Speculation on Quantum Bound States and Radiating Charges

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The classical Maxwell equations may be used to describe charges which radiate photons if the source (say a point charged particle) accelerates. This process of radiation has even been applied even to quantum systems. For example, some argue that a quantum accelerating charge may absorb the photons it radiates. In (1), a phenomenological Hamiltonian is established to describe a diatomic molecule which is driven by a laser and behaves as an oscillator radiating in a manner linked to Larmor radiation.

In (2), it is argued that Maxwell's equations for a photon (no sources) represent a quantum mechanical equation similar to Dirac's equation. If this is the case, one may argue that Maxwell's equations with sources present should also represent quantum equations, yet classical values are used for the velocity and acceleration of a point charge.

In previous notes, we have described a quantum bound state in a statistical manner as a momentum distribution of free particle states exp(ipx) i.e. W(x)=wavefunction = Sum over p a(p)exp(ipx). For the case of a potential, we argue acceleration is caused by stochastic hits V(x)=Sum over k Vk exp(ikx) which change the momentum distribution form point to point. The stochastic hits may knock the particle to the right or left at any time, so it seems there is no notion of overall forward or backwards motion for any prolonged period of time as there is in classical physics. When a particle is in a state exp(ipx), it has constant velocity and is not radiating. Potential knocks occur in the forward and backward direction with equal probability if a(p)=a(-p). If Maxwell's equations use average values such as v(x) and acceleration which do not really apply to quantum mechanics (except in an rms manner for a bound state), then one may argue that a source term j=qv(x) is zero if "v(x)' is in both the forward or backward direction. Even for a quantum particle moving in one direction, exp(ipx) seems to indicate constant velocity on average i.e. there may be zitterbewegung or some other type of periodic physical motion, but there is no radiation. Thus, we argue that the classical Maxwell's equations may not really be applied to a bound quantum state if there is no distinct forward or backwards motion. It is still possible that the potential knocks could lead to radiation emission, but experimentally bound states are stable or quasi-stable, so radiation emission from the system does not occur except in the case that a photon is emitted when an excited state drops to a lower one. In such a case, it appears that a resonance decouples into two quantum resonances, one being the photon and the other being the lower energy state.

Maxwell's Equations and a Radiating Charge

Lamor's expression yields the energy of photons (em radiation) emitted by an accelerating charge. It follows from Maxwell's equations of motion by using retarded time in the point charge source i.e. t-r/c. For example, A(r,t) = q v(t-r/c). dA/dr= q dv/dt (-1/c). In such a calculation, low powers of r appear in the denominator of expressions for B and E and so the Poynting vector 1/uo ExB normal to the radius of a sphere and integrated over the sphere as r→ infinite does not vanish.

Lamor’s expression was already known when quantum mechanics was formulated and there was a puzzle as to why an accelerating electron (in a hydrogen atom for example) would not radiate away its energy. Quantum mechanical bound states were experimentally verified to be stable, but only for certain energies, even though there was “apparently” still acceleration of charged particles.

Today, the idea of charged particles radiating in quantum scenarios is still used. There exist theories which try to calculate a quantum particle moving in a circle which radiates em waves and then reabsorbs them. In (1), a diatomic molecule driven by a laser has been phenomenologically treated as an oscillator which radiates more or less in keeping with the Lamor process. The results of such a treatment match experiment quite closely.

For a bound quantum state, however, one has stability for energy eigenstates. In this note, we try to investigate this.

Maxwell’s Equations for a Photon

In (2), it is argued that Maxwell’s equations for a photon are really quantum mechanical equations similar in nature to the Dirac equation. If one has source terms in Maxwell’s equations, however, these are treated in a purely classical manner. For example $A(r,t)=qv(t-r/c)$ and $dA/dr = q\, dv/dt\, (-1/c)$. One does not have velocity as a function of time or acceleration of a particle in quantum mechanics. Thus, there seems to be somewhat of a strange situation. The sourceless Maxwell equations represent a resonance with E and B behaving as $\exp(-ipx-wt)$ where $p=w$ which is statistical in nature (we argue) just as a quantum particle. When a source is added, it is treated as classical, yet the photons emitted by the accelerating charge are quantum mechanical. The velocity and acceleration are classical notions.

Stochastic Quantum Bound State

We argue that a quantum bound state is described statistically by a momentum distribution of free particle wavefunctions $\exp(ipx)$ (each occurring at a different time) at each x. This distribution changes with x accounting for average kinetic energy KE = [Sum over p a(p) pp/2m exp(ipx)]/ W(x) = Sum over p pp/2m P(p/x) changing exactly as classical kinetic energy KE(x). The big difference, we argue, is that there is no sense of continuous forward or backward motion in a quantum system. In a classical system, a particle moves from left to right in time and then backwards at a later time. In a stochastic system, there are forward and backward motions so one has only overall averages at x. Thus, KE(x) i.e. -1/2m d/dx d/dx W / W includes both forward and backward motion. In such a case, it is not clear how one may consider a particle to accelerate in the sense of Maxwell’s equations. In a statistical model, a particle is in one exp(ipx) state or another and these have constant momentum p and do not radiate. There is no acceleration in the sense of Lamor’s formula, it seems. It seems that if radiation were to be emitted, it would have to occur during a stochastic hit which knocks an exp(ip1 x) into exp(i p2 x) as there is a very rapid change from p1 to p2, but this does not seem to occur experimentally. It seems that applying Maxwell’s equations using velocity=vrms(x) from quantum mechanics and trying to suggest that this represents acceleration in one direction is a problem because vrms(x) includes both forward and backwards motion in quantum mechanics.

If one considers the time-independent Schrodinger, equation and expresses W(x) (wavefunction) and V(x) as Fourier series then:

pp/2m a(p) + Sum over k Vk a(p-k) = E a(p)

This suggests a resonance between the charged particle's momentum p (which should include E and B field contributions) with k the momentum of a virtual photon from V(x). This seems to be a different picture from an accelerating source leading to low r denominator fields at large distances (i.e. to a Poynting vector dot normal vector integrated over a large sphere) which does not vanish and so represents radiation out of a system.

As argued in (1), there may be cases where there is actual driven motion i.e. a laser driving a diatomic molecule. In such a case, there may be the idea of forward motion for half a cycle followed by backward motion for the other half. Then, radiation is emitted as verified experimentally. This, however, does not seem to be the same as a quantum bound state.

Given that it is argued in (2) that a photon is a solution of a quantum mechanical type of equation (represented by Maxwell's equations) with no source, it seems that a photon represents a resonance. An absorbed photon which is a resonance couples with an electron in a certain energy state (another resonance) to form a combined quantum mechanical resonance i.e. a higher energy state. The photon may decouple to form its own E, B resonance again dropping the electron into a lower energy quantum resonance. Traditionally, a photon is portrayed as simply a source of energy, not as a resonance with a particular frequency.

Conclusion

In conclusion, we argue in this speculative note that a bound quantum charged particle does follow a continuous route to the left for half a cycle and then to the right for the other half, thus the idea of velocity and acceleration in the classical sense do not seem to make sense. The particle receives stochastic hits and may move to the right at one instant and then to the left at the next. The system is described statistically in terms of a momentum distribution of states exp(ipx) which do not radiate because velocity is constant. If exp(ipx), however, only represents constant velocity on average and there is a "zitterbewegung" or some other physical type of periodic motion, then one would expect this to cause radiation, but it doesn't.

Thus, it seems it would be difficult to apply Larmor's formula of radiation to a quantum bound charged particle which is based on Maxwell's equation (Poynting vector) with a source which distinctly moves and accelerates in a certain direction. The quantum particle is stochastic and emitted radiation would need to occur when a particle is hit from one momentum p state to another, but this does not happen, unless the radiation is then reabsorbed. The stochastic hits come from a potential V(x)=Sum over k exp(ikx) Vk which itself is in resonance with the exp(ipx) of the particle, but exp(ikx) may represent a virtual photon.

When a photon, which is apparently a quantum resonance, is absorbed by a quantum system (e.g. atom), there exists a resonance of the charged particle. A new higher energy resonance is formed, but the photon may decouple and form its own resonance again and be emitted. This emission seems to be of a different nature than that of the accelerating charge described by Maxwell's equations.

(As noted, it is possible to have radiation from quantum systems, i.e. a diatomic molecule driven by a laser, and this does seem to emit radiation in a way linked to Larmor's formula. In (1), the acceleration of such a system is given as: d/dt d/dt <W(x,t)|r|W(x,t)>. For a bound state, exp(iEt)exp(-iEt) causes time dependence to disappear and hence acceleration to be zero.)

Reference

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