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Robust MPC-Based Gait Generation in Humanoids

Filippo M. Smaldone, Nicola Scianca, Leonardo Lanari, Giuseppe Oriolo

***Abstract*— We introduce a robust gait generation framework for humanoid robots based on our Intrinsically Stable Model Predictive Control (IS-MPC) scheme, which features a stability constraint to guarantee internal stability. With respect to the original version, the new framework adds multiple components addressing the robustness problem from different angles: an observer-based disturbance compensation mechanism; a ZMP constraint restriction that provides robustness with respect to bounded disturbances; and a step timing adaptation module to prevent the loss of feasibility. Simulation and experimental results are presented.**

I. Introduction

The fundamental requirement for successful humanoid locomotion is balance, commonly expressed by the condition that the Zero Moment Point (ZMP) must remain in the robot support polygon. Due to the complexity of the full humanoid dynamics, simplified models such as the Linear Inverted Pendulum (LIP) are adopted to relate the motion of the Center of Mass (CoM) to that of the ZMP. These models can be used for gait generation via Model Predictive Control (MPC), encoding the balance requirement through ZMP constraints [1].

MPC schemes work as long as a feasible solution exists; in particular, a desirable property is recursive feasibility. However, this is typically proven for the nominal case, and may be lost in the presence of perturbations. Many papers have therefore proposed techniques to increase the robustness of MPC-based gait generation [2], [3], [4], [5], [6].

Since humanoid dynamics are inherently unstable, we developed an Intrinsically Stable MPC (IS-MPC) method that uses a stability constraint to ensure that the CoM trajectory will be bounded with respect to the ZMP [7]. Thanks to this constraint, IS-MPC is guaranteed to be both recursively feasibile and internally stable.

In this paper we robustify the IS-MPC scheme using three main components:

1) observer-based disturbance compensation through a modified stability constraint;
2) ZMP constraint restriction for recursive feasibility against bounded disturbances;
3) step timing adaptation to avoid imminent losses of feasibility, e.g., due to impulsive pushes.

We briefly recall the nominal IS-MPC scheme before describing its robust version and presenting some simulation/experimental results.

The authors are with the Dipartimento di Ingegneria Informatica, Automatica e Gestionale, Sapienza Università di Roma, via Ariosto 25, 00185 Roma, Italy. E-mail: {lastname}@diag.uniroma1.it.

II. IS-MPC

IS-MPC provides CoM/ZMP trajectories and footsteps that realize as closely as possible a set of candidate footsteps, provided by a footstep planner along with step timings. The scheme operates over time intervals of duration δ. At a generic t_k, the candidate footsteps are known over a *preview horizon*, assumed to be not smaller than the MPC *control horizon* $T_c = C\delta$.

Denoting the CoM and ZMP position respectively by $\boldsymbol{p}_c = (x_c, y_c, z_c)$ and $\boldsymbol{p}_z = (x_z, y_z, 0)$, the LIP model is

$$\ddot{x}_c = \eta^2(x_c - x_z),$$

where $\eta = \sqrt{g/\bar{z}_c}$, with g the gravity acceleration and $\bar{z}_c$ the constant CoM height (we focus on the x component as an identical equation holds for the y component). More specifically, the prediction model used in IS-MPC is a LIP with dynamic extension where the ZMP velocity is the input.

Let us collect the decision variables over T_c as

$$\begin{aligned} \dot{X}_z^k &= (\dot{x}_z^k \ \dots \ \dot{x}_z^{k+C-1})^T \\ X_f^k &= (x_f^1 \dots x_f^F)^T, \end{aligned}$$

where x_f^j is the j-th footstep within the control horizon.

The ZMP constraints require that the ZMP lies at all times within the support polygon:

$$x_z^m(t, X_f^k) \leq x_z(t) \leq x_z^M(t, X_f^k), \tag{1}$$

for $t \in (t_k, t_{k+C}]$, where $x_z^m(t, X_f^k)$ and $x_z^M(t, X_f^k)$ are the upper and lower bounds, which depend on the footsteps.

Kinematic constraints are also imposed to ensure that the robot can actually execute the generated footsteps. Along x, this translates to a maximum step length.

The stability constraint ensures that the scheme is internally stable. Indeed, the transformed coordinate

$$x_u = x_c + \dot{x}_c/\eta,$$

called *capture point* or *divergent component of motion*, highlights the unstable dynamics inside the LIP:

$$\dot{x}_u = \eta\,(x_u - x_z).$$

Despite this instability, x_c will remain bounded with respect to x_z provided that the following *stability condition* is satisfied

$$x_u^k = \eta \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau. \tag{2}$$

The latter condition depends on the future ZMP trajectory, and is thus non-causal. To obtain a causal constraint, we split the above integral in the integral from t_k to $t_k + T_c$, which

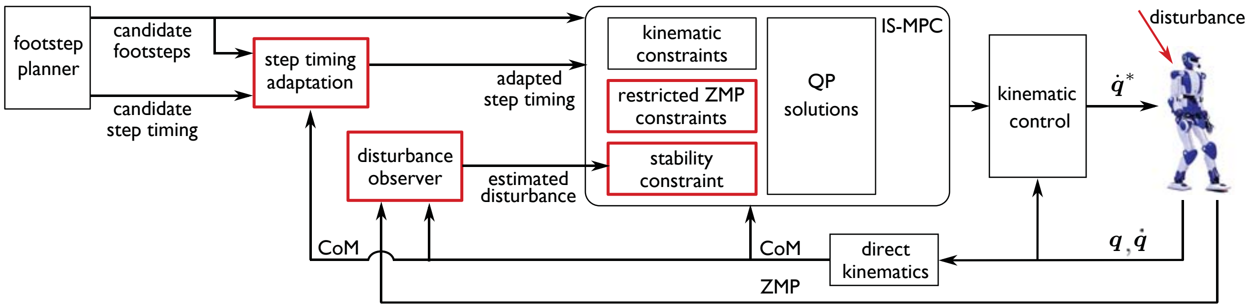

Fig. 1. A block scheme of the robust IS-MPC method for gait generation. Components added with respect to the original scheme are shown in red.

depends on $\dot{X}_z^k$, plus the integral from $t_k + T_c$ to ∞, in which we use a ZMP trajectory $\tilde{x}_z$ conjectured on the basis of the preview information (*anticipative tail*). This leads to the *stability constraint*

$$\eta \int_{t_k}^{t_{k+C}} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau = x_u^k - \tilde{c}_x^k \tag{3}$$

where

$$\tilde{c}_x^k = \eta \int_{t_{k+C}}^{\infty} e^{-\eta(\tau - t_k)} \tilde{x}_z(\tau) d\tau.$$

The integral in (3) can be written in closed form as a linear function of the ZMP velocity inputs $\dot{x}_z^k, \ldots, \dot{x}_z^{k+C-1}$.

We can now formulate the QP-MPC problem:

$$\min_{\dot{X}_z^k, X_f^k} \|\dot{X}_f^k\|^2 + \mu \|X_f^k - \hat{X}_f^k\|^2$$

subject to:

- ZMP constraints (1)
- kinematic constraints
- stability constraint

where $\hat{X}_f^k = (\hat{x}_f^1 \ldots \hat{x}_f^F)^T$ is a vector collecting the candidate footstep position. The resulting CoM trajectory, together with a swing foot trajectory landing at the next footstep, are sent to the kinematic control module.

III. Robust IS-MPC

Consider the following perturbed LIP model

$$\ddot{x}_c = \eta^2 (x_c - x_z) + w, \tag{4}$$

where w collects the effect on CoM acceleration of unmodeled dynamics, parametric uncertainties and/or external forces. In particular, we assume that

$$w(t) = w_m + \Delta w(t), \tag{5}$$

with w_m a constant representing the persistent component of the disturbance, referred to as *mid-range disturbance*, and $\Delta w(t)$ a deviation from the mid-range, for which we assume $|\Delta w(t)| \leq \Delta^{\max}$. The mid-range disturbance may be a constant acceleration due, say, to an unknown ground slope; whereas the deviation could account for a slowly varying force, e.g., produced by the robot carrying a swinging load.

With this model of disturbance in mind, a robust version of the IS-MPC scheme can be devised combining three complementary approaches, see Fig. 1.

The first novelty is the disturbance observer block. This is mainly designed for providing an estimate of the mid-range disturbance w_m. This estimate is then used to perform an indirect compensation of these disturbances through a modified stability constraint.

A second layer of robustness is added by means of the ZMP constraint restriction. This consists in progressively reducing the admissible region for the ZMP as time proceeds in the control horizon, and offers robustness against the deviation term $\Delta w(t)$, by guaranteeing that recursive feasibility of QP-MPC is maintained up to the bound $\Delta^{\max}$.

Finally, step timing adaptation is performed in the presence of impulsive perturbations that exceed the deviation bound $\Delta^{\max}$. If an imminent loss of feasibility is detected for QP-MPC, the duration of the current step is modified so as to recover feasibility.

The rest of this section will provide some detail about the above three components.

A. Indirect disturbance compensation

For the perturbed LIP model (4), the stability condition (2) must be modified as

$$x_u^k = \eta \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau - \frac{1}{\eta} \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} w(\tau) d\tau. \tag{6}$$

Enforcing this condition implies indirect disturbance compensation, but requires knowledge of the future evolution of $w(t)$. To maintain causality, we perform compensation of the mid-range term w_m, reconstructed through a disturbance observer. In particular, we set up a simple linear observer by extending the LIP with the disturbance model $\dot{w}_m = 0$ and assuming the availability of x_c (CoM), x_z (ZMP) as measurements.

As in the nominal case, a causal stability constraint is derived from condition (6) by conjecturing an anticipative tail. In accordance with the idea of performing compensation of the mid-range disturbance, we replace $w(\tau)$ in the second integral with the current mid-range disturbance estimate $\hat{w}_m^k$, leading to the modified stability constraint

$$\eta \int_{t_k}^{t_{k+C}} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau = x_u^k + \frac{\hat{w}^k}{\eta^2} - \tilde{c}_x^k.$$

More details can be found in [8].

B. ZMP constraint restriction

The deviation Δw from the mid-range disturbance is not taken into account in the modified stability constraint. To achieve robustness with respect to this residual component of the disturbance (as well as to the observation error), we resort to ZMP constraint restriction as a way to guarantee recursive feasibility under bounded disturbances.

Consider a *restriction function* $R(t)$, defined as a non-decreasing function over $[0, T_c]$ such that

$$|R(t)| \leq d/2,$$

with d the size of the ZMP constraint (1). The restricted ZMP constraint for $t \in (t_k, t_{k+C}]$ is then expressed as

$$x_z^m(t, X_f^k) + R(t) \leq x_z(t) \leq x_z^M(t, X_f^k) - R(t). \qquad (7)$$

It is possible to show that a suitable restriction can be computed such that IS-MPC with the restricted constraint (7) remains recursively feasible in the presence of deviations up to $\Delta^{\max}$ around the observed mid-range disturbance $\hat{w}^k$. For example, we can adopt a linear restriction function $R(t) = rt$, and derive an expression for r as a function of $\Delta^{\max}$, see [9] for details.

C. Feasibility-driven step timing adaptation

Fixed step timings are an obvious limitation, especially when perturbations are present. Physical intuition suggests that if the robot is pushed, anticipating or delaying the next step can be beneficial for maintaining balance. However, including step timings as decision variables in QP-MPC leads to a problem that is either nonlinear or involves discrete optimization, with considerable detriment in terms of computational efficiency and convergence guarantees.

To maintain the linearity of our scheme, step timing adaptation is performed prior to IS-MPC gait generation. The step timings coming from the footstep planner are thus regarded as candidates, and they will be definitive only after adaptation.

The strategy upon which adaptation relies is based on the *feasibility region* $\mathcal{F}_k$ of IS-MPC. In particular, IS-MPC is feasible at t_k if and only if $(x_u^k, y_u^k) \in \mathcal{F}_k$, where

$$\mathcal{F}_k = \{(x_u, y_u) : x_u^{k,m} \leq x_u \leq x_u^{k,M}, y_u^{k,m} \leq y_u \leq y_u^{k,M}\}$$

where $x_u^{k,m}$, $x_u^{k,M}$, $y_u^{k,m}$, $y_u^{k,M}$ denote the bounds of the region along x and y, whose expressions are rather complex. However, by encoding the duration of the current step as $\Delta\lambda = e^{-\eta(t_s^1 - t_k)}$, where t_s^1 is the time at which the (single support of the) current step ends, we can obtain a conservative estimate $\mathcal{F}_{\text{est}}^k$ of the feasibility region which is linear in $\Delta\lambda$. We can then solve the following QP-STA

$$\min_{\Delta\lambda} (\Delta\lambda - \widehat{\Delta\lambda})^2$$

subject to:
- feasibility constraints for x_u and y_u
- timing constraints

where $\widehat{\Delta\lambda} = e^{-\eta(\hat{t}_s^1 - t_k)}$ is the time-to-step (in exponential encoding) according to the current candidate timing. The feasibility constraints encode the requirement that QP-MPC should be feasible after adaptation, while the timing constraint ensures that the adapted step timing can be physically realized by the robot. Whenever adaptation is not necessary, the solution of QP-STA will return the candidate step timing.

The step timing adaptation module is described in detail in [10].

IV. Results

We now present some simulation and experimental results obtained using the robust IS-MPC framework. For comparison, for every scenario we provide also results obtained with nominal IS-MPC.

Experiments were ran on a NAO, a small-sized humanoid. In the first, shown in Fig. 2, the robot carries a bag of mass of 0.67 Kg hanging from its right elbow. The observer generates an estimate of the mid-range disturbance (which is an acceleration towards the right due to the additional mass) to be used in the modified stability constraint. This partial disturbance compensation produces a slight displacement of the CoM trajectory towards the left, as if 'leaning' against an equivalent push coming from that direction to counteract its effect. Moreover, the ZMP constraint restriction is used to provide robustness to the remaining part of the disturbance, which is due to the oscillatory motion of the bag and therefore time-varying. In this experiment, step timing adaptation was never activated because no imminent loss of feasibility loss was detected. Overall, the robust IS-MPC scheme effectively copes with the disturbances by maintaining balance and accurately realizing the planned footsteps. For comparison, results using nominal IS-MPC are also shown: the disturbance due to the swinging payload negatively affects the gait, resulting in the a significant deviation from the planned path and almost leading to a fall.

In the second scenario the humanoid is subject to an impulsive push while walking forward; for this reason, only the step timing adaptation module is active, while indirect disturbance compensation and ZMP constraint restriction are not used. To obtain a reproducible push, a 0.265 kg ball is attached to a rope so to act as a pendulum that hits the robot. In the experiment of Fig. 3, the push occurs during the single support phase when the remaining time-to-step is 0.25 s. Robust IS-MPC is able to maintain balance by immediately reducing the time-to-step to 0.1 s. This is motivated by the fact that as a shortening of the step duration has the effect of enlarging the feasibility region in the x-direction. On the contrary, nominal IS-MPC becomes unfeasible after the push, leading the robot to a loss of balance.

To prove the applicability of the proposed method to different platforms, we present, in the DART environment, a

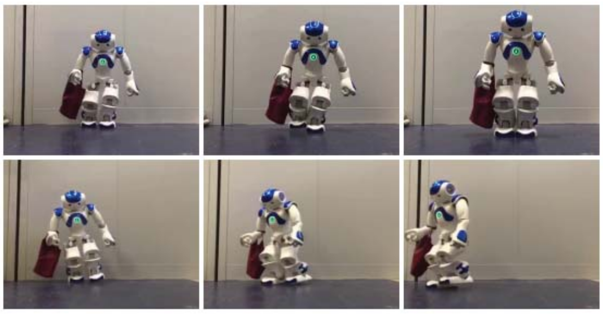

Fig. 2. NAO walks carrying a swinging bag: robust IS-MPC (top), nominal IS-MPC (bottom). With robust IS-MPC, compensation of the mid-range disturbance produces a slight leaning action against the disturbance, while ZMP constraint restriction provides robustness to the time-varying components of w. On the contrary, nominal IS-MPC struggles to maintain balance and visibly deviates from a straight line path.

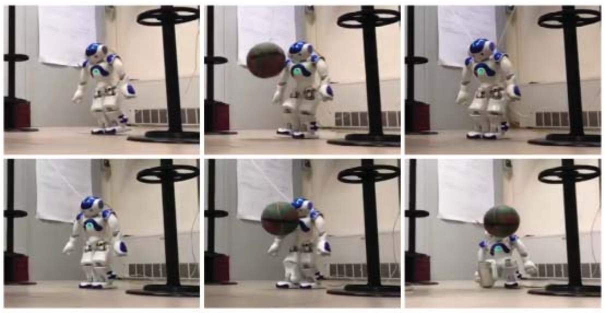

Fig. 3. NAO is hit by a ball while walking: robust IS-MPC (top), nominal IS-MPC (bottom). Thanks to the step adaptation module, IS-MPC is able to preserve feasibility by reducing the step duration, while nominal IS-MPC fails causing the robot to fall.

dynamic simulation for HRP-4, a full-sized humanoid robot. The robot must walk under a constant push of 45 N along the sagittal direction; at some point, an impulsive push of 85 N is added in the coronal direction for 0.1 s. As the results of Fig. 4 clearly show, robust IS-MPC effectively tolerates these disturbances. The persistent component of the disturbance is in fact estimated by the observer and compensated via the modified stability constraint, working together with the ZMP constraint restriction. The step timing adaptation module only changes step timing immediately after the impulsive push. Once again, the nominal scheme fails causing the robot to fall.

V. Conclusions

This paper presents an MPC-based scheme for robust humanoid gait generation. It is based on the IS-MPC scheme and it relies on three different modules which can operate simultaneously or independently. The proposed approach is general enough to work on different humanoid platforms such as NAO and HRP-4, allowing them to walk in the presence of disturbances of different nature. The framework is validated through dynamic simulations and experiments.

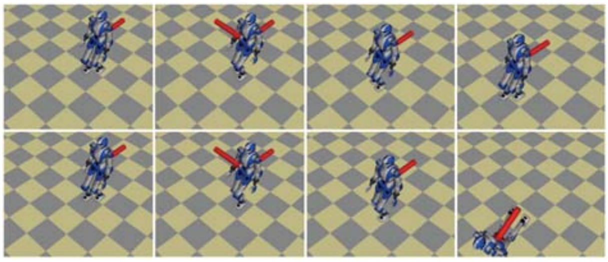

Fig. 4. HRP-4 walking under multiple pushes: robust IS-MPC (top), nominal IS-MPC (bottom). The robot is walking under a constant disturbance in the x direction and an impulsive disturbance in the y direction. Robust IS-MPC is able to successfully withstand the disturbances, while the nominal scheme fails.

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