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INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI: 10.21474/IJAR01/12466

DOI URL: <http://dx.doi.org/10.21474/IJAR01/12466>



RESEARCH ARTICLE

COMPARISON BETWEEN YOUNG'S DOUBLE SLIT AND $\Lambda - \delta$ AND $V - \delta$ TYPE THREE LEVEL ATOMS PRODUCING QUANTUM INTERFERENCE

A. Hazarika¹, L.K. Rajkhowa², J. Saikia³ and G.D. Baruah²

1. District Institute of Education and Training, Darrang, Dalgaoon.
2. Centre for Laser & Optical Science, New Uchamati, Doomdooma.
3. Department of Physics, J.B. College (Autonomous), Jorhat.

Manuscript Info

Manuscript History

Received: 10 December 2020

Final Accepted: 14 January 2021

Published: February 2021

Key words:-

Double Slit, $\Lambda - \delta$ Type Atom, $V - \delta$
Type Atom

Abstract

We make a comparison between the concepts and physics behind Young's double slit experiment and $\Lambda - \delta$ and $V - \delta$ type three level atoms producing quantum interference.

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Introduction:-

In the present work we shall concern ourselves with two topics of considerable interest. Young's double slit experiment leading to interference. $\Lambda - \delta$ and $V - \delta$ type atoms leading to interference resulting lasing without inversion (LWI). These two topics are well known in quantum optics. Interference is the phenomenon of superposition of two or more waves resulting in a new wave pattern. In the seventeenth century, Sir Issac Newton (1642-1726) proclaimed that light consisted of particles. In the early part of seventeenth century, Christian Huygens (1629-1695) enunciated a convenient working principle to describe how the progress of the primary wave from a source of light is due to the generation of the secondary wavelets from every point of the primary wave front. We may indicate here that Sir Issac Newton could not measure the mass of the corpuscles, but Huygens could measure the wavelength of the light waves. Thomas Young devised the double slit experiment to prove that light consisted of waves. It is worthwhile to introduce the topic of quantum interference which is so important from quantum mechanical point of view. Nowadays the experiment which was performed by Thomas Young in 1801 for light waves is also used for electrons, neutrons and even molecules as big as the soccer ball like fullerene C_{60} . All these cases exhibit the same kind of interference. In all these cases interference is observed even if the particles are shot one at a time. According to Richard Feynman [1] double slit experiment is in the heart of quantum mechanics. The concept of quantum interference states that the elementary particles such as photons can be at more than one place at a given time (through the principle of superposition) and the individual particles such as photon can cross its own trajectory and interfere with itself with the direction of its path. Although the implications of Young's double slit experiment are somewhat difficult to accept it has produced reliable proof of quantum interference through repeated experiments. The topic of quantum interference has already led to many counterintuitive phenomena like lasing without inversion (LWI), coherent trapping etc [2 – 7]. The central idea of lasing without inversion is that absorption cancellation provides the possibility of generating light amplification even if there is no population inversion. Such a situation can be realized in a three level system, when two coherent atomic transitions destructively interfere and as a result, cancel absorption. The three level schemes may be either a $V - \delta$ scheme or a $\Lambda - \delta$ scheme.

Corresponding Author:- A. Hazarika

Address:- District Institute of Education and Training, Darrang, Dalgaoon.

In the present work we make an attempt to establish a satisfactory analogy between double slit experiment and three level atomic schemes leading to LWI in actual experiment. The root of the analogy is the fact that both double slit and LWI three level schemes are, to a conceptual idealization, are interference experiments. The paper is organized in the following manner. In section 2 classical interference of light waves and quantum interference is briefly described to emphasize the characteristic features of two slit experiment. In section 3 we describe the basic physics involved in $V - \delta$ and $\Lambda - \delta$ scheme of three level atoms leading to lasing without inversion (LWI). In section 4 a comparative analysis of double slit scheme and three level schemes is presented. Section 5 presents an outlook and concluding remark.

Classical interference of light waves:

For convenience we describe the standard double slit interference experiment which will be related to the three level atomic schemes. The standard double slit experiment is shown in Fig. 1.

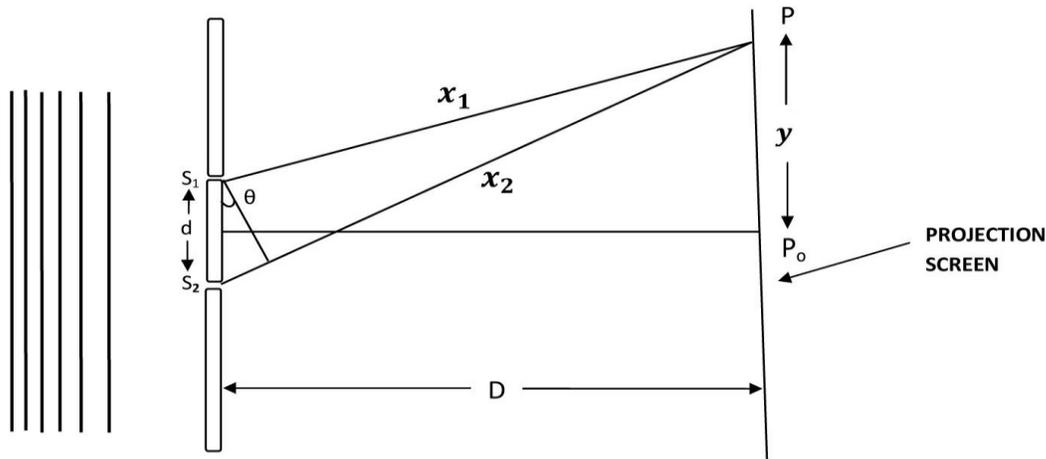


Fig 1. Classical double slit interference

Let us consider a point P on the screen which is located at a distance y from the central line. Monochromatic light from the first slit S₁ travels a distance x₁ to reach this point, and light from second slit S₂ travels a slightly longer distance to reach this point. It is easily shown that

$$x_2 - x_1 = \Delta x = \frac{d}{D}y \tag{2-1}$$

Provided $d \ll D$ and the wave function at the point P can be written as

$$\Psi(y, t) \equiv \Psi_1(t)e^{ikx_1} + \Psi_2(t)e^{ikx_2} \tag{2-2}$$

where $\Psi_1(t)$ = wave function at the first slit

$\Psi_2(t)$ = wave function at the second slit

Since the two slits are assumed to be illuminated by in phase light waves of equal

$$\Psi_1 = \Psi_2 \tag{2-3}$$

It is worthwhile to note we are ignoring the difference in amplitude of the waves from the two slits at the screen, due to the slight difference between x_2 and x_1 compared to the difference in their phase. For $D \gg \lambda$ this is a reasonable approximation. The energy flux or the intensity of the light at some point on the screen is approximately equal to the energy density of the light at this point times the velocity of light provided $y \ll D$. Hence it follows that the intensity on the screen at a distance y from the central line is given by

$$I(y) \equiv |\Psi(y, t)|^2 \tag{2-4}$$

Using Eqn. (1) - (4) we obtain

$$I(y) \propto \cos^2\left(\frac{k \Delta x}{2}\right) = \cos^2\left(\frac{kd}{2D}y\right) \tag{2-5}$$

Fig. 2:- shows the well known characteristic interference pattern which corresponds to the expression (5).

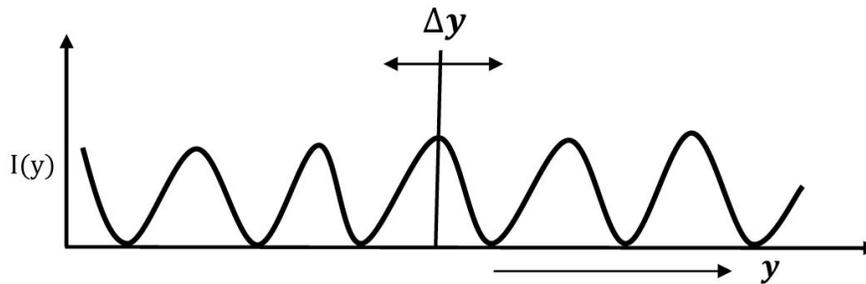


Fig 2 : Double slit interference pattern

Let us now consider the phenomenon of quantum interference in a double slit. According to Feynman the essentials of quantum mechanics could be grasped from an explanation of the double slit experiment. What is actually happening in the double slit experiment is clearly described by Feynman [1] and also by others. In the double slit, if one slit is covered the pattern is what would be expected, a single line of light (the image of the slit) aligned with whatever slit is open. One would expect that if both slits are open, the pattern of light would reflect the fact, the two lines of light, aligned with the slits. In fact, however, what happens is that the photographic film or the recorder as the projection screen is entirely separated into multiple lines with alternate dark and bright intensities. This is what is known as interference, taking place between waves or particles going through the slits, in what seemingly should be two non crossing trajectories. We would also expect that if the beam of photons is slowed enough to ensure that individual photons (or so called anti-bunched photons) are hitting the photographic film (like the experiment of Taylor [8] performed in the year 1909) there would be no interference and the pattern of light would be two lines of light, aligned with the slits. In fact, however, the resulting pattern still indicates interference which means that somehow, the singles photons are interfering with themselves. Many believe that Dirac's famous remark [9] that "each photon interferes only with itself, interference between two photons does not occur" originated from Taylor's experiment. In any case this seems impossible because we expect that a single photon will go through one slit or other and end up in one of the two possible light line areas. But this is not what is happening. According to Feynman each photon not only goes through both the slits simultaneously but traverses every possible trajectory on way to the target, just not in theory but in fact. In order to see how this might possibly occur, experiments have focused on tracking the paths of individual photons. However, in these cases what happens is that measurements in some way disrupts the photon trajectories (in accordance with uncertainty principle), and somehow the results of experiments become what would be predicted by classical physics. The two-slit interferometer was examined by Sudarshan and Rothman [10] and they showed that the standard exposition of the double slit experiment is incorrect because it treats the interference as arising from the photon wave function Ψ whereas the interference is really between the coherent states of the field which do not correspond to the single photon states. According to Sudarshan and Rothman [10] a close examination of the standard exposition of two-slit experiment reveals several ambiguities and conceptual errors and a correct treatment requires a field theoretic approach. According to them a coherent state is constructed in much the same way as state vectors are assembled in quantum mechanics

$$|\Phi(r)\rangle = \sum_k u_k |z_k\rangle,$$

Where the coherent state $|\Phi\rangle$ propagates, the mode function propagates to new mode frequencies. Since each mode function behaves independently of the other, Φ itself propagates as if it were classical as it did in the one mode way. The case of many sources – in phase or out of phase – is treated in the same way because any state of illumination can be obtained by a suitable weighted average of coherent states. If the sources are in transient phase, they must show transient interference. Sudarshan [10] particularly indicates this to remind us that many are under the spell of Dirac's famous statement which states that each photon interferes with itself-interference between two photons never occurs. That this is not true can be verified by anyone who has turned on a car radio to listen to a jammed BBC in Eastern Europe. If one regards the interference as taking place between coherent states then the question about what constitute a photon disappears. Sudarshan and Rothman [10] further concludes that an experiment carried out by reducing the intensity of a light beam will never produce an electric field that corresponds to a single photon and the single photons produced from decays or atomic transitions do not have an electric field associated with them. They also remarked that entire discussion is concerned with far field intensity pattern. If the screen were

placed immediately behind the slits one merely records two localized intensities that in no way distinguish between particles and waves. If the screen were placed very close to the slits one finds only two localized regions of uniform illumination with a small interference pattern between them. In this connection it is worthwhile to discuss the relevant paragraphs regarding photon in the text of Laser physics by Sargent, Scully and Lamb Jr.[11]. They write “photons are quanta of single(monochromatic) mode of the radiation field and are not localized at any particular position and time within the cavity like fuzzy balls; rather, they are spread out over the entire cavity. In fact no satisfactory quantum theory of photons as particles has ever been given”. As regards Dirac’s statement they further adds that there is an apparent contradiction in Dirac’s statement when it is well known that two separated radio transmitters can produce interference effects and for that matter, two lasers can as well. The difficulty disappears when one remembers that both transmitters are coupled to the modes of the universal radiation field. A photon is simply a particular energy eigen-state of one such radiation mode.

Quantum interference in Lasing without inversion:

In this section we shall concern ourselves with the topic of lasing without inversion (LWI) which is the result of quantum interference. It is worthy of remark that the main idea of (LWI) is that absorption is cancelled and the process leads to amplification (and laser) even if the population inversion is not present. Such a situation can be realized in three-level system, when two coherent atomic transitions interfere destructively and leads to cancellation of absorption.

The configuration of a $\Lambda - \delta$ three level system is shown in Fig 3. It is formed by an upper level $|a\rangle$ which is connected to two closely lying lower levels $|b\rangle$ and $|c\rangle$ through interaction with electromagnetic fields E_1 and E_2 respectively, in such a way that only atomic transitions $|a\rangle - |c\rangle$ and $|a\rangle - |b\rangle$ are allowed. The physical reason for cancelling absorption in this case is the uncertainty in atomic transitions $|c\rangle - |a\rangle$ and $|b\rangle - |a\rangle$ which results in destructive interference between them. Since both these transitions are directed to the same atomic state $|a\rangle$ it is impossible to find out along which path, $|c\rangle - |a\rangle$ or $|b\rangle - |a\rangle$ such a transition is made. This situation is similar to Young’s double slit interferometer, where interference is a consequence of uncertainty in finding through which of the two slits the photon passed[12].

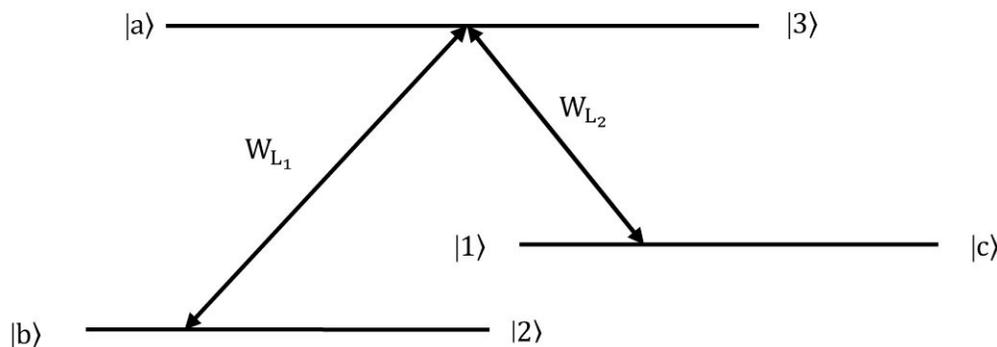


Fig 3: $\Lambda - \delta$ configuration of a three level atom interacting with two fields.

Referring to Fig 3 we note that the absorption probability will be equal to the squared sum of the probability amplitudes corresponding to $|c\rangle - |a\rangle$ and $|b\rangle - |a\rangle$ transitions. When there is a correlation between these probability amplitudes it will lead to an interference term which, under appropriate phase condition can make the absorption produce equal to zero. This is the clue to the phenomenon of Lasing without Inversion (LWI). That is, it is generally accepted that one needs population inversion in order to overcome absorption to get laser action. But the emission probability, which is the sum of the transition probabilities $|a\rangle - |c\rangle$ and $|a\rangle - |b\rangle$ and is independent of their mutual correlation. This results from different final states $|b\rangle$ and $|c\rangle$. In this case it is known exactly along which path the atom makes a transition to the lower levels $|a\rangle$ and $|b\rangle$, so there is no uncertainty in atomic routes and as a result there is no interference between these transitions. Thus there is an asymmetry between up and down transitions and this leads to amplification.

How this is taking place. To know this it is worthwhile to follow the approach adopted by Scully and Zubairy [13]. The method adopted by them is semi-classical where the electromagnetic field is considered classically but the atom is treated quantum mechanically. The main idea is to calculate the time dependent probability amplitudes for each level and then show that the probability of transition to the upper level can vanish for particular initial conditions but the transition probability to a lower level does not vanish. The Hamiltonian for the atomic system as shown in Fig 3 is written as

$$H = H_0 + H_1 \quad (3-1)$$

$$\text{where } H_0 = \hbar\omega_a|a\rangle\langle a| + \hbar\omega_b|b\rangle\langle b| + \hbar\omega_c|c\rangle\langle c| \quad (3-2)$$

$$H_1 = -\frac{\hbar}{2}(\Omega_{R_1} e^{-i\Phi_1} e^{-iW_{L_1}t}|a\rangle\langle b| + \Omega_{R_2} e^{-i\Phi_2} e^{-iW_{L_2}t}|a\rangle\langle c|) + H.C \quad (3-3)$$

H_0 and H_1 represent the unperturbed and interaction part of Hamiltonian respectively, and

$$\Omega_{R_1} e^{-i\Phi_1} = \frac{\wp_{ba} E_1}{\hbar}; \quad \Omega_{R_2} e^{-i\Phi_2} = \frac{\wp_{ca} E_2}{\hbar} \quad (3-4)$$

$\Omega_{R_1} e^{-i\Phi_1}$ and $\Omega_{R_2} e^{-i\Phi_2}$ are complex Rabi frequencies associated with the interaction of the electromagnetic fields E_1 and E_2 of frequencies W_{L_1} and W_{L_2} with atomic transitions $|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$ respectively. The matrix elements of the electric dipole moment corresponding to atomic transitions $|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$ are

$$\wp_{ba} = e\langle b|r|a\rangle; \quad \wp_{ca} = e\langle c|r|a\rangle \quad (3-5)$$

The wave function of the three level atomic system is

$$\Psi(t) = C_a(t)|a\rangle + C_b(t)|b\rangle + C_c(t)|c\rangle \quad (3-6)$$

To find the probability amplitudes $C_a(t)$, $C_b(t)$, $C_c(t)$ we solve the Schrödinger equation

$$i\hbar|\dot{\Psi}(t)\rangle = H|\Psi(t)\rangle \quad (3-7)$$

Solving Schrödinger equation the probability amplitudes for the atomic states can be found for an arbitrary choice of initial conditions $C_a(t=0)$, $C_b(t=0)$, $C_c(t=0)$. To calculate the absorption probability we consider the initial state of the system for which the population is equally distributed with fixed phases between the two lower states $|b\rangle$ and $|c\rangle$, i.e. atomic system in low states at $t=0$. Mathematically this statement can be written as

$$C_a(0) = 0, C_b(0) = \frac{1}{\sqrt{2}}, C_c(0) = \frac{1}{\sqrt{2}} e^{-i\Psi} \quad (3-8)$$

The solution of the Schrödinger equation for this set of initial condition under assumption of resonance $W_{ab} = W_{L_1}$ and $W_{ac} = W_{L_2}$ to the lowest order in time gives the following result for the probability amplitude of the upper level

$$C_a(t) = i \frac{t}{2\sqrt{2}} (\Omega_{R_1} e^{-i\Phi_1} + \Omega_{R_2} e^{-i(\Phi_2 + \Psi)}) \quad (3-9)$$

In this equation the first and second terms of the sum represent the probability amplitudes corresponding to transitions from $|b\rangle - |a\rangle$ and $|c\rangle - |a\rangle$ respectively. The absorption probability in this case is

$$|C_a(t)|^2 = P_a = t^2 \Omega_R^2 [1 + \cos(\Phi_1 - \Phi_2 - \Psi)]/4 \quad (3-10)$$

where we have taken $\Omega_{R_1} = \Omega_{R_2} = \Omega_R$

From Eqn.(3-10) we find that the absorption probability $P_a = |C_a(t)|^2 = 0$ when $\Phi_1 - \Phi_2 - \Psi = \pm \Pi$. The atomic system will stay at low energy levels $|c\rangle$ and $|b\rangle$ at all times for these specific phase conditions. Since there are no transitions to higher energy levels the system will have no absorption. Now let us find the emission probability. Suppose that initially the population is in the upper state i.e. $C_a(0) = 1$, and $C_b(0) = 0$, $C_c(0) = 0$. The solution of the Schrödinger equation (3-7) for these initial conditions assuming

$$(\Omega_{R_1}^2 + \Omega_{R_2}^2)^{1/2} t = \Omega_t \ll 1$$

gives the following approximate results

$$C_b(t) = \frac{i\Omega_{R_1}^* t}{2}, C_c(t) = \frac{i\Omega_{R_2}^* t}{2} \quad (3-11)$$

The emission probability is equal to the sum of the squared probability amplitudes associated with atomic states $|b\rangle$ and $|c\rangle$, i.e.,

$$P_e = P_b + P_c = |C_b(t)|^2 + |C_c(t)|^2 = \frac{\Omega^2 t^2}{4} \quad (3-12)$$

Thus we observe that the emission probability is independent of the relative phase between atomic states $|b\rangle$ and $|c\rangle$, because first the probability amplitudes are squared and only then summed. But for the probability of absorption, first the probability amplitudes are summed and only then squared. That is why the absorption probability is mathematically dependent on the relative phase between the atomic transitions. One can identify from Eqn (3-12) that the emission probability is always non zero for $t > 0$. Therefore if the atomic system is prepared with phase conditions as described earlier; it is possible to have net gain even when there is no population inversion. This leads to the process of lasing without inversion (LWI) which has been demonstrated experimentally.

The Comparison:

The basic facts involved in a double slit interferometer leading to interference and also the basic concepts involved in a three level atom in a $\Lambda - \delta$ configuration leading to lasing without inversion as a result of destructive interference; have been considered in earlier sections. We now turn to a comparison between the two situations. Before placing the comparison in a tabular form let us discuss the salient features of the comparison. The slit width is the basic topic. We observe here that a simple demonstration of Young’s experiment can be made by constructing a double slit in an exposed film by drawing the point of a penknife across the film guided by a straightedge. A source of light such as a bulb is now viewed by holding the double slit close to the eye and looking at the source. If the slits are close together, for example 0.2 mm apart they give widely spaced fringes, whereas slits further apart, for example 1 mm, give very narrow fringes. When the double slit separation is about 1.5 mm one can observe still narrower fringes. The retina of the eye acts as the projection screen. A piece of red glass placed adjacent to and above another of green glass in front of the lamp source will show that the red waves produce wider fringes than the green which is due to their greater wavelengths. This situation is described by Klauder and Sudarshan [14] in a similar manner. According to them, on the basis of the standard classical wave theory a single wave incident on the two slits undergoes subsequent self interference which may lead to complete destructive interference at various points of observation in the point of observation of the projection screen. The possibility of such destructive interference is closely related to the existence of a definite phase relationship between the constituent signals and under these circumstances we may say that these two beams are coherent. But this picture of complete destructive interference may not be in full agreement with the experimental results of observations for usual thermal source the angle subtended by the source at the projection screen is not too small. When such a source is moved inward toward the screen, thus increasing the angle subtended, it usually occurs that the interference pattern gradually washes out and under this circumstances the maximum and minimum intensities become less pronounced. As a quantitative measure of this aspect Sudarshan [14] has introduced, following Michelson the parameter visibility given by

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Where I_{\max} and I_{\min} are the intensities at the maxima and minima of fringe pattern. The more slowly V decreases with increasing path difference, the sharper the line with red cadmium line, it dropped to 0.5 at a path difference of 10 cm or at $d=5$ cm. With certain lines, the visibility does not decrease uniformly but fluctuates with more or less regularly. This behaviour shows that the line has a fine structure, consisting of two or more lines.

Let us now try to analyse how the discussions as above are compatible with the observations in a $\Lambda - \delta$ type (or $V - \delta$ type) atomic configurations. First consider the slit width. In case of Young’s experiment the slit width is about 0.2 mm to 1 mm. But the energy level separation between two closely lying levels in an actual $\Lambda - \delta$ atomic configuration, such as the hyperfine energy level separation in Na atom is 1770 MHz as shown in Fig 4. In this figure the relevant energy levels of Na atom showing hyperfine structure in the presence of a weak magnetic field are shown. Appropriate levels of $F = 1$, $F = 2$ and $F = 1'$ would each correspond to the levels $|1\rangle, |2\rangle$ and $|3\rangle$ of the $\Lambda - \delta$ scheme of the Fig 3.

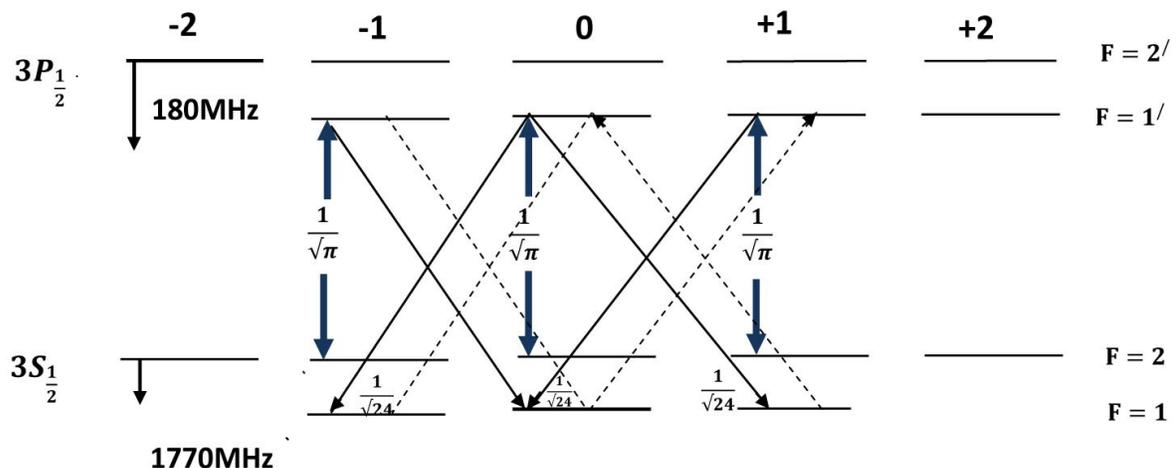


Fig. 4:- Relevant energy levels of Na atom showing hyperfine structure in a weak magnetic field. Appropriate levels of $F=1$, $F=2$ and $F=1'$ would each correspond to the levels $|1\rangle, |2\rangle$ and $|3\rangle$ of the $\Lambda - \delta$ schemes.

From Fig. 4 we infer that the separation of the energy levels corresponding to the separation of the slits in the double slit experiment is very small. From the discussion made by Feynman [1] we reasonably infer that the interference observed in Young's double slit is quantum interference. Now the question arises about the projection screen in the $\Lambda - \delta$ type atomic configuration.

In the case of Young's double slit experiment we have the projection screen where interference pattern appears. Where is the corresponding projection screen in the $\Lambda - \delta$ configuration? As an analogous situation in the $\Lambda - \delta$ configuration we may reasonably infer that the projection screen is at the position of the energy level $|a\rangle$ which is at a distance $|a\rangle - |b\rangle$ or $|a\rangle - |c\rangle$. As $|a\rangle$ comes closer and closer to the levels $|b\rangle$ or $|c\rangle$ quantum interference will be less pronounced and when $|a\rangle$ is very close to the low lying levels quantum interference is completely destroyed. This is identical to the parameter V which is referred to as visibility of fringes by Sudarshan [14] (following Michelson). Similarly for a fixed position of $|a\rangle$ if the separation of the energy levels $|b\rangle$ and $|c\rangle$ increases quantum interference gradually washes out. In the double slit experiment it was observed that as the slit width increases fringes of interference become narrower and narrower. From what has been discussed above it is reasonable to infer that quantum interference in an atomic configuration such as $\Lambda - \delta$ or $V - \delta$ schemes may be used to manipulate spontaneous emission. We have observed earlier that Eqn. (3-10) shows the probability of absorption $|C_a(t)|^2 = 0$ when $\Phi_1 - \Phi_2 - \Psi = \pm \Pi$. Under these conditions population is trapped in the lower states $|b\rangle$ and $|c\rangle$ and there is no absorption even in the presence of the field. This is what is known as dark states. Independently of these speculations Alzetta et al [15] from Pisa reported the first experimental evidence of atomic interference in agreement with the simple model discussed in section 3. It is worthy of remark that Eqn. (3-10) can be put in the form of Eqn. (2-5) of the double slit experiment. But the information carried by Eqn. (3-10) cannot be obtained in Eqn. (2-5). Cosine term indicates interference in both the cases.

It now appears appropriate to conclude the comparison between Young's double slit and $\Lambda - \delta$ atomic configuration in a tabular form with some comments on the applications of such comparisons. This is shown in Table 1.

Table 1. Summary of comparison between the Double slit experiment and three level atomic configurations ($\Lambda - \delta$ scheme).

Topic	Young's Double Slit	$\Lambda - \delta$ atomic configuration
System	To straight narrow opening placed in front of a source and a projection screen	Three level atomic system with lower two levels are very close together
Slit separation(d)	$\sim .2 \text{ mm}^{-1} \text{ mm}$	$\sim 1770 \text{ MHZ}$
Intensity	$I(y) = \cos^2\left(\frac{kd}{2D}y\right)$ $= \cos^2\left(\frac{\delta}{2}\right)$	$ C_a(t) ^2 = t^2 \Omega_R^2 [1 + \cos(\Phi_1 - \Phi_2 - \Psi)]/4$
Visibility	$V_f = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$ ($V_f = \delta$ Visibility of fringes)	$V_q = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$ ($V_q = \delta$ Visibility of quantum interference)
Phase	$I(y) = 0$ for $\delta = \Pi, 3\Pi, 5\Pi \dots$	$ C_a(t) ^2 = 0$, when $\Phi_1 - \Phi_2 - \Psi = \pm \Pi$ population is trapped in the lower states $ b\rangle$ and $ c\rangle$
Conservation of energy	No violation of the law of conservation of energy is involved in the interference experiment.	No violation of the law of conservation of energy is involved.
Projection Screen	As Young's experiment is performed $D \gg d$ (Usually thousand times larger than d)	In $\Lambda - \delta$ configuration for Na atom ($^3P_{1/2} - ^3S_{1/2}$) $> 1770 \text{ MHZ}$
Application	Young's double slit experiment leading to interference patterns has numerous applications that include Michelson's Stellar interferometer, correlation interferometer and many more.	$\Lambda - \delta$ type atomic configuration leading to quantum interference via atomic coherence has application as Lasing with Inversion.

Conclusion:-

We may appropriately conclude this work with some comments on the comparison between double slit and $\Lambda - \delta$ or $V - \delta$ atomic configuration. Both are quite analogous. We believe that such comparison of analogous situations will help us to understand better the atomic coherence effects so that they may be used for practical applications.

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