

$$f(t) = \cos(\omega t) = \cos(2\pi a t)$$

$$F(\nu) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \nu t} dt$$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

$$\cos(2\pi a t) = \frac{e^{i2\pi a t} + e^{-i2\pi a t}}{2}$$

$$F(\nu) = \int_{-\infty}^{\infty} \left(\frac{e^{i2\pi a t} + e^{-i2\pi a t}}{2} \right) e^{-2\pi i \nu t} dt$$

$$F(\nu) = \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{i2\pi t(a-\nu)} + \int_{-\infty}^{\infty} e^{i2\pi t(-a-\nu)} \right] dt$$

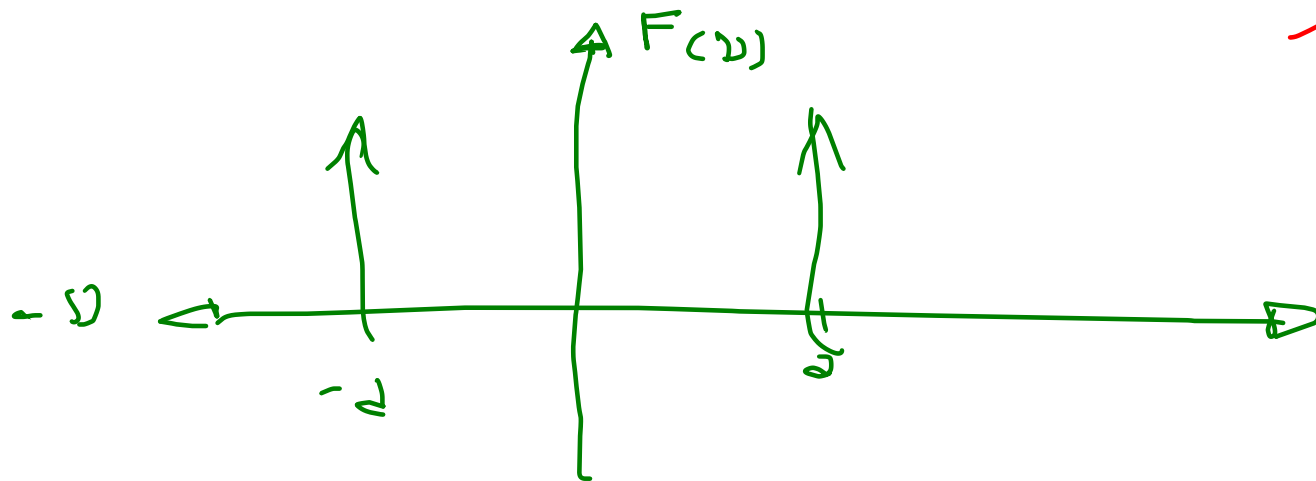
$$F(\nu) = \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{i2\pi t(a-\nu)} + \int_{-\infty}^{\infty} e^{i2\pi t(-a-\nu)} \right] dt$$

we know $\delta(t-a) = \int_{-\infty}^{\infty} e^{2\pi i \nu (t-a)} d\nu$

$$F(\nu) = \frac{1}{2} [\delta(-\nu+a) + \delta(-\nu-a)]$$

$$2\pi a = \omega$$

$$a = \omega / 2\pi$$



Lap. Trans

Time

Laplace

$$f(t) \longleftrightarrow F(s)$$

$$\dot{f}(t) \longleftrightarrow sF(s) - f(0)$$

$$\ddot{f}(t) \longleftrightarrow s^2 F(s) - s\dot{f}(0) - \ddot{f}(0)$$

Ex 4 $m\ddot{u} + c\dot{u} + ku = f(t)$

$$m \mathcal{L}[\ddot{u}(t)] + c \mathcal{L}[\dot{u}(t)] + k \mathcal{L}[u(t)] = \mathcal{L}[f(t)]$$

$$m [s^2 U(s) - s u(0) + \dot{u}(0)] + c [s U(s) - u(0)] + k U(s) = F(s)$$

$$U(s) [ms^2 - cs - k] - [ms \cancel{u(0)} + m \cancel{\dot{u}(0)} + c \cancel{u(0)}] = F(s)$$

$$U(s) [ms^2 - cs - k] = [\cancel{ms^2(0)} + m\cancel{z(0)} + \cancel{c(0)}] = F(s)$$

$$U(s) [ms^2 + cs + k] = F(s)$$

$$H(s) = \frac{U(s)}{F(s)}$$

$$H(s) = \frac{1}{ms^2 + cs + k}$$

$$H(i\omega) = \frac{1}{-m\omega^2 + i\omega c + k}$$

$$A_s = \frac{A_o}{A_i}$$

$$|H(i\omega)| = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

i] $A_o = 1$ & $\phi_o = 0$

$$A_i = \sqrt{\text{Re}^2 + \text{Im}^2}$$

$$A_i = \sqrt{(k - m\omega^2)^2 + (\omega c)^2}$$

$$A_i = \sqrt{(1 - \beta^2)^2 + (2\xi\beta)^2}$$