An Antinomy

Ralf Wüsthofen

Notations. Let $\mathbb{N}$ denote the natural numbers starting from 1, let $\mathbb{N}_n$ denote the natural numbers starting from n > 1 and let $\mathbb{P}_3$ denote the prime numbers starting from 3.

SSGB: *Every even integer greater than 6 can be expressed as the sum of two different primes.*

Theorem. *Both SSGB and the negation ¬SSGB hold.*

Proof. $S_g := \{ (pk, mk, qk) \mid k, m \in \mathbb{N}; p, q \in \mathbb{P}_3, p < q; m = (p + q) / 2 \}$.

SSGB <=> $\forall x \in \mathbb{N}_4 \;\; \exists (pk, mk, qk) \in S_g \quad x = m$.

¬SSGB <=> $\exists n \in \mathbb{N}_4 \;\; \forall (pk, mk, qk) \in S_g \quad n \neq m$.

(C): $\forall k \in \mathbb{N} \;\; \exists (pk', mk', qk') \in S_g \quad nk = pk' \lor nk = mk' = 4k'$.

(M): $\nexists\, p, q \in \mathbb{P}_3, p < q \quad n = (p + q) / 2$.

$\forall y \in \mathbb{N}_3 \quad S_g = S_g+(y) \cup S_g-(y)$, where

$S_g+(y) := \{ (pk', mk', qk') \in S_g \mid \exists k \in \mathbb{N} \quad pk' = yk \lor mk' = yk \lor qk' = yk \}$,

$S_g-(y) := \{ (pk', mk', qk') \in S_g \mid \forall k \in \mathbb{N} \quad pk' \neq yk \land mk' \neq yk \land qk' \neq yk \}$.

¬SSGB => (($S_g = S_g+(n) \cup S_g-(n)$) or ¬(C) or ¬(M)).

¬SSGB => $S_g = S_g+(n) \cup S_g-(n)$.

(1): $\forall y \in \mathbb{N}_3$ ¬SSGB => $S_g = S_g+(y) \cup S_g-(y)$.

(2): $\forall y \in \mathbb{N}_3$ SSGB => $S_g = S_g+(y) \cup S_g-(y)$.

(3): $\forall y \in \mathbb{N}_3$ ((SSGB => $S_g = S_g+(y) \cup S_g-(y)$) and (¬SSGB => $S_g = S_g+(y) \cup S_g-(y)$)).

(3) => $\forall y \in \mathbb{N}_3$ ($\forall$ S (¬SSGB => S_g = S) <=> ($S_g+(y) \cup S_g-(y)$ = S))

<=> ($\forall$ S (¬SSGB => S_g = S) <=> (S_g = S)) <=> ¬SSGB.

(3) => $\forall y \in \mathbb{N}_3$ ($\forall$ S (SSGB => S_g = S) <=> ($S_g+(y) \cup S_g-(y)$ = S))

<=> ($\forall$ S (SSGB => S_g = S) <=> (S_g = S)) <=> SSGB. □