Potential Game for Energy-Efficient RSS-based Positioning in Wireless Sensor Networks

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***Abstract*—Positioning is a key aspect for many applications in wireless sensor networks. In order to design practical positioning algorithms, it is crucial to employ efficient algorithms that maximize the battery lifetime while achieving a high degree of accuracy. The number of participating anchor nodes and their transmit power have an important impact on the energy consumption of positoning a node. This paper proposes a game theoretical algorithm to optimize resource usage in obtaining location information in a wireless sensor network. The proposed method provides positioning and tracking of nodes using RSS measurements. We use the Geometric Dilution of Precision as an optimization metric for our algorithm, with the aim of minimizing the number and power of anchor nodes that collaborate in positioning, thus saving energy. The algorithm is shown to be a potential game, therefore convergence is guaranteed. A distributed, low complexity solution for the implementation is presented. The game is applied to WSN and results show the trade-off between power saving and positioning error.**

***Index Terms*—Wireless sensor networks, distributed algorithms, game theory, potential games, positioning, resource planning.**

I. Introduction

Wireless Sensor Networks (WSN) consist of autonomous low-complexity sensor nodes to collect, analyze and transmit data [1]. The main requirement of such systems is the reduction of the power consumption due to the limited battery lifetime. Therefore, it is important to employ energy-efficient algorithms and save resources. At the same time, accuracy is important for positioning applications. However, energy efficiency and accuracy are related issues and strategies for saving energy could lead to reduce accuracy in positioning.

The legacy system for positioning devices is the Global Navigation Satellite System (GNSS). However, on the one hand, for indoor environments the satellite signal has poor coverage and GNSS is unsuitable for indoor location estimation. On the other hand, the GNSS module is known to be power hungry, which prevents its usage in low-complexity, energy-efficient WSN devices. Therefore, positioning methods based on cooperation among sensors are used in those cases where GNSS is not suitable [2]. With cooperative methods, the known position of some nodes (referred to as anchor nodes) is used to estimate the position of the unknown nodes (referred to as target nodes).

The focus of this work is on RSS-based positioning systems. This approach uses the Received Signal Strength (RSS) measurements from the anchor nodes to the target nodes to determine the location of the device. The advantage of the RSS approach with respect to other techniques is that it requires no additional hardware. The main disadvantage is that it is affected by multipath fading and other propagation effects. Typically, the RSS measurements are modeled with the log-normal path loss model [3], [4]. In [5], it is showed that the Cramér-Rao Lower Bound (CRLB) for distance estimation with RSS measurements is proportional to real distance and also depends on the channel parameters.

In this paper, we consider a deployed, IEEE 802.15.4 compliant, WSN that consists of anchor and target nodes. The positioning of the target nodes is performed with the RSS measurements from anchor nodes. Within this context, we address the problem of distributed optimization of energy consumption while maintaining a certain quality of the positioning measure at the target nodes. We cast the problem in the form of a potential game.

1) Related work: Energy expenditure in WSNs can be classified under data transmission/reception, data processing, and data acquisition or sensing. Data acquisition and transmission/reception consume significantly more energy than data processing as it is shown in [6].

In order to conserve power and energy there are different methods in WSN [7]. In the literature, the problem of energy efficiency while maintaining a given accuracy for positioning of WSN has been addressed. Several works treated data acquisition conservation methods to achieve energy saving by minimizing the energy expenditure in data transmission/reception rates and sensing by adapting a sampling problem. Node selection strategies also save energy because they avoid the use of a large number of cooperative anchor nodes and hence, they reduce the packet exchange saving energy. Some approaches use CRLB to select nodes or select the anchor nodes based on a distance metric [8]. The main disadvantage of distance based criterion is that geometry of the selected cooperating nodes is not contemplated. For positioning with trilateration method, as deeply studied in GNSS positioning, the geometry of the satel-

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lites affects the final position estimation of the receiver [9]. Geometric Dilution of Precision (GDOP) metric is a measure of the goodness of a certain geometry for positioning purposes. In WSN positioning with trilateration, GDOP based strategies were also used for node selection. A trivial selection method is an exhaustive search that evaluates the GDOP for all the possible active sets given a set of possible sensors. However, since the number of combinations grows exponentially with the number of anchor nodes, the algorithm is only viable if the set of sensors is small. Otherwise, suboptimal approaches have been presented in [10].

Node selection strategies have also been dealt with cooperative games that require agreements between devices. The idea of forming the best group of anchor nodes for positioning was addressed in [11]. The work presents a distributed, cooperative, game-theoretic scheme for energy-efficient data acquisition in bearings-only localization. Another example is [12], that presents a RSS-based localization and tracking scheme using cooperative game-theoretic tools in which the best anchor coalition is kept while the other coalitions are allowed to enter low power mode. However, cooperative games for coalition selection need information exchange between anchor nodes until they reach an agreement that might lead to a high communication cost.

2) Why potential games? In non-cooperative game theory, devices have potentially conflicting interests and they try to maximize their payoff. Non-cooperative game theory has been applied to the allocation of resources such as power. In distributed power control, non-cooperative games have been used for avoiding collisions and energy saving. In [13] a unified framework based on potential games is proposed to deal with power control problems for avoiding interferences. In general, from a physical layer perspective, in power control problems the quality of service (QoS) requirements are formulated as constraints on the signal-to-interference-and-noise ratio of each user. In this work, we propose a distributed power control and distributed data transmission/reception conservation method with node selection, with the goal of saving energy in WSN with RSS-based positioning capabilities. The proposed algorithm minimizes the transmit power of anchor nodes as well as performs the selection of a set of anchor nodes for positioning of the target node, while using a positioning error metric based on the GDOP as QoS to maintain an adjustable level of accuracy.

Since we are dealing with a decentralized system, non-cooperative game theory provides appropriate models to study such scenarios [14]. In the considered problem the advantage of non-cooperative games, in front of cooperative approaches, is that nodes do not have to reach an agreement and hence the effects derived from cooperation such as communications costs are not present. The problem of power control can be addressed in a distributed fashion with potential games. Therefore, an optimal solution can be reached when the players play an iterated algorithm in a distributed way. The algorithm avoids the exhaustive search that evaluates the GDOP for all the possible active sets of anchor nodes that leads to a combinatorial problem.

Distributed power control for positioning has also been studied in combination with time-of-arrival (TOA)-based ranging. In [15], a network with TOA-based positioning capabilities is considered, which allows to pose the power control problem as a supermodular game. In contrast to TOA-based ranging, the case of RSS-based ranging has the particularity that increasing the power level does not impact on obtaining better range estimates. The effect of the received power level is that above a certain threshold (where anchor and target are in range) ranging is feasible and below it is not. This prevents the use of the supermodular game developed in [15] as the assumptions made there do not hold anymore. In [16], the performance of a potential game for RSS-based ranging was presented in a static scenario. In the present work, these results are extended and completed in a dynamic scenario on a distributed fashion, including an Extended Kalman filter for tracking the moving node. Moreover, the computational complexity of the algorithm is analyzed and more results and simulations are presented. Our results are summarized as follows.

3) Main results:

- Formulation of the distributed power control problem as a potential game. In this work, we use as QoS metric the GDOP which assesses the goodness of a network geometry for positioning purposes [9]. We address the problem of assigning a minimum transmission power to each anchor node and minimize the number of anchor nodes that assists in the ranging process, while maintaining a certain quality in the positioning solution of the target nodes. This problem is formulated as a potential game that reaches an equilibrium. The case of multiple target nodes is addressed.
- Distributed error metrics based on GDOP are presented to avoid costly information exchanges between nodes. Moreover, a solution to implement the game is presented and analyzed in terms of its computational complexity.
- Results show that the algorithm reaches equilibrium, thus saving energy in a setup where anchor nodes help in positioning target nodes with RSS measurements. The equilibrium reached with the distributed algorithm based on the potential game is compared to a centrally global solution with exhaustive search.

The reminder of the paper is organized as follows. In Section II, the system model, log-normal ranging model for RSS measurements, both static and dynamic scenarios, and the GDOP metric are presented. In Section III, we explain our potential game for energy saving RSS-based positioning. In Section IV, distributed error metrics to calculate the GDOP are detailed. In Section V a possible solution to implement the game is explained and analyzed. In Section VI, simulation and numerical results are presented to illustrate the behavior of the proposed algorithm. Section VII concludes the work with final remarks.

II. Problem Formulation and System Model

The problem under study involves the energy-efficient positioning of nodes in a WSN that is applied to two setups. First setup, namely static scenario, is composed of static target nodes while in the second setup, namely dynamic scenario, the target nodes are moving in the area.

A. Scenario Definition

The static and dynamic scenarios are respectively composed of a set of M nodes, that aim at estimating their position; and a set of N anchor nodes with known locations, emitting ranging signals to allow positioning of the former nodes.

For the mobile scenario, we define the two-dimensional coordinates of the nodes at time t as

$$\mathbf{x}^{(j)}(t) = [x^{(j)}(t), y^{(j)}(t)]^\top \qquad j = 1, \dots, M \tag{1}$$

$$\mathbf{x}_a^{(i)} = [x_a^{(i)}, y_a^{(i)}]^\top \qquad i = 1, \dots, N \ . \tag{2}$$

For the static scenario, these definitions hold with $\mathbf{x}^{(j)} \triangleq \mathbf{x}^{(j)}(t)$. The geometrical distance between the j-th node and the i-th anchor is defined as

$$\rho_{j,i} = \| \mathbf{x}^{(j)}(t) - \mathbf{x}_a^{(i)} \| \ , \tag{3}$$

with $\| \cdot \|$ being the Euclidean norm on $\mathbb{R}^2$ and where we omitted the time-dependence of $\rho_{j,i}$ for the dynamic scenario.

We define the set of anchor nodes that provide coverage to the j-th node as $\mathcal{N}_j$, and its dimension as $|\mathcal{N}_j|$. Moreover, we define the set of target nodes whose messages are received at the i-th anchor node as $\mathcal{T}_i$, with dimension being $|\mathcal{T}_i|$. These sets might be time varying for the dynamic setup.

B. Ranging Model

We further assume that the physical layer of the nodes is capable of estimating the RSS of an incoming signal. In particular, the IEEE 802.15.4 physical layer has this capability.

The target node uses the RSS value to estimate $\rho_{j,i}$. The RSS-based ranging measures are commonly modeled using the log-normal path loss model [3], defined as

$$L_{j,i} = L_o - 10p \log_{10}\left(\frac{\rho_{j,i}}{\rho_o}\right) \ , \tag{4}$$

where ρ_o is a reference distance, L_o is the attenuation at such reference distance in dB, $\rho_{j,i}$ is as in (3), $L_{j,i}$ the path loss for the distance $\rho_{j,i}$ in dB, and p the path loss exponent (typ. 3 in our scenarios). Notice that $L_{j,i} = P_{Tx,i} - P_{Rx,j}$, where $P_{Tx,i}$ and $P_{Rx,j}$ are the transmitted and received powers in dBm for the pair $\{j, i\}$, respectively. The channel has a random contribution, modeled in dB by $\upsilon_{j,i} \sim \mathcal{N}(0, \sigma_{j,i}^2)$. Then

$$P_{Rx,j} = P_{Tx,i} - L_o + 10p \log_{10}\left(\frac{\rho_{j,i}}{\rho_o}\right) + \upsilon_{j,i} \ , \tag{5}$$

where $\sigma_{j,i}$ is due to the fading effects in static and dynamic environments (node movement or environment changes, e.g. people movement). It is known that multipath fading can be addressed by switching the communication carrier frequency. Channel hopping was studied for WSN and it is applied in the IEEE 802.15.4e standard. The averaging of the RSS values reduces its standard deviation when samples are collected over different frequency channels in a short time period, rather than on a single channel but over a longer time interval [17].

Rearranging terms in (5) we obtain a distance estimate

$$\hat{\rho}_{j,i} = \rho_o \cdot 10^{\frac{L_o - L_{j,i} + \upsilon_{j,i}}{10 \cdot p}} \ , \tag{6}$$

and using (4)

$$\hat{\rho}_{j,i} = \rho_{j,i} \cdot 10^{\frac{\upsilon_{j,i}}{10 \cdot p}} \ . \tag{7}$$

It becomes clear that it depends on a log-normal random variable ω as $\hat{\rho}_{j,i} = \rho_{j,i} \cdot \omega$. Recall that the logarithm of a log-normal random variable is normally distributed. If $\omega \sim Log - \mathcal{N}(\mu_\omega, \sigma_\omega^2)$ is distributed log-normally, then $\ln(\omega) \sim \mathcal{N}(\mu_\xi, \sigma_\xi^2)$ is a normal random variable ξ. Therefore,

$$\xi = \ln \omega \sim \mathcal{N}\left(0, \left(\frac{\ln 10 \cdot \sigma_{j,i}}{10 \cdot p}\right)^2\right) \ , \tag{8}$$

where $\mu_\xi = 0$ and $\sigma_\xi = \frac{\ln 10 \cdot \sigma_{j,i}}{10p}$. The variance of the log-normal random variable ω is $\sigma_\omega^2 = (e^{\sigma_\xi^2} - 1)e^{2\mu_\xi + \sigma_\xi^2} = (e^{\sigma_\xi^2} - 1)e^{\sigma_\xi^2}$. Therefore, the variance of the distance estimation $\hat{\rho}_{j,i}$ may be

$$\sigma_{\hat{\rho}_{j,i}}^2 = \rho_{j,i}^2 \cdot (e^{\sigma_\xi^2} - 1)e^{\sigma_\xi^2} \ . \tag{9}$$

From the previous equation, it can be noticed that the variance of the distance estimation $\hat{\rho}_{j,i}$ between target j and anchor node i is proportional to the distance between both nodes. Therefore, larger distances cause higher error in distance estimation.

C. Positioning Equations

In the static scenario, a target node could estimate its position with linear Least Squares (LS) estimator [18]. Considering the above setup, a mobile target node could estimate its position with an Extended Kalman Filter (EKF). Following [19], we assume that the position $\mathbf{x}^{(j)}(t)$ and velocity $\mathbf{v}^{(j)}(t) = [v_x^{(j)}(t), v_y^{(j)}(t)]^\top$ evolve in time as

$$\mathbf{s}_j(t) = \mathbf{A} \ \mathbf{s}_j(t-1) + \mathbf{G} \ \mathbf{w}_j(t) \ , \tag{10}$$

where $\mathbf{w}_j \sim \mathcal{N}(0, \sigma_{\mathrm{w}}^2 \cdot \mathbf{I})$, Δ is the time interval between samples,

$$\mathbf{s}_j(t) = \begin{pmatrix} x^{(j)}(t) \\ y^{(j)}(t) \\ v_x^{(j)}(t) \\ v_y^{(j)}(t) \end{pmatrix} \ , \quad \mathbf{A} = \begin{pmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \ , \tag{11}$$

$$\mathbf{G} = \begin{pmatrix} \Delta^2/2 & 0 \\ 0 & \Delta^2/2 \\ \Delta & 0 \\ 0 & \Delta \end{pmatrix} \ . \tag{12}$$

The covariance matrix of the driving and observation noise is given by

$$\mathbf{Q}_j = \sigma_{\mathrm{w}}^2 \cdot \mathbf{G}\mathbf{G}^\top \ . \tag{13}$$

We consider the measurements are the received powers $P_{Rx,j}$ at the j-th node from the set of anchor nodes within range, $\mathcal{N}_j$. The measurements are related to the unknown parameters $\mathbf{s}_j(t)$ according to (5), where

$$\rho_{j,i} = \sqrt{(x^{(j)}(t) - x_a^{(i)})^2 + (y^{(j)}(t) - y_a^{(i)})^2} \ . \tag{14}$$

Then the observation equation is

$$\mathbf{y}_j(t) = \mathbf{h}(\mathbf{s}_j(t)) + \boldsymbol{\nu}_j(t) \ , \tag{15}$$

with

$$\mathbf{h}(\mathbf{s}_j(t)) = \begin{pmatrix} P_{Tx,1} - L_o + 10p\log_{10}\left(\frac{\rho_{j,1}}{\rho_o}\right) \\ \vdots \\ P_{Tx,|\mathcal{N}_j|} - L_o + 10p\log_{10}\left(\frac{\rho_{j,|\mathcal{N}_j|}}{\rho_o}\right) \end{pmatrix} \quad (16)$$

and

$$\boldsymbol{\nu}_j(t) = \left(\upsilon_{j,1}, \ldots, \upsilon_{j,|\mathcal{N}_j|}\right)^\top , \quad (17)$$

with covariance matrix given by

$$\mathbf{C}_j(t) = \mathrm{diag}\left(\sigma^2_{j,1}, \ldots, \sigma^2_{j,|\mathcal{N}_j|}\right) . \quad (18)$$

Since the measurement function is nonlinear in the signal parameters, to estimate the state vector with the EKF we apply a linearization. The Jacobian is given by

$$\mathbf{H}_j(t) = \left.\begin{pmatrix} H_j^{(1,1)}(t) & H_j^{(1,2)}(t) & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ H_j^{(|\mathcal{N}_j|,1)}(t) & H_j^{(|\mathcal{N}_j|,2)}(t) & 0 & 0 \end{pmatrix}\right|_{\mathbf{s}=\hat{\mathbf{s}}(t|t-1)} , \quad (19)$$

where

$$\begin{aligned} H_j^{(i,1)}(t) &= \frac{10p\ (x^{(j)}(t) - x_a^{(i)})}{\ln 10 \cdot \rho^2_{j,i}} \\ H_j^{(i,2)}(t) &= \frac{10p\ (y^{(j)}(t) - y_a^{(i)})}{\ln 10 \cdot \rho^2_{j,i}} . \end{aligned}$$

In summary, with the above definitions, the EKF equations [20] for our problem are given by

$$\begin{aligned} \hat{\mathbf{s}}_j(t|t-1) &= \mathbf{A}\hat{\mathbf{s}}_j(t-1|t-1) && (20) \\ \mathbf{M}_j(t|t-1) &= \mathbf{A}\mathbf{M}_j(t-1|t-1)\mathbf{A}^\top + \mathbf{Q}_j && (21) \\ \mathbf{K}_j(t) &= \mathbf{M}_j(t-1|t-1)\mathbf{H}_j(t)^\top(\mathbf{C} \\ &+ \mathbf{H}_j(t)\mathbf{M}_j(t|t-1)\mathbf{H}_j(t)^\top)^{-1} && (22) \\ \hat{\mathbf{s}}_j(t|t) &= \hat{\mathbf{s}}_j(t|t-1) + \mathbf{K}_j(t)(\mathbf{y}_j(t) \\ &- \mathbf{h}(\hat{\mathbf{s}}(t|t-1))) && (23) \\ \mathbf{M}_j(t|t) &= (\mathbf{I} - \mathbf{K}_j(t)\mathbf{H}_j(t))\mathbf{M}_j(t|t-1) , && (24) \end{aligned}$$

where $\mathbf{y}_j(t)$ are the measurements of the received power. For the j-th node, $\mathbf{K}_j(t)$ is the so-called Kalman gain, $\mathbf{M}_j(t|t-1)$ is the covariance of the predicted state, and $\mathbf{M}_j(t|t)$ is the covariance of the estimated state.

D. Geometric Dilution of Precision

In this section we present the GDOP metric, which we use as the error metric in our game. The origin of the GDOP measure comes from the trilateration procedure, from which a receiver computes its position based on range measurements to a set of transmitters. Trilateration involves solving a geometrical problem, whose solution is given by the intersection of spheres centered at the transmitters and radii equal to the measured ranges. TOA ranging error can be modeled with a Gaussian random variable, although this might not hold in non-line-of-sight conditions in which case TOA measurements are typically biased. With this assumption, the problem is nonlinear and typically solved by a LS algorithm after linearization.

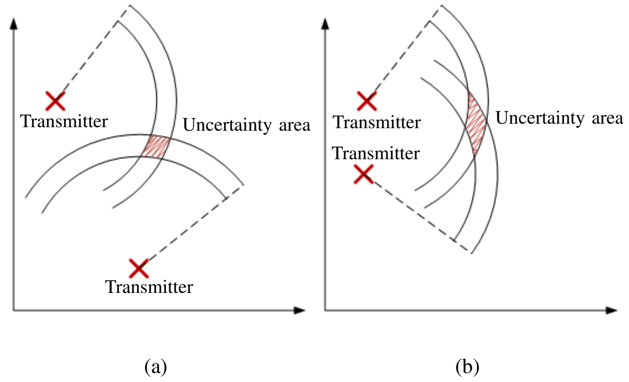

Fig. 1. Conceptual representation of *good* and *bad* two-dimensional geometries, 1(a) and 1(b) respectively. Two transmitters are located in a plane, the resulting hyperbolic positioning solution of a range-based receiver has an uncertainty area that depends on the relative location of transmitters.

Such linearization is the Jacobian of the distance function that relates changes in the position domain to changes in range values, resulting in the so-called *visibility matrix*. The GDOP is constructed from the covariance of the inverse visibility matrix, and thus it relates the covariance of range-errors to that of position solution (in fact, it is highly related to the CRLB on the variance of a position estimator). Larger values of GDOP imply worse positioning accuracy than a geometry that provides a low GDOP.

A conceptual representation of the concept behind GDOP is depicted in Figure 1, where two transmitters are used to solve for a two-dimensional position in the plane using range measurements. In the presence of noisy ranges the uncertainty is visualized as the two concentrical circles, with the true range lying in between. The intersection of the two circles, in the noisy case, provides an area in which the receiver is estimated to be. Comparing Figures 1(a) and 1(b) it becomes evident that the geometrical situation of the transmitters affects the size of this area, which is indeed quantified by the GDOP. Thus the GDOP can be thought of as a value that measures the effect of network geometry on the position solution. Larger GDOP values imply worse positioning solutions, and vice versa.

However, there are differences between the Gaussian error model for TOA and the log-normal model for RSS techniques. In [21], the CRLB of variance of the position estimation has been studied for both models. The main difference between TOA and RSS models is that in the case of RSS techniques, the CRLB scales with the size of the system even if geometry is kept the same. In [18], the GDOP expression has been derived from the received power estimation (5) for the RSS log-normal model as

$$\mathrm{GDOP}_j = \frac{\ln(10)}{10}\sqrt{\frac{\sum_{i=1}^{|\mathcal{N}_j|}\rho_{j,i}^{-2}}{\sum_{i=1}^{|\mathcal{N}_j|-1}\sum_{k=i+1}^{|\mathcal{N}_j|}\frac{\sin^2\phi_{ik}}{\rho^2_{j,i}\rho^2_{j,k}}}} \quad (25)$$

where ϕ_{ik} is the angle between the two vectors from target node j to the ith and kth anchor nodes. From (25) we can observe that the GDOP depends not only on the angular distribution of reference nodes (geometry), but also on the distances of the target node to the reference nodes.

Notice that anchor nodes in (25) are those of the set $\mathcal{N}_j$. $\mathcal{N}_j$ depends on the number of anchor nodes whose beacons are received with enough power by node j. Therefore, it depends on the transmit powers of the anchor nodes. For a given receiver sensitivity s and distance-dependent path loss function $f_L(d)$, an anchor node with transmit power p_a at distance d_a from node j belongs to set $\mathcal{N}_j$ if $p_a > s/f_L(d_a)$. Therefore, given the dependence of GDOP_j on $\mathcal{N}_j$, we may in turn express the GDOP as a function of the power vector of the anchor nodes $\mathbf{p}$ explicitly as $\mathrm{GDOP} \triangleq \mathrm{GDOP}(\mathbf{p})$. Due to the dependence of $\mathcal{N}_j$ on $\mathbf{p}$, we can observe that any error metric based on the covariance of the estimator for RSS model will be a non convex function on $\mathbf{p}$. In particular, the $\mathrm{GDOP}(\mathbf{p})$ shows discontinuities because of the inclusion or exclusion of a node in $\mathcal{N}_j$. Moreover, let us define the mean GDOP over the entire network as

$$\overline{\mathrm{GDOP}}(\mathbf{p}) = \frac{1}{M}\sum_{j=1}^{M}\mathrm{GDOP}_j(\mathbf{p}) \ , \tag{26}$$

where, rearranging terms in (25) for convenience, we obtain

$$\mathrm{GDOP}_j = \frac{\ln(10)}{10}\sqrt{\frac{\sum_{i=1}^{|\mathcal{N}_j|}\prod_{\substack{k=1\\k\neq i}}^{|\mathcal{N}_j|}\rho_k^2}{\sum_{i=1}^{|\mathcal{N}_j|-1}\sum_{k=i+1}^{|\mathcal{N}_j|}(\prod_{\substack{l=1\\l\neq i,k}}^{|\mathcal{N}_j|}\rho_l^2)\sin^2\phi_{ik}}} \ . \tag{27}$$

III. Game Theoretical Power Allocation for Positioning with Wireless Sensors

Game Theory is a collection of models and analytic tools used to study interactive decision processes [14], [22]. We limit our discussion to non-cooperative models that address the interaction among individual decision makers. Such models are called *games* and the decision makers are referred to as *players* which are assumed to be rational in this work. A strategic non-cooperative game $\Gamma(\Omega, \mathcal{A}, u)$ has three main components: $i)$ Ω is the set of N players; $ii)$ $\mathcal{A}$ is the set of pure strategies and $\mathbf{a} = [a_1, \ldots, a_N]^\top \in \mathcal{A} \subseteq \mathbb{R}^N$ the chosen strategies, where $a_i \in \mathcal{A}_i$ represents the strategy of the i-th player over the set of its possible strategies $\mathcal{A}_i$. Thus, $\mathcal{A} = \times_{i=1}^N \mathcal{A}_i$ and $\mathbf{a}_{-i} \in \mathcal{A}_{-i} = \times_{j\neq i}^N \mathcal{A}_j$ represents the strategies of all players but the i-th; $iii)$ $u_i : \mathcal{A} \mapsto \mathbb{R}$ is the utility function of the i-th player. The utility function (or payoff) quantifies the preferences of each player to a given strategy, provided the knowledge of other's strategies. Then, $u \triangleq \{u_i\}_{i\in\Omega}$ is the set of all N utility functions.

Then, a non-cooperative game is a procedure where players choose the strategy that maximizes their utility function. The *Nash equilibrium* (NE) is a stable solution of the game in which no player may improve its utility function by unilaterally deviating from it.

Definition 1 (Nash Equilibrium). *A strategy profile* $\mathbf{a}^*$ *is a Nash equilibrium if,* $\forall i \in \Omega$ *and* $\forall a_i \in \mathcal{A}$, $u_i(\mathbf{a}^*) \geq u_i(a_i, \mathbf{a}^*_{-i})$.

In general, games may have a large number of NE or may not have any. Thus, it is of interest to design the utility function in a way such that the game has at least one equilibrium point. It is proved in [23] that under certain conditions of the utility function, the existence and uniqueness of a NE is ensured. However, the utility function may be designed according to a criteria which could eventually yield to non-convex functions. In those cases, there is another way for deriving sufficient conditions for existence and uniqueness of the NE in a game based on the so-called *potential games* [24]. In this type of games the incentive of all players to change their strategy can be expressed by a global utility function (called potential function) $V(\mathbf{a})$. We use the name *exact potential game* (EPG) when the game admits an exact potential function, i.e., a player-independent real valued function that measures the marginal payoff when any player deviates unilaterally.

Definition 2 (EPG). *A strategic game* $\Gamma(\Omega, \mathcal{A}, u)$ *is an exact potential game if there exist an exact potential function* $V : \mathcal{A} \to \mathbb{R}$ *s.t.* $\forall i \in \Omega, \forall \mathbf{a}_{-i} \in \mathcal{A}_{-i}$ *and* $\forall a_i, b_i \in \mathcal{A}_i$ *such that*

$$V(a_i, \mathbf{a}_{-i}) - V(b_i, \mathbf{a}_{-i}) = u_i(a_i, \mathbf{a}_{-i}) - u_i(b_i, \mathbf{a}_{-i}) \ . \tag{28}$$

An important result due to [24] is that the optima of the potential function of an EPG correspond to the Nash equilibria of the game.

A. Game Theoretical Algorithm

In our problem, players are the anchor nodes and the game is that of finding a NE such that each anchor node is transmitting at a minimal power while maintaining a certain positioning quality for the M target nodes. As a metric to assess such quality we use the GDOP. With this setup, Ω is the set of anchor nodes in the network. The set of strategies that the i-th reference node can choose are the set of its possible discrete power levels $\mathcal{P}_i$. We define $\mathbf{p} = [p_1, \ldots, p_N]^\top \in \mathcal{P} = \times_{i=1}^N \mathcal{P}_i$ as the vector containing the strategies of each node. We also assume that, at the beginning of the game, anchor nodes transmit with their maximum power level in order to gather information and allow initial positioning of nodes.

We adopt a dynamic game with iterative *best response* algorithm to achieve a NE of the game defined by $\Gamma(\Omega, \mathcal{P}, u)$. Anchor nodes decide iteratively its power transmission by maximizing its utility function,

$$\hat{p}_i = \arg\max_{p_i\in\mathcal{P}_i}\{u_i(p_i, \hat{\mathbf{p}}_{-i})\} \ . \tag{29}$$

After each iteration, the selected power level may modify the geometry of the network, thus impacting on the maximization of other players' utility. The design of a utility function and the existence of a potential function is crucial for the task of identifying NE in the game. In our algorithm the goal is to attain a desired positioning quality for the M target nodes, as well as reducing the total power of the N anchor nodes. As presented in Section II-D, the GDOP provides an appealing metric to assess such quality. Therefore, the algorithm accepts a strategy if condition $\overline{\mathrm{GDOP}}(\mathbf{p}) \leq \gamma$ is fulfilled, with γ being a design parameter. Recall that the initial topology is such that all nodes transmit at maximum power. Following the result in [25], the utility function stated in Proposition 1 is considered.

Proposition 1. *The game $\Gamma(\Omega, P, u)$ where the individual utilities are given by*

$$u_i(p_i, \mathbf{p}_{-i}) = \begin{cases} p_{init} - p_i & \text{if} \quad \overline{\text{GDOP}}(p_i, \mathbf{p}_{-i}) \le \gamma \\ -p_i & \text{otherwise} \end{cases} \tag{30}$$

is an EPG and the exact potential function is

$$V(\mathbf{p}) = \begin{cases} p_{init} - \sum_{i\in\Omega} p_i & \text{if} \quad \overline{\text{GDOP}}(p_i, \mathbf{p}_{-i}) \le \gamma \\ -\sum_{i\in\Omega} p_i & \text{otherwise,} \end{cases} \tag{31}$$

where $p_{init} = p_{max}$ is the maximum power of the sensor node.

Proof: We prove it by applying the concept of EPG in Definition 1. Consider $p_i, p_i' \in \mathcal{P}_i \mid p_i < p_i'$, therefore

$$\Delta u_i = u_i(p_i, \mathbf{p}_{-i}) - u_i(p_i', \mathbf{p}_{-i}) = p_i' - p_i \tag{32}$$

regardless $\overline{\text{GDOP}}(p_i, \mathbf{p}_{-i}) \le \gamma$ or $\overline{\text{GDOP}}(p_i, \mathbf{p}_{-i}) > \gamma$. Similarly, the potential variational may be

$$\begin{aligned} \Delta V &= V(p_i, \mathbf{p}_{-i}) - V(p_i', \mathbf{p}_{-i}) \\ &= -\left(p_i + \sum_{j\in\Omega; j\neq i} p_j\right) + \left(p_i' + \sum_{j\in\Omega; j\neq i} p_j\right) \\ &= p_i' - p_i \ . \end{aligned} \tag{33}$$

Thus, $\Delta u_i \equiv \Delta V$ therefore V is an exact potential function and the game $\Gamma(\Omega, \mathcal{P}, u)$ is an EPG. ■

The designed game falls into the category of EPG games, and thus finding the NE point of (30) is equivalent to maximizing the potential function in (31). We note that GDOP is not a convex function on $\mathbf{p}$. Therefore, we cannot claim that $V(\mathbf{p})$ has a single optimum, and thus the game might have several NE that satisfy $\overline{\text{GDOP}}(\mathbf{p}) \le \gamma$. However, simulations of Section VI-A reveal that the distributed algorithm obtains results which are comparable to a global approach.

IV. Distributed Error Metric

The game presented above has several challenges when it comes to implementation. A major concern relates to the amount of information exchange required in the networks, as anchor nodes require knowledge of global information of target nodes' in order to calculate $\overline{\text{GDOP}}(\mathbf{p})$. Our goal here is to minimize the information exchange requirements in order to preserve the benefits from power savings, due to reduced transmission power at the reference nodes. To that aim we propose to use other metrics, instead of $\overline{\text{GDOP}}(\mathbf{p})$, that only require transmission of information from in-range target nodes to anchors at each game iteration. This information includes the target's own position estimate and the set $\mathcal{N}_j$.

We propose to modify the discontinuity condition in (30)-(31) so as to use only local GDOP estimates. Two alternatives are presented. Similarly to the game using global information, we consider that at the beginning of both games players transmit with maximum power in order to allow initial positioning of target nodes and information gathering. The algorithms proceed in an iterative best response fashion until convergence.

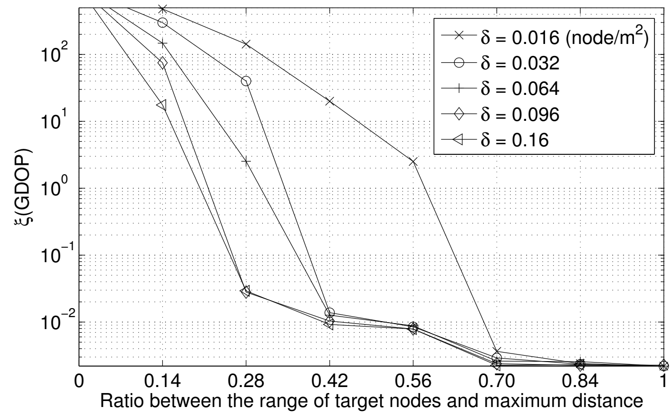

Fig. 2. RMSE in (35) for a number of target node densities δ (node/m^2) and 100 Monte Carlo trials.

A. Local GDOP Average

In this case a local estimate of the average GDOP is considered, defined as $\overline{\text{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ in (34) for the i-th anchor node. Recall that $\mathcal{T}_i$ is the set of target nodes from which the i-th anchor nodes receives status information, as they are within its range. Then, each anchor can compute

$$\overline{\text{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) = \frac{1}{|\mathcal{T}_i|} \sum_{j\in\mathcal{T}_i} \text{GDOP}_j(\mathbf{p}) \ . \tag{34}$$

The resulting utility function for the i-th player is then modified to take values as $p_{\max} - p_i$ if $\overline{\text{GDOP}}_{\mathcal{T}_i}(p_i, \mathbf{p}_{-i}) \le \gamma$. With this setup, it is possible that the overall $\overline{\text{GDOP}}$ value exceeds the threshold eventually, since the average used by each player is local. In other words, a certain strategy might lead to $\overline{\text{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) \le \gamma$ but $\overline{\text{GDOP}}_{\mathcal{T}_{i'}}(\mathbf{p}) > \gamma$, forcing the i'-th node to increase its power in next game iteration.

Notice that this distributed solution approximates the previous game when transmission powers of target nodes are such that one can consider $\overline{\text{GDOP}} \simeq \overline{\text{GDOP}}_{\mathcal{T}_i}$, $\forall i$. For the static scenario, Figure 2 shows the Root Mean Square Error (RMSE) between $\overline{\text{GDOP}}$ and $\overline{\text{GDOP}}_{\mathcal{T}_i}$, defined as

$$\xi(\text{GDOP}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N} |\overline{\text{GDOP}} - \overline{\text{GDOP}}_{\mathcal{T}_i}|^2} \ , \tag{35}$$

versus the ratio range of target nodes over the maximum distance in the network (thus being independent of a particular node's power levels). The approximation is valid for increasing target node's power and density.

B. Worst Case GDOP

We propose here an alternative design where worst-case is addressed. In this configuration, the condition to maximize $u_i(p_i, \mathbf{p}_{-i})$ is to ensure that all target nodes have the specified GDOP. That is, the condition for the i-th player can be formulated as

$$\text{GDOP}_j(\mathbf{p}) \le \gamma, \forall j \in \mathcal{T}_i \ , \tag{36}$$

and the utility in (30) should be modified accordingly. It can be easily seen that a game implementing such utility yields to a steady state solution. Recall that game starts with all players transmitting with maximum power. Notice that a player has no incentives to decrease its power if it causes at least one target node increase its GDOP. Same applies to the rest of players when iterating, and thus a stable solution is eventually achieved when no player can modify further its strategy.

Although the achieved solution is not optimal (from an energy-efficient point of view), it provides a strategy set which ensures the specified target GDOP. This might be useful in applications where this is the most restrictive issue, rather than proper power control.

V. Distributed Implementation and Computational Resources

A. Potential Game with Positioning Algorithm

In this section, we present implementation challenges of the proposed game theoretical algorithm. We discuss a possible solution, which minimizes the information exchange between target and anchor nodes, as well as the number of operations.

For each game, anchors play N_{it} rounds of best response iterations. Once all anchors have played the first round, they play again another round and successively. At each iteration, the corresponding anchor node has to compute GDOP values, which depend on the estimated positions of the target nodes within range. Such position estimate is performed at target nodes using the set of RSS values. On one hand, the information exchange required between target and anchor nodes is presented in Figure 3 for the dynamic scenario. The algorithm starts when one or more target nodes broadcast a ranging request ($\overrightarrow{\text{RRq}}$). Then, $|\mathcal{N}_j|$ iterations of the game are performed. At each iteration, a ranging reply $\overleftarrow{\text{RRi}}$ and a confirmation frame $\overrightarrow{\text{CFi}}$ are interchanged between a target and the corresponding anchor nodes from $i = 1$ to $i = |\mathcal{N}_j|$. Following iterations of the game can be performed until $n = N_{it}|\mathcal{N}_j|$ iteration, in which the game reaches NE. Therefore, we consider that the number of best response iterations is N_{it} and the total number of algorithm iterations is $N_{it}|\mathcal{N}_j|$. The iterations of the best response algorithm are executed by anchor nodes iteratively. Once a game is finished, the target moves and after a while another game can start. Note that the right arrow ($\rightarrow$) over the frame name indicates a frame transmission from target to anchor i and the left arrow ($\leftarrow$) a frame transmission from anchor i to target. On the other hand, algorithm 1 shows the detailed pseudo-code description of the proposed algorithm for the dynamic scenario. It shows the operations performed by target and anchor nodes in each iteration of the game. Once the algorithm starts when one or more target nodes broadcast a ranging request ($\overrightarrow{\text{RRq}}$), then the prediction phase of the EKF is performed in the target node. Therefore the prediction for the position of the target node is known. Based on this prediction, the game runs and the GDOP can be calculated. In each iteration of the game, the following information exchange is required between the corresponding anchor node and target node:

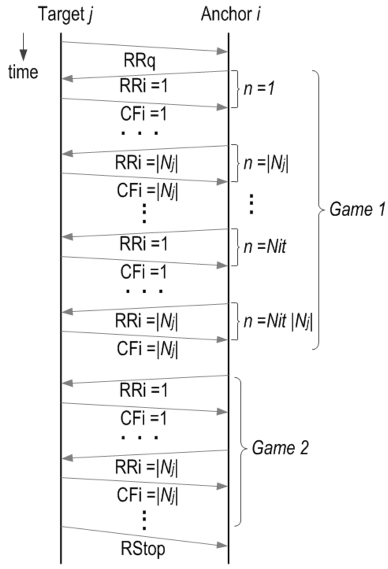

Fig. 3. Time diagram of the algorithm.

- $\overleftarrow{\text{RRi}}$: the ranging reply with the chosen transmit power p_i is transmitted from anchor to target node, then the target nodes estimates the distance with RSS-based technique and an averaging can be done with previous RSS.
- $\overrightarrow{\text{CFi}}$: the target nodes respond to the anchor node with the confirmation frame or acknowledgement that contains needed information for the game ($\hat{\rho}_{j,i},\ i = 1...|\mathcal{N}_j|$). Once the corresponding anchor i receives this $\overrightarrow{\text{CFi}}$, it plays choosing a new p_i maximizing its utility function depending on the used GDOP metric. To analyze the maximization of its utility function, the corresponding anchor i analyzes the condition GDOP $\leq \gamma$ for each value p_i of the set of transmit powers $P_{Tx,i}$ with the following steps: i) Anchor i estimates GDOP metric for p_i. Therefore anchor i analyzes if its contribution to GDOP estimation is required for p_i. Anchor i contributes when $P_{Rx,j}(p_i, \hat{\rho}_{j,i}) > s$, where s is the sensibility and $P_{Rx,j}$ is estimated with (5). ii) For p_i, the condition GDOP $\leq \gamma$ is analyzed. iii) Once steps i) and ii) are performed for the set $P_{Tx,i}$, anchor i chooses the p_i. If the contribution is required, anchor i chooses p_i minimum such that GDOP $\leq \gamma$; but if the contribution is not required, anchor i turns off thus saving energy.

Once the game is over each target has the information to execute the update phase of the EKF. Then the target estimates its position. The algorithm can start again with the prediction phase and the process runs again. The time interval between game performances could be controlled by parameters as battery and error metric of the target node.

For the static scenario, the same information exchange may apply but only for one game in Figure 3. Moreover, in Algorithm 1 the positioning procedure may be performed with linear LS algorithm.

The amount of message exchange $\overleftarrow{\text{RRi}}$ and $\overrightarrow{\text{CFi}}$ between anchor and target nodes can be justified as follows. $\overleftarrow{\text{RRi}}$ is used to estimate the distance with RSS-based techniques in the target node. This frame is necessary in any positioning algorithm. $\overrightarrow{\text{CFi}}$ is required to transmit needed information to execute our resource planning game. However, $\overrightarrow{\text{CFi}}$ can be seen as an acknowledgement (ACK) frame in terms of communication protocol with a data payload. Note that IEEE 802.15.4 protocol supports accesses to the medium with ACK mode. Moreover, the $|\mathcal{N}_j|$ times that an anchor plays in a game is justified by the fact that each time a RSS is received and the node can average $|\mathcal{N}_j|$ RSS values. This averaging reduces the RSS standard deviation (see Section II-B).

Algorithm 1 Potential game for energy efficient positioning for $M = 1$ and dynamic setup

Target node j computes the predicted position with the Extended Kalman Filter.
Initialization: $\overrightarrow{\text{RRq}}$.
Set $\mathbf{p} = \mathbf{P}_{max}$.
Game iterations phase: $n = 1$, $i = 1$.
while $n < (N_{it}|\mathcal{N}_j|)$ **do**
  $\overleftarrow{\text{RRi}}$ is sent from anchor i to target node j:
  · It contains: if $n = 1$, $\mathbf{x}_a^{(i)}$ and initial $\mathbf{p}$.
  · Operations of target node: distance estimation $\hat{\rho}_i$.
  $\overrightarrow{\text{CFi}}$ is sent from target j to anchor node i:
  · It contains: $\hat{\rho}_i$, $i = 1 ... |\mathcal{N}_j|$.
  · Operations of anchor node: if $i > 1$, update p_i $|$ $\text{GDOP}(p_i, \mathbf{p}_{-i}) \leq \gamma$.
  if $i = |\mathcal{N}_j|$ **then**
    $i = 1$
  else
    $i = i + 1$ {Next anchor node}
  end if
  $n = n + 1$
end while
End: $\overleftarrow{\text{RStop}}$.
Target node j computes the update phase of the EKF: position $\hat{\mathbf{x}}$.

B. Computational Resources

In this section we analyze the required computational resources of the presented solution. From Algorithm 1, the computational complexity can be obtained by calculating the number of basic operations involved. In Table I the number of operations is summarized for $M = 1$ target node. The operations are shown with respect to the number of anchor nodes $|\mathcal{N}_j|$. Taking into account that the upper bound is $|\mathcal{N}_j| \leq N$, then $\mathcal{O}(|\mathcal{N}_j|) \leq \mathcal{O}(N)$.

From Table I, we calculate the total asymptotical computational cost for the target node $\mathcal{C}_j$ and we obtain

$$\begin{aligned} \mathcal{C}_j &= \mathcal{O}((N_{it}|\mathcal{N}_j| - 1)(|\mathcal{N}_j|^3 + 8|\mathcal{N}_j|^2 + 43|\mathcal{N}_j| + 216)) \\ &\leq \mathcal{O}(N_{it}N^4). \end{aligned} \tag{37}$$

The cost for the anchor node i, $\mathcal{C}_i$, is

$$\begin{aligned} \mathcal{C}_i &= \mathcal{O}\left((N_{it}|\mathcal{N}_j| - 1)\left(\frac{2|\mathcal{N}_j|^3 + 3|\mathcal{N}_j|^2 - 5|\mathcal{N}_j| + 24}{4}\right)\right) \\ &\leq \mathcal{O}\left(\frac{N_{it}N^4}{2}\right). \end{aligned} \tag{38}$$

Note from (38) that $\mathcal{C}_{\text{target}}$ scales with $|\mathcal{N}_j|^4$. One of the most demanding operations is the inverse matrix for $\mathbf{K}(t)$, because the size of the matrix is $|\mathcal{N}_j| \times |\mathcal{N}_j|$. For the anchor node i, the most complex operations are part of the GDOP computation (see (b) operations in Table 1) that scales with $|\mathcal{N}_j|^4$.

For the case $M > 1$ each target node has to execute the same operations detailed in Table I while the anchor node i has to calculate the operations of Table I for each target node. Therefore, the computational complexity also scales with M and is given by

$$\begin{aligned} \mathcal{C}_i &= \mathcal{O}\left(M(N_{it}|\mathcal{N}_j| - 1)\left(\frac{2|\mathcal{N}_j|^3 + 3|\mathcal{N}_j|^2 - 5|\mathcal{N}_j| + 24}{4}\right)\right) \\ &\leq \mathcal{O}\left(\frac{MN_{it}N^4}{2}\right). \end{aligned} \tag{39}$$

The quartic relation of the computational complexity with the number of anchor nodes might be an issue in large-scale networks, mostly in sensor networks due to the limited power processing of the motes. A possible workaround is to limit the total number of anchor nodes used for positioning.

VI. Simulation Results

The proposed algorithm was tested in the static and dynamic scenarios that were introduced in Section II-A. Each node had a 2.4GHz IEEE 802.15.4 ready RF Transceiver based on a CC2420 from Texas Instruments. The set of transmit powers of the CC2420 is $P_{Tx,i} = \{1, 0.79, 0.50, 0.31, 0.1, 0.032, 0.0015, 0\}$ mW.

While the static scenario aims to show the convergence to a NE for one game, the mobile scenario shows the convergence of several games together with the tracking operation of the target nodes. In both cases, target nodes estimated its position with the set of RSS values. In the static scenario, target nodes estimated its position by a LS algorithm whereas in the mobile scenario a EKF was used for tracking. Results obtained with the static scenario were published in [16]. The density of target nodes is higher for the static scenario than for the mobile scenario. In both cases, we compare the performance of the distributed $\overline{\text{GDOP}}_j(\mathbf{p})$ and $\overline{\text{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ metrics with the global metric $\overline{\text{GDOP}}(\mathbf{p})$.

A. Static Scenario

The considered static scenario was composed of $M = 20$ nodes that aim at locating themselves using RSS signal to a set of $N = 8$ anchor nodes (Figure 7). Anchor nodes were distributed at known positions in a 25×25 meters area, whereas the M nodes were placed randomly in the space.

In one game, anchors play N_{it} rounds (best response iterations). In each round all anchors play in an ordered sequential fashion. Thus, once all anchors have played the first round, they play again another round and successively. The number of rounds is N_{it}, therefore each player plays N_{it} times or iterations. At each iteration, the corresponding anchor node has to compute GDOP values, which depend on the estimated positions of the target nodes within range. Such position estimate is performed at target nodes using the set of RSS values. At each iteration, the random error $v_{j,i}$ is different in each RSS measurement, it affects the distance estimation $\hat{\rho}_{j,i}$. Thanks to the game iterations, the RSS values are averaged N_{it} times, thus decreasing its error.

TABLE I
COMPLEXITY OF ALGORITHM 1

Node	Computation	Operation	Size	Cost
Target j	Prediction phase of EKF: $\hat{\mathbf{s}}(t\vert t-1) = \mathbf{A}\hat{\mathbf{s}}(t-1\vert t-1)$ $\mathbf{M}(t\vert t-1) = \mathbf{A}\mathbf{M}(t-1\vert t-1)\mathbf{A}^\top + \mathbf{Q}$	 Matrix product Matrix products, addition	 $4\times\vert\mathcal{N}_j\vert\times 1$ $4\times 4\times 4$	$\leq \mathcal{O}(5N+84)$ $\mathcal{O}(5\vert\mathcal{N}_j\vert+4)$ $\mathcal{O}(4^3+16)$
Target j	Update phase of EKF: $\mathbf{H}(t)$, see (19) $\mathbf{M}(t\vert t) = (\mathbf{I}-\mathbf{K}(t)\mathbf{H}(t))\mathbf{M}(t\vert t-1)$ $\mathbf{K}(t) = \mathbf{M}(t-1\vert t-1)\mathbf{H}(t)^\top$ $\cdot\,(C+\mathbf{H}(t)\mathbf{M}(t\vert t-1)\mathbf{H}(t)^\top)^{-1}$ $\hat{\mathbf{s}}(t\vert t) = \hat{\mathbf{s}}(t\vert t-1)$ $+\,\mathbf{K}(t)(\mathbf{x}(t)-\mathbf{h}(\hat{\mathbf{s}}(t\vert t-1)))$	 Additions, products Matrix products and addition Matrix products, addition and inverse Matrix product and additions	 $26\vert\mathcal{N}_j\vert$ $4\times\vert\mathcal{N}_j\vert\times 1$	$\leq \mathcal{O}(N^3+8N^2$ $+38N+132)$ $\mathcal{O}(26\vert\mathcal{N}_j\vert)$ $\mathcal{O}(16\vert\mathcal{N}_j\vert+4^3)$ $\mathcal{O}(\vert\mathcal{N}_j\vert^3+8\vert\mathcal{N}_j\vert^2$ $+17\vert\mathcal{N}_j\vert+4^3)$ $5\vert\mathcal{N}_j\vert+4$
Anchor i	Compute $N_{Tx}^{(*)}$ $P_{Rx,j}$: $P_{Rx,j} = P_{Tx,i} - L_o + 10p\log_{10}\left(\frac{\hat{\rho}_{j,i}}{\rho_o}\right)$	$N_{Tx}^{(*)}$ (2 Additions, $\log_{10}$, 2 products, division)	6	$\mathcal{O}(6)$
Anchor i	Compute GDOP (see 27) with and without i contribution $\sum_{i=1}^{\vert\mathcal{N}_j\vert}\prod_{k=1, k\neq i}^{\vert\mathcal{N}_j\vert}\rho_k^2$; (a) $\sum_{i=1}^{N-1}\sum_{k=i+1}^{N}(\prod_{l\neq i,k}^{N}\rho_{j,l}^2)\sin^2\phi_{ik}$; (b) Choose minimum $P_{Tx,i}$ $\mid$ GDOP $\leq \gamma$	Operations: 2·(a, b, c) (a) Product, addition (b) Product, addition, sin (c) Product, division, root	1 $2(\vert\mathcal{N}_j\vert-1)\vert\mathcal{N}_j\vert$ $\frac{(2\vert\mathcal{N}_j\vert-3)\vert\mathcal{N}_j\vert!}{(\vert\mathcal{N}_j\vert-2)!(2!)}$ 3	$\leq \mathcal{O}\left(\frac{2N^3+3N^2-5N}{4}\right)$ $\mathcal{O}\left(2\vert\mathcal{N}_j\vert^2-2\vert\mathcal{N}_j\vert\right)$ $\mathcal{O}\left(\frac{2\vert\mathcal{N}_j\vert^3-5\vert\mathcal{N}_j\vert^2+3\vert\mathcal{N}_j\vert}{4}\right)$ $\mathcal{O}(3)$

(*) Number of transmit powers.

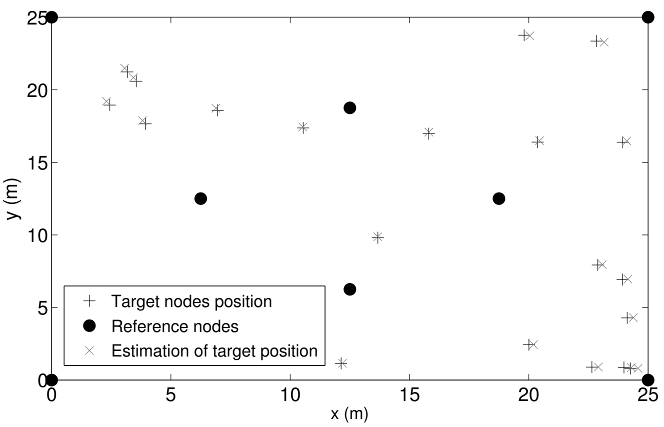

Fig. 4. Simulation scenario consisting of a 25×25 meter region where a set of anchor sensor nodes (big dots) are distributed at known positions, whereas the target sensor nodes (black crosses) are placed randomly. Position estimates (grey crosses) are also shown for the last iteration of the game.

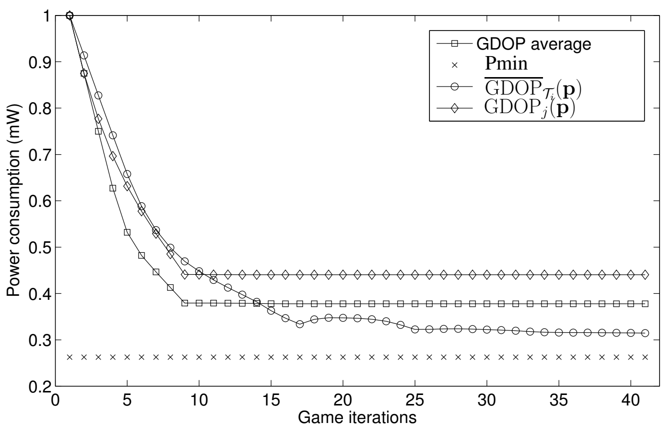

Fig. 5. Mean power of anchor nodes versus iterations of the game.

We considered $N_{it} = 5$ iterations of the game as stopping rule and the mean GDOP value was $\gamma = 1.3$. For RSS based range model we considered $p = 3$ and $\sigma_{j,i} = 0.1$ dB for all possible $\{i, j\}$ pairs.

Recall that initially all nodes transmit at their maximum power. Results of the proposed algorithm were averaged over 100 Monte Carlo independent trials and compared to those obtained by an algorithm that globally optimizes the set of power levels $\mathbf{p}$. That is, the solution of the coordination game that finds the global optima of the potential function $V(\mathbf{p})$. This solution, implemented by exhaustive search, explores all combinations of power levels for the N nodes ($\dim\{\mathcal{P}\}^N$) and obtains the set of strategies with lower mean power ($\bar{\mathbf{p}}_{\text{min}}$) over the network, with the condition on the GDOP holding. In the simulation results, we compared the average results of our method with the GDOP average of all the target nodes $\overline{\text{GDOP}}(\mathbf{p})$, the distributed game with local GDOP average $\overline{\text{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and the distributed game with worst case GDOP, as well as $\bar{\mathbf{p}}_{\text{min}}$.

Figure 5 shows the evolution of the mean power of the network versus the iterations of the game. We can observe that this value decreases and tends to $\bar{\mathbf{p}}_{\text{min}}$. Of interest is the comparison of these results with those in Figure 6, where we can identify that although our algorithm might yield larger mean power values, we experience a tradeoff in the final GDOP achieved. Results of the case with local GDOP average come closer to $\bar{\mathbf{p}}_{\text{min}}$ than for worst case GDOP. This is because worst case GDOP assures that each node's GDOP is below the threshold.

B. Dynamic Scenario

The considered mobile scenario was composed of $M = 2$ mobile target nodes that aim at locating themselves using the RSS from a set of anchor nodes (Figure 7). Anchor nodes

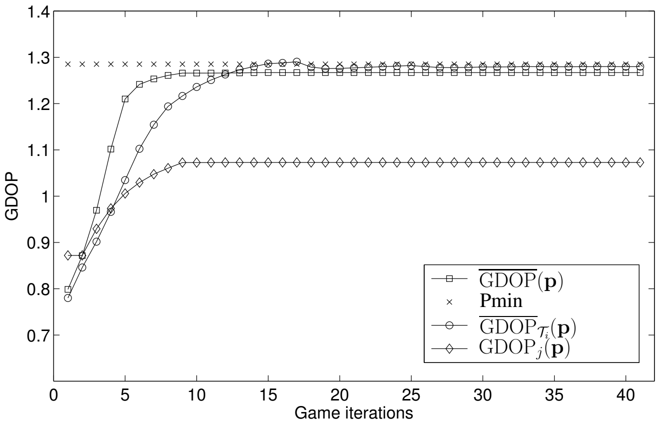

Fig. 6. $\overline{\mathrm{GDOP}}$ versus iterations of the game.

were distributed at known, regular positions every 15m in a $800 \times 450\mathrm{m}^2$ area. They were numbered starting at origin in Figure 7. The $M = 2$ target nodes were placed in a close position initially, but their trajectories diverge. Target nodes were mobile and they sent requests to receive ranging signals at intervals of $\Delta = 0.8$s. In each request a game started until the NE was achieved. In order to show how the algorithm works, Δ equals for each node was considered. The playing anchor nodes computed the GDOP of the target nodes for the three cases previously presented: $\overline{\mathrm{GDOP}}(\mathbf{p})$, $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and $\mathrm{GDOP}_j(\mathbf{p})$. Therefore, as a game was executed every Δ s and the number of games was $N_{games} = 120$, the simulation duration in time was $\Delta \cdot N_{games} = 96$ s. Results of the proposed algorithm were averaged over 200 Monte Carlo independent trials.

For the simulations, the used values of channel model parameters are from [17]. These parameters values have been obtained from an experimental campaign to collect the RSSI measurements with sensor nodes equipped with CC2420 transceiver. Thus, we consider $L_0 = -20$ dB, $\rho_0 = 0.1$ m, $p = 3$ and $\sigma_{j,i} = 4$ dB. The initial conditions of the first target node were the following: the position was $x^{(1)}(0) = y^{(1)}(0) = 401$ m and the velocity was $v_x^{(1)}(0) = v_y^{(1)}(0) = 0$ m/s. While for the second node the position was $x^{(2)}(0) = y^{(2)}(0) = 403$ m and the velocity was $v_x^{(2)}(0) = v_y^{(2)}(0) = 0$ m/s. Also, $\sigma_{\mathrm{w}}^2 = 5$ for both target nodes.

Every game was run every Δ s and the different games were played by different anchors due to target node movement. In Figure 8, the mean power of anchor nodes at the NE of the played games ($N_{games} = 120$) is showed. The power value is minimized to values close to 0.001mW. Of interest is the comparison of these results with Figure 9. We can identify that, for each played game, the proposed algorithm maintains the global metric $\overline{\mathrm{GDOP}}(\mathbf{p}) < \gamma$ as well as the distributed metric $\mathrm{GDOP}_j(\mathbf{p}) < \gamma$, ($\gamma = 4$). The differences with the threshold are due to the errors in the RSS measurements. For the less restrictive case $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ of Figure 9 the values are clearly different to the other two metrics, when the two trajectories of the target nodes are close and anchor nodes performs the GDOP averaging $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$. Figure 10 shows the resulting RMSE after the power control games were executed, with consistent results.

As previously commented, in Section IV-A, the RMSE between $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and $\overline{\mathrm{GDOP}}(\mathbf{p})$ depends on target node density. The approximation $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) \simeq \overline{\mathrm{GDOP}}(\mathbf{p})$ is valid for increasing density of target nodes (considering maximum transmit power for target nodes). In the mobile scenario there are $M = 2$ target nodes. Thus, effect due to low density of target nodes for $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ can be observed, mostly in the part of the figures that corresponds to close trajectories of the target nodes (from 1st to 25th games). As earlier mentioned, in this part of Figure 9 the estimated GDOP metric $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ is less accurate than the other metrics. Also, target trajectories are close and share anchor nodes, however the number of target nodes within range τ_i is different for each anchor node i. In Figure 11, the percentage of anchor nodes with $\tau_i = 1$ is shown (before playing the game). At the beginning, target nodes share the majority of anchor nodes, but when target trajectories separate, the number of anchor nodes that have one target node within range increases. For example in game 17th, 50% of anchor nodes have $\tau_i = 1$. Thus, the playing anchor nodes decide the new transmit power taking into account the GDOP average of τ_i, but this average $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ can change for each player i as τ_i is different for each player. Comparing Figures 9 and 11, we can observe that the difference between $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and γ is low when the percentage of anchor nodes with $\tau_i = 1$ is $< 15\%$, meaning that the local metric approximates properly the global metric. The difference increases with larger percentages up to approx. 50% (corresponding to the 10th to 20th games). Then, the difference decreases again for larger percentages since, once the trajectories of target nodes are distant enough, $\mathrm{GDOP}_j(\mathbf{p})$ and $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ values are similar (Figure 9). This is because target nodes use different set of anchor nodes and thus the GDOP averaging $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ ($\tau_i = 1$ for all i) is equivalent to the worst-case GDOP.

In conclusion, distributed $\mathrm{GDOP}_j(\mathbf{p})$ and $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ metrics are valid quantities, taking into account that the density of target nodes affects its performance. The results show that for low density of target nodes ($M = 2$ in this case), the approximation $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) \simeq \overline{\mathrm{GDOP}}(\mathbf{p})$ depends on τ_i. Thus, $\mathrm{GDOP}_j(\mathbf{p})$ metric is a better option to approximate to $\overline{\mathrm{GDOP}}(\mathbf{p})$. For $M = 20$ target nodes (static scenario), the metric $\mathrm{GDOP}_j(\mathbf{p})$ is more conservative than $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ as it was shown in Section VI-A, being the approximation $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p}) \simeq \overline{\mathrm{GDOP}}(\mathbf{p})$ valid for increasing density of target nodes.

Finally, in Figure 12 the activity of the anchor nodes depending on the trajectory of the target nodes is showed. The relation between the ID number of each anchor node and its position in the scenario can be checked in Figure 7. There are anchor nodes that contribute to positioning at a certain instant and when they are no longer necessary, they do not contribute, thus saving energy. Moreover, the figure shows the set $\mathcal{N}_j$ of anchor nodes that provide ranging signals to target node 1 and target node 2. At the beginning of the trajectories, target nodes share anchor nodes for positioning, however when their trajectories separate, different anchor nodes help in positioning each target node.

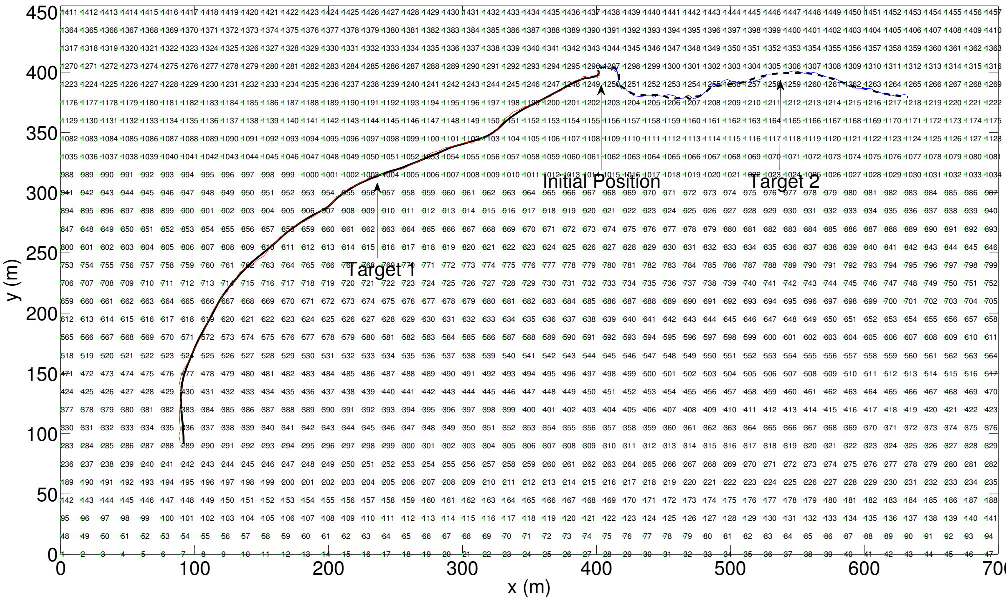

Fig. 7. Scenario consisting of anchor nodes (green points) and M=2 target nodes with different trajectories. The initial position of target 1 is (401, 401) and for target 2 (403, 403). The anchor nodes are numbered with and ID number starting from the origin. The trajectory of target node 1 is represented with a solid line (black color for real trajectory and red for estimated one). The trajectory of target node 2 is represented with a dashed line (black: real trajectory; blue: estimated trajectory).

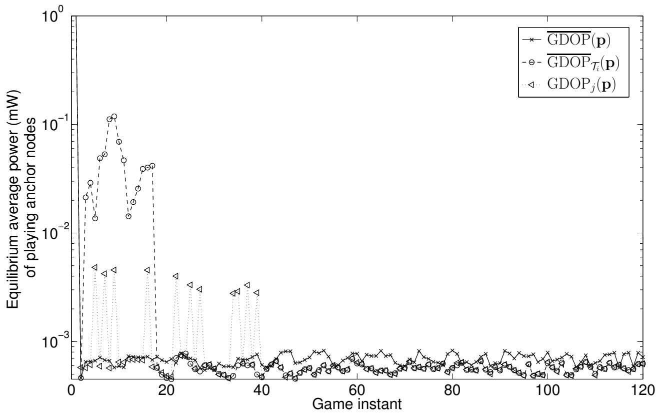

Fig. 8. Average power of playing anchors in Nash equilibrium of the games for metrics $\overline{\mathrm{GDOP}}(\mathbf{p})$, $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and $\mathrm{GDOP}_j(\mathbf{p})$.

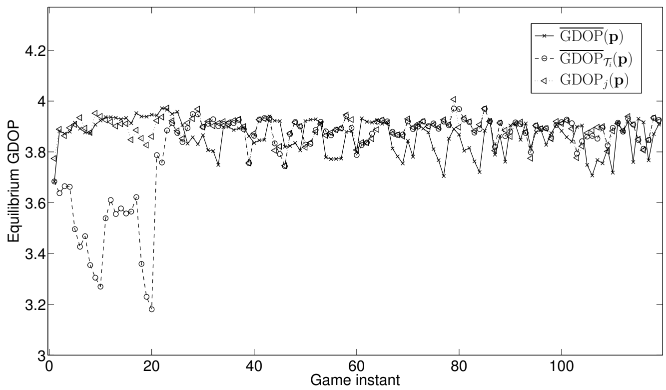

Fig. 9. Real GDOP in Nash equilibrium of the games for metrics $\overline{\mathrm{GDOP}}(\mathbf{p})$, $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and $\mathrm{GDOP}_j(\mathbf{p})$.

VII. Conclusions

In this paper we presented an algorithm for distributed power control and node selection, with the goal of saving energy in WSN with RSS-based positioning capabilities. The proposed algorithm minimizes the transmit power of anchor nodes as well as performs the selection of a set of anchor nodes for positioning target nodes, while using the GDOP to maintain an adjustable level of positioning accuracy. The anchor nodes that are not necessary for positioning purposes are turned to low power mode, saving energy.

We used the framework provided by non-cooperative potential games to design and analyze our algorithm. The game falls into the category of EPG potential games. The proposed solution provides a distributed approach to select the power levels of anchor nodes such that a predefined positioning quality is ensured, as quantified by the GDOP metric. Two distributed metrics have been proposed to estimate the average GDOP using merely the local information available at each anchor node. We discussed a possible solution for a fully-distributed implementation of this game. This solution was analyzed in terms of its asymptotical computational complexity. Performance was assessed by means of computer simulations in two scenarios, an static setup and a dynamic one.

For the sake of simplicity in the simulations of the mobile scenario, we have considered that the position of the target node is fixed during a small time window in which the iterations of a game are performed. It is a reasonable approximation since the communications rate is much smaller

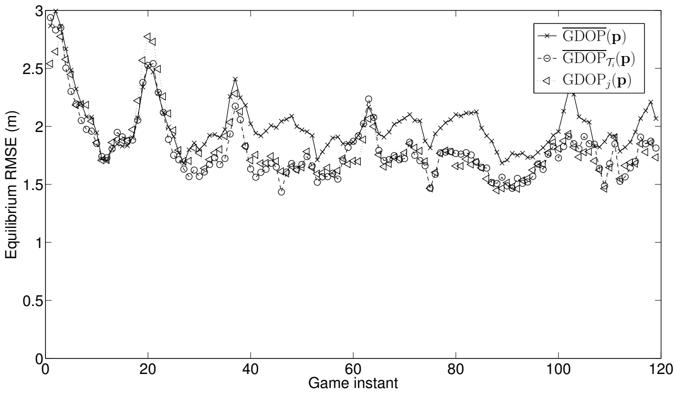

Fig. 10. RMSE (m) in Nash equilibrium iterations of the games for metrics $\overline{\mathrm{GDOP}}(\mathbf{p})$, $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$ and $\mathrm{GDOP}_j(\mathbf{p})$.

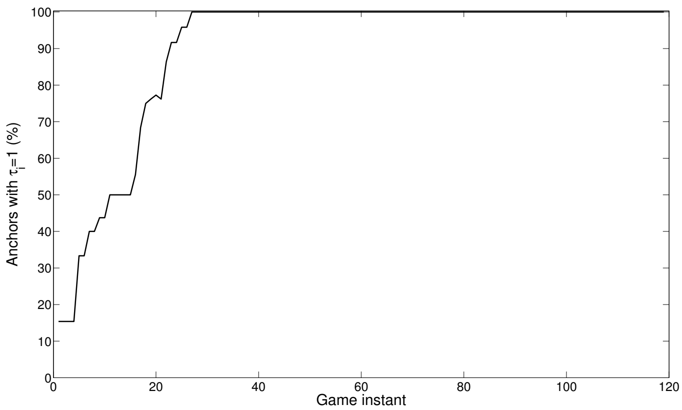

Fig. 11. When the trajectories of the target nodes depart, the percentage of nodes with $\tau_i = 1$ increase. Case plotted for metric $\overline{\mathrm{GDOP}}_{\mathcal{T}_i}(\mathbf{p})$.

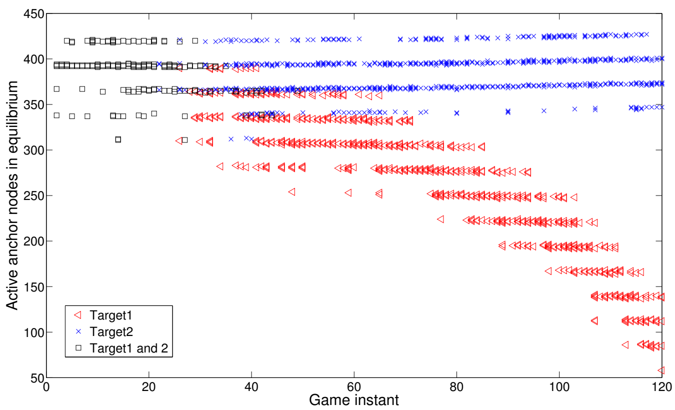

Fig. 12. Active anchor nodes (ID number) in Nash equilibrium iterations of the games for $\mathrm{GDOP}_j(\mathbf{p})$ case (no Monte Carlo trials).

than the velocity of the mobile node. However, in a future work we will study the convergence of the game taking into account the change of the position between iterations of the algorithm. For a given trajectory, this latter approach may reduce the number of games and hence the amount of required information exchange.

Finally, results revealed that the distributed algorithm obtains results which are comparable to a global approach, as well as requiring much less computational resources. The complexity is on the order of $\mathcal{O}(n_p^N)$ and $\mathcal{O}(n_p)$ for the global and proposed solutions, respectively, with n_p being the number of available power levels and N the number of anchor nodes to adjust.

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