, according to Remark 3.3.

Definition 3.9:

Let $A(a + bI, c + dI), B(x + yI, z + tI)$ be two neutrosophic points from $R(I)^2$, we define:

(a) The midpoint of $[AB]$ is $C\left(\frac{a+bI+x+yI}{2}, \frac{c+dI+z+tI}{2}\right)$.

(b) The neutrosophic distance between A and B is equal to $\|\overrightarrow{AB}\|$.

Example 3.10:

Consider the following neutrosophic points $A(1 + I, 2 - 3I), B(-I, -1 + 2I)$,

The neutrosophic vector $\overrightarrow{AB} = (-1 - 2I, -3 + 5I)$, the square of the neutrosophic distance between A and B is $\|\overrightarrow{AB}\|^2 = 1 + 9 + I[9 + 4 - 1 - 9] = 10 + 3I$.

Hence the neutrosophic distance is equal to $\sqrt{10 + 3I} = \sqrt{10} + I[\sqrt{13} - \sqrt{10}]$. We can find easily that $(\sqrt{10 + 3I})^2 = (\sqrt{10} + I[\sqrt{13} - \sqrt{10}])^2 = 10 + 3I$.

Let C be the neutrosophic midpoint of $[AB]$, then $C\left(\frac{1}{2}, \frac{1}{2} - \frac{1}{2}I\right)$.

Now, we list some geometrical and algebraic properties of the classical space $R^2 \times R^2$. We will need them in the forthcoming sections.

Remark 3.11:

Let $V = R^2 \times R^2$ be the Cartesian product of the classical Euclidean plane with itself, we have

(a) V has a module structure over the ring $R \times R$, with respect to the following operations:

Addition: $((a, b), (c, d)) + ((x, y), (z, t)) = ((a + x, b + y), (c + z, d + t))$,

Multiplication by a duplet scalar from $R \times R$: $(m, n) \cdot ((a, b), (c, d)) = ((m \cdot a, n \cdot b), (m \cdot c, n \cdot d))$.

(b) The norm of any vector in V can be defined as a duplet number from $R \times R$, as follows:

$$\|((a, b), (c, d))\| = (\sqrt{a^2 + c^2}, \sqrt{b^2 + d^2}).$$

Example 3.12:

Consider the following two points from the space V , $A((1,2), (2,5)), B((-1,4), (3, -2))$, we have:

(a) $\overrightarrow{AB} = ((-2,2), (1, -7))$.

(b) $\|\overrightarrow{AB}\| = (\sqrt{(-2)^2 + (1)^2}, \sqrt{(2)^2 + (-7)^2}) = (\sqrt{5}, \sqrt{53})$.

(c) Let $r = (5,8) \in R \times R$ be a duplet scalar, we have:

$$r \cdot \overrightarrow{AB} = ((-10,16), (5, -56)), \text{ it is clear that } \|r \cdot \overrightarrow{AB}\| = r \cdot \|\overrightarrow{AB}\|.$$

4. The connection between neutrosophic and classical geometry

This section is devoted to clarifying the relationships between neutrosophic coordinates defined above, and between classical geometrical coordinates.

Many important questions arise according to section 3. The first one is about famous relations in classical geometry: Does the midpoint of $[AB]$ have the same neutrosophic distance from A and B ? If the answer is no, then our geometrical system is weak and has no importance because it contradicts logical statements.

The second, do the neutrosophic points have relationships with classical points. This question is the most important one, that is because if it has a positive answer, then we can study geometrical shapes in the neutrosophic plane.

The third is about how can we define neutrosophic lines, circles, elliptic curves... etc.

We try to answer these important questions by using algebra since the neutrosophic plane with two N-dimensions is a module over the ring $R(I)$.

Definition 4.1: Let $M = R(I)^2 = R(I) \times R(I), V = R^2 \times R^2$ be the neutrosophic plane with two N-dimensions and the Cartesian product of the classical Euclidean space R^2 with itself, we define the AH-isometry map as follows:

$$f: M \rightarrow V; f(a + bI, c + dI) = ((a, a + b), (c, c + d)).$$

We can define the one-dimensional isometry between $R(I)$ and the space $R \times R$ as follows:

$$g: R(I) \rightarrow R \times R; g(a + bI) = (a, a + b).$$

Remark: The one-dimensional isometry is an algebraic isomorphism between $R(I)$ and $R \times R$.

Proof: Let $a + bI, c + dI$ be two neutrosophic real numbers, then

$$\begin{aligned} f(a + bI + c + dI) &= f([a + c] + [b + d]I) = (a + c, a + c + b + d) = (a, a + b) + (c, c + d) \\ &= f(a + bI) + f(c + dI). \end{aligned}$$

$$\begin{aligned} f([a + bI]. [c + dI]) &= f(ac + I[ad + bc + bd]) = (ac, ac + ad + bc + bd) = (a, a + b). (c, c + d) \\ &= f(a + bI). f(c + dI). \end{aligned}$$

f is a correspondence one-to-one, that is because $Ker(f) = \{0\}$, and for every pair $(a, b) \in R \times R$, there exists $a + (b - a)I \in R(I)$ such that $f(a + [b - a]I) = (a, b)$. Thus, f is an isomorphism.

Example 4.2:

Consider the following neutrosophic point $A(1 + I, 3 - 6I)$, its isometric image is $((1, 2), (3, -3))$.

Consider the following neutrosophic vector $\vec{u} = (2 - I, 4 + I)$, its isometric vector is

$$\vec{v} = ((2, 1), (4, 5)).$$

The idea behind the AH-isometry is to deal with neutrosophic points as classical points and to explore their properties using classical Euclidean geometry.

The following theorem is considered as the fundamental theorem in neutrosophic Euclidean geometry since it describes the relation between neutrosophic space with two N-dimensions and the classical module generated by the Cartesian product of the classical Euclidean space by itself.

Theorem 4.3: (Fundamental Theorem in neutrosophic Euclidean Geometry)

Let $f: M \rightarrow V; f(a + bI, c + dI) = ((a, a + b), (c, c + d))$ be the AH-isometry defined above, we have:

- (a) f preserves addition operation between vectors.
- (b) f preserves distances between points.
- (c) f is a bijection one-to-one between M and V .
- (d) Multiplying a neutrosophic vector by a neutrosophic real number is preserved up to isometry, i.e. The direct image of a neutrosophic vector multiplied by a neutrosophic real number is exactly equal to its AH-isometric image multiplied by the one-dimensional isometric image of the corresponding neutrosophic real number.

Proof:

(a) Let $\vec{u} = (a + bI, c + dI), \vec{v} = (x + yI, z + tI)$ be two neutrosophic vectors, we have

$$\begin{aligned} f(\vec{u} + \vec{v}) &= f(a + x + I[b + y], c + z + I[d + t]) = ((a + x, a + x + b + y), (c + z, c + z + d + t)) \\ &= ((a, a + b), (c, c + d)) + ((x, x + y), (z, z + t)) = f(\vec{u}) + f(\vec{v}) \end{aligned}$$

(b) We must prove that the norm of the classical vector $\overrightarrow{f(u)}$,

is exactly equal to the one-dimensional isometric image of the norm of the neutrosophic vector \vec{u} .

$\|f(\vec{u})\|^2 = (a^2 + c^2, (a + b)^2 + (c + d)^2)$, on the other hand, we have

$$g(\|\vec{u}\|^2) = g(a^2 + c^2 + I[(a + b)^2 + (c + d)^2 - a^2 - c^2]) = (a^2 + c^2, (a + b)^2 + (c + d)^2) = \|f(\vec{u})\|^2.$$

(c) Suppose that $f(a + bI, c + dI) = f(x + yI, z + tI)$, hence $((a, a + b), (c, c + d))$

$$= ((x, x + y), (z, z + t)), \text{ thus } x = a, b = y, z = c, d = t, \text{ so that } f \text{ is injective.}$$

It is clear that f is surjective, thus it is a bijection.

(d) Consider the following neutrosophic vector $\vec{u} = (a + bI, c + dI)$

With the following neutrosophic real number $m + nI$, we have

$$(m + nI) \cdot \vec{u} = ((m + nI)(a + bI), (m + nI)(c + dI)) = ((ma + I[mb + na + nb]), (mc + I[md + nc + nd])), \text{ on the other hand, we have}$$

$$\begin{aligned} f((m + nI) \cdot \vec{u}) &= ((ma, (ma + mb + na + nb)), (mc, mc + md + nc + nd)) \\ &= (m, m + n) \cdot ((a, a + b), (c, c + d)) = g(m + nI) \cdot f(a + bI, c + dI). \end{aligned}$$

Example 4.4:

Consider the following two neutrosophic points $A(1 + 2I, I), B(3I, -2 + I)$, we have:

(a) The isometric points of A, B are $A' = ((1,3), (0,1)), B' = ((0,3), (-2, -1))$.

(b) $\vec{AB} = (-1 + I, -2)$, the corresponding isometric vector is $\vec{A'B'} = ((-1,0), (-2, -2)) = f(\vec{AB})$.

(c) The neutrosophic distance $[AB] = \sqrt{1 + 4 + I[0 + 4 - 1 - 4]}$

$$= \sqrt{5 - I} = \sqrt{5} + I[4 - \sqrt{5}]. \text{ The classical distance between isometric images is}$$

$$[A'B'] = (\sqrt{(-1)^2 + (-2)^2}, \sqrt{(0)^2 + (-2)^2}) = (\sqrt{5}, 4) = g([AB]).$$

Theorem 4.3 introduces an algorithm to transform any neutrosophic point to a classical Cartesian product of two classical points. The following theorem describes the inverse relation between classical coordinates and neutrosophic coordinates, i.e. It clarifies how to go back from classical coordinates to neutrosophic coordinates.

Theorem 4.5:

Let $A((a, b), (c, d))$ be a Cartesian product of two classical points, then the inverse isometric image (the corresponding neutrosophic point) is

$$B(a + (b - a)I, c + (d - c)I).$$

Proof:

It holds directly by taking the image of B with respect to AH-isometry, the point A is obtained.

Example 4.6:

Consider the following classical point $A((1,2), (-1,4))$, its corresponding neutrosophic point is $B(1 + I, -1 + 5I)$.

As a result of Section 4, we can find that all geometrical famous properties are still true in neutrosophic Euclidean geometry, that is because we can transform any neutrosophic point to a corresponding classical point by preserving addition, distances, and multiplication by scalars.

5. Some neutrosophic geometrical shapes with two N-dimensions

Definition 5.1: (Neutrosophic circle)

Let $M(a + bI, c + dI)$ be a fixed neutrosophic point, we define the neutrosophic circle with center M and radius $R = r_1 + r_2I \geq 0$ to be the set of all two N-dimensional points $N(X, Y) = N(x_0 + x_1I, y_0 + y_1I)$; $dist(M, N) = R = const.$

Theorem 5.2:

Let $M(a + bI, c + dI)$ be a fixed neutrosophic point, $R = r_1 + r_2I$ be a neutrosophic real positive number, we have:

(a) The equation of the circle with center M and radius R is $([(x_0 + x_1I) - [a + bI]])^2 + ([y_0 + Iy_1] - [c + dI])^2 = R^2$.

(b) The previous neutrosophic circle is equivalent to the following direct product of two classical circles

$$C_1: (x_0 - a)^2 + (y_0 - c)^2 = r_1^2, C_2: ((x_0 + x_1) - [a + b])^2 + ([y_0 + y_1] - [c + d])^2 = (r_1 + r_2)^2.$$

Proof:

(a) By using the neutrosophic distance form defined in Definition 3.8 and Definition 3.9, we get is $([(x_0 + x_1I) - [a + bI]])^2 + ([y_0 + Iy_1] - [c + dI])^2 = R^2$.

(b) To obtain the classical equivalent geometrical system of the neutrosophic circle, it is sufficient to take its isometric image as follows:

$$f(((x_0 + x_1I) - [a + bI]))^2 + f((([y_0 + Iy_1] - [c + dI]))^2) = f(R^2), \text{ hence}$$

$$((x_0 - a)^2, (x_0 + x_1 - [a + b])^2) + ((y_0 - c)^2, (y_0 + y_1 - [c + d])^2) = (r_1^2, (r_1 + r_2)^2), \text{ thus}$$

$$(((x_0 - a)^2 + (y_0 - c)^2), ((x_0 + x_1 - [a + b])^2 + (y_0 + y_1 - [c + d])^2)) = (r_1^2, (r_1 + r_2)^2),$$

Thus, we get $(x_0 - a)^2 + (y_0 - c)^2 = r_1^2$ and

$$((x_0 + x_1) - [a + b])^2 + ([y_0 + y_1] - [c + d])^2 = (r_1 + r_2)^2.$$

Example 5.3:

Consider the following neutrosophic circle: $C: (X - I)^2 + (Y - (2 - 3I))^2 = (2 + I)^2$

It is equivalent to the direct product of the following two classical circles:

$$C_1: (x_0 - 0)^2 + (y_0 - 2)^2 = 2^2, C_2: ((x_0 + x_1) - [-1])^2 + ([y_0 + y_1] - [-1])^2 = (2 + 1)^2.$$

Definition 5.4: (Neutrosophic line)

We define the neutrosophic line by the set of all two N-dimensional points (X, Y) with the property

$$AX + BY + C = 0; X = x_0 + x_1I, Y = y_0 + y_1I, A = a_0 + a_1I, B = b_0 + b_1I, C = c_0 + c_1I.$$

Theorem 5.5:

Let $AX + BY + C = 0$ be an equation of a neutrosophic line d , this line is equivalent to the direct product of the following two classical lines:

$$d_1: a_0x_0 + b_0y_0 + c_0 = 0, d_2: (a_0 + a_1)(x_0 + x_1) + (b_0 + b_1)(y_0 + y_1) + c_0 + c_1 = 0.$$

Proof:

By taking the isometric image to the equation $AX + BY + C = 0$, we get the proof.

Example 5.6:

Consider the following neutrosophic line $(1 + I)X + (2 - 4I)Y + 1 - 3I = 0$, it is equivalent to the following two classical lines

$$d_1: x_0 + 2y_0 + 1 = 0, d_2: 2(x_0 + x_1) - 2(y_0 + y_1) - 2 = 0.$$

Remark 5.7:

(a) If we have two classical circles $C_1: (x_0 - a)^2 + (y_0 - c)^2 = (r_1)^2, C_2: (x_1 - b)^2 + (y_1 - d)^2 = (r_2)^2$, then we can transform the set of their direct product $C_1 \times C_2$, into one neutrosophic circle by using the inverse image of the AH-isometry as follows:

$$C: (X - M)^2 + (Y - N)^2 = r^2; X = x_0 + (x_1 - x_0)I, Y = y_0 + (y_1 - y_0)I, M = a + (b - a)I, N = c + (d - c)I, r = r_1 + (r_2 - r_1)I.$$

The proof holds easily by taking the inverse image with respect to AH-isometry.

(b) By the same argument, if we have two classical lines:

$a_0x_0 + b_0y_0 + c_0 = 0, a_1x_1 + b_1y_1 + c_1 = 0$, we can transform the set of their direct product into one neutrosophic line as follows:

$$AX + BY + C = 0; A = a_0 + (a_1 - a_0)I, B = b_0 + (b_1 - b_0)I, X = x_0 + (x_1 - x_0)I, Y = y_0 + (y_1 - y_0)I, C = c_0 + (c_1 - c_0)I.$$

4. Conclusions

In this article, we presented a general definition of neutrosophic Euclidean geometry. We studied the mechanism of comparison between neutrosophic numbers. In the luminosity of our findings, we make a connection between neutrosophic geometrical concepts and classical Euclidean geometry. Further, the concept of the neutrosophic plane with n neutrosophic dimensions is obtained. Also, Euclidean geometric concepts are extended neutrosophically such as neutrosophic distance, neutrosophic midpoint, neutrosophic vectors, neutrosophic circles, and lines.

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References

- [1] Abobala, M., "A Study of AH-Substructures in n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85, 2020.
- [2] Smarandache F., and Abobala, M., " n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54, 2020.
- [3] Hatip, A., Alhamido, R., and Abobala, M., " A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76, 2019.
- [4] Abobala, M, "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 72-75, 2020.
- [5] Abobala, M., " Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78, 2020.
- [6] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings ", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66, 2020.
- [7] N. Olgun and A. Hatip, "The Effect Of The Neutrosophic Logic On The Decision Making," in *Quadruple Neutrosophic Theory And Applications*, Belgium, EU, Pons Editions Brussels, 2020, pp. 238-253.

- [8] Abdel-Basst, Mohamed, Rehab Mohamed, and Mohamed Elhoseny. "<? covid19?> A model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans." *Health Informatics Journal* (2020): 1460458220952918.
- [9] J. Anuradha and V. S, "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka," *Neutrosophic Sets and Systems*, vol. 31, pp. 179-199, 2020.
- [10] A. Chakraborty, B. Banik, S. P. Mondal, and S. Alam, " Arithmetic and Geometric Operators of Pentagonal Neutrosophic Number and its Application in Mobile Communication Service-Based MCGDM Problem," *Neutrosophic Sets and Systems*, vol. 32, pp. 61-79, 2020.
- [11] S. K. Patro and F. Smarandache, "THE NEUTROSOPHIC STATISTICAL DISTRIBUTION, MORE PROBLEMS, MORE SOLUTIONS," *Neutrosophic Sets and Systems*, vol. 12, pp. 73-79, 2016.
- [12] Hatip, A., "Neutrosophic Special Functions", *International Journal of Neutrosophic Science*,
- [13] Alhamido, R., and Gharibah, T., "Neutrosophic Crisp Tri-Topological Spaces", *Journal of New Theory*, Vol. 23, pp.13-21, 2018.
- [14] Agboola, A.A.A., and Akinleye, S.A., "Neutrosophic Vector Spaces", *Neutrosophic Sets and Systems*, Vol. 4, pp. 9-17, 2014.
- [15] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", *International Journal of Neutrosophic Science*, Vol. 6, pp. 80-86, 2020
- [16] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", *International Journal of Neutrosophic Science*, Vol. 7, pp. 79-86, 2020.
- [17] Hatip, A., and Olgun, N., " On Refined Neutrosophic R-Module", *International Journal of Neutrosophic Science*, Vol. 7, pp.87-96, 2020.
- [18] Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", *International Journal of Neutrosophic Science*, Vol. 9, pp. 110-116, 2020.
- [19] Sankari, H., and Abobala, M., "n-Refined Neutrosophic Modules", *Neutrosophic Sets and Systems*, Vol. 36, 2020.
- [20] Abdel-Basset, M., Gamal, A., Son, L. H., & Smarandache, F. (2020). A Bipolar Neutrosophic Multi-Criteria Decision Making Framework for Professional Selection. *Applied Sciences*, 10(4), 1202.
- [21] G. Shahzadi, M. Akram and A. B. Saeid, "An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis," *Neutrosophic Sets and Systems*, vol. 18, pp. 80-88, 2017.
- [22] Abdel-Basset, M., Gunasekaran M., Abdullah G., and Victor C., "A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT", *IEEE Internet of Things Journal* , 2019.
- [23] Abdel-Basset, Mohamed, et al. "An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries." *Risk Management* (2020): 1-27.
- [24] T.Chalapathi and L. Madhavi,. "Neutrosophic Boolean Rings", *Neutrosophic Sets and Systems*, Vol. 33, pp. 57-66, 2020.
- [25] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., Gamal, A., & Smarandache, F. (2020). Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In *Optimization Theory Based on Neutrosophic and Plithogenic Sets* (pp. 1-19). Academic Press.
- [26] Smarandache, F., and Abobala, M., "n-Refined neutrosophic Rings", *International Journal of Neutrosophic Science*, Vol. , pp. , 2020.

- [27] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces I", *International Journal of Neutrosophic Science*, Vol. 7, pp. 97-109, 2020.
- [28] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces II", *International Journal of Neutrosophic Science*, Vol. 9, pp. 22-36, 2020.
- [29] Olgun, N., and Khatib, A., "Neutrosophic Modules", *Journal of Biostatistic and Biometric Application*, Vol. 3, 2018.
- [30] W. B. V. Kandasamy and F. Smarandache, *Neutrosophic Rings*, Hexis, Phoenix, Arizona: Infinite Study, 2006.
- [31] F. Smarandache, *Introduction to Neutrosophic Statistics*, USA: Sitech & Education Publishing, 2014.
- [32] F. Smarandache, "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets," *Inter. J. Pure Appl. Math.*, pp. 287-297, 2005.
- [33] Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", *Neutrosophic Sets and Systems*, Vol. 39, 2021.
- [34] M. Ali, F. Smarandache, M. Shabir and L. Vladareanu, "Generalization of Neutrosophic Rings and Neutrosophic Fields," *Neutrosophic Sets and Systems*, vol. 5, pp. 9-14, 2014.
- [34] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, *Journal of Fuzzy Extension and Applications*, Vol. 2, pp. 16-22, 2021.
- [35] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, *Journal of Mathematics*, Hindawi, 2021.
- [36] Abobala, M., Partial Foundation of Neutrosophic Number Theory, *Neutrosophic Sets and Systems*, Vol. 39, 2021.
- [37] Sankari, H., and Abobala, M., " AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", *Neutrosophic Sets and Systems*, Vol. 38, pp. 101-112, 2020.

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