Performance Evaluation of Power Demand Scheduling Scenarios in a Smart Grid Environment

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Abstract

Smart grid technology is considered as the ultimate solution to challenges that emerge from the increasing power demands, the subsequent increase in pollution, and the outmoded power grid infrastructure. The successful implementation of the smart grid is mainly driven by the utilization of modern communication technologies, which aim at the provision of advanced demand side management mechanisms, such as demand response. In this paper, we present and analyze four power-demand scheduling scenarios that aim to reduce the peak demand in a smart grid infrastructure. The proposed scenarios consider that each consumer is equipped with a certain number of appliances of different power demands and different operational times, while the percentage of consumers that agree to participate in the demand scheduling program is also incorporated in our models. We provide the analysis for the determination of the peak demand in a residential area, based on recursive formulas. The proposed analysis is validated through simulations; the accuracy of the analytical models is found to be quite satisfactory. Moreover, we unveil the consistency and necessity of the proposed scenarios and corresponding analytical models.

Keywords: smart grid, power demand, demand scheduling, performance evaluation

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Nomenclature

B_m	probability of exceeding P after the acceptance of type-m request
$b_m(j)$	control function for CDS
$b_{m,t}(j)$	control function for CDS
$c_{m,0}(j)$	control function for DRS
$c_{m,t}(j)$	control function for DRS and PRS
d_m^{-1}	mean appliance operational duration
$d_{m,t}^{-1}$	mean appliance operational duration when $P_{t-1} \leq j < P_t$
e	predefined upper bound of the blocking probabilities
j	total number of PU in use
M	number of appliances
M_1	number of appliances that are able to compress their demands
M_2	number of appliances that are able to postpone their operation
P	maximum number of supported p.u. in the real system
p_m	power demand of type-m appliance
$p_{m,t}$	compressed power demand when the total power consumption is $P_{t-1} \leq j < P_t$
P_t	power threshold for the scheduling scenarios
Q	distribution normalization constant
$s_{m,t}$	percentage of type-m appliances that participate in the scheduling program, when $P_{t-1} \leq j < P_t$
T	number of thresholds for CDS and DRS
Greek symbols	
λ_m	power request arrival rate of type-m appliances
$\lambda_{m,t}$	power request arrival rate of type-m appliances when $P_{t-1} \leq j < P_t$
Subscripts	
m	appliance type from the M appliance set
m'	appliance type from the expanded $2M$ appliance set
t	power threshold

1. Introduction

The present electric power grid has persisted for several decades and its capability to address the future demand for electricity is doubtful. The evolution of electric power systems should be a next generation infrastructure that provides reliability, efficiency, and resilience to equipment failures. The smart grid aims at enhancing the flexibility and consistency of the electric power grid, through the utilization of communication technologies that provide intelligent control over power consumption [1], [2]. In this *smart* energy consumption environment, users' appliances are able to adjust their power demands according to their practical needs, while contributing to the reduction of the total power consumption in peak-power demand hours [3], [4].

The successful implementation of the demand management in a smart grid environment is mainly driven by exploiting Demand Response (DR) programs that are applicable to either industrial/commercial or residential consumers. This customer-enabled power consumption management is the key smart-grid feature that enables the adaptation of power demands to time variable prices, while it improves the efficiency and the reliability of the power grid and achieves peak demand reduction [5]. Consequently, a DR scheme that provides a fair charging scheme would not only benefit the participants that can save more money, but it would also enable the energy provider to meet its pollution obligations and reduce the power generation cost by eliminating the need for activating expensive-to-run power plants during peak demand hours [6], [7]. This peak demand reduction target can be achieved by applying DR programs not only to industrial/commercial consumers, but also to residential consumers, since both sectors can mutually mitigate the grid's congestion during peak hours [8], [9].

The design of efficient DR algorithms is a crucial issue for the deployment of the smart grid. These algorithms can be classified according to the offered motivations into price-based and incentive-based programs [10], and based on the decision variable into task scheduling and energy scheduling [11]. In price-based DR programs, consumers are granted time-varying prices that are defined based on the electricity cost in different time periods, while in incentive-based DR programs, consumers are offered fixed or time-varying payments, in order to motivate their electricity usage reduction during periods of system stress, but they are also penalized for not participating in the program. Furthermore, in task scheduling DR [12], the key function is the control on the activation time of the requested load, which can be shifted from peak-demand to low-demand periods. The reduction of the total power consumption in peak-demand hours can be also achieved by applying energy scheduling DR programs [13], which target the power consumption reduction of specific loads, through the control of their operation to consume less power during system stress. Both task and energy scheduling DR programs are considered as the most effective strategies that can be applied to households for the reduction of peak-to-average ratio in load demand [14], while they can be combined with price-based or incentive-based schemes, in order to make the DR program attractive to the consumers. A detailed discussion on the challenges and requirements of load scheduling methods can be found in [15].

There is a significant number of research articles that study the implementation of scheduling DR programs. Most of these research efforts use simulation [16], [17] or optimization methods [18], [19], [20] to deal with the power-demand control problem. Analytical models have provided solution to the same load management problem. The current power consumption is used in [21], in order to decide the power request scheduling. Two power demand control policies are proposed and analyzed: the first policy assumes that a power controller activates immediately or postpones power requests, based on the current power consumption. In the second policy, a new request is activated immediately, if the total power consumption is lower than a threshold, else it is queued. Similar power demand control policies are presented in [22]. Furthermore, in [23] an analytical model for the peak and total energy consumption reduction under Interruptible/Curtailable service (I/C) and Capacity Market Programs (CAP) is proposed. In all cases, the power requirement of each power request equals to 1 power unit. Multiple appliances with diverse power demands are considered in [24] for the efficient determination of the peak demand in the residential area, by considering either energy or task scheduling policies. However, the proposed models do not consider the percentage of consumers that refuse to participate in the program, while the computational complexity of the analytical model for the task-scheduling scheme is high, due to the absence of a recursive formula for the determination of state probabilities.

In this paper, we study both task and energy scheduling programs, by considering a smart grid architecture where each end-user is connected to a Central Load Controller (CLC), located at a substation of the Distribution Network Operator (DNO). We propose and analyze four power demand scheduling scenarios for the control of power requests by the CLC. Each scenario tackles a different approach on the control of the users' power demands, and achieves a different performance regarding the total required power consumption in a residential area. Compared to the state of the art, the key advantages of the proposed scenarios and corresponding analytical models are: 1) the consideration of a set of consumers, each one equipped with a specific number of appliances with different power demands, different operational times and different arrival rates of power requests, 2) the inclusion of the percentage of consumers that wish to contribute to the program for each appliance, 3) the low computational complexity of the proposed analytical models due to the utilization of recursive formulas. Therefore, the proposed models are more realistic and provide more accurate peak-demand results, compared to models that do not consider consumers' participation percentages, while the proposed models can be applied to DR programs that require near real-time decisions, due to their significantly low complexity. More precisely, the four proposed scenarios and their key features are:

- The first scenario, named the "default scenario", is introduced in order to determine the upper bound of the total power consumption in the residential area under study.
- The second scenario, named the "Compressed Demand Scenario" (CDS), is an energy scheduling scenario, where a number of appliances are able to compress their power demands and simultaneously expand their operational times. This compression mode (also known as load curtailing [25]) is applied to specific types of appliances, and it is only activated when the power consumption in the residential area exceeds predefined thresholds.
- In the third scenario, named the "Delay Request Scenario" (DRS), power requests are delayed in buffers for a predefined time period, which is different for each appliance's type and is a function of the current power consumption and predefined power thresholds, while after this delay the power requests attempt to access the system. This power-request

delay is also known as demand shifting and may reduce the power consumption by waiting for the termination of the operation of already activated appliances, without accepting any new power requests.

- Finally, in the "Postponement Request Scenario" (PRS), power requests that arrive at the CLC when the power consumption exceeds a threshold P_1, are not postponed for a constant time period as in the DRS case, but until the power consumption drops below a second threshold P_2, with $P_2 \leq P_1$.

Examples of the operation of the three scheduling scenarios (CDS, DRS and PRS) are presented in Fig. 1. The CDS is more suitable for appliances with an elastic load component (e.g. appliances that have heating elements), while the demand-shifting scenarios (DRS and PRS) can be applied to a variety of appliances that can handle operational delays. A combination of the proposed scenarios can achieve maximum peak-demand reduction, through the demand compression of an appliance's set and the demand shifting of another set of appliances. Table 1 summarizes the four basic scenarios together with 2 combined scenarios that consider both demand compression and demand shifting, and it presents the appliances' types that can be applied to each scenario and the corresponding scheduling parameters. It should be noted that the application of the demand-shifting scenarios eliminates the probability of higher load levels, mainly due to the consideration of power thresholds: under DRS (Fig. 1b), the gradual increase of power-request delays results in less accepted power requests while more appliances terminate their operation, whereas under PRS (Fig. 1c), power requests are delayed until the total power consumption drops below a power threshold that is smaller than the threshold that is considered for the shifting activation. Furthermore, the participation of the consumers to the scheduling scenarios should be motivated through the provision of incentives, such as lower electricity prices for consumers that decide to participate in the program, in order to change their power consumption habits and contribute to the peak demand reduction.

The main contribution of this paper is the derivation of recursive formulas for each scenario that determine the distribution of power units in the residential area. The utilization of recursive formulas is a computationally efficient method that minimizes the complexity of the required calculations; therefore the near real-time peak-demand calculations can be used in order to provide fast decisions for the efficient application of the scheduling programs. The validation of the proposed analytical models is realized through the comparison of analytical results with corresponding results from simulation. The analytical results are obtained by solving the proposed recursive formulas, while simulation results are obtained from our objected-oriented simulator, which executes the rules of each scheduling scenario without considering any equations. Through this validation process, the accuracy of all models is found to be quite satisfactory. Therefore, the proposed analytical models can be efficiently used for the peak demand determination, which is realized in a very short time, in comparison to simulations, which are typically time-consuming and are generally performed by using troublesome and expensive simulation tools. Moreover, the proposed analytical models are pattern agnostic; therefore they can be applied to a wide range of applications, while considering both demand compression and demand-request postponement. Finally, we compare analytical results from the proposed scenarios with corresponding analytical results from [22] and [24], since these models also incorporate power thresholds for the activation of the load scheduling scheme. We show that the proposed analytical models are more realistic, since they consider multiple power requests with diverse power requirements, while they achieve better performance regarding the total power consumption.

The organization of this paper is as follows. In Section 2, we introduce the modeling principles for the default scenario, while in Section 3 we present and analyze the three proposed scheduling scenarios. In Section 4, we discuss the applicability of the proposed scenarios, when both power compression and request delay are jointly applied. Section 5 is the evaluation section, where both analytical and simulation results are displayed and discussed. We conclude our paper in Section (6).

2. Modeling Principles of the Default Scenario

In this section, we present the basic principles for the modeling of the smart grid infrastructure under study. We also present the default scenario, which is introduced in order to determine the upper bound of the power consumption in the residential area under study. These modeling principles are also considered in the scheduling scenarios that are presented in the following section.

We consider a residential area where each residence is equipped with up to M appliances (Fig. 2). For each residence there is an Energy Consumption Controller (ECC) connected to all appliances. Each residence is connected to the power line coming from the energy source, while the ECC is connected to the CLC through a Local Area Network (LAN). Each appliance requires a certain amount of power in order to operate properly. The power demand of appliance m ($m = 1, \dots, M$) is denoted as p_m power units (PUs), while the maximum number of PUs that the DNO can support in the specific area is denoted as P. The ECC receives the required number of PUs from each appliance and reports these requirements to the CLC, by using load control messages that are sent through the LAN control channel. The controller activates the power requests immediately upon the reception of the load control message, i.e. no request scheduling occurs. The arrival process of the requests for p_m PUs from all residences follows a Poisson distribution, with a mean arrival rate denoted as r_m. The operation of the $m-$type appliance is generally distributed with a mean duration of d_m^{-1}. The Poisson distribution has been considered as a suitable solution for modeling the power requests' arrival process ([22], [26]). Furthermore, appliances' operation times have been considered to follow a general distribution, which is a more widespread solution compared to exponential distribution followed in several research schemes ([27], [28]). Based on the above assumptions, we determine the distribution of the PUs in use by using the following recursive formula:

$$jq(j) = \sum\nolimits_{m=1}^{M} (r_m d_m^{-1}) p_m q(j - p_m) \tag{1}$$

for $j = 1, ..., P$. Eq. (1) provides the distribution of the probabilities $q(j)$ that j PUs are in use in the residential area. A similar recursive formula is used for the distribution of the occupied bandwidth in multi-rate communication networks [29], which also assumes Poisson arrivals and generally distributed service times. Eq. (1) is solved by using an iterative method, where we set $q(0) = 1$ and $q(j) = 0$, for $j < 0$ and $j > P$. In this way, we calculate the unnormalized probabilities $q(j)$; these probabilities are normalized over the summation $Q = \sum_{i=0}^{P} q(i)$. It should be noted that the assumption of discrete power consumption can provide efficient results, especially when 1 PU is considered equivalent to a very small value of the (continuous) power consumption (e.g. 1 $PU \Leftrightarrow 0.01\ W$).

Due to the finite nature of P, there is a probability B_m that after the acceptance of a power request, the total number of PUs exceeds P. From Eq. (1), B_m can be calculated as the sum of the probabilities of all states that makes the total number of PUs in use to exceed P:

$$B_m = \sum_{j=P-p_m+1}^{P} (q(j)/Q) \tag{2}$$

Eq. (2) can be used to determine the minimum value of P, which guarantees that the requested PUs don't suffer an outage probability larger than a predefined maximum value e. Therefore, by considering a very small value e for the outage probability (e.g. $e = 10^{-5}$, since power requests should not be blocked) we can use Eq. (2) together with Eq. (1) in order to determine the minimum value of P. This calculation is realized by considering the following steps: (i) set an initial value for P, (ii) determine the distribution of PUs in use from Eq. (1) and outage probabilities from Eq. (2), (iii) repeat step (ii) by constantly increasing the value of P until all results of Eq. (2) for all M appliances are below the threshold e. Therefore, since the proposed policies do not consider blocked power requests, the peak demand for a specific set of power requests can be determined by considering the aforementioned method for a very small value for the parameter e, so that the number of blocked power requests is negligible.

3. The Scheduling Scenarios

3.1. The Compressed Demand Scenario

In the Compressed Demand Scenario (CDS), appliances are prompted to gradually reduce their power demands when the total power consumption exceeds predefined power thresholds. The consideration of multiple power thresholds minimizes the effect of an abrupt power reduction that could result in significant decrease in consumers' convenience and comfort. We consider T thresholds for the total number of PUs in use. Upon the arrival of an m-type power demand, if the total number of PUs in use is less than the first threshold P_0, the demand is accepted with its initial requirements p_m and operational time d_m^{-1}. If the total number of PUs in use exceeds P_0, then the CLC sends a message to all consumers, which prompts that the power requests of a specific appliance set should be compressed, so that the total power consumption is reduced. Specifically, the message informs that if consumers wish to contribute to the demand compression mode, then a request for type-m appliance will be accepted with a compressed power demand $p_{m,1} < p_m$ and an extended operational time $d_{m,1}^{-1} > d_m^{-1}$, $m = 1, \ldots, M$. Correspondingly, when the total number j of PUs in use is $P_{t-1} \leq j < P_t$ $(t = 1, \ldots, T)$, consumers that agree to participate in the program are informed that a request for a type-m appliance will be accepted with a compressed power demand $p_{m,t}$ and an extended operational time $d_{m,t}^{-1}$, with $p_m > p_{m,1} > \ldots > p_{m,T}$ and $d_m^{-1} < d_{m,1}^{-1} < \ldots < d_{m,T}^{-1}$. It should be noted that for the reduction of the energy consumption at peak demand hours, the product $(p_{m,t} \times d_{m,t}^{-1})$ should be gradually reduced with the increase of the power consumption, so that $(p_{m,t-1} \times d_{m,t-1}^{-1}) > (p_{m,t} \times d_{m,t}^{-1})$.

The message sent by the CLC refers only to appliances that are able to reduce their power demands and at the same time extend their operation times, e.g. water heaters or air conditioners. Furthermore, the message also contains information for the incentives provided to consumers in order to accept the power request compression (e.g. lower electricity rates for consumers that agree to contribute to the program). These incentives should be adjusted based on the total power consumption, so that more consumers would be motivated to participate in the program when the total power consumption is high; e.g. by considering a price function that is a decreasing function of the current power consumption. Based on the consumers' preferences, the ECC sends a new message to the CLC that contains the response of the consumer on the acceptance or the rejection of the program, while then consumers that agree to participate in the scheduling program adjust the appliance's operation either manually or automatically through a home energy management system [30]. We consider that the probability that consumers will agree to compress their demands for type-m appliances when the total power consumption is $P_{t-1} \leq j < P_t$, is denoted by $s_{m,t}$, while the probability that the consumers will continue to use their appliances with their initial power demands is denoted as $1 - s_{m,t}$. The assumption of variable acceptance probabilities $s_{m,t}$ is used in order to take into account that consumers may react differently to the scheduling messages that contain different pricing signals, depending on the current power consumption. It should be noted that power compression is only activated when the power consumption exceeds the first threshold P_0, while it is deactivated when the power consumption drops below P_0. In contrast, appliances that are not able to compress their power demands (e.g. home entertaining sets or computers) continue to require p_m PUs even if the total number of PUs in use exceeds the first threshold P_0. The sequence of messages exchanged between the ECC and the CLC are illustrated in Fig. 3, while the procedure that takes place at the CLC upon the arrival of a power demand for this scenario is depicted in Fig. 4.

The consideration of the acceptance probabilities $s_{m,t}$ affects the power requests' arrival rate. Specifically, since a percentage of consumers agree to compress their demands, two groups of the same appliance type should be considered: the first group will operate with compressed power demands, while the second group will continue to operate under their nominal power demands. To this end, the following analysis considers $2\dot{M}$ types of appliances; the first group comprises of appliances that agree to participate in the program, together with half of appliances that are not able to compress their demands, while the second group consists of appliances that refuse to compress their demands, together with the other half of appliances that are unable to compress their demands. Based on these considerations, the power requests' arrival rate $R_{m'}(j)$ of the m'-th appliance's type ($m' \in 2\dot{M}$) is denoted as:

$$R_{m'}(j) = \begin{cases} \frac{r_m}{2} & \text{if } \gamma_{m'} = 0, m' \in 2M,\ j \in P \\ \frac{r_m}{2} & \text{if } \gamma_{m'} = 1,\ m' \in 2M,\ j \leq P_0 \\ r_m \mathrm{s}_{m',t} & \text{if } \gamma_{m'} = 1,\ m' \leq M,\ P_{t-1} \leq j - p_{m',t} < P_t \\ r_m(1 - \mathrm{s}_{m',t}) & \text{if } \gamma_{m'} = 1,\ m' > M,\ P_{t-1} \leq j - p_{m',t} < P_t \end{cases} \tag{3}$$

where $\gamma_{m'}$ denotes the ability of the appliances to compress their demands ($\gamma_{m'} = 0$ for appliances that are unable to compress their demands, while

$\gamma_{m'} = 1$ for appliances that are able to participate in the program) and r_m is the power requests' arrival rate of the original m ($m = 1, \ldots, M$) appliances' type. Therefore, for $\gamma_{m'} = 0$ the arrival rate is $r_m/2$, since half of these appliances belong to the first group ($m' \leq 2M$) and the other half belongs to the second appliances' group ($m' > 2M$). The latter rule is also valid for appliances that are able to compress their demands ($\gamma_{m'} = 1$) and the current power consumption is below the first threshold ($j \leq P_0$), where no scheduling occurs. However, when the current power consumption is $P_{t-1} \leq j < P_t$, a $s_{m',t}$ percentage of consumers will agree to compress their demands, since they belong to the first appliances' group ($m' \leq 2M$), while a $1 - s_{m',t}$ percentage that belong to the second appliances' group ($m' > 2M$) will refuse to participate in the program. It should also be noted that due to the consideration of the two appliances' groups, the probabilities $s_{m',t}$ are defined so that $s_{m',t} = s_{m'+M,t} = s_{m,t}$, for $m' \leq M$.

The calculation of the probabilities distribution $q(j)$ of the PUs in use is based on the following recursive formula:

$$\begin{aligned} jq(j) = {} & \textstyle\sum_{m'=1}^{2M} R_{m'}(j) d_{m'}^{-1} p_{m'} b_{m'}(j) q(j - p_{m'}) + \\ & \textstyle\sum_{m'=1}^{2M} \sum_{t=1}^{T} R_{m'}(j) d_{m',t}^{-1} p_{m',t} b_{m',t}(j) q(j - p_{m',t}) \end{aligned} \tag{4}$$

$$\text{where} \qquad b_{m'}(j) = \begin{cases} 1 & (\text{if } 1 \leq j - p_{m'} \leq P_0 \text{ and } \gamma_{m'} = 1) \\ & \text{or } (\text{if } 1 \leq j \leq P \text{ and } \gamma_{m'} = 0) \\ & \text{or } (\text{if } 1 \leq j \leq P \text{ and } \gamma_{m'} = 1 \text{ and } m' > M) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$\text{and} \qquad b_{m',t}(j) = \begin{cases} 1 \ \text{ if } \ P_{t-1} < j \leq P_t \text{ and } \gamma_{m'} = 1 \text{ and } m' \leq M \\ 0 \ \text{ otherwise} \end{cases} \tag{6}$$

The function $b_{m'}$ activates the recursive formula when a type-m' appliance is capable of reducing its power demands, while the function $b_{m',t}$ is used in order to include the compressed power demands in the calculation of the distribution $q(j)$. The parameters $p_{m'}$, $p_{m',t}$ and $d_{m'}^{-1}$, are defined based on the corresponding values of the parameters for the original appliances' set ($m \in M$), so that $p_{m'} = p_{m'+M} = p_m$ for $m' \leq M$, $p_{m',t} = p_{m,t}$ for $m' \leq M$, $p_{m',t} = p_m$ for $m' > M$ (since power demands from the second appliances' group are not compressed), while $d_{m',t}^{-1} = d_{m,t}^{-1}$ for $m' \in M$ and $d_{m',t}^{-1} = d_m^{-1}$ for $m' > M$.

The proof of Eq. (4) is provided in Appendix A, where initially a single threshold is considered and a corresponding recursive formula is derived. This formula is then extended in order to cover the case of multiple thresholds.

For the appliances that are not able to compress their power demands, the outage probability $B_{m'}$ that the total power consumption will exceed P upon the arrival of a power demand for p_m PUs can be calculated by using Eq. (2), while the outage probability $B_{m',t}$ for requests from appliances that compress their power demands is given by:

$$B_{m',t} = \sum\nolimits_{j=P-p_{m',t}+1}^{P} (q(j)/Q) \tag{7}$$

The computation of the minimum value of P so that the outage probability will not exceed a predefined maximum value e is realized by considering both B_m and the set of $B_{m',t}$. A method for solving the set of equations Eq.

(3) - Eq. (7) is presented in Fig. 4. Furthermore, the proposed analytical model can be used in order to determine the number T of power thresholds and the value P_t of each threshold that achieves an optimal peak demand reduction. This optimization procedure can be realized through an iterative method, where in each step the peak demand is calculated by using the algorithm presented in Fig. 4 for a given set of (T, P_t), and at the end of the iterations the optimal value set that achieves the minimum peak demand is derived. It should be noted that if a single threshold is considered and P_0=P, then the CDS coincides with the default scenario.

3.2. The Delay Request Scenario

The Delay Request Scenario (DRS) uses the same set of thresholds as in the case of the CDS. Under the DRS, power requests are delayed in one of M buffers (one for each type of appliance) that are installed in the CLC. After the delay in the buffer, the power request immediately tries to access the system. In this way, when the total power consumption exceeds a power threshold, power demands are not accepted for a specific time period (since new power requests are delayed in the buffer) and therefore the total power consumption is not increased. Furthermore, during this time period a number of already accepted requests are terminated (since a number of appliances terminate their operation), and therefore the total power consumption is reduced. The power request delay also causes the reduction of the final arrival rate of requests, due to the increase of the inter-arrival time as a result of the delay in the buffer, and consequently the probability to reach high-power consumption states is also reduced. For the activation of each appliance a number of messages are exchanged between the ECC and the

CLC, in a way similar to the one that was described for the CDS case (Fig. 3). In addition, the procedure that takes place at the CLC upon the arrival of a power demand for the DRS is illustrated in Fig. 5.

As in the case of CDS, the analysis for the derivation of the peak demand for the case of DRS also considers that a percentage $s_{m,t}$ of consumers will agree to postpone their demands, when the current power consumption is $P_{t-1} \leq j < P_t$, while $(1 - s_{m,t})\%$ of the consumers will refuse to participate in the program. Specifically, the delay that a power demand of the m-th appliances' type has to suffer is denoted as $1/\lambda_{m,t}$, $m = 1, \ldots, M$, for $P_{t-1} \leq j < P_t$. The values of the parameters $1/\lambda_{m,t}$ are defined by considering the capability of the appliance to tolerate a delay for its operation. For example, lighting during evening hours cannot endure operation delays, while the operation of devices such as the washing machine could be postponed.

The calculation of the arrival rate of the power requests, when the number of PUs exceeds the first threshold P_0, is based on the inter-arrival time of the requests after their postponement at the buffer, and also on the probability $s_{m,1}$ that the consumer will agree to participate the program. This time is equal to the inter-arrival time of requests $1/r_m$ that arrive at the buffer plus the time $1/\lambda_{m,1}$ that these requests detain at the buffers; therefore, for the general case where the current power consumption is $P_{t-1} \leq j < P_t$, the final arrival rate is given by the inverse of the aforementioned summation, multiplied by the percentage $s_{m,t}$ of consumers that agree to postpone their power requests. However, the arrival rate of power requests from consumers that refuse to postpone their demands is only a function of the percentage $1 - s_{m,t}$, while the power requests' arrival rate from appliances that cannot endure delays is defined in the same way as in the case of CDS:

$$R_{m'}(j) = \begin{cases} \frac{r_m}{2} & \text{if } \gamma_{m'} = 0,\ m' \in 2M \\ s_{m',t}\frac{r_m \lambda_{m',t}}{r_m + \lambda_{m',t}} & \text{if } \gamma_{m'} = 1,\ P_{t-1} \leq j - p_{m'} < P_t,\ m' \leq M \\ (1 - s_{m',t}) r_m & \text{if } \gamma_{m'} = 1,\ P_{t-1} \leq j - p_{m'} < P_t,\ m' > M \end{cases} \tag{8}$$

where $\lambda_{m'} = \lambda_m$ and $s_{m',t} = s_{m'+M,t} = s_{m,t}$, for $m' \leq M$.

The distribution $q(j)$, for $j = 1, ..., P$, of the PUs in use is given by the following recursive formula:

$$\begin{aligned} jq\left(j\right) = & \textstyle\sum_{m'=1}^{2M} R_{m'}(j) d_{m'}^{-1} c_{m',0}(j) p_{m'} q(j - p_{m'}) + \\ & \textstyle\sum_{m'=1}^{2M} \sum_{t'=1}^{T} R_{m'}(j) d_{m'}^{-1} c_{m',t}(j) p_{m'} q(j - p_{m'}), \end{aligned} \tag{9}$$

$$\text{where } \ c_{m',0}(j) = \begin{cases} 1 \quad (\text{if } j - p_{m'} \leq P_0 \text{ and } \gamma_{m'} = 1) \\ \quad \text{or } (\text{if } 1 \leq j \leq P \text{ and } (\gamma_{m'} = 0) \\ \quad \text{or } (\text{if } 1 \leq j \leq P \text{ and } (\gamma_{m'} = 1) \text{ and } (m' > M) \\ 0 \quad \text{otherwise} \end{cases} \tag{10}$$

$$c_{m',t}(j) = \begin{cases} 1 \text{ if } (P_{t-1} \leq j - p_{m'} < P_t) \text{ and } (\gamma_{m'} = 1) \text{ and } (m' <= M) \\ 0 \text{ otherwise} \end{cases} \tag{11}$$

The proof of Eq. (9) is presented in Appendix B. Furthermore, similarly to the function $b_{m'}(j)$ of the CDS, the function $c_{m',1}(j)$ is used in order to control the recursive formula of Eq. (9), in order to include the types of appliances that are able to endure delays, while the function $c_{m',2}(j)$ activates the formula of Eq. (9) for the same types of appliances, when the current power consumption j exceeds the threshold P_0.

The probability that the total power consumption will exceed P upon the arrival of a power demand for p_m PUs can be calculated by Eq. (2). A method for solving the set of equations Eq. (8)-Eq. (11) for the calculation of the P minimum value is presented in Fig. 5. As in the case of CDS, the proposed analytical model for the DRS can be used in order to determine the number T of thresholds and the value P_t of each threshold that achieves optimal peak demand reduction. In addition, if the delay $1/\lambda_m$ is set to zero for all M types of appliances, the DRS coincides with the default scenario.

3.3. The Postponement Request Scenario

The Postponement Request Scenario (PRS) is a special case of the DRS, since both of these scenarios assume that power requests are delayed, in order to reduce the peak demand. However, in the case of PRS, only two thresholds are considered: Above the threshold P_2 the user is prompted that the appliance operation should be delayed, until the total number of PUs in use is dropped below a second threshold P_1, with $P_1 \leq P_2$ (Fig. 3). The probability that the user will agree is denoted as s_m and the probability of refusal is 1-s_m. This procedure should be based on a dynamic pricing model, in order to provide the incentive to the end-user to agree on postponing the power request. Furthermore, the consideration of the different thresholds P_2 and P_1 for the scheduling activation and the deactivation, respectively, affect the power-request arrival rate from only task scheduling appliances, while the arrival rate of power requests from appliances that cannot endure delays is not a function of the current power consumption; for the latter appliances' types we consider that $s_m = 0$. Based on the PRS aforementioned assumptions, the requests arrival rate for p_m PUs is a function of the total number of PUs in use:

$$r_m(j)=\begin{cases} r_{m,1}(j)=r_m+s_m r_m & \text{if } j \leq P_1 \\ r_{m,2}(j)=r_m & \text{if } P_1 < j \leq P_2 \\ r_{m,3}(j)=(1-s_m)r_m & \text{if } j > P_2 \end{cases} \tag{12}$$

The CLC implements the PRS by checking the conditions of 12, as it is depicted in Fig. 6. The distribution $q(j)$, for $j=1,...,P$, can be calculated by the recursive formula:

$$jq(j)=\sum_{m=1}^{M}\sum_{n=1}^{3} r_{m,n} d_m^{-1} c_{m,n}(j) p_m q(j-p_m) \tag{13}$$

$$\text{with } \; c_{m,1}(j)=\begin{cases} 1 & \text{if } 0 \leq j \leq P_1+p_m \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

$$c_{m,2}(j)=\begin{cases} 1 & \text{if } P_1+p_m < j \leq P_2+p_m \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$c_{m,3}(j)=\begin{cases} 1 & \text{if } P_2+p_m < j \leq P \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

Proof: see Appendix C.

As in the cases of the other three scenarios, the functions $c_{m,1}(j)$, $c_{m,2}(j)$ and $c_{m,3}(j)$ are used in order to control the recursive formula, based on the current power consumption j and the corresponding value of the arrival rate function $r_m(j)$. Furthermore, the probability that the total power consumption will exceed P upon the arrival of a power demand for p_m PUs can be calculated by using Eq. (2). Eqs. (12)-(16) can be solved by using the algorithm presented in Fig. 6. It should be noted that if the probability s_m is set to zero for all M types of appliances, the PRS coincides with the default scenario.

4. Applicability of the proposed scenarios

The application of the CDS is based on the fact that peak demand reduction is achieved by considering appliances that are able to compress their demands. Furthermore, the DRS and PRS are applied to appliances that are able to postpone their requests, in order to reduce the peak demand. However, maximum demand reduction can be achieved by compressing the demands of an appliances' set, while at the same time, postponing the requests of a different set of appliances. In general, household appliances can be divided into a group of appliances that are able to compress their demands, another appliance group that can endure request delays and a third group that cannot tolerate any demand compression or request delay. This categorization can be used in order to utilize the proposed scenarios for the derivation of the peak demand, when both power compression and request postponement are considered.

A combined scenario that assumes households equipped with $M_1 + M_2 = M$ types of appliances can be considered in order to achieve maximum peak demand reduction. The first appliance set M_1 comprises of appliances that are able to compress their demands, while the second set M_2 comprises of appliances that can tolerate request delays, together with appliances that cannot endure demand compression or request delays. Alternatively, the latter group of appliances can be considered as part of the first appliance set M_1. Based on these assumptions, the CDS can be applied to the first appliance set M_1, while the DRS or the PRS is suitable for the second set M_2, since both scenarios consider appliances that tolerate delays, together with scheduling-inelastic appliances. Therefore, under this combined scenario the peak demand can be determined as the sum of the total power consumption, which is derived by the CDS equations, plus the total consumption that is calculated by using the equations of DRS or PRS. The selection between the DRS or PRS should be based on the resulting peak-demand reduction of both scenarios, while also on the grid's communication infrastructure, since DRS introduces more overhead mainly due to the multiple threshold consideration, which requires continuous communication for the execution of the scheduling program.

The application of the proposed analytical models should also consider the specific characteristics of the household appliances. In general, appliance manufacturers focus on the energy efficiency of their products, while little interest is given on the peak demand reduction. In terms of the DRS or PRS, most appliances can handle operation delays; it is up to the consumers' convenience to decide for the demand-request postponement, in order to contribute in the peak demand reduction. However, the shifting demand scenario should also consider that a number of appliances may have specific restrictions regarding the activation deadline [31]. On the other hand, demand compression should be applied only to appliances that have an elastic load component that results in the decrease of its instantaneous power draw, but at the expense of an increased operational time [32]. Such an elastic load component can be found in appliances that have heating elements, such as air-conditioners, laundry pairs and electric stoves; in these appliances power compression can be achieved by reducing the heating temperature, while increasing the operational time of the appliance. It should be noted that some appliances' types can either compress their demands or schedule their operation to an upcoming time-slot. These specific appliances should follow a single scenario (either the CDS or one of the DRS or PRS) and the best solution can be derived by applying the proposed analytical models and selecting the scenario with the lowest peak demand. Furthermore, power requests' arrival rates should be carefully defined based on the characteristics of the residential area under study (appliance population, typical power consumption patterns, etc.), while for the appliances' operation times other factors, such as weather conditioners (e.g. for air conditioners or water heaters) or time of day (e.g. for electric stoves or lightning) should be considered, in order to effectively apply the proposed analytical models.

It should be noted that the reduction of the peak demand is not the only objective of a DR program; these demand management programs should also aim for the efficient utilization of energy surplus that is produced by renewable energy sources. A typical strategy for the consumption of this excess energy is the provision of incentives to consumers to activate their appliances during energy surplus periods. The proposed analysis may be used for the efficient calculation of the additional number of power requests that should be arrived at the CLC, in order to consume this surplus energy. More specifically, by using Eq. (1) and Eq. (2) of the default scenario, a set of power-requests' arrival rates can be calculated, for a given value of the parameter P, which is the power provided by the renewable energy sources.

This set denotes the additional power requests that the system can handle due to the energy surplus and can be used by the power utility in order to define the number of consumers that should be informed to increase their power consumption. The latter procedure can be realized through messages that are sent to the consumers, in order to inform them for any kind of incentives (based on the pricing policy of the utility), that will motivate them to activate their appliances. However, by using the aforementioned method, various arrival rate sets can be derived, for the same value of P. It is therefore up to the power utility to select the appropriate set, by considering other factors, such as the time of day and the customers' consuming behavior, in order to motivate the activation of a specific set of appliances.

5. Results and Discussion

In this section we provide analytical and simulation results for the evaluation of the proposed analytical models of the corresponding scheduling scenarios. To this end, we consider a residential area where each residence has $M = 10$ major appliances: 1) an electric stove, 2) a laundry pair, 3) a water heater, 4) a dishwasher, 5) a refrigerator, 6) an air conditioner, 7) a home office set, 8) an entertainment set 9) lighting and 10) a plug-in hybrid electric vehicle (PHEV). The power demands of these appliances are (p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}) = (20, 15, 40, 10, 6, 25, 5, 7, 4, 100) PUs, with corresponding operational times (d_1^{-1}, d_2^{-1}, d_3^{-1}, d_4^{-1}, d_5^{-1}, d_6^{-1}, d_7^{-1}, d_8^{-1}, d_9^{-1}, d_{10}^{-1}) = (40, 30, 30, 40, 60, 40, 40, 50, 60, 30) minutes. These values are derived by considering typical values for appliances' power demands and operational times [32], [33] and by assuming that 1 PU = 100 Watt. We consider that the electric stove, the dishwasher and the PHEV are task scheduling appliances, the laundry pair, the water heater and the air-conditioner are energy scheduling appliances, while the refrigerator, the home office set, the entertainment set and lighting are not participating in any scheduling scheme. Based on this appliance categorization, we consider two cases: the first case considers the combined application of CDS and DRS, and the second case considers the utilization of CDS and PRS. In both cases, the energy scheduling appliances together with the refrigerator and the home-office set are applied to the model of CDS, while the task scheduling devices together with the entertainment set and lighting are considered for the DRS or PRS models.

The evaluation of the accuracy of the proposed analysis is realized through the comparison of analytical results with corresponding results from simulation. To this end, we built an object-oriented simulator by using the C++ programming language that executes the rules of the scheduling scenarios, while it creates events (power requests) based on random numbers. The simulator considers a large number of residences (in order to simulate the Poisson request arrivals), while each residence is equipped with the aforementioned set of $M = 10$ appliances. More precisely, the simulator generates 3×10^6 power requests from 2×10^4 residences, while a stabilization time that corresponds to the first 2×10^5 requests is assumed, in order for the simulator to reach the steady state. Simulation results are obtained as mean values of 8 runs, with 95% confidence interval, while only the mean values are used in the following figures, since the reliability ranges are found to be very small (therefore 8 runs per result are more than enough to produce efficient mean values). In each run the simulator records the current power consumption after the acceptance of each power request and returns the highest value of these records as the peak demand; an example of a simulation run for the default scenario is illustrated in Fig. 7, where the arrival rate of all appliances is assumed to be equal to 0.12 requests per minute. The consideration of a large number of power requests in the simulator enables the frequent activation and deactivation of the scheduling mechanism of each scenario, which is important for the derivation of accurate simulation results. It should be noted that the presented analytical results, which are derived by solving the proposed analytical models, are obtained in a less than 2 s. in average, which is a significantly shorter time compared to 14 min. in average that is required in order to obtain the simulation results. This fact proves the necessity of the proposed analytical models for the efficient execution of a load scheduling scheme, especially when near real-time scheduling decisions are required.

In Fig. 8 we evaluate the performance of CDS and DRS by comparing analytical and simulation results for maximum requested number of PUs, versus the demand-request arrival rate. In Fig. 8 we also present analytical and simulation results of the baseline policy that considers all 10 types of appliances, in order to show the achieved peak demand reduction under the combined scenario of CDS and DRS. The analytical results are obtained by solving the proposed equations (Eq. (1) - Eq. (2) for the default scenario, Eq. (3) - Eq.(7) for CDS, Eq. (2), Eq. (8) - Eq. (11) for DRS, and the iterative methods presented in Fig. 4 and Fig. 5 for CDS and DRS results, respectively), while simulation results are obtained from the simulator. Two thresholds are considered, which are set to be 60% and 75% of P, respectively, in order to provide a fair comparison between CDS, DRS and PRS, since the latter scenario considers two thresholds. When the current power consumption exceeds the first threshold, consumers are prompted to reduce their power demands by 15% and at the same time expand their operational time by 15%, while these values are both changed to 25%, when power consumption exceeds the second threshold. For the DRS case, power requests are delayed for 4 and 8 minutes, when the power consumption exceeds the first and the second threshold, respectively. For presentation purposes, we assume that the arrival rate is the same for all appliances (indicated in the x-axis of Fig. 8); evidently, the proposed analytical model can be applied to any arrival-rate set, since the power-requests arrival rates are used in the proposed analytical models in a parametric way. We also assume that the percentage of consumers that agree to participate in the program when the power consumption surpasses the first threshold is 60% for all appliances, whereas for the second threshold this percentage is increased to 70% for all appliances (due to more encouraging incentives offered to consumers). The values of P are calculated so that the probability that the total power consumption will not exceed P is below $e = 10^{-5}$. The comparison of analytical and simulation results of Fig. 8 reveals that the accuracy of both CDS and DRS analytical models is very satisfactory, since the maximum difference between the analysis and simulation is 1.8%. As it was anticipated, the increase of the power-requests arrival rate result in the increase of the peak demand, since high arrival-rate values correspond to larger number of activated appliances. Furthermore, comparing the results of the default scenario and the combined scenario of CDS and DRS, we notice that there is an average reduction of 20.7% of the peak demand. It should be also pointed out that the analytical results of Fig. 8 are exactly the same as the ones obtained by considering that 1 PU = 0.01 W, without a significant increase of the computation time, due to the use of recursive formulas.

The evaluation of the PRS is realized by considering the same assumptions as in the DRS case, regarding the two thresholds; however, since PRS considers a single percentage of consumers that agree to participate in the program, this value is set to 70% for all appliances, as in the case of DRS, when the second threshold (set to 75% of the maximum power consumption) is exceeded. In Fig. 9 we present analytical and simulation results for the combined CDS-PRS scenario, together with results from the baseline policy. Fig. 9 also includes analytical and simulation results for the CDS and PRS scenarios, in order to highlight the contribution of the two scenarios to the peak demand. The analytical results for the PRS are obtained by solving Eq. (2) and Eq. (12) - Eq. (16) through the iterative method presented in Fig. 6. As the results of Fig. 9 reveal, the accuracy of the proposed analytical models is very satisfactory, since the maximum difference between analytical and simulation results is 2.1%. Furthermore, the comparison of the results for the baseline policy and the combined CDS-PRS shows that the average peak demand reduction is 21.8%, which is higher than the corresponding value under the combined CDS-DRS scenario. In addition, by comparing the DRS results from Fig. 8 and the PRS results from Fig. 9, we notice that PRS performs better in terms of peak demand reduction, since it results in 14.7% reduction in the power consumption, compared to 12.9% reduction achieved by the application of DRS. What is interesting is that the performance of PRS in terms of peak demand reduction is increased for high arrival-rate values, since more power requests arrive at the ECC and therefore more requests are postponed and for longer periods. Therefore, PRS can be applied to task scheduling appliances of large residential areas, since PRS can effectively reduce the peak demand when a significant number of power requests are generated, compared to DRS. On the other hand, DRS can offer a gradual increase of the power-requests' delay, so that consumers' convenience and comfort are not highly affected. For example, if 6 thresholds were assumed instead of 2 for the DRS case, a gradual increase of the requests' delay can be applied, instead of an unknown delay as in the PRS case (since requests are postponed until power consumption drops below the thresholf P_2). However, in the latter example, by using the assumption of 60% consumers' participation, the DRS model results in 2472 PUs peak demand, instead of 2186 PUs under the PRS.

A significant parameter of the proposed scenarios is the consumers' percentage that agree to participate in the scheduling program. To this end, Fig. 10 presents analytical results for the peak demand under the CDS, DRS and PRS, versus the consumers' percentage that agree to compress their demands or postpone their requests. To provide a fair comparison between the three scenarios, we consider a single power threshold for CDS and DRS (set to 60% of maximum power consumption), so that a single value of the percentage $s_{m,1}$ is considered for CDS and DRS, as in the case of PRS. Furthermore, all $M = 10$ appliances are applied to the scenarios, while CDS, DRS or PRS are applied to both task- and energy-scheduling appliances. For any case, the power-requests' arrival rate is set to 2 requests/minute for all appliances, while the values for the other parameters are the same as the ones used for the derivation of the results of Fig. 8 and Fig. 9. As it was anticipated, higher participation percentages results in lower peak demand values for all scenarios, since more consumers compress their demands or postpone their power requests. The best performance in terms of peak demand reduction is achieved by the CDS, while it is followed by PRS and DRS. This outcome is a result of the selection of the values of the parameters of each scenario. For example, CDS and DRS have the same performance, if the power request delay for DRS is increased from 8 to 9.4 minutes. Therefore, for task scheduling purposes, PRS achieves lower peak demand values, compared to DRS. However, under PRS consumers are not aware of the duration of postponement of their appliances' operation. Simulation results indicate that the average postponement is 11.7 minutes under PRS, which is significantly higher than 8 minutes that were applied to appliances under DRS.

The main advantage of CDS and DRS is the utilization of multiple power thresholds that target the minimization of the demand scheduling effect on consumers' comfort. Under CDS, the number of thresholds that are applied to the scheduling program does not affect the peak demand value. For example, if two thresholds are considered (60% and 75% of the maximum power consumption), for consumers' participation percentage of 60%, the resulted peak demand is 2604 PUs; this same value is determined by the application of 5 thresholds (60%, 64%, 68%, 72% and 75% of the maximum power consumption). The same conclusion, in terms of the effect of the number of thresholds on the peak demand, is derived for the case of DRS. Therefore, both CDS and DRS can be applied by using a high number of thresholds, in order to provide a gradual application of the scheduling program. However, a high number of power thresholds requires real-time information for the total power consumption and its relation to the power thresholds ($P_{t-1} \leq j < P_t$), which is the decision parameter for the accurate selection of the scheduling parameters (power compression under CDS, or request delay under DRS); this requirement can be satisfied through an efficient communication infrastructure that guarantees minimum transmission delays and packet losses. On the other hand, PRS utilizes only two power thresholds. The selection of these two thresholds is crucial, since they not only affect the amount of peak demand reduction, but also the duration of delay that power requests suffer. To this end, in Fig. 11 we provide analytical results for the peak demand versus the power-requests' arrival rate, for various values of the scheduling de-activation threshold P_2, while the participation percentage is set to 0.6 for all appliances. The scheduling activation threshold P_1 is kept constant and equal to 75% of the maximum power consumption, in order to study the effect of the relation between P_1 and P_2 to the peak demand reduction. The study of Fig. 11 reveals that lower values of P_2 results in lower peak demand values, since the time period until the current power consumption drops below P_2 is higher than the corresponding time period for high values of P_2 and therefore more power requests are delayed; however, when the threshold P_2 is low, longer request delays arise. Therefore, the selection of the two power thresholds of PRS should not only target the minimization of the peak demand, but also consider the consumers' tolerance on long power requests' delays.

The main advantage of the proposed analytical models is their pattern agnostic nature, since the various features of the system (power-request arrival rates, appliances operational times, number of appliances, number of power thresholds) are considered in a parametric way; therefore the proposed analysis may be applied to a variety of cases, in order to efficiently calculate the peak demand. To this end, we considered a more realistic evaluation scenario, which takes into account the typical usage profile of residential appliances ([34]), based on demand patterns from the island La Palma in Spain during the first day of May 2014 [35], for three different power consumption periods: morning (8:00 - 10:00), afternoon (14:00 - 16:00) and evening (20:00 - 22:00). The arrival rates of power requests were calculated by considering the load demand patterns from [35] and the population of residential users in the island La Palma. Furthermore, we consider that the appliances' types that are applied to each scheduling scenario, the percentage of consumers that agree to participate in the program, as well as the power thresholds are the same as ones that were used for the derivation of the results presented in Fig. 8 and Fig. 9. In Table 2 we present analytical and simulation results for the baseline scenario, the CDS together with DRS, and the CDS together with PRS, for the three power consumption periods. The comparison of analytical to corresponding simulation results of Table 2 reveals the high accuracy of the proposed analysis. Finally, we compare the proposed analytical models with corresponding models of [24] and [22]. The models presented in [24] aim at reducing the peak demand by considering different appliances per consumer, with diverse power requirements. Since the consumers' participation percentage is not considered in the models in [24], a comparison can only be achieved by considering that in the proposed scenarios all appliances either compress their demands or postpone their requests, when the power consumption exceeds predefined power thresholds. Under this assumption, the proposed models and the corresponding models from [24] produce the same peak demand results. However, the computational complexity of the task scheduling scheme in [24] is significant. Precisely, the the computational time for the derivation of the results of the task scheduling model in [24] is significantly higher (26 minutes in average, using a quad core 2.53 GHz CPU and 4GB RAM), compared to the computational time for the derivation of results from DRS (less than 2 seconds). Therefore, the proposed analytical models can be applied to DR programs that require near real-time decisions that could be made based on fast peak-demand calculations. On the other hand, the power demand control policies that are presented in [22] assume that the controller activates immediately or postpones power requests, based on the current power consumption, while the power requirement of each power request equals to 1 power unit. To this end, for the comparison of the proposed scenarios with the control policies of [22], we consider the following equivalence, since the analysis in [22] considers unit power requests:

$$(\alpha/\mu) = \sum_{m=1}^{M} p_m(r_m/d_m) \tag{17}$$

Eq. (17) assumes that the total ratio of the arrival rate to the operation time of M appliances is equivalent to the ratio of the arrival rate α to the service time μ of [22]. The two power control policies in [22] named Threshold Postponement (TP) and Controlled Release (CR) consider that power requests are postponed; therefore we compare the performance of these policies only with the proposed DRS and PRS. For a fair comparison, we set the TP threshold P_b of [22] as $P_b = P_0$ for the DRS, while for the PRS we set

P_b=P_1=P_2. Furthermore, the deadline of the power requests for the TP and CR policies equals to the delay $1/\lambda_m$ that requests suffer, under DRS. The consumers' participation percentages are set to 1 for all appliances in the proposed models, since the control policies of [22] consider that all requests are postponed. The values of all other parameters are the same as the ones used in the application examples for the evaluation of the proposed scenarios. In Fig. 12 we present analytical results for the total requested number of PUs versus the power requests' arrival rate, under TP and CR policies of [22] and the proposed DRS and PRS. The study of Fig. 12 reveals the superiority of the proposed scenarios over the policies of [22]. Furthermore, in this evaluation example PRS performs worse than DRS in terms or peak demand reduction; this outcome is inconsistent with the results of Fig. 10 (where PRS outperforms DRS), since the assumption of $P_1 = P_2$ results in 5.6 min. average power-request delay values, which is lower than the 8 min. average delay of power requests under DRS. Evidently, the overestimations of the demand policies of [22] proves the necessity of implementing the proposed scenarios that consider multiple types of requests with diverse power requirements, which also take into account that a percentage of consumers may refuse to participate in the scheduling program, while they are also based on simple recursive formulas.

6. Conclusion

We present and analyze four power demand control scenarios in a smart grid environment. All scenarios assume that each residence is equipped with a specific number of appliances, each with different power demands and operational times, and take into account the percentage of consumers that wish to participate in the program. For each scenario we propose a recursive formula for the determination of the distribution of PUs in use, which is used to calculate the total power consumption in the residential area. The accuracy of the proposed models is quite satisfactory, as it is verified by simulations. The evaluation of the proposed scenarios indicate that a significant peak demand reduction can be achieved by scheduling the appliances' operation, while this reduction is highly affected by the choice of the values for the system's parameters. Furthermore, the proposed analytical models generate peak demand results in a small computational time, compared to simulations and other analytical models in the literature. In our future work we will study the case where each appliance type has a finite number of devices and may alter its operation between ON and OFF states, while also the case where consumers are induced to increase their power consumption, when power excess is available, due to the utilization of renewable energy sources.

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Appendix A.

In order to prove Eq. (4), we consider that the power compression and the change of the requests' arrival rate when the total number of the PUs in use exceeds the first power threshold P_0 and the subsequent thresholds leads to a non-product form solution. To this end, we firstly study the system model and construct the one dimensional Markov chain of the system with the state transition diagram of Fig. 13a, where each state j represents the number of PUs in use, when $j - p_{m'} \leq P_0$. The construction of this state transition diagram is inspired by the call-level analysis of a multi-rate communication network presented in [36]. In these states, no power compression occurs. The local balance equation of the transition diagram of Fig 13a is:

$$\begin{aligned} &q(j - p_{m'})R_{m'}(j) = q(j)y_{m',0}(j)d_{m'} \Leftrightarrow \\ &q(j - p_{m'})\tfrac{R_{m'}(j)}{d_{m'}}p_{m'} = q(j)y_{m',1}(j)p_{m'} \end{aligned} \tag{A.1}$$

for $j - p_{m'} \leq P_0$ and $m' = 1, \ldots, 2M$. The function $y_{m',0}(j)$ is the mean number of type-m' appliances in use in the grid that require $p_{m'}$ PUs, when the total number of PUs in use is $j > P_0 + p_{m'}$.

We also construct the one dimensional Markov chain of the system with the state transition diagram of Fig. 13b, where each state j represents the number of PUs in use, when $P_{t-1} \leq j - p_{m'} < P_t$, for the m'-th appliance's type. In this case, power compression occurs with parameters $p_{m',t}$ and $d_{m',t}^{-1}$. The local balance equation of the state transition diagram of Fig 13b is:

$$\begin{aligned} &q(j - p_{m',t})R_{m'}(j) = q(j)y_{m',t}(j)d_{m',t} \Leftrightarrow \\ &q(j - p_{m',t})\tfrac{R_{m',t}(j)}{d_{m',t}}p_{m',t} = q(j)y_{m',t}(j)p_{m',t} \end{aligned} \tag{A.2}$$

The function $y_{m',t}(j)$ is the mean number of appliances in use that require $p_{m'}$ PUs, when the total number of PUs in use in the residential area is $P_{t-1} \leq j - p_{m'} < P_t$. In both cases, the power-requests' arrival rate $R_{m'}(j)$ is given by Eq. (3). By using Eq. (A.1) for all $2M$ appliances' types we obtain:

$$\sum_{m'=1}^{2M} q(j - p_{m'}) \frac{R_{m'}(j)}{d_{m'}} p_{m'} = q(j) \sum_{m'=1}^{2M} y_{m',0}(j) p_{m'} \tag{A.3}$$

for $j \leq P_0 - p_{m'}$. Similarly, from Eq. (A.2) and for all $2M$ appliances' types and T thresholds, we obtain:

$$\sum_{m'=1}^{2M} \sum_{t=1}^{T} q(j - p_{m',t}) \frac{R_{m'}(j)}{d_{m',t}} p_{m',t} = q(j) \sum_{m'=1}^{2M} \sum_{t=1}^{T} y_{m,t}(j) p_{m',t} \tag{A.4}$$

The total number j of the PUs in use in any state $j \in [0,P]$ is given by the sum of the products of the mean number $y_{m,t}(j)$ of appliances in use by the number $p_{m,t}$ (with $p_{m,0} = p_m$) of the PUs that these appliances demand, for all $2M$ appliances' types and for all thresholds:

$$j = \left[\sum_{m'=1}^{2M} y_{m',0}(j) p_{m'} + \sum_{m'=1}^{2M} \sum_{t=1}^{T} y_{m',t}(j) p_{m',t} \right] \tag{A.5}$$

for $j = 0, ..., P$. Therefore, in order for the summation of the Right Hand Side (RHS) of Eq. (A.3) to be equal to j, we have to assume that $y_{m',0}(j) \cong 0$ for $j > P_0 - p_{m'}$. Similarly, in order for the summation of RHS of Eq. (A.4) to be equal to j, we have to assume that $y_{m',t}(j) \cong 0$ outside the region $[P_{t-1}, P_t]$. These two assumptions are expressed by Eq. (5) and Eq. (6), respectively. By using these two assumptions, Eq. (A.5), and by summing up side by side Eq. (A.3) and Eq. (A.4), we obtain Eq. (4).

Appendix B.

In order to prove Eq. (9), we follow the same procedure as in the case of the proof of Eq. (4). Specifically, since in both cases the same threshold set is assumed, when the total power consumption j is less than the first threshold P_0, Eq. (A.3) is also valid for the case of DRS. However, when the total power consumption exceeds the first power threshold, power requests are postponed, while this delay is a function of the power thresholds P_t. Therefore, for the general case where the current power consumption is $P_{t-1} \leq j - p_{m'} < P_t$, the local balance equation of the corresponding Markov chain for type-m' appliances is expressed by:

$$\begin{aligned} &q(j-p_{m'})R_{m'}(j) = q(j)y_{m',t}(j)d_{m'} \Leftrightarrow \\ &q(j-p_{m'})\tfrac{R_{m',t}(j)}{d_{m'}}p_{m'} = q(j)y_{m',t}(j)p_{m'} \end{aligned} \tag{B.1}$$

which is converted to the following expression for all $2M$ appliances' types and T thresholds:

$$\sum\nolimits_{m'=1}^{2M} \sum\nolimits_{t=1}^{T} q(j-p_{m'}) \frac{R_{m'}(j)}{d_{m'}} p_{m'} = q(j) \sum\nolimits_{m'=1}^{2M} \sum\nolimits_{t=1}^{T} y_{m,t}(j) p_{m'} \tag{B.2}$$

Therefore, by following the same procedure as in the case of the proof of Eq.(4), we assume that the mean number $y_{m',0}(j) \cong 0$ for $j > P_0 - p_{m'}$ and $y_{m',t}(j) \cong 0$ outside the region $[P_{t-1}, P_t]$. These assumptions are expressed by Eq. (10) and Eq. (11), respectively. By using these assumptions and by summing up side by side Eq. (A.3) and Eq. (B.2), we derive Eq. (9).

Appendix C.

In order to prove Eq. (13), we also follow the analysis for the proof of Eq. (4) and we define the local balance equations from the equivalent state transition diagrams:

$$\begin{aligned} &q(j-p_m)r_{m,1} = q(j)y_{m,1}(j)d_m \Leftrightarrow \\ &q(j-p_m)\tfrac{r_{m,1}}{d_m}p_m = q(j)y_{m,1}(j)p_m \end{aligned} \tag{C.1}$$

where $y_{m,1}(j)$ is the mean number of appliances that require p_m PUs when j PUs are in use, for $j - p_m \leq P_1$. Also,

$$\begin{aligned} &q(j-p_m)r_{m,2} = q(j)y_{m,2}(j)d_m \Leftrightarrow \\ &q(j-p_m)\tfrac{r_{m,2}}{d_m}p_m = q(j)y_{m,2}(j)p_m \end{aligned} \tag{C.2}$$

where $y_{m,2}(j)$ is the mean number of appliances that require p_m PUs when $P_1 + p_m < j \leq P_2 + p_m$, and

$$\begin{aligned} &q(j-p_m)r_{m,3} = q(j)y_{m,3}(j)d_m \Leftrightarrow \\ &q(j-p_m)\tfrac{r_{m,3}}{d_m}p_m = q(j)y_{m,3}(j)p_m \end{aligned} \tag{C.3}$$

where $y_{m,3}(j)$ is the mean number of appliances that require p_m PUs when j PUs are in use in the grid, for $P_2 + p_m < j \leq P$. By using Eqs. (C.1), (C.2) and (C.3) and by summing up for all M power levels, we obtain:

$$\begin{aligned}
&\sum_{m=1}^{M} q(j-p_m)\tfrac{r_{m,1}}{d_m}p_m = q(j) \sum_{m=1}^{M} y_{m,1}(j)p_m,\ j\in[0,P_1-p_m]\\
&\sum_{m=1}^{M} q(j-p_m)\tfrac{r_{m,2}}{d_m}p_m = q(j)\sum_{m=1}^{M} y_{m,2}(j)p_m,\ j-p_m\in[P_1,P_2]\\
&\sum_{m=1}^{M} q(j-p_m)\tfrac{r_{m,3}}{d_m}p_m = q(j) \sum_{m=1}^{M} y_{m,3}(j)p_m,\ j\in[P_2-p_m,P]
\end{aligned} \tag{C.4}$$

As in the cases of the CDS and DRS, we need to assume that $y_{m,1}(j)\cong 0$ for $j>P_1-p_m$, $y_{m,2}(j)\cong 0$ outside the region $P_1-p_m<j\le P_2-p_m$ and that $y_{m,3}(j)\cong 0$ for $j<P_2-p_m$. By using the three assumptions and by summing up side by side the three equations of Eq. (C.4) we derive Eq. (13), while the aforesaid assumptions are expressed by Eq. (14)-(16).

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