# Computer-Aided Analysis of Three-Phase Rectifiers

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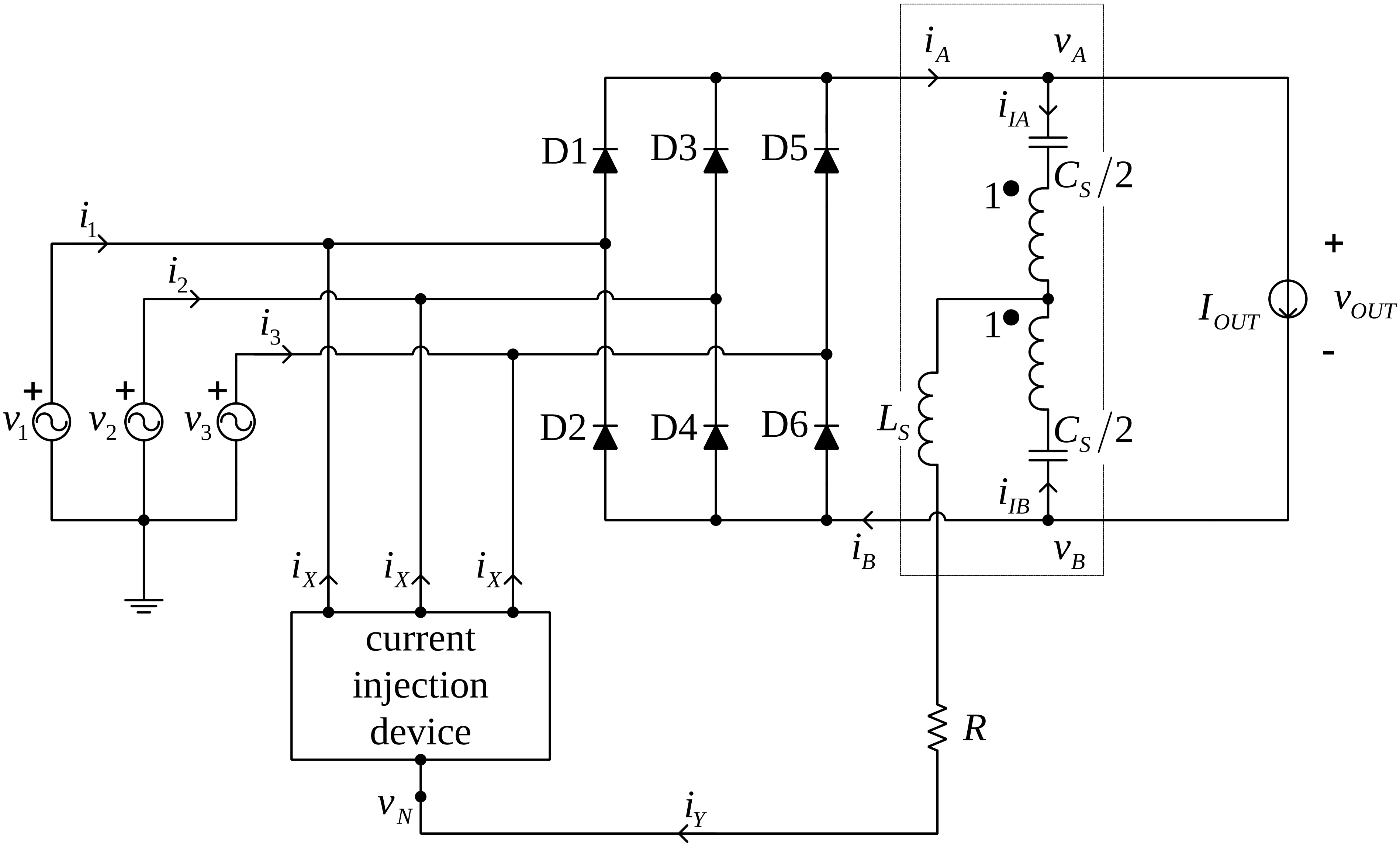
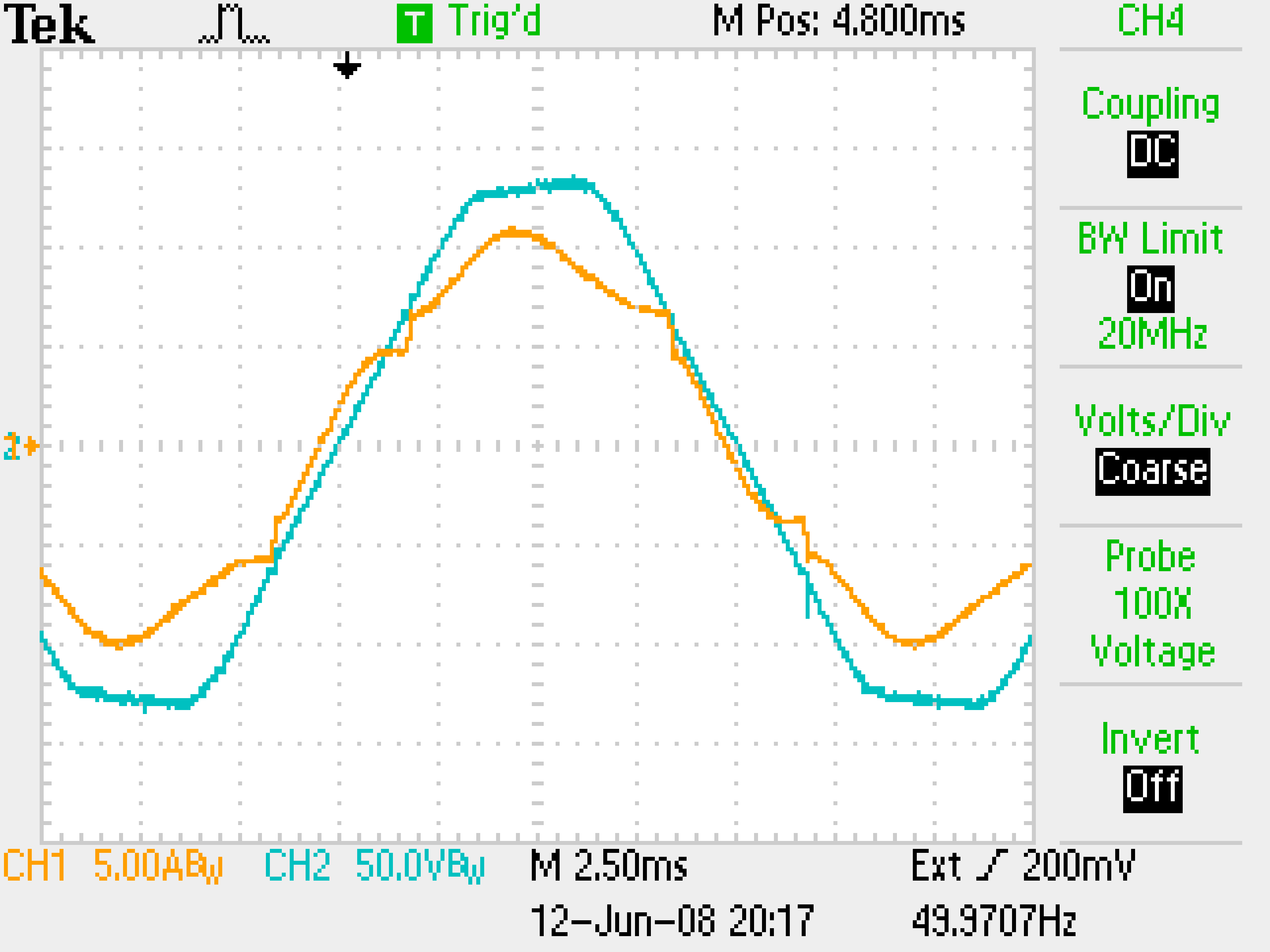
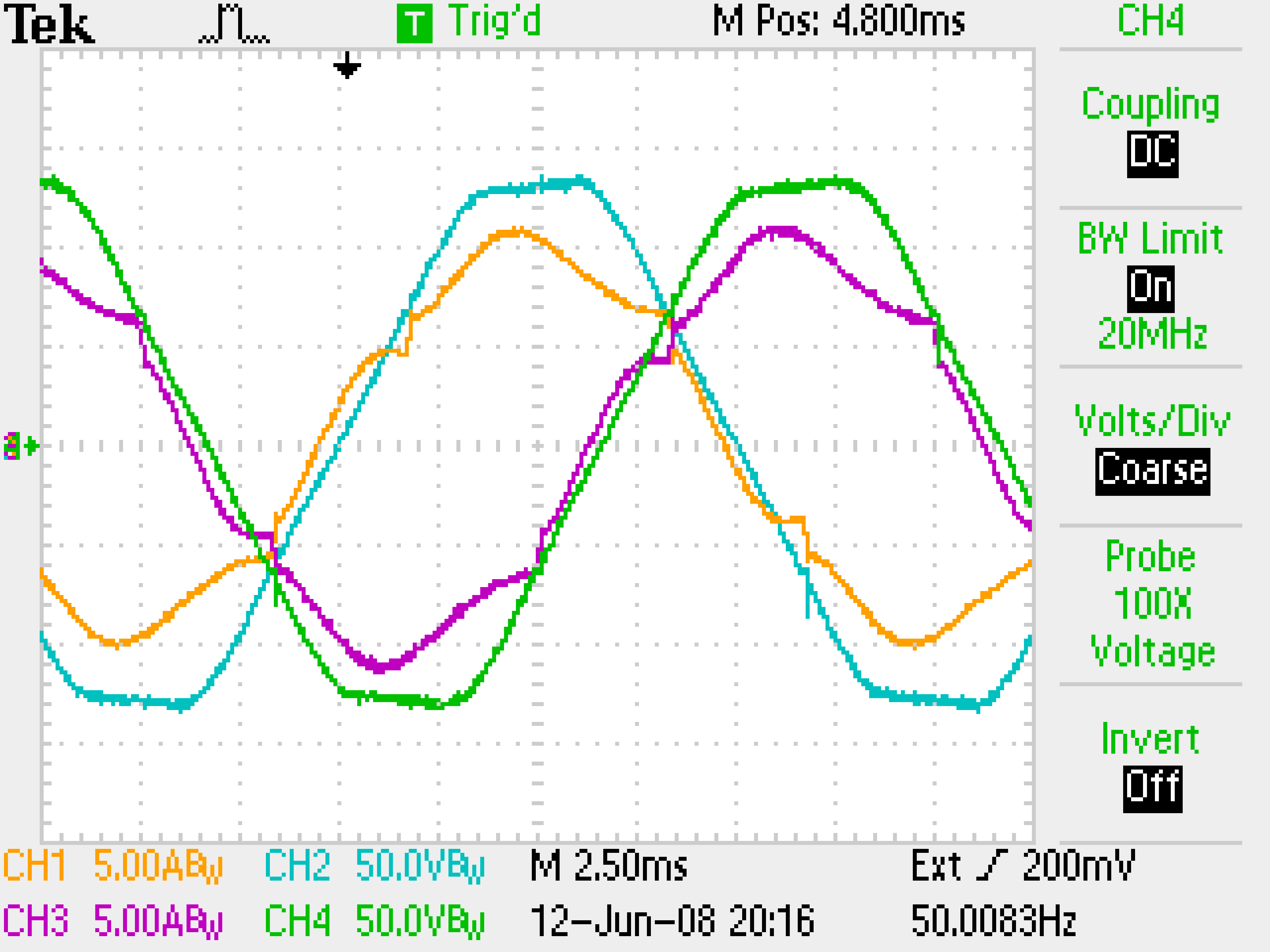
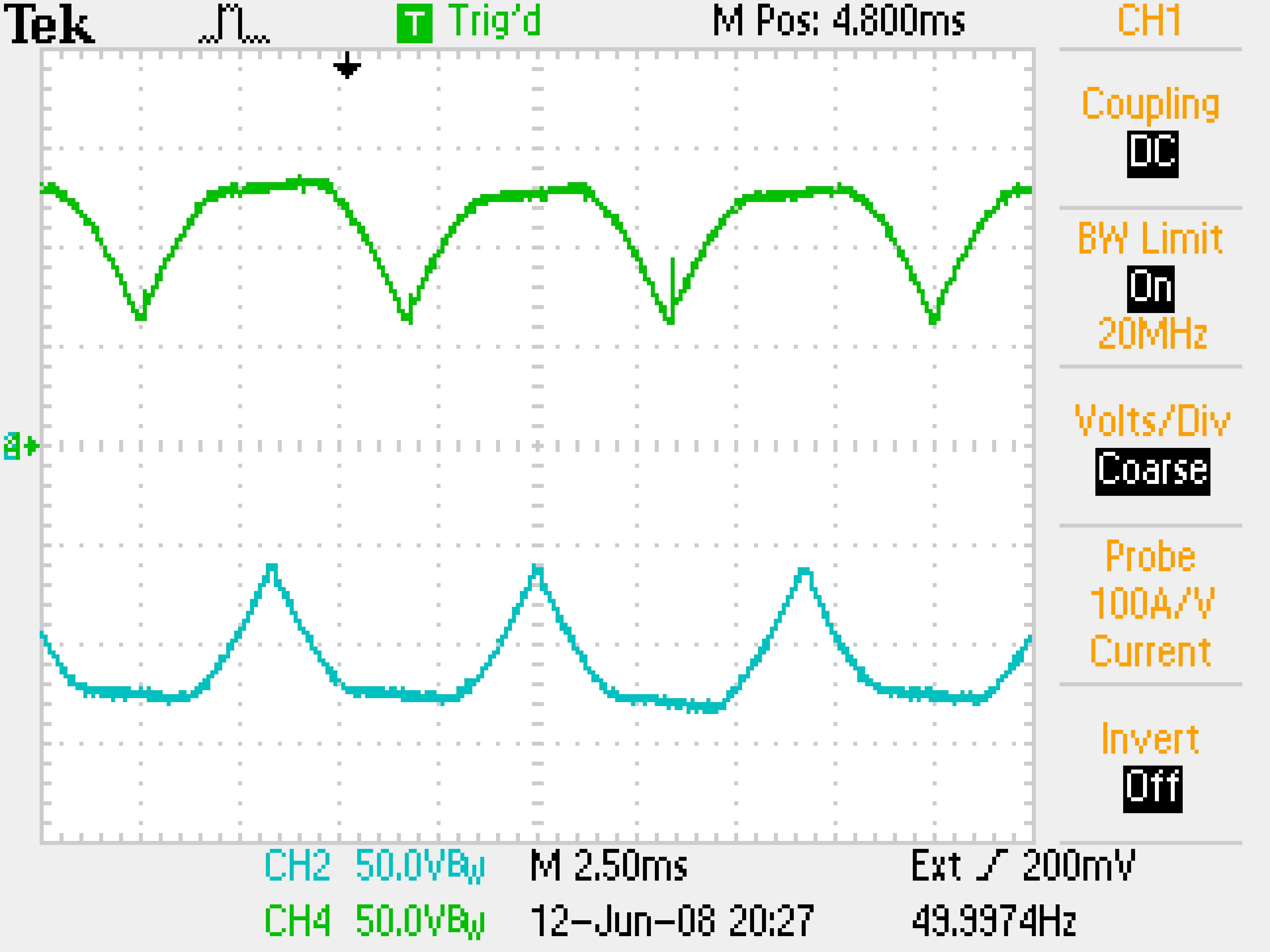
a problem:



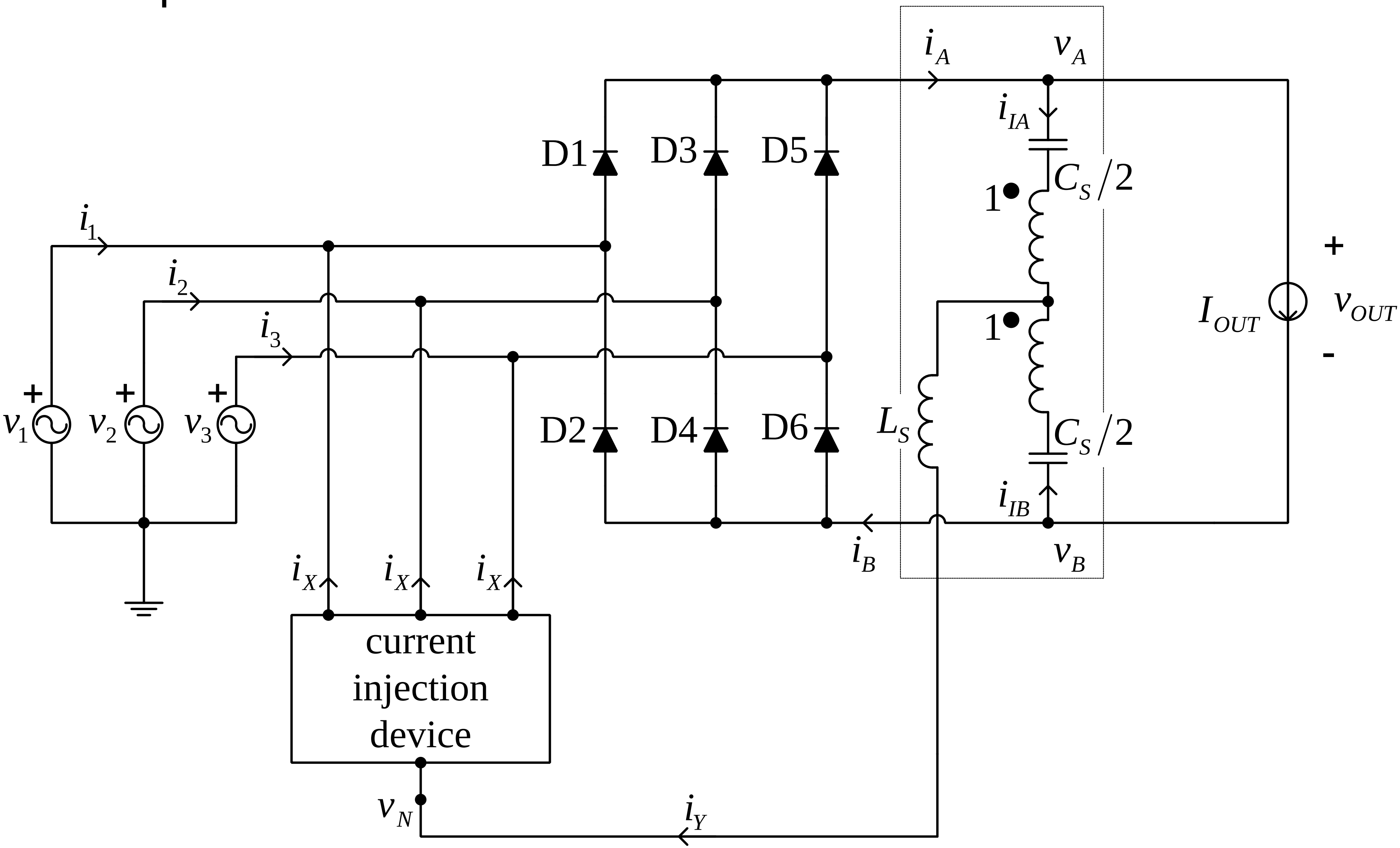
$i_A$
$v_A$
$i_{IA}$
D1
D3
D5
$C_S/2$
1
$i_1$
$i_2$
$i_3$
1
+
$I_{OUT}$
$v_{OUT}$
-
+
+
+
$v_1$
$v_2$
$v_3$
D2
D4
D6
$L_S$
$C_S/2$
$i_{IB}$
$i_B$
$v_B$
$i_X$
$i_X$
$i_X$
current
injection
device
$R$
$v_N$
$i_Y$

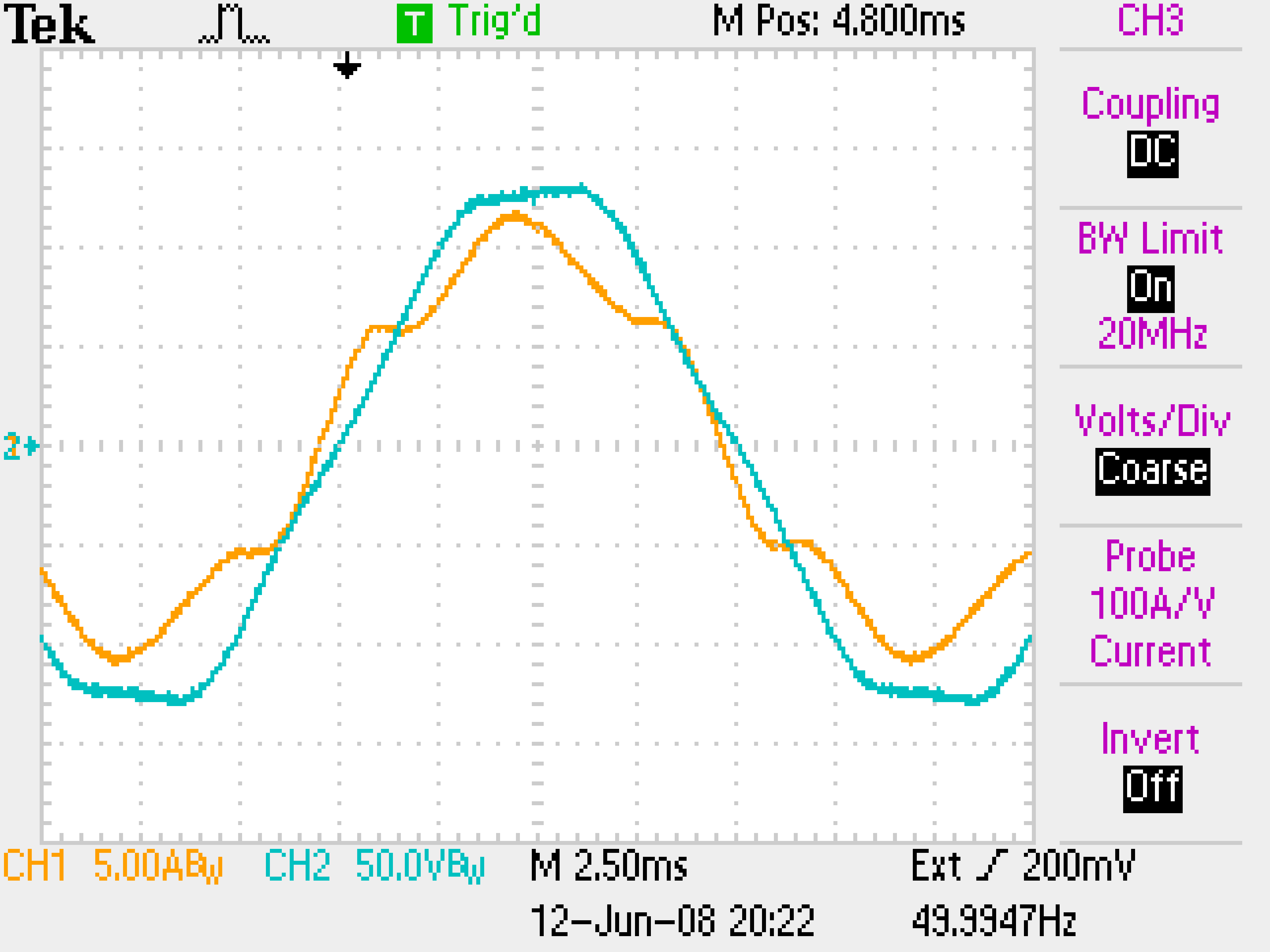

Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00AΩ
CH2 50.0VΩ
M 2.50ms
Ext ⟋ 200mV
12-Jun-08 20:17
49.9707Hz


Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
CH3 5.00A
CH4 50.0V
12-Jun-08 20:16
50.0083Hz

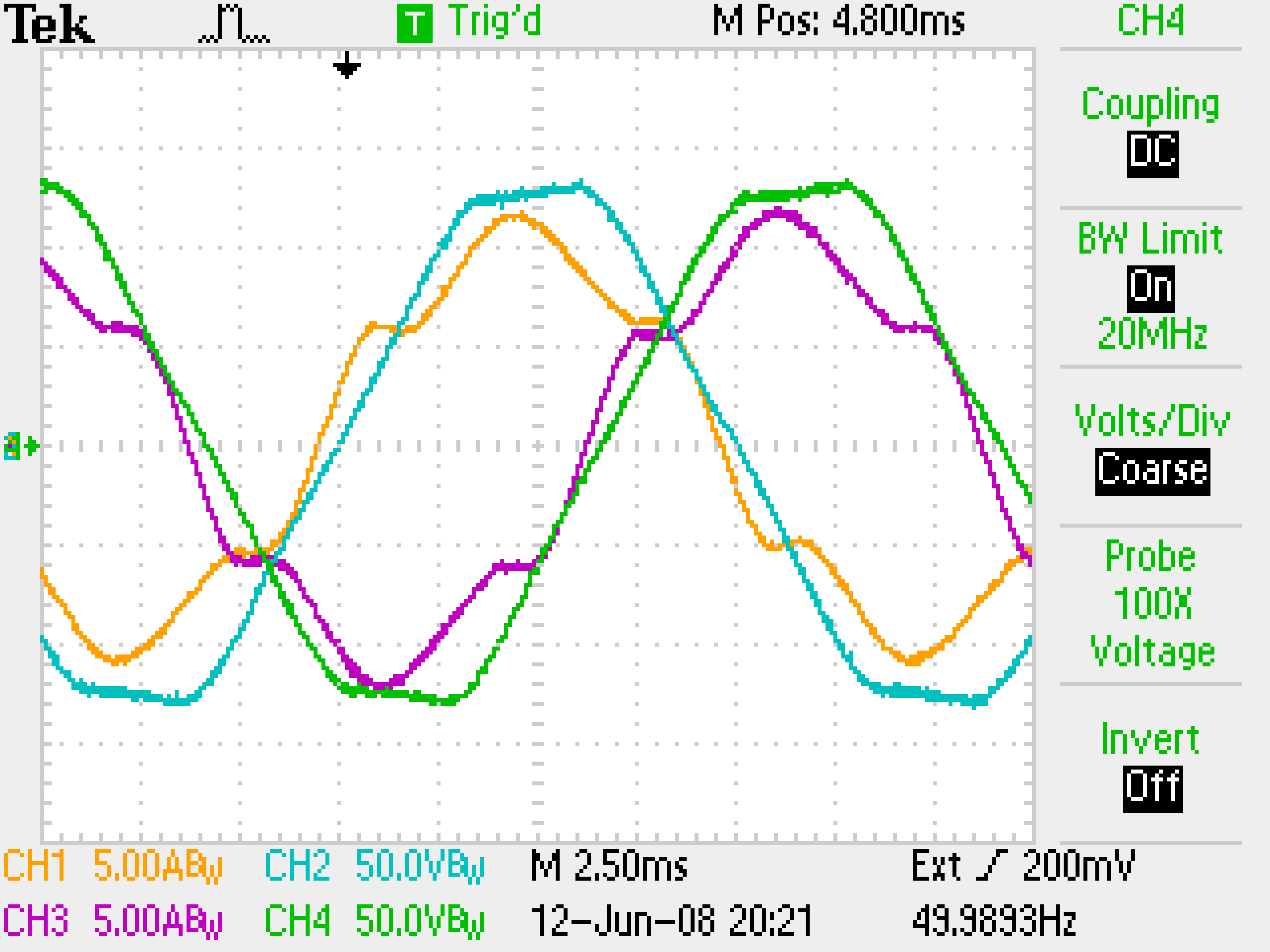

Tek
T Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100A/V
Current
Invert
Off
CH2 50.0V BW
M 2.50ms
Ext 200mV
CH4 50.0V BW
12-Jun-08 20:27
49.9974Hz

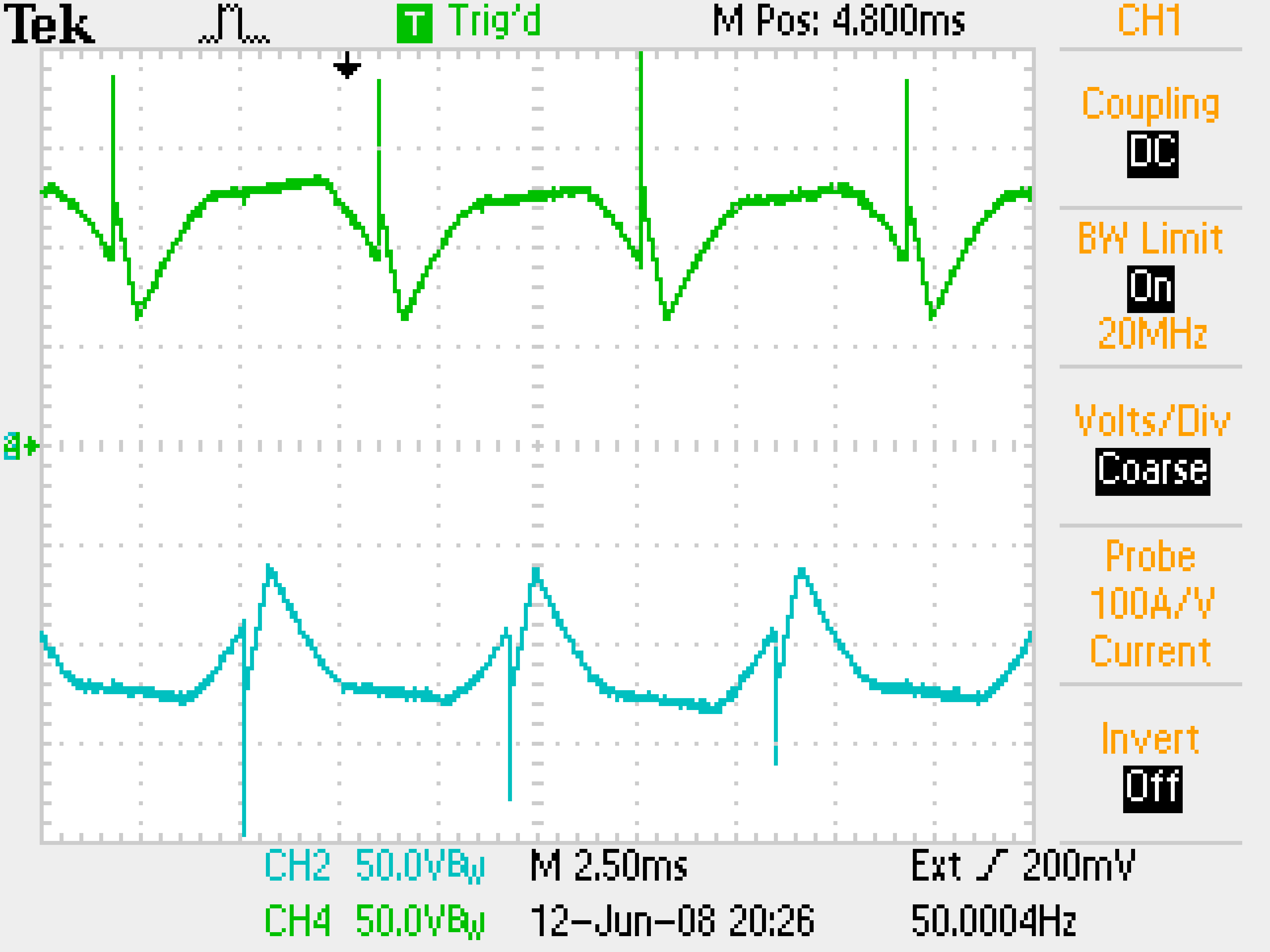
real problem:





Tek
Trig'd
M Pos: 4.800ms
CH3
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100A/V
Current
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 20:22
49.9947Hz

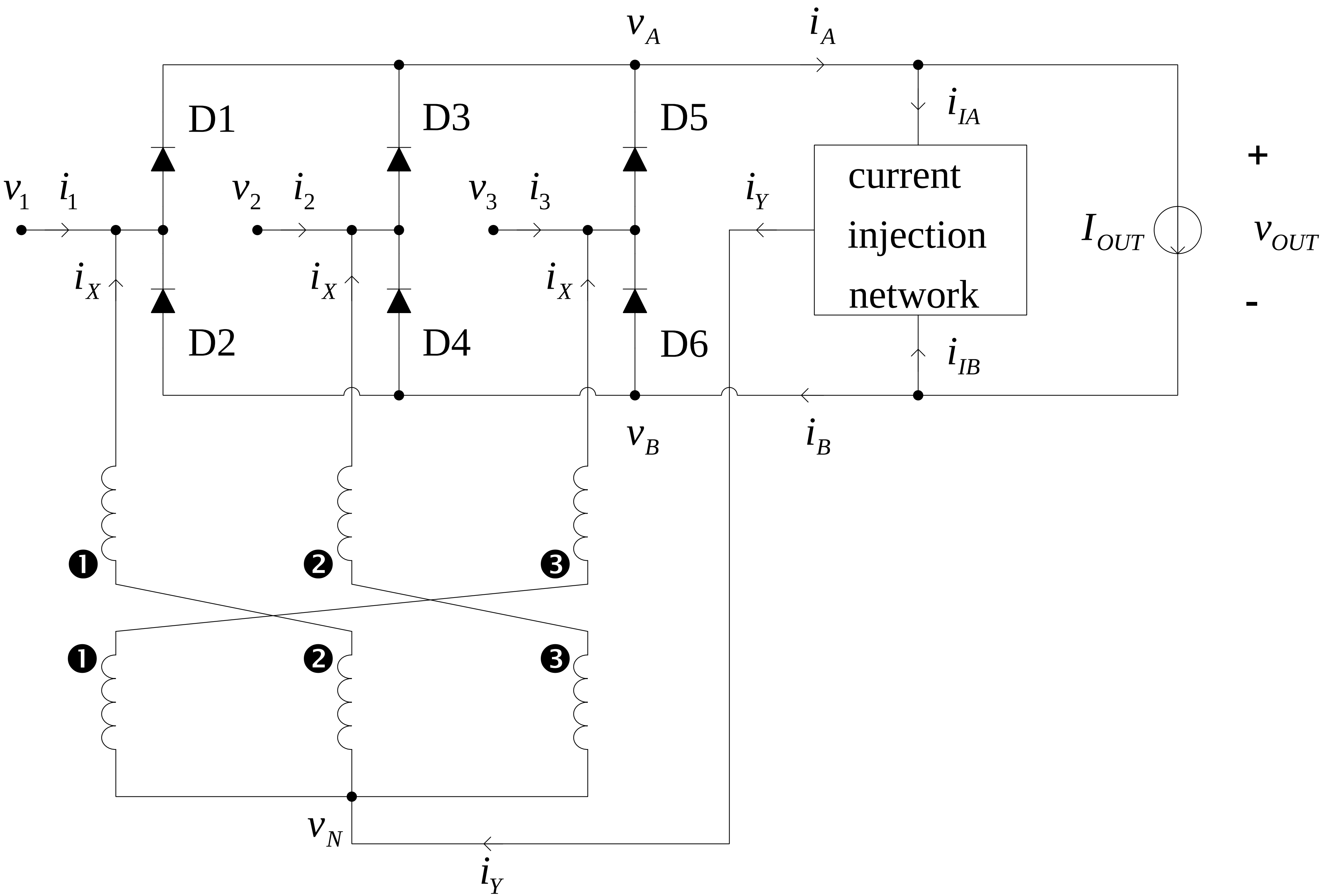

Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
CH3 5.00A
CH4 50.0V
12-Jun-08 20:21
49.9893Hz



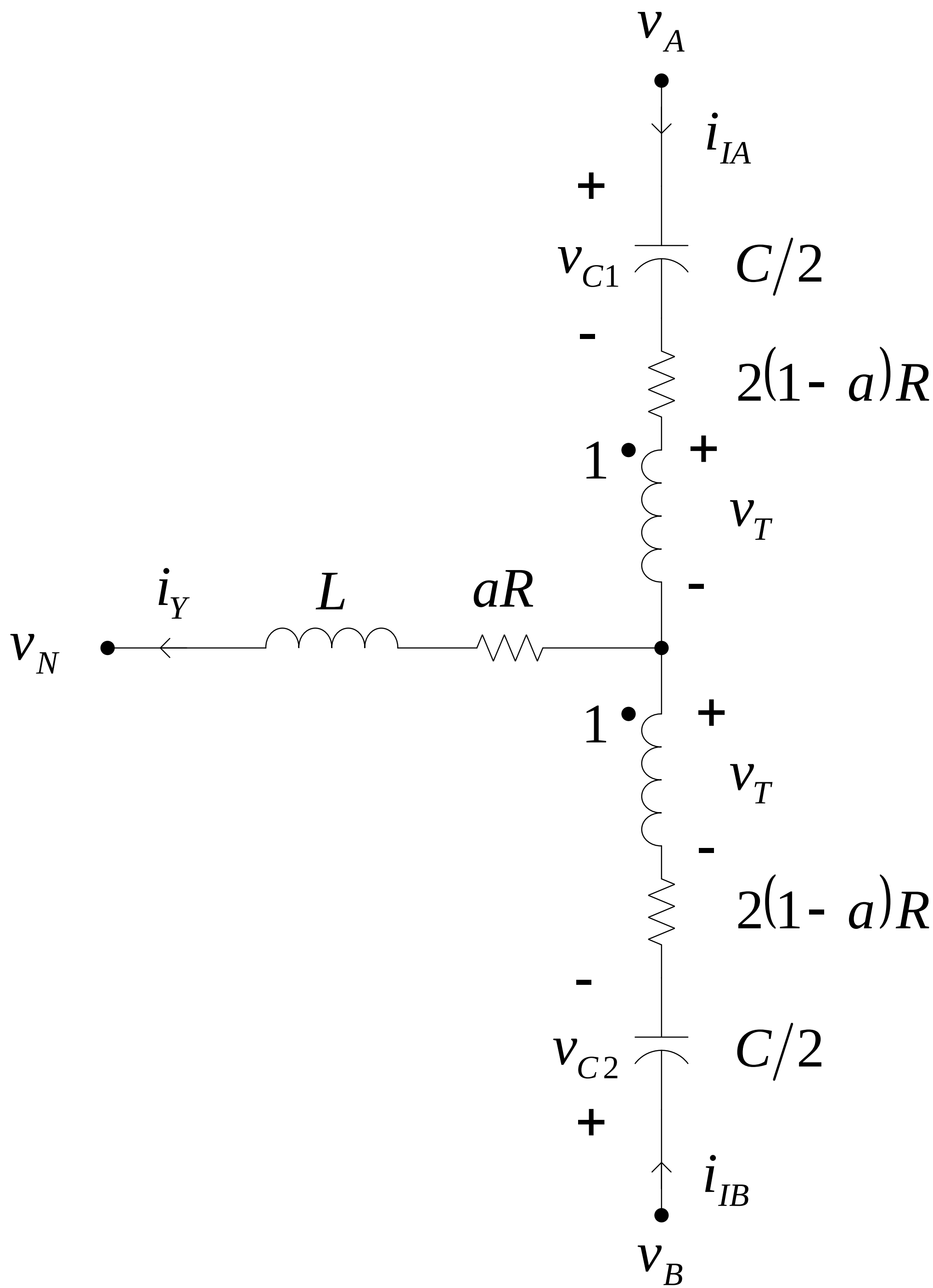
Tek
Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100A/V
Current
Invert
Off
CH2 50.0V
CH4 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 20:26
50.0004Hz

- How to analyze the circuit?
- There is no closed-form solution
- Even if there is a closed-form solution, would it be helpful?
- Numerical methods cannot be avoided
- How to make the numerical results general enough?
- Soldering iron thinking – > SPICE thinking
- What is the minimum number of parameters that affect the rectifier operation?
- How to introduce proper normalization?

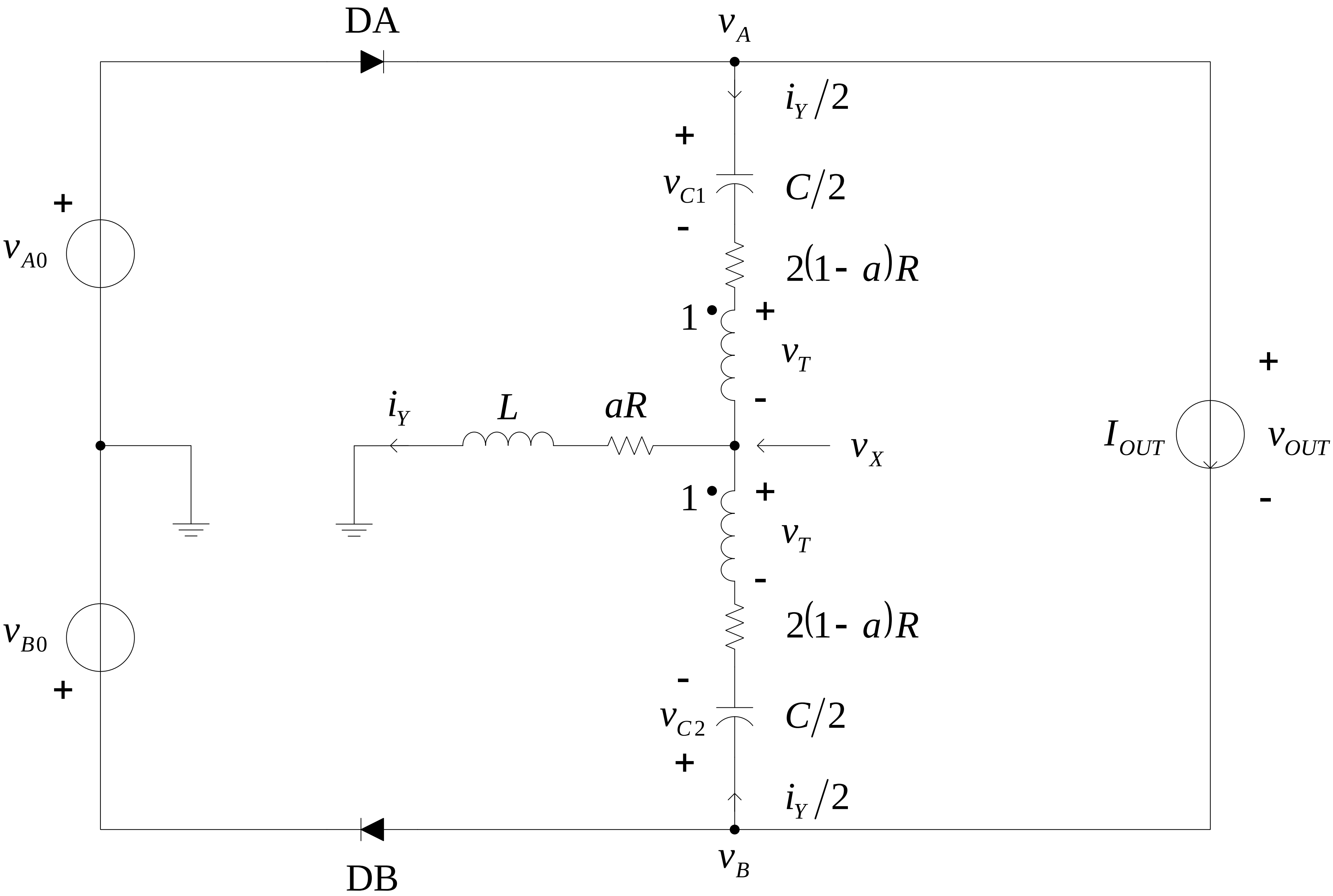
Too many questions . . .



$v_A$
$i_A$
D1
D3
D5
$i_{IA}$
current
injection
network
+
$v_1$
$i_1$
$v_2$
$i_2$
$v_3$
$i_3$
$i_Y$
$I_{OUT}$
$v_{OUT}$
$i_X$
$i_X$
$i_X$
-
D2
D4
D6
$i_{IB}$
$v_B$
$i_B$
❶
❷
❸
❶
❷
❸
$v_N$
$i_Y$



$v_A$
$i_{IA}$
+
$v_{C1}$
$C/2$
-
$2(1-a)R$
1
+
$v_T$
-
$i_Y$
$L$
$aR$
$v_N$
1
+
$v_T$
-
$2(1-a)R$
-
$v_{C2}$
$C/2$
+
$i_{IB}$
$v_B$



DA
$v_A$
$i_Y/2$
+
$v_{C1}$
$C/2$
-
$2(1-a)R$
+
$v_{A0}$
1
+
$v_T$
-
$i_Y$
$L$
$aR$
$v_X$
+
$I_{OUT}$
$v_{OUT}$
-
1
+
$v_T$
-
$2(1-a)R$
$v_{B0}$
+
-
$v_{C2}$
$C/2$
+
$i_Y/2$
$v_B$
DB

| state | equations | conditions | transition if the condition is violated |
| --- | --- | --- | --- |
| 1 | $L\frac{di_Y}{dt} = -Ri_Y - v_C + \frac{v_{A0} + v_{B0}}{2}$ | $-2I_{OUT} \le i_Y$ | to state 2 |
| | $C\frac{dv_C}{dt} = i_Y$ | $i_Y \le 2I_{OUT}$ | to state 3 |
| 2 | $i_Y = -2I_{OUT}$ | $v_C > \frac{v_{A0} + v_{B0}}{2} + 2RI_{OUT}$ | to state 1 |
| | $C\frac{dv_C}{dt} = -2I_{OUT}$ | | |
| 3 | $i_Y = 2I_{OUT}$ | $v_C < \frac{v_{A0} + v_{B0}}{2} - 2RI_{OUT}$ | to state 1 |
| | $C\frac{dv_C}{dt} = 2I_{OUT}$ | | |

$$m = \frac{v}{V_m}$$

$$R_0 = \sqrt{\frac{L}{C}} \qquad \omega_R = \frac{1}{\sqrt{LC}}$$

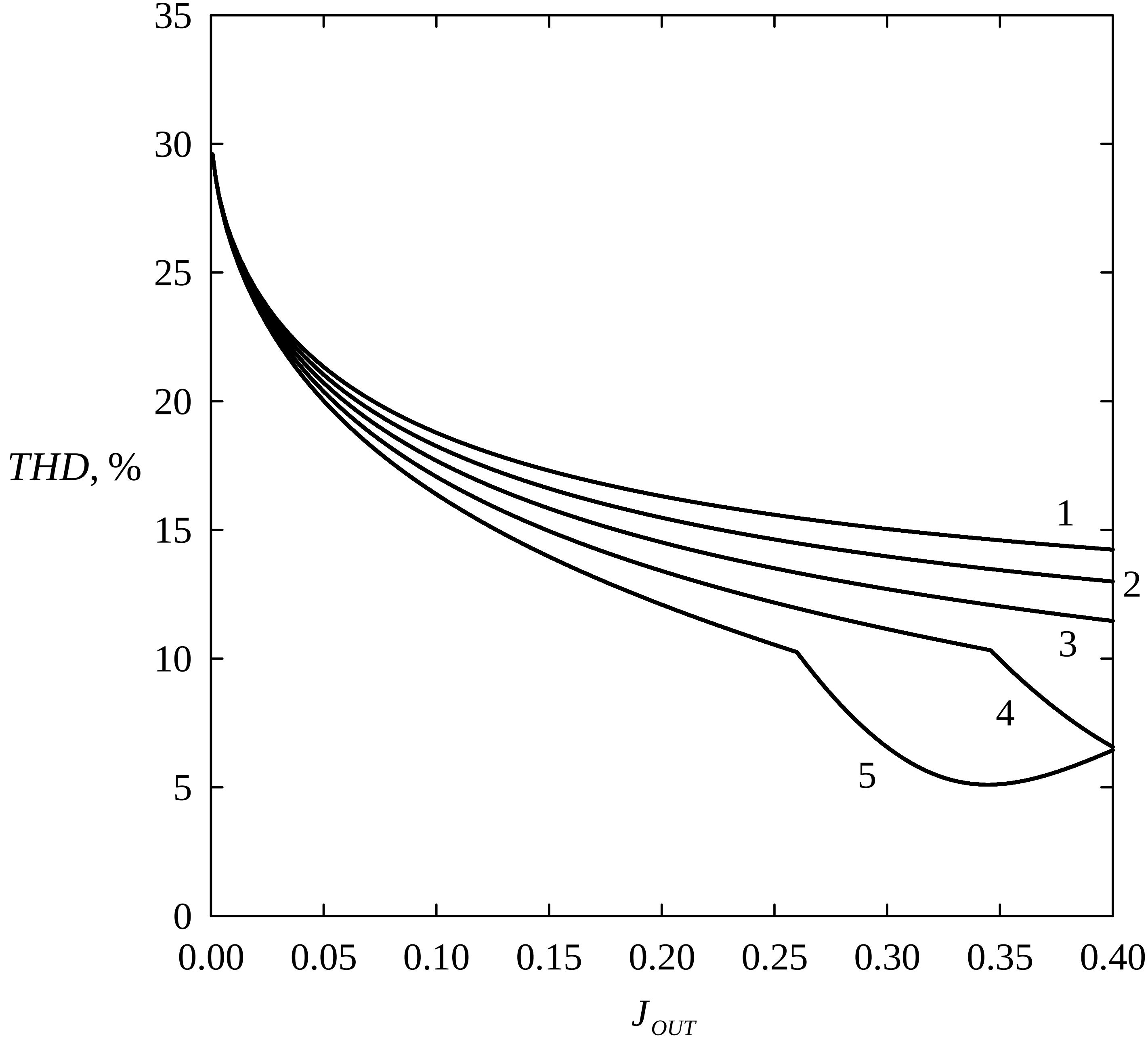
$$\rho = \frac{R}{R_0}$$

$$j = \frac{R_0}{V_m} i$$

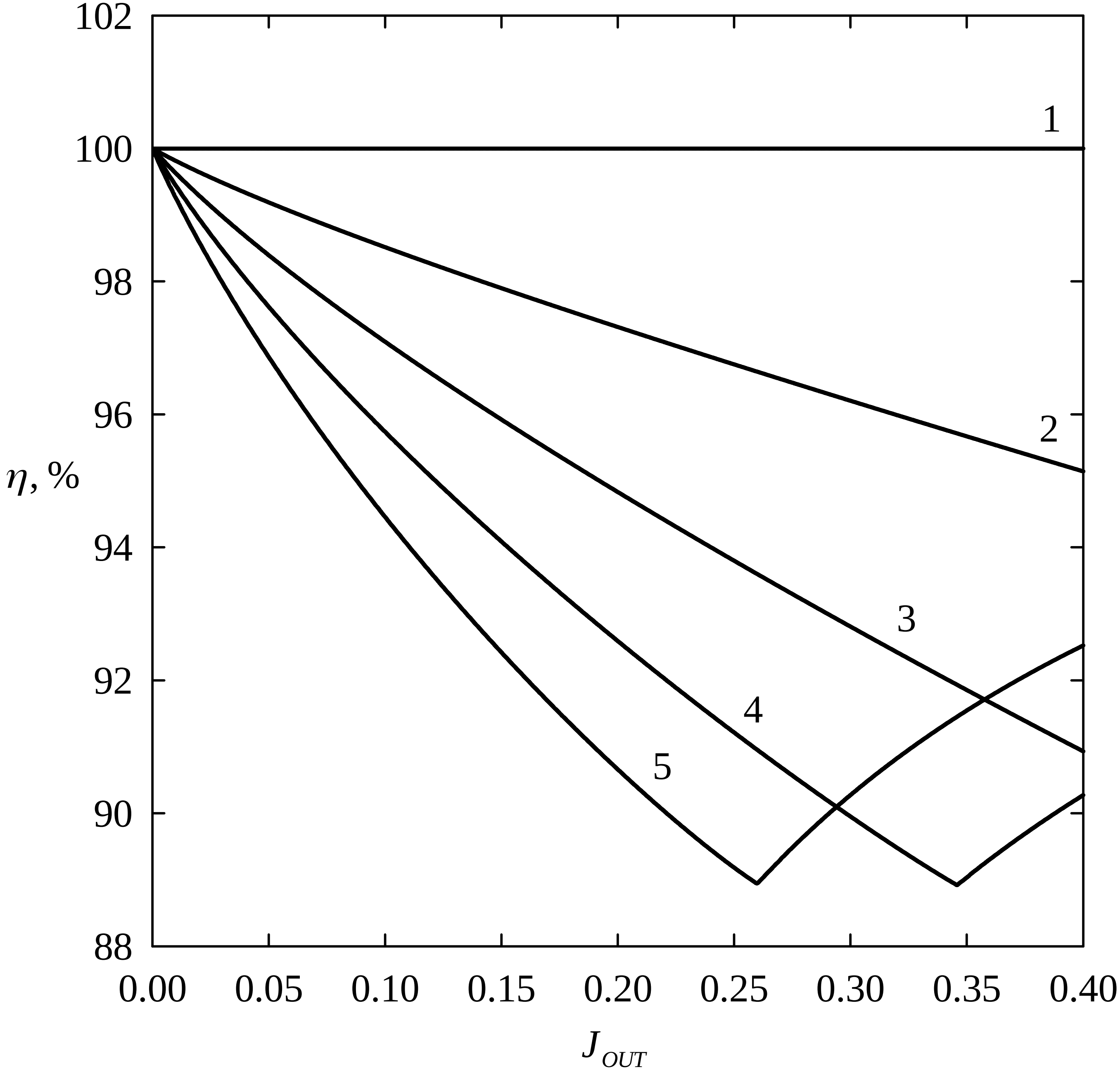
$$\phi = \omega_0 t$$

$$r = \frac{\omega_R}{3\,\omega_0}$$

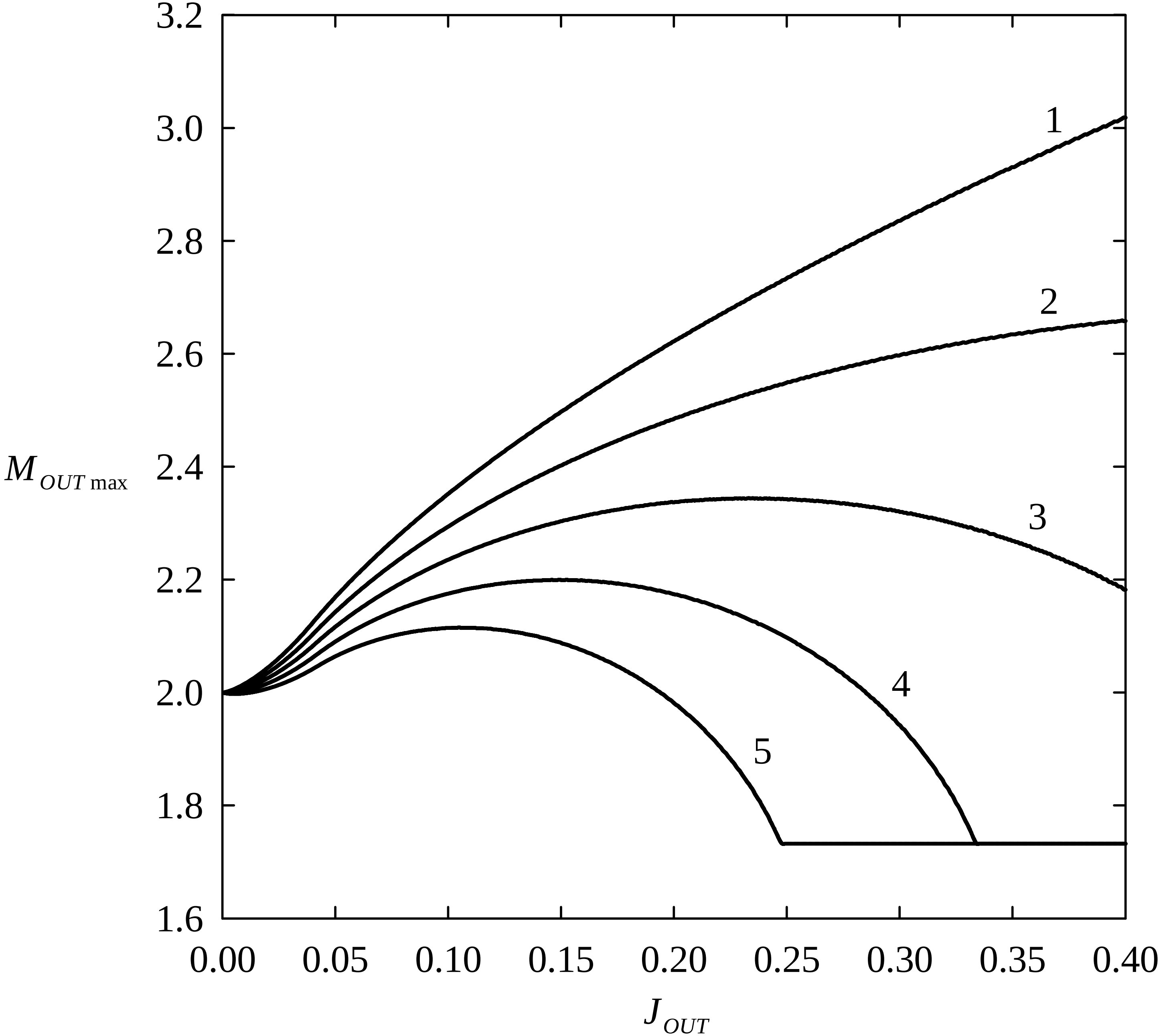
| state | equations | conditions | transition if the condition is violated |
|---|---|---|---|
| 1 | $\frac{dj_Y}{d\varphi} = 3r\left(-\rho j_Y - m_C + \frac{m_{A0} + m_{B0}}{2}\right)$ | $-2J_{OUT} \leq j_Y$ | to state 2 |
| | $\frac{dm_C}{d\varphi} = 3rj_Y$ | $j_Y \leq 2J_{OUT}$ | to state 3 |
| 2 | $j_Y = -2J_{OUT}$ | $m_C > \frac{m_{A0} + m_{B0}}{2} + 2\rho J_{OUT}$ | to state 1 |
| | $\frac{dm_C}{d\varphi} = -6rJ_{OUT}$ | | |
| 3 | $j_Y = 2J_{OUT}$ | $m_C < \frac{m_{A0} + m_{B0}}{2} - 2\rho J_{OUT}$ | to state 1 |
| | $\frac{dm_C}{d\varphi} = 6rJ_{OUT}$ | | |



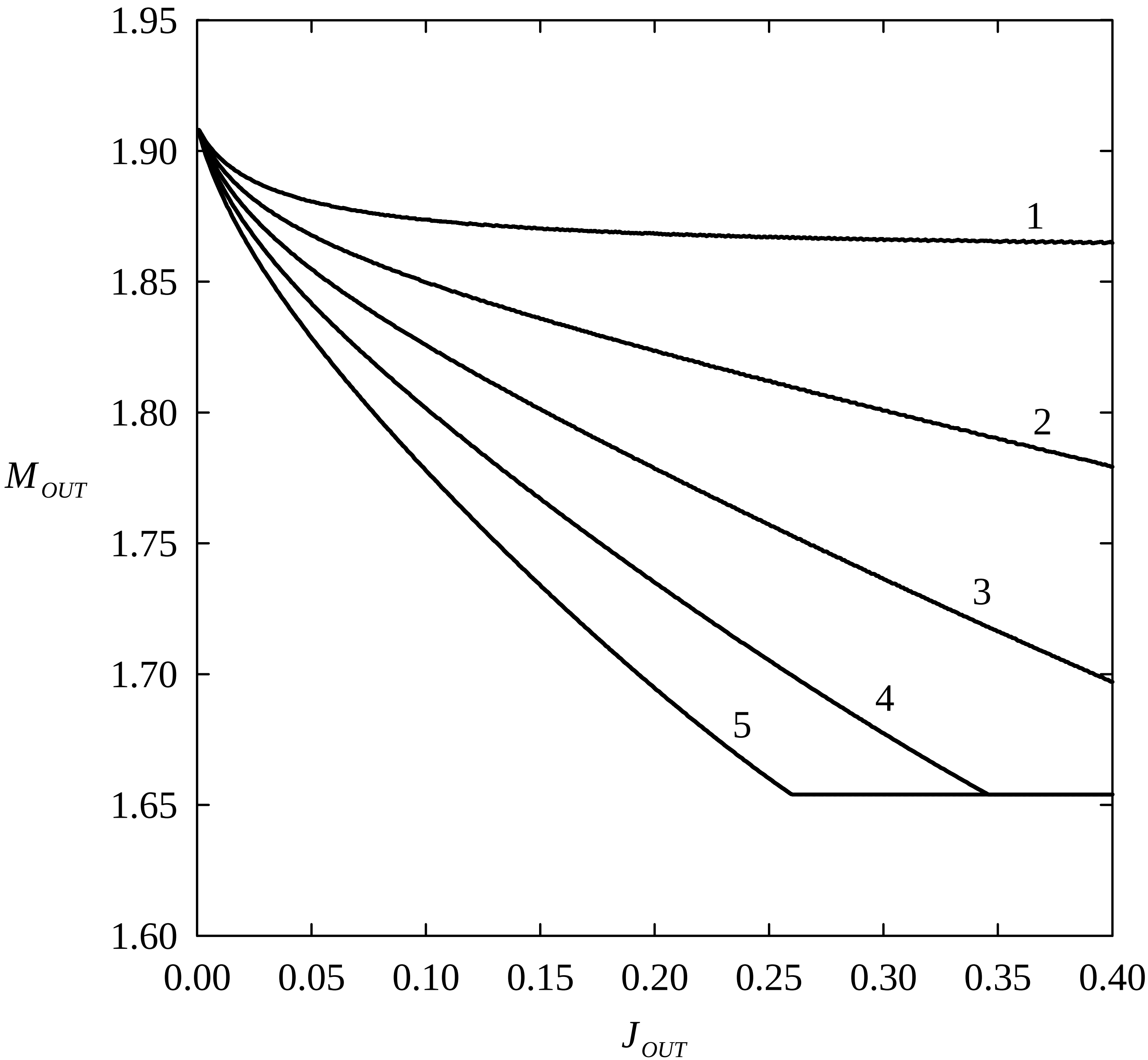
THD, %
35
30
25
20
15
10
5
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$J_{OUT}$
1
2
3
4
5



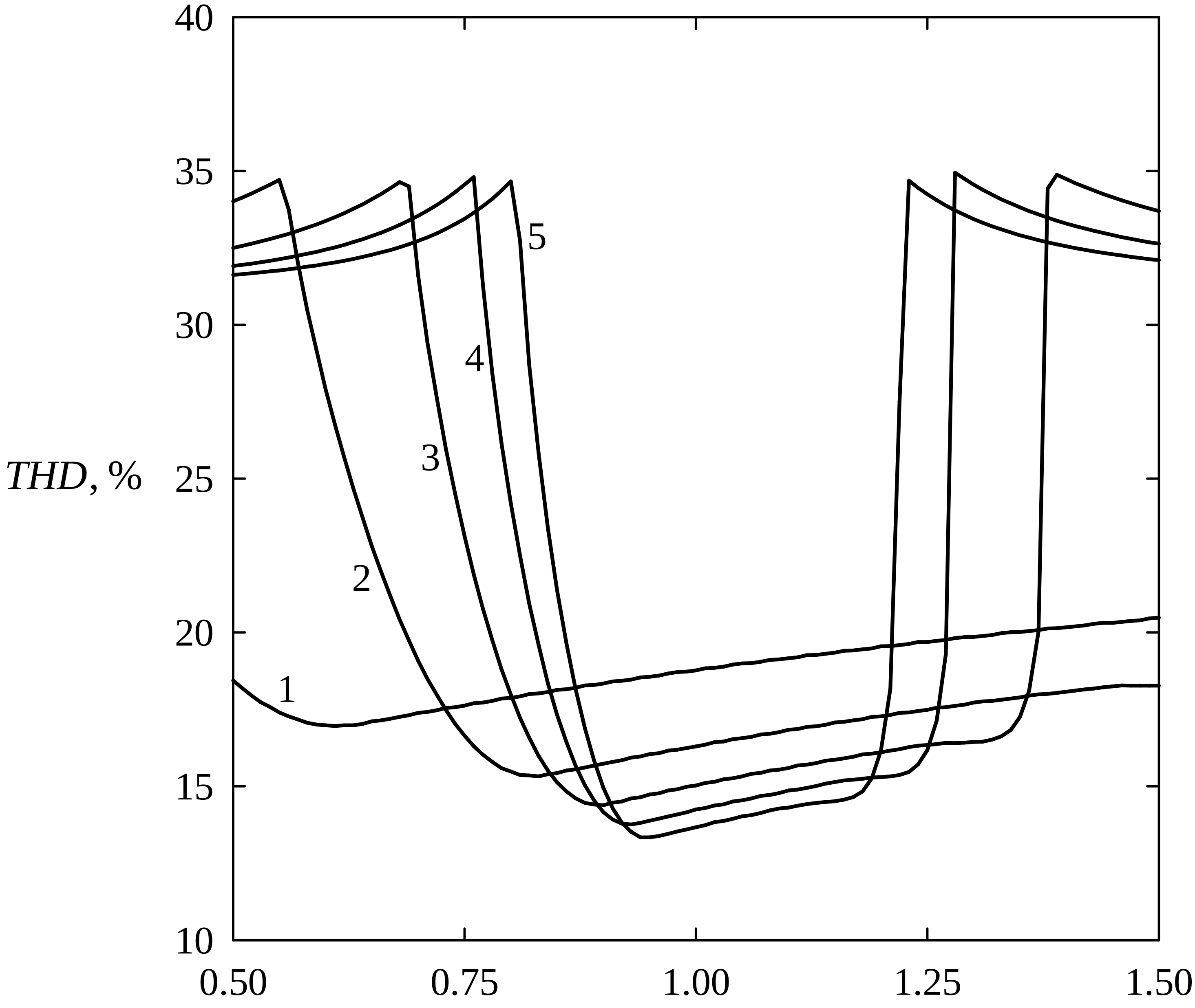
102
100
98
96
94
92
90
88
η, %
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$J_{OUT}$
1
2
3
4
5



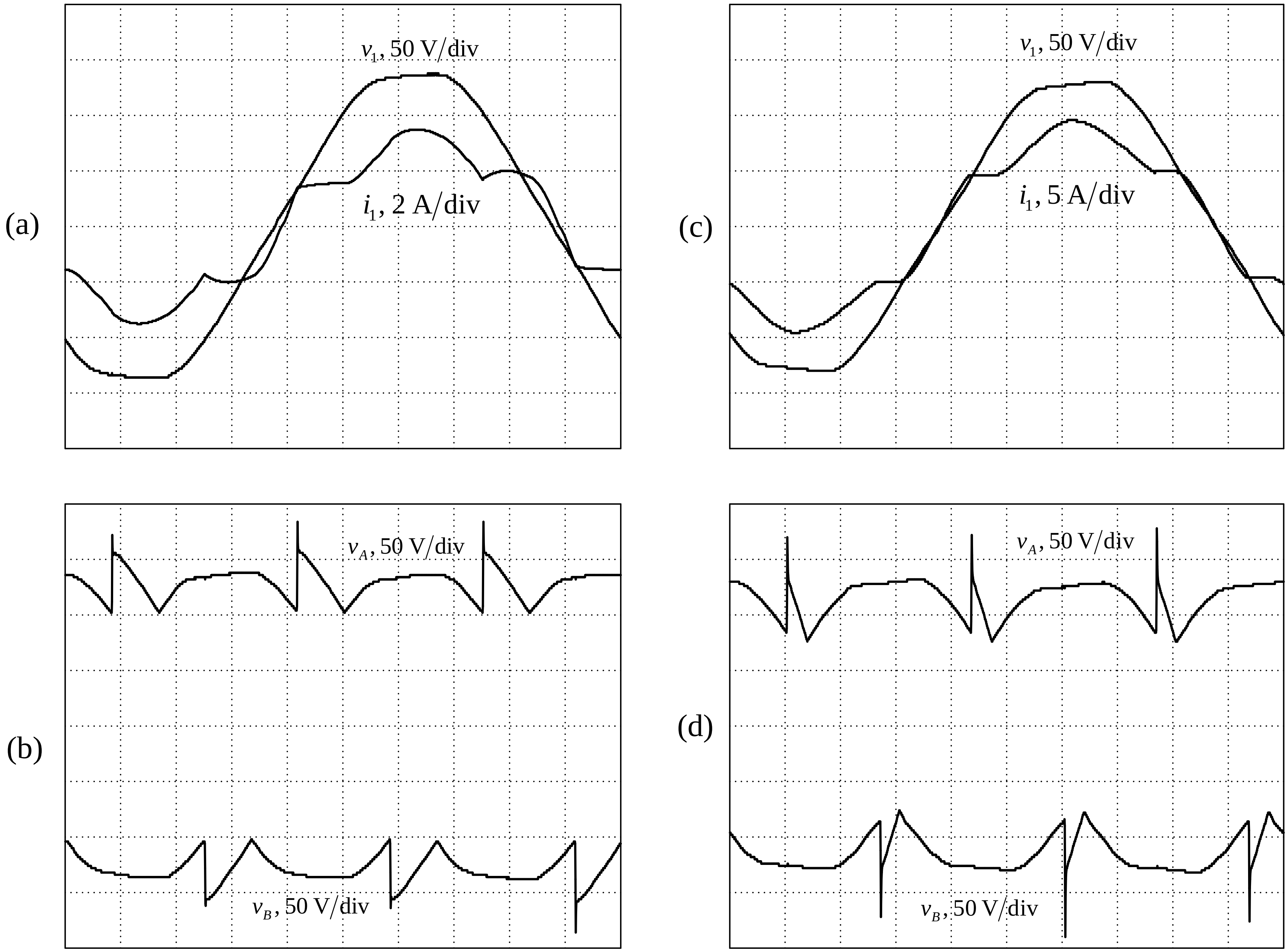
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
$M_{OUT\,\max}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$J_{OUT}$
1
2
3
4
5



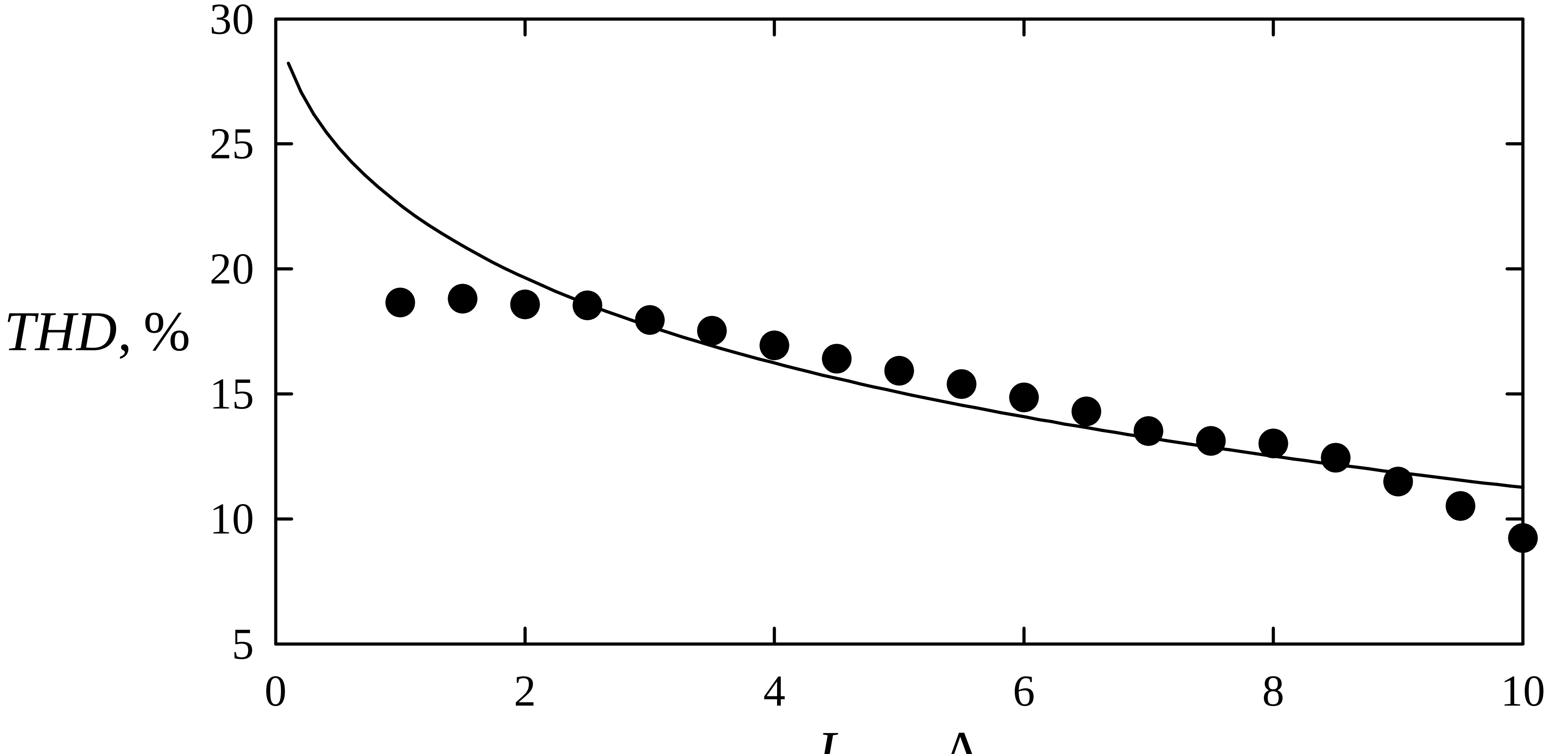
1.95
1.90
1.85
1.80
1.75
1.70
1.65
1.60
$M_{OUT}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$J_{OUT}$
1
2
3
4
5

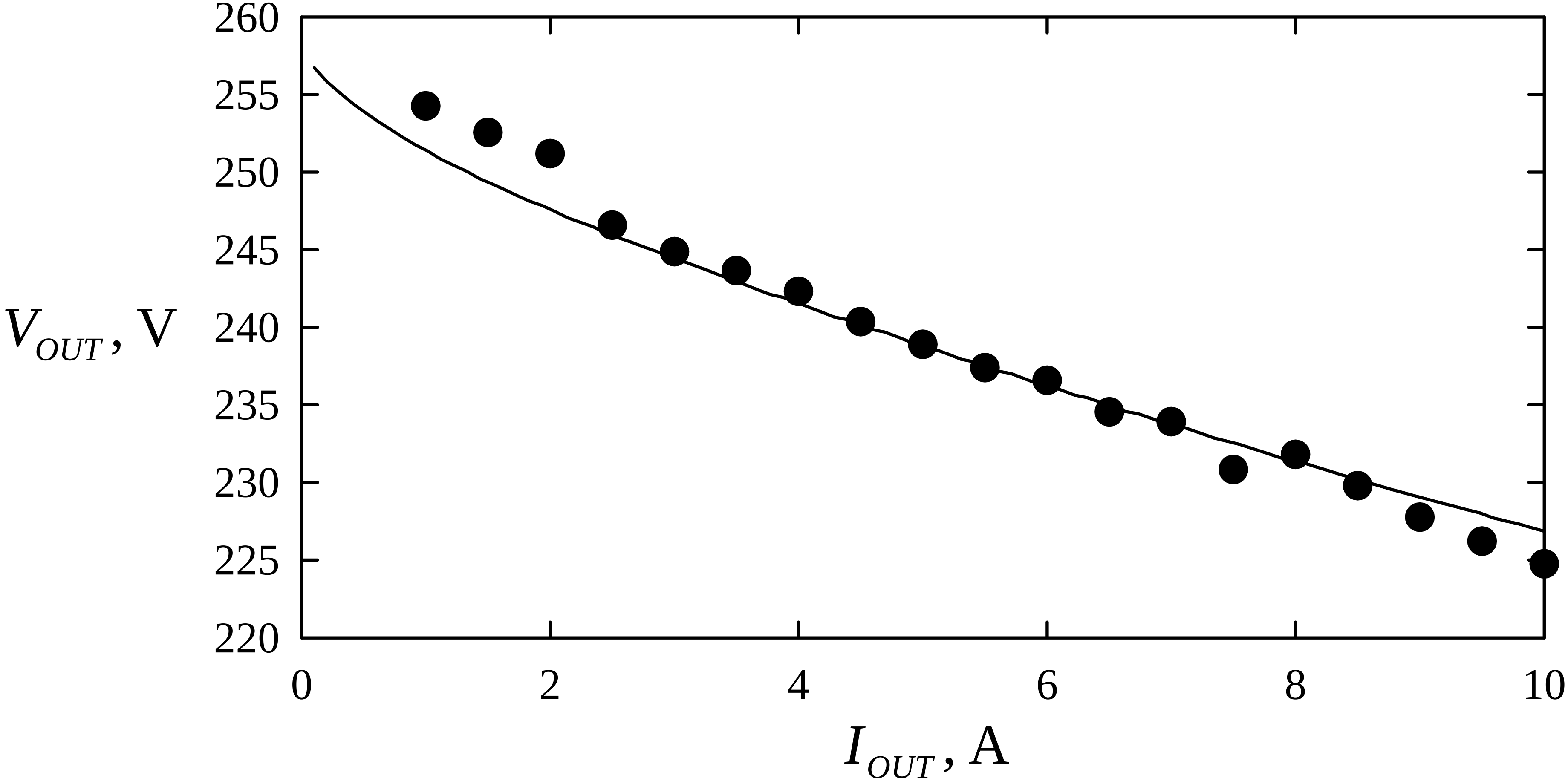


40
35
30
25
20
15
10
THD, %
0.50
0.75
1.00
1.25
1.50
r
1
2
3
4
5



(a)
$v_1$, 50 V/div
$i_1$, 2 A/div
(b)
$v_A$, 50 V/div
$v_B$, 50 V/div
(c)
$v_1$, 50 V/div
$i_1$, 5 A/div
(d)
$v_A$, 50 V/div
$v_B$, 50 V/div


30
25
20
15
10
5
THD, %
0
2
4
6
8
10
I_OUT, A



260
255
250
245
240
235
230
225
220
$V_{OUT}$, V
0
2
4
6
8
10
$I_{OUT}$, A

# An Extrapolation Method For Accelerated Convergence to Steady State Solution of Power Electronics Circuits

## Problem Statement

$$x_{k+1} = f\left(x_k\right)$$

$$\xi = f\left(\xi\right)$$

## Generalized Aitken's Method

$$f\left(x\right) = \mathbf{Ax} + B$$

$$\xi = f\left(\xi\right) = \mathbf{A\xi} + B$$

$$x_{k+1} = f\left(x_k\right) = \mathbf{Ax}_{\boldsymbol{k}} + B$$

$$x_{k+1} - \xi = A\left(x_k - \xi\right)$$

$$\xi = \left(I_n - A\right)^{-1}\left(x_{i+1} - \mathbf{Ax}_{\boldsymbol{i}}\right)$$

$$e_i = x_i - x_{i-1}$$

$$x_{i+1} - x_i = A\left(x_i - x_{i-1}\right)$$

$$e_2 = \mathbf{Ae_1}$$
$$e_3 = \mathbf{Ae_2}$$
$$\cdots$$
$$e_{n+1} = \mathbf{Ae}_{\boldsymbol{n}}$$

$$\begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{n+1} \end{bmatrix} = \begin{bmatrix} A & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & A & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & A \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

$$\begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{n+1} \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \begin{bmatrix} A_1^T \\ A_2^T \\ \vdots \\ A_n^T \end{bmatrix}$$

$$E_i = \begin{bmatrix} e_i^T & 0_{1\times n} & \cdots & 0_{1\times n} \\ 0_{1\times n} & e_i^T & \cdots & 0_{1\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{1\times n} & 0_{1\times n} & \cdots & e_i^T \end{bmatrix}$$

determine **A**, determine **ξ**, start again

Problem with dynamic degeneration, common to the DCM

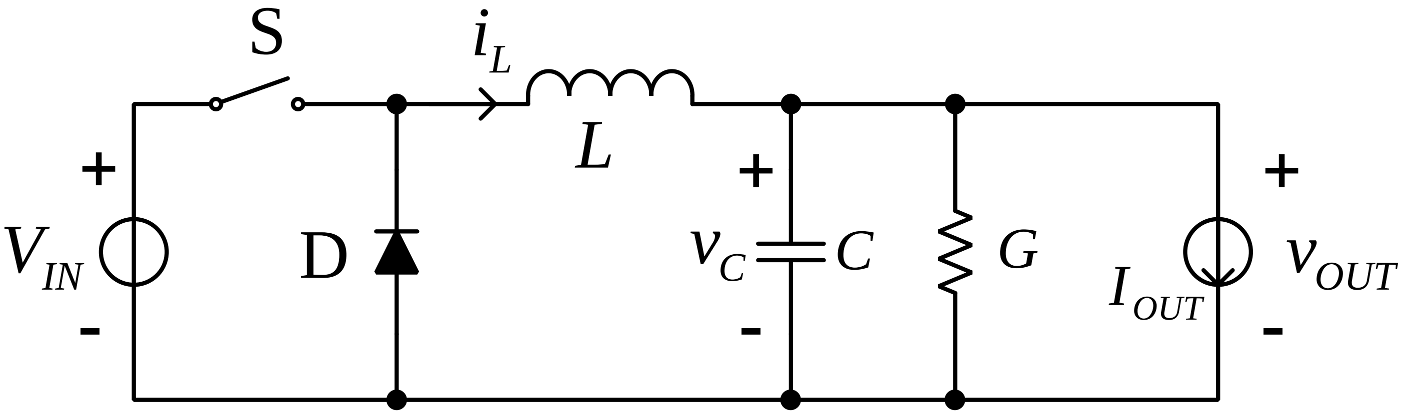
$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} e_{1,1} & e_{1,2} & 0 & 0 \\ 0 & 0 & e_{1,1} & e_{1,2} \\ e_{2,1} & e_{2,2} & 0 & 0 \\ 0 & 0 & e_{2,1} & e_{2,2} \end{bmatrix}$$

$$x_{0,2} = x_{1,2} = x_{2,2}$$

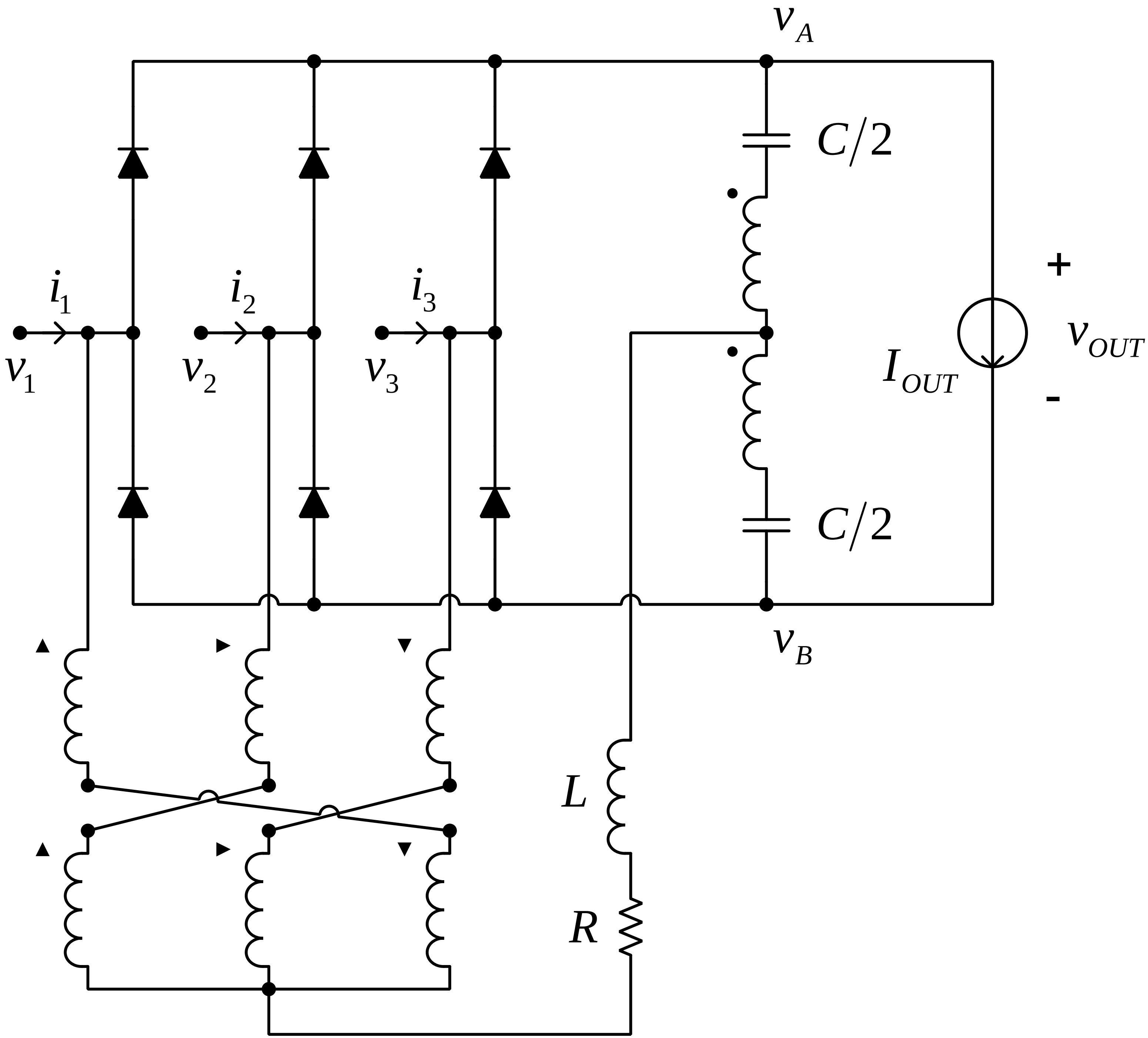
$$e_{1,2} = e_{2,2} = 0$$

Convergence?

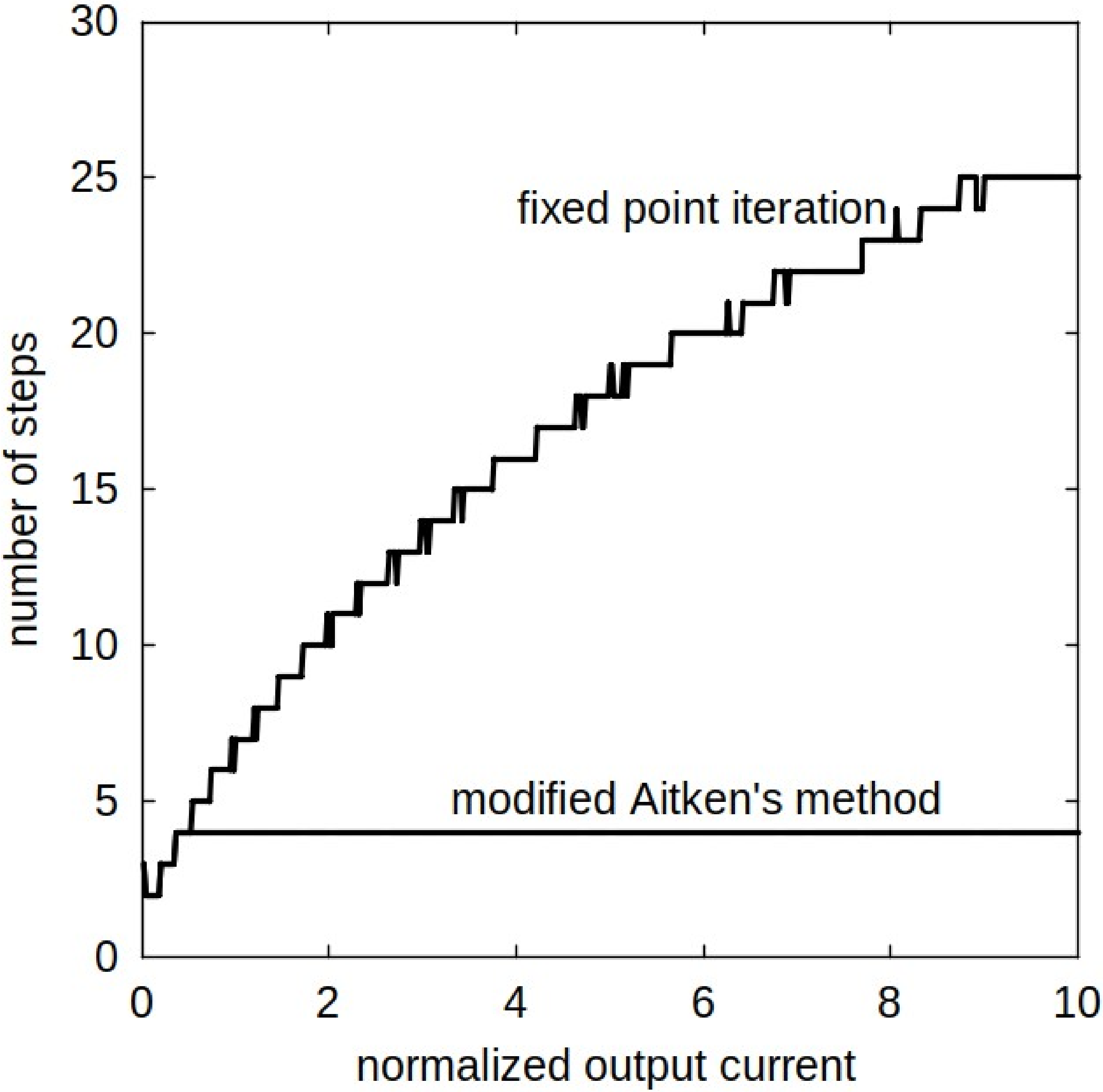
# Simulation Results



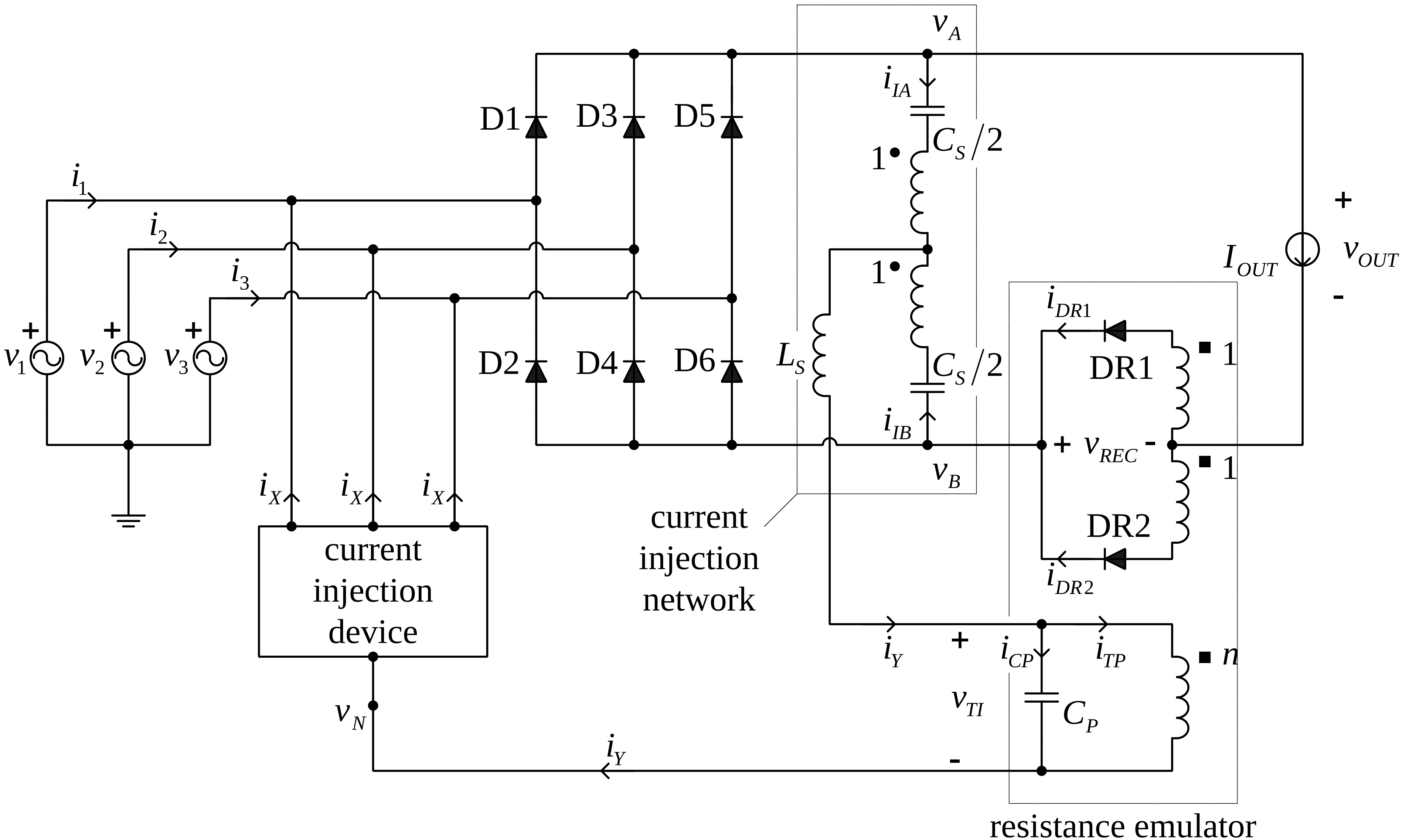

| example | fixed point iteration | Aitken's method | modified Aitken's method |
|---|---|---|---|
| PWM, CCM | 736 | 4 | 4 |
| PWM, DCM | 109 | 55 | 10 |
| CMC, DCM | 137 | 89 | 13 |
| CMC, CCM | diverges | 19 | 19 |



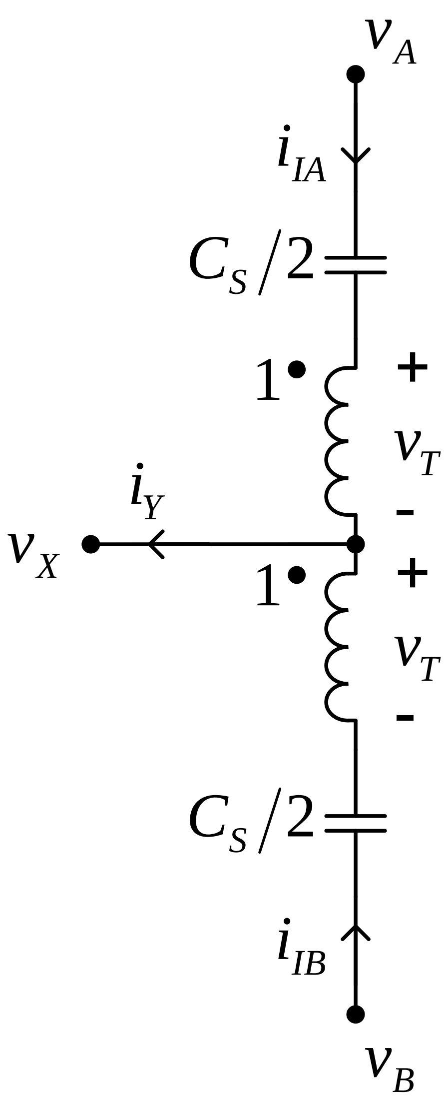
$v_A$
$C/2$
$i_1$
$i_2$
$i_3$
$v_1$
$v_2$
$v_3$
+
$v_{OUT}$
$I_{OUT}$
-
$C/2$
$v_B$
$L$
$R$


30
25
20
15
10
5
0
0
2
4
6
8
10
number of steps
normalized output current
fixed point iteration
modified Aitken's method

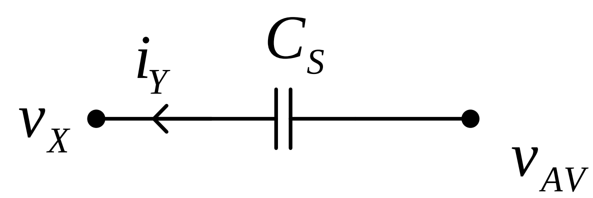
another problem:

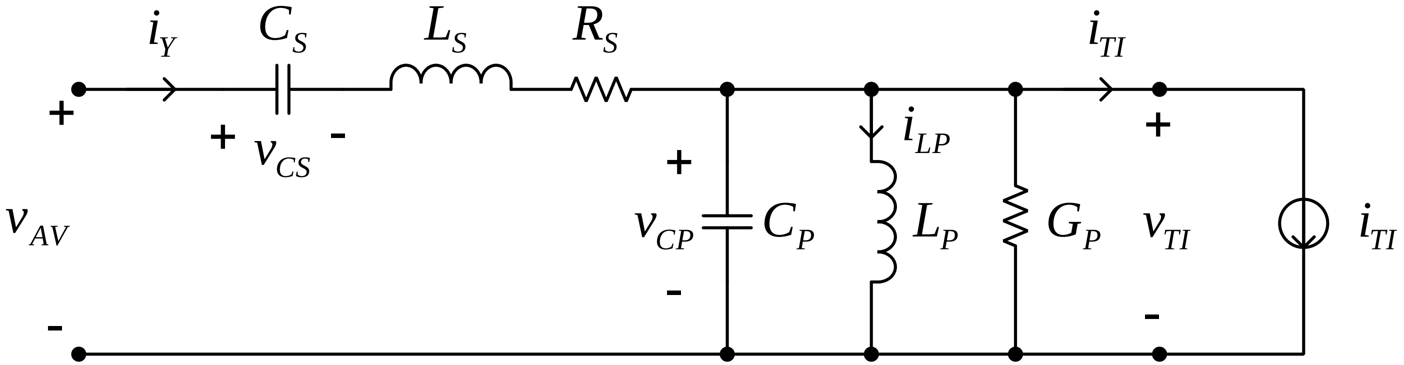


analytical preparation, pre-analysis




→

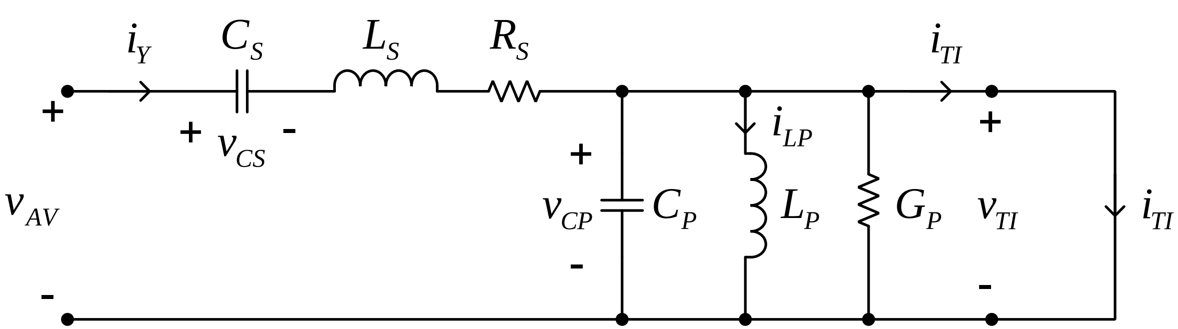





$$i_{TI} = \frac{1}{n} I_{OUT} \operatorname{sgn}(v_{CP})$$

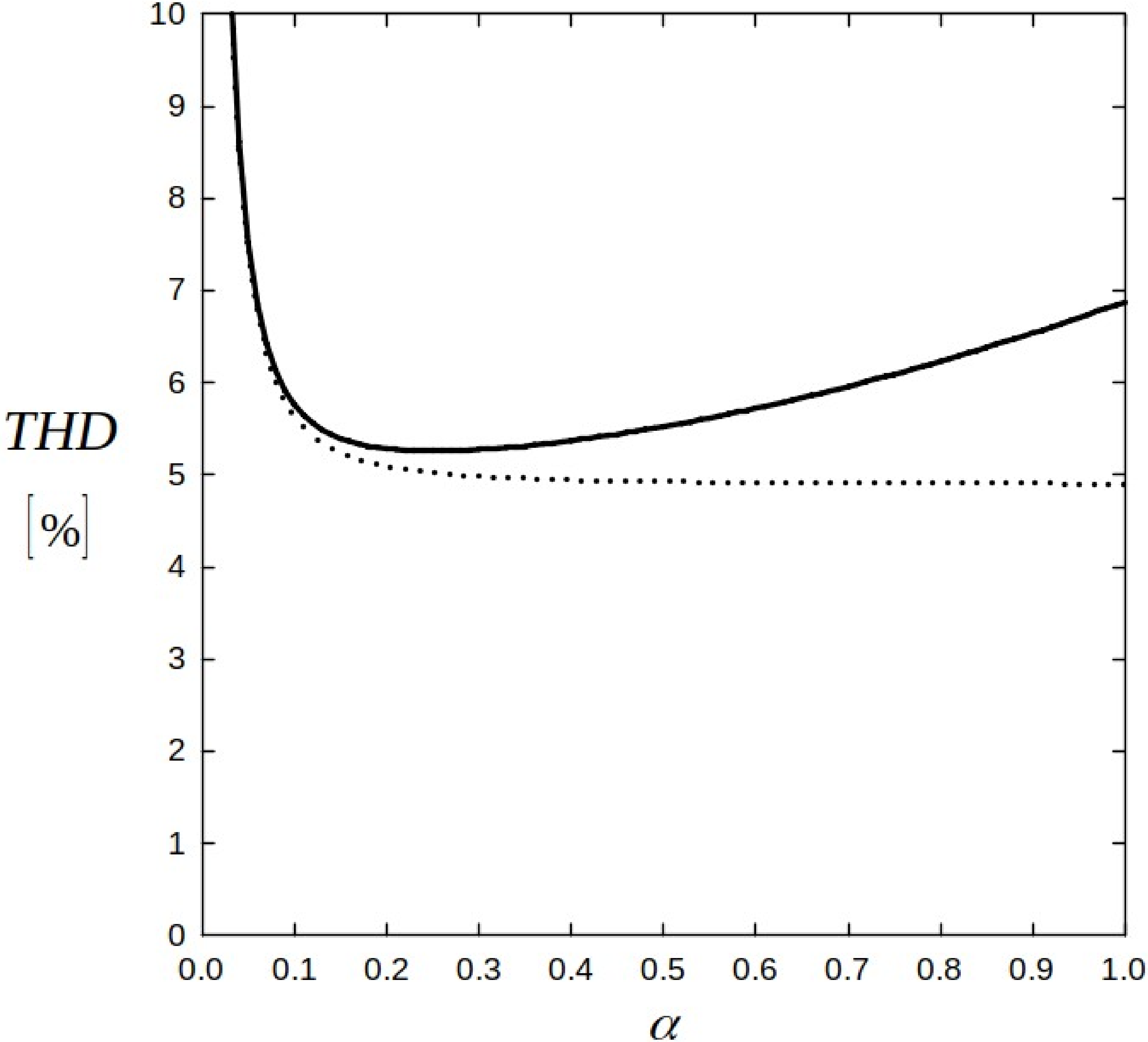
$$
\frac{d}{dt}\begin{bmatrix} i_Y \\ v_{CS} \\ i_{LP} \\ v_{CP} \end{bmatrix} = \begin{bmatrix} -\frac{R_S}{L_S} & -\frac{1}{L_S} & 0 & -\frac{1}{L_S} \\ \frac{1}{C_S} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_P} \\ \frac{1}{C_P} & 0 & -\frac{1}{C_P} & -\frac{G_P}{C_P} \end{bmatrix} \begin{bmatrix} i_Y \\ v_{CS} \\ i_{LP} \\ v_{CP} \end{bmatrix} + \begin{bmatrix} \frac{v_{AV}}{L_S} \\ 0 \\ 0 \\ -\frac{1}{nC_P} I_{OUT}\,\mathrm{sgn}\left(v_{CP}\right) \end{bmatrix}
$$

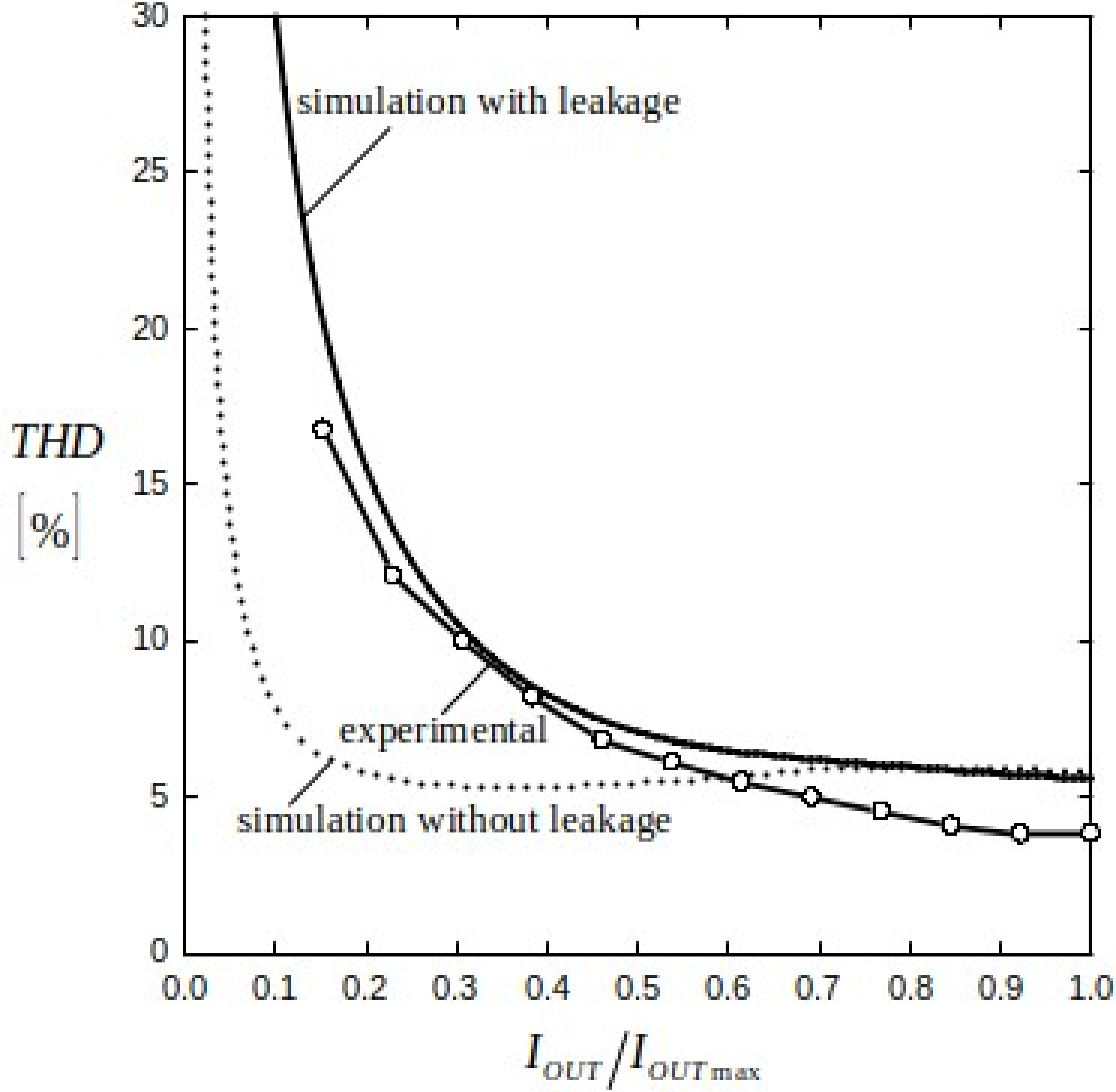
$$
\frac{d}{dt}\begin{bmatrix} i_Y \\ v_{CS} \\ i_{LP} \end{bmatrix} = \begin{bmatrix} -\frac{R_S}{L_S} & -\frac{1}{L_S} & 0 \\ \frac{1}{C_S} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_Y \\ v_{CS} \\ i_{LP} \end{bmatrix} + \begin{bmatrix} \frac{v_{AV}}{L_S} \\ 0 \\ 0 \end{bmatrix}
$$



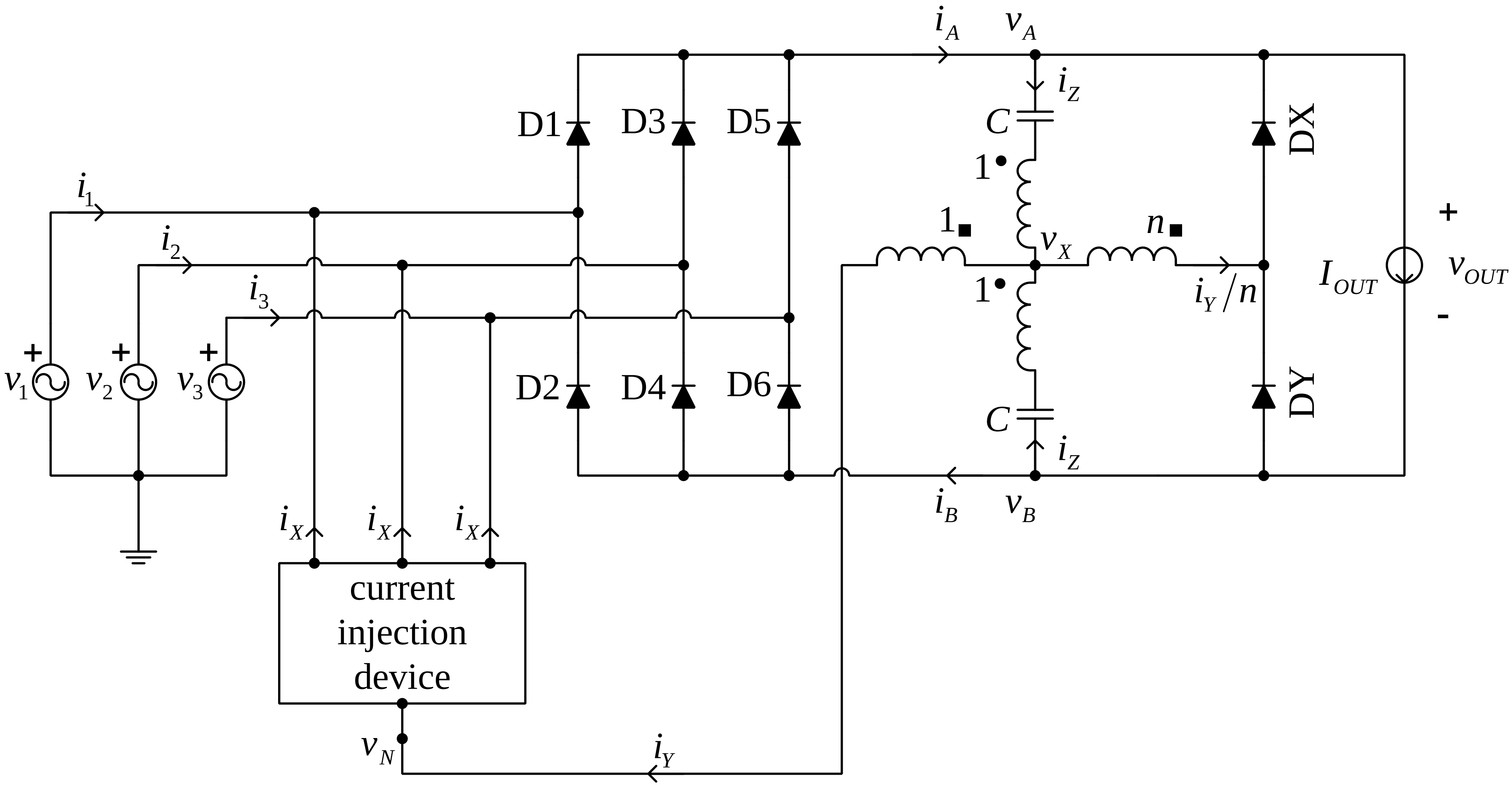
$$\frac{d}{d\phi}\begin{bmatrix} j_Y \\ m_{CS} \\ j_{LP} \\ m_{CP} \end{bmatrix} = \begin{bmatrix} -\frac{3r_S}{Q_S} & -\frac{3r_S}{\alpha\rho_S} & 0 & -\frac{3r_S}{\alpha\rho_S} \\ 3r_S\alpha\rho_S & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3r_P}{\alpha\rho_P} \\ 3r_P\alpha\rho_P & 0 & -3r_P\alpha\rho_P & -\frac{3r_P}{Q_P} \end{bmatrix} \begin{bmatrix} j_Y \\ m_{CS} \\ j_{LP} \\ m_{CP} \end{bmatrix} + \begin{bmatrix} \frac{3r_S}{\alpha\rho_S} m_{AV} \\ 0 \\ 0 \\ -3r_P\alpha\rho_P \frac{\operatorname{sgn}(m_{CP})}{n} \end{bmatrix}$$

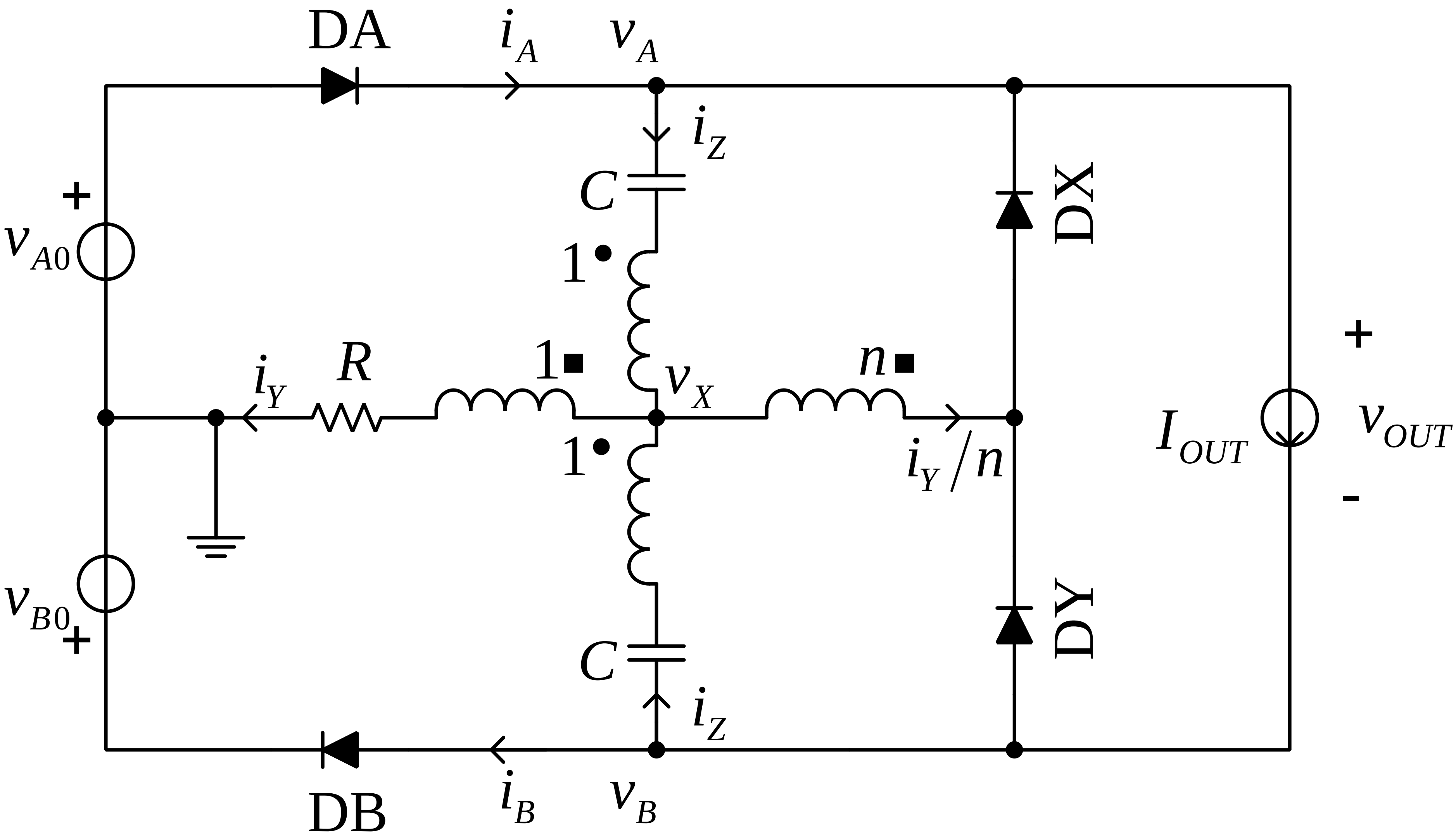
$$\frac{d}{d\phi}\begin{bmatrix} j_Y \\ m_{CS} \\ j_{LP} \\ m_{CP} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{\pi\sqrt{3}}{\alpha} & 0 & -\frac{\pi\sqrt{3}}{\alpha} \\ \frac{3\sqrt{3}}{\pi}\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\pi^2\sqrt{3}}{\alpha} \\ \frac{3\sqrt{3}}{\pi^2}\alpha & 0 & -\frac{3\sqrt{3}}{\pi^2}\alpha & 0 \end{bmatrix} \begin{bmatrix} j_Y \\ m_{CS} \\ j_{LP} \\ m_{CP} \end{bmatrix} + \begin{bmatrix} \frac{\pi\sqrt{3}}{\alpha} m_{AV} \\ 0 \\ 0 \\ -\frac{9\sqrt{3}}{8\pi}\alpha \operatorname{sgn}(m_{CP}) \end{bmatrix}$$


THD
[%]
10
9
8
7
6
5
4
3
2
1
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\alpha$


simulation with leakage
experimental
simulation without leakage
THD
[%]
$I_{OUT}/I_{OUTmax}$
30
25
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

and another problem (a fresh one, PESC 2008):






analyzed as a resistive circuit!

normalization:

$$m_X = \frac{v_X}{V_m}$$

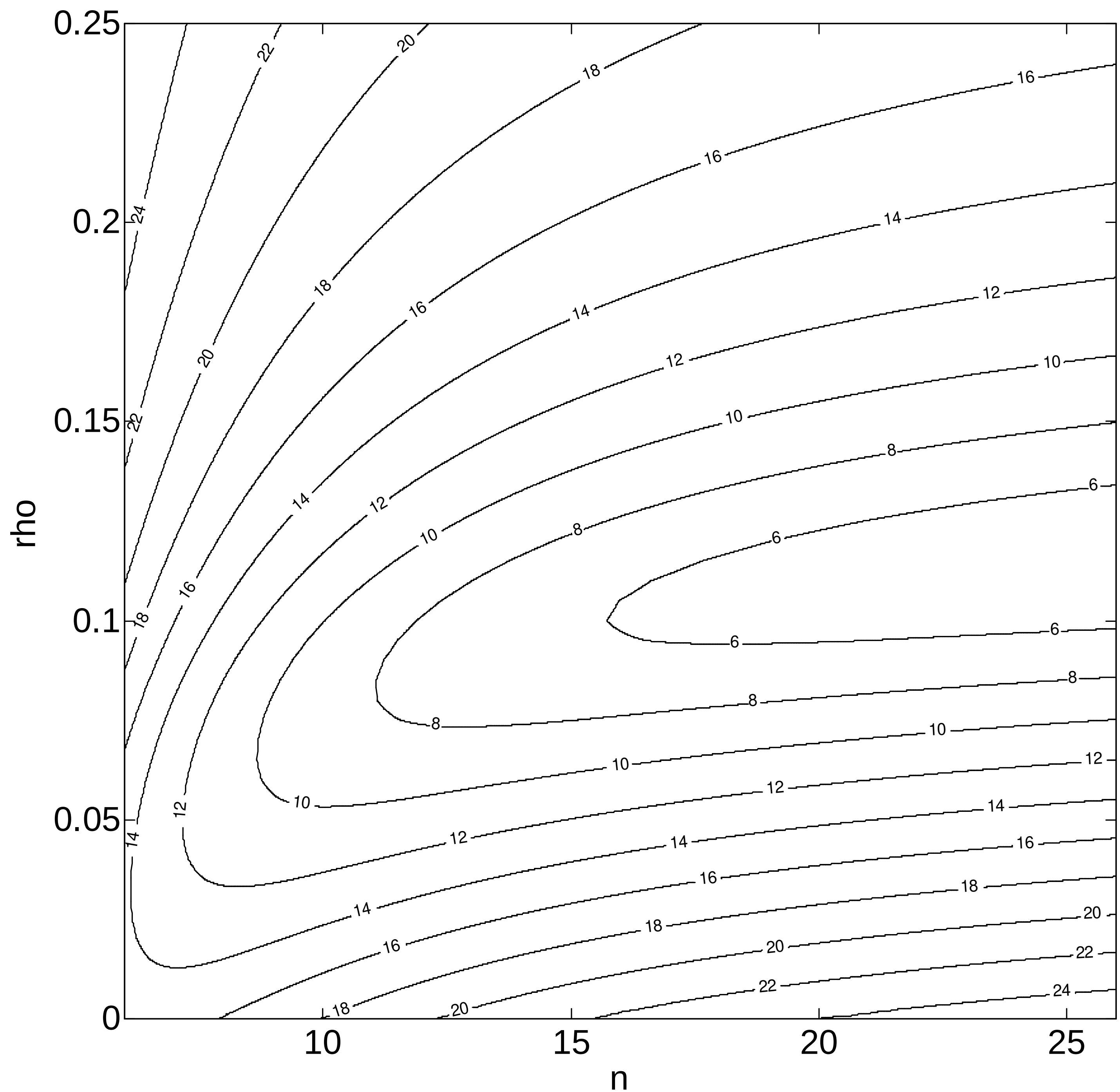
$$j_X = \frac{i_X}{I_{OUT}}$$

$$\rho = R\,\frac{I_{OUT}}{V_m}$$

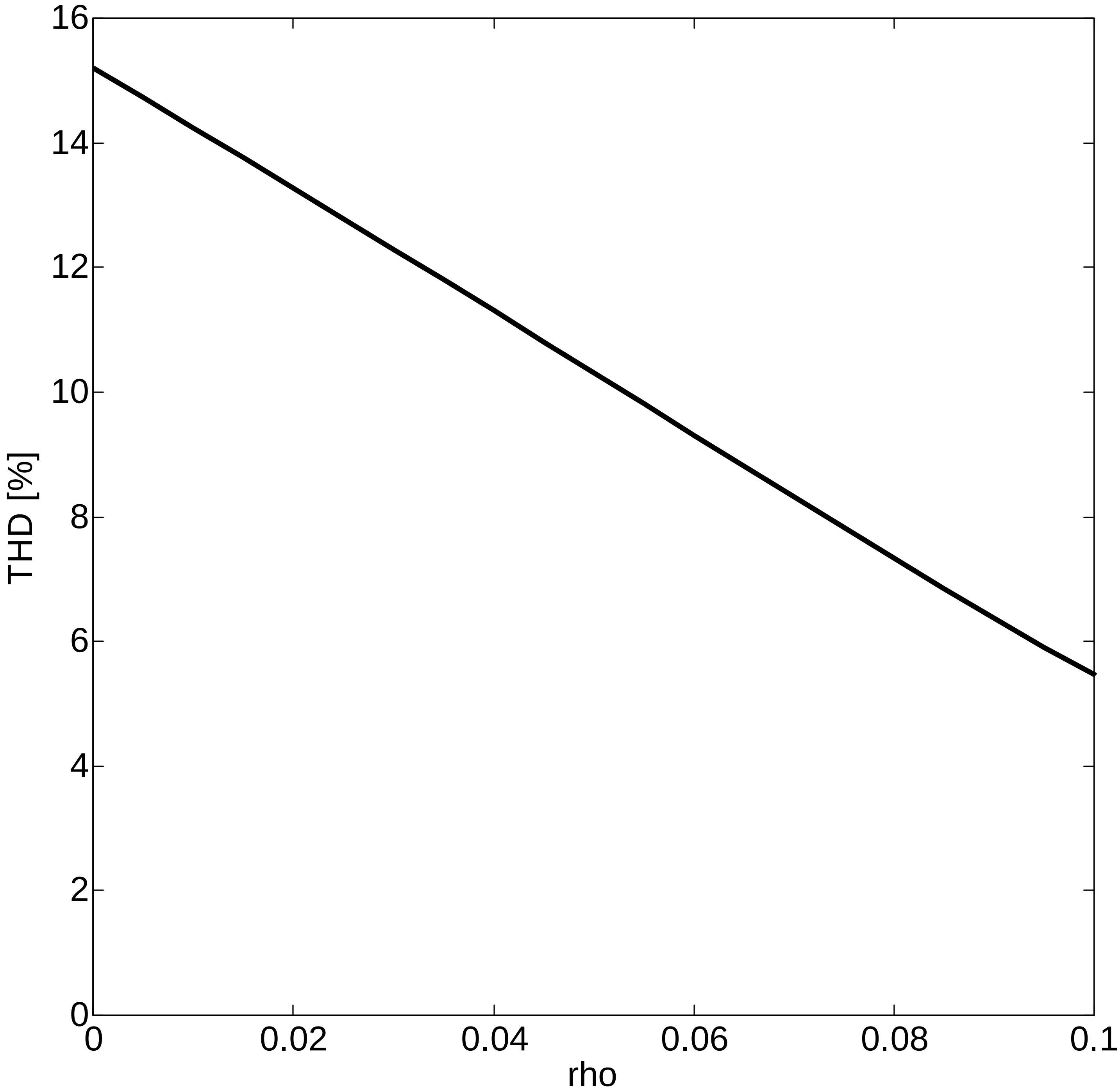
| state | DA | DB | DX | DY | $j_Y$ | $j_A$ | $j_B$ |
|---|---|---|---|---|---|---|---|
| –2 | 0 | 1 | 0 | 1 | $j_Y = -\frac{2n}{n+1}$ | 0 | $-j_Y$ |
| –1 | 1 | 1 | 0 | 1 | $j_Y = \frac{1}{2n\rho}\left((n+1)m_{A0} + (n-1)m_{B0}\right)$ | $1+\frac{n+1}{2n}j_Y$ | $1-\frac{n-1}{2n}j_Y$ |
| 0 | 1 | 1 | 0 | 0 | $j_Y = 0$ | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | $j_Y = \frac{1}{2n\rho}\left((n-1)m_{A0} + (n+1)m_{B0}\right)$ | $1+\frac{n-1}{2n}j_Y$ | $1-\frac{n+1}{2n}j_Y$ |
| 2 | 1 | 0 | 1 | 0 | $j_Y = \frac{2n}{n+1}$ | $j_Y$ | 0 |

| state | condition 1 | transition | condition 2 | transition |
|---|---|---|---|---|
| $-2$ | $(n+1)m_{A0}+(n-1)m_{B0}+\frac{4n^2}{n+1}\rho<0$ | $-1$ | | |
| $-1$ | $(n+1)m_{A0}+(n-1)m_{B0}<0$ | $0$ | $(n+1)m_{A0}+(n-1)m_{B0}+\frac{4n^2}{n+1}\rho>0$ | $-2$ |
| $0$ | $(n-1)m_{A0}+(n+1)m_{B0}<0$ | $1$ | $(n+1)m_{A0}+(n-1)m_{B0}>0$ | $-1$ |
| $1$ | $(n-1)m_{A0}+(n+1)m_{B0}>0$ | $0$ | $(n-1)m_{A0}+(n+1)m_{B0}-\frac{4n^2}{n+1}\rho<0$ | $2$ |
| $2$ | $(n-1)m_{A0}+(n+1)m_{B0}-\frac{4n^2}{n+1}\rho>0$ | $1$ | | |

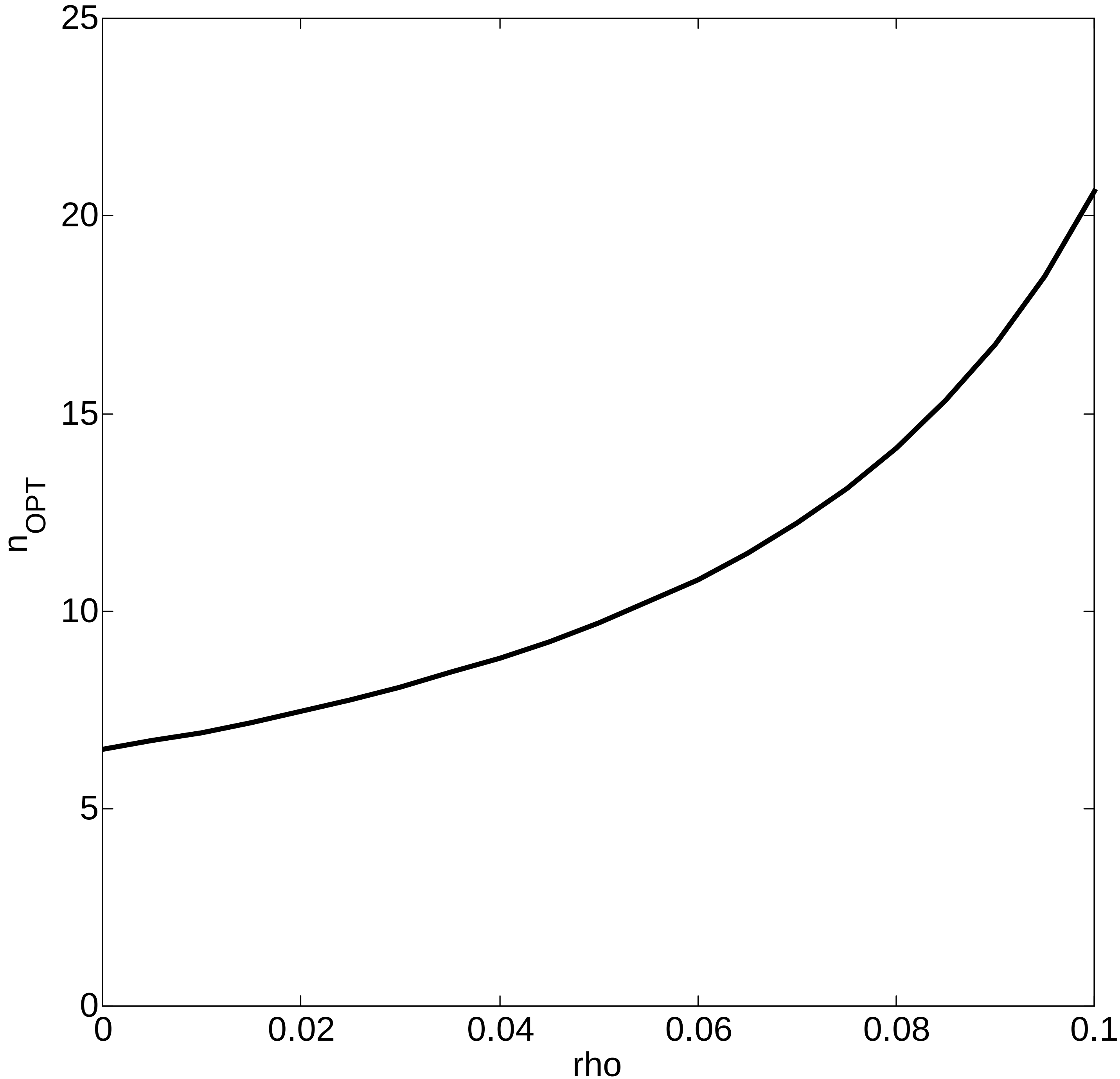
| state | $m_A$ | $m_B$ |
|---|---|---|
| $-2$ | $-\frac{n-1}{n+1}m_{B0}-\frac{4n^2}{(n+1)^2}\rho$ | $m_{B0}$ |
| $-1$ | $m_{A0}$ | $m_{B0}$ |
| 0 | $m_{A0}$ | $m_{B0}$ |
| 1 | $m_{A0}$ | $m_{B0}$ |
| 2 | $m_{A0}$ | $-\frac{n-1}{n+1}m_{A0}+\frac{4n^2}{(n+1)^2}\rho$ |



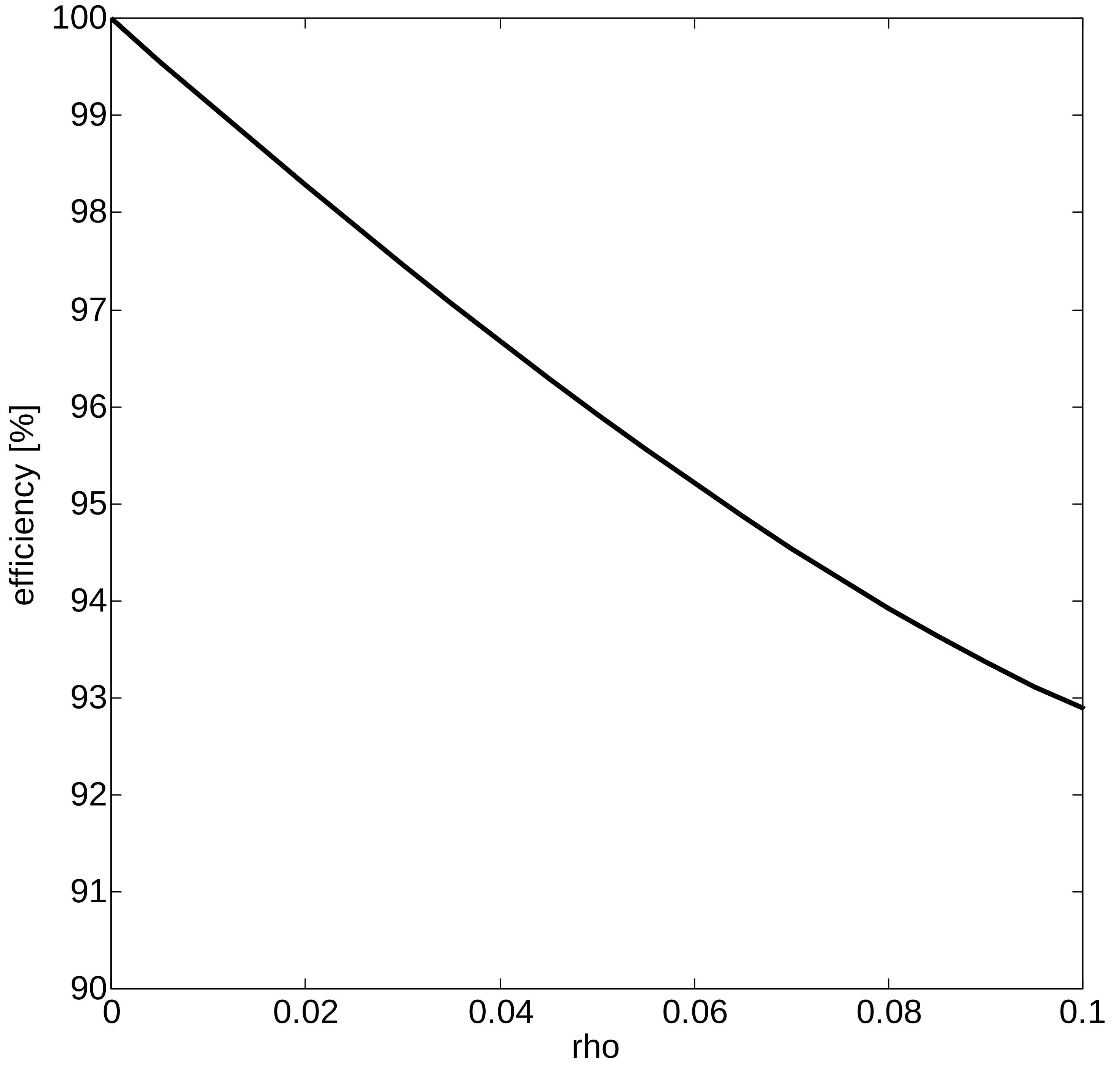
rho
n
0.25
0.2
0.15
0.1
0.05
0
10
15
20
25
24
22
20
18
16
14
12
10
8
6

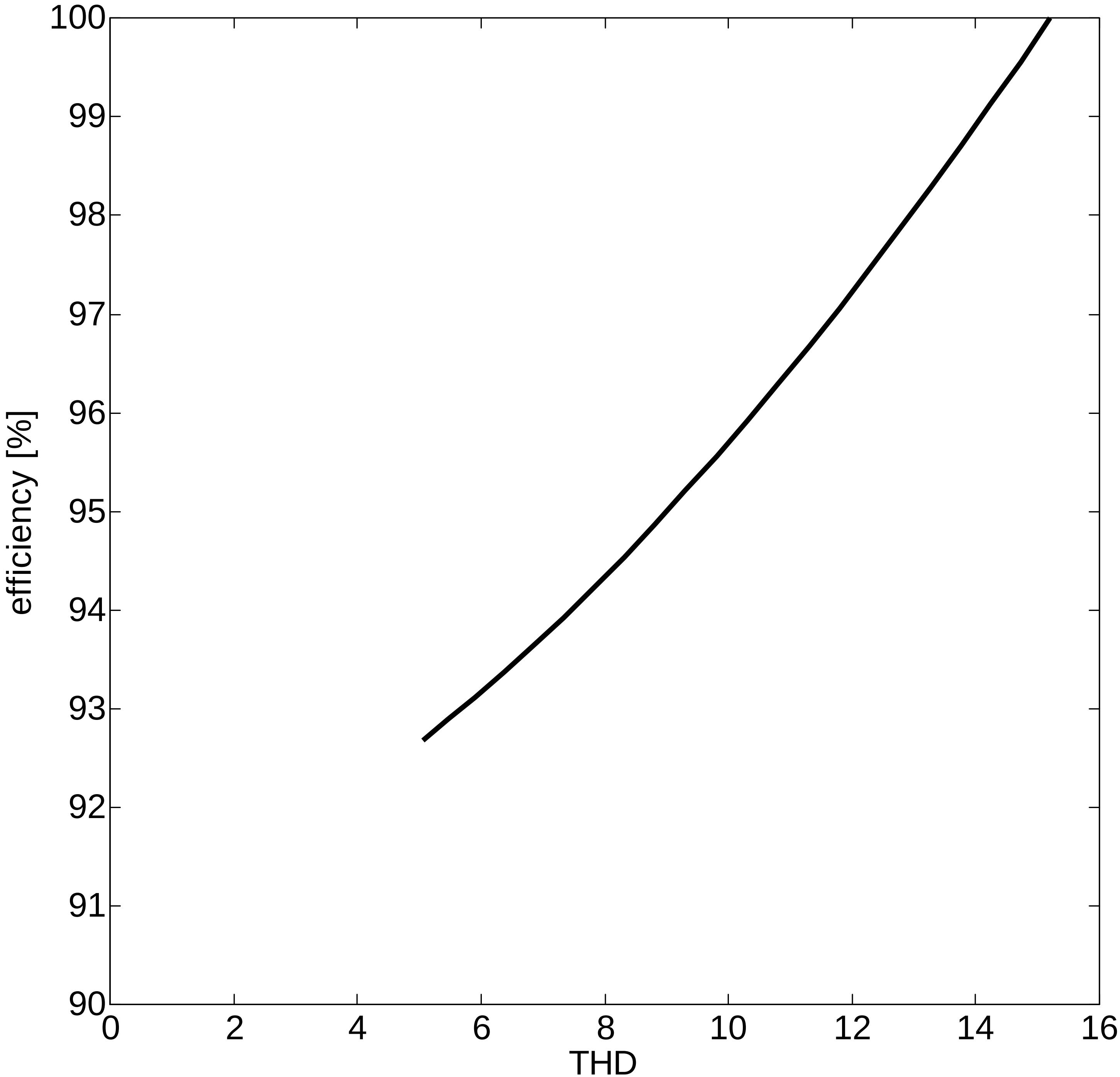


16
14
12
10
8
6
4
2
0
THD [%]
0
0.02
0.04
0.06
0.08
0.1
rho

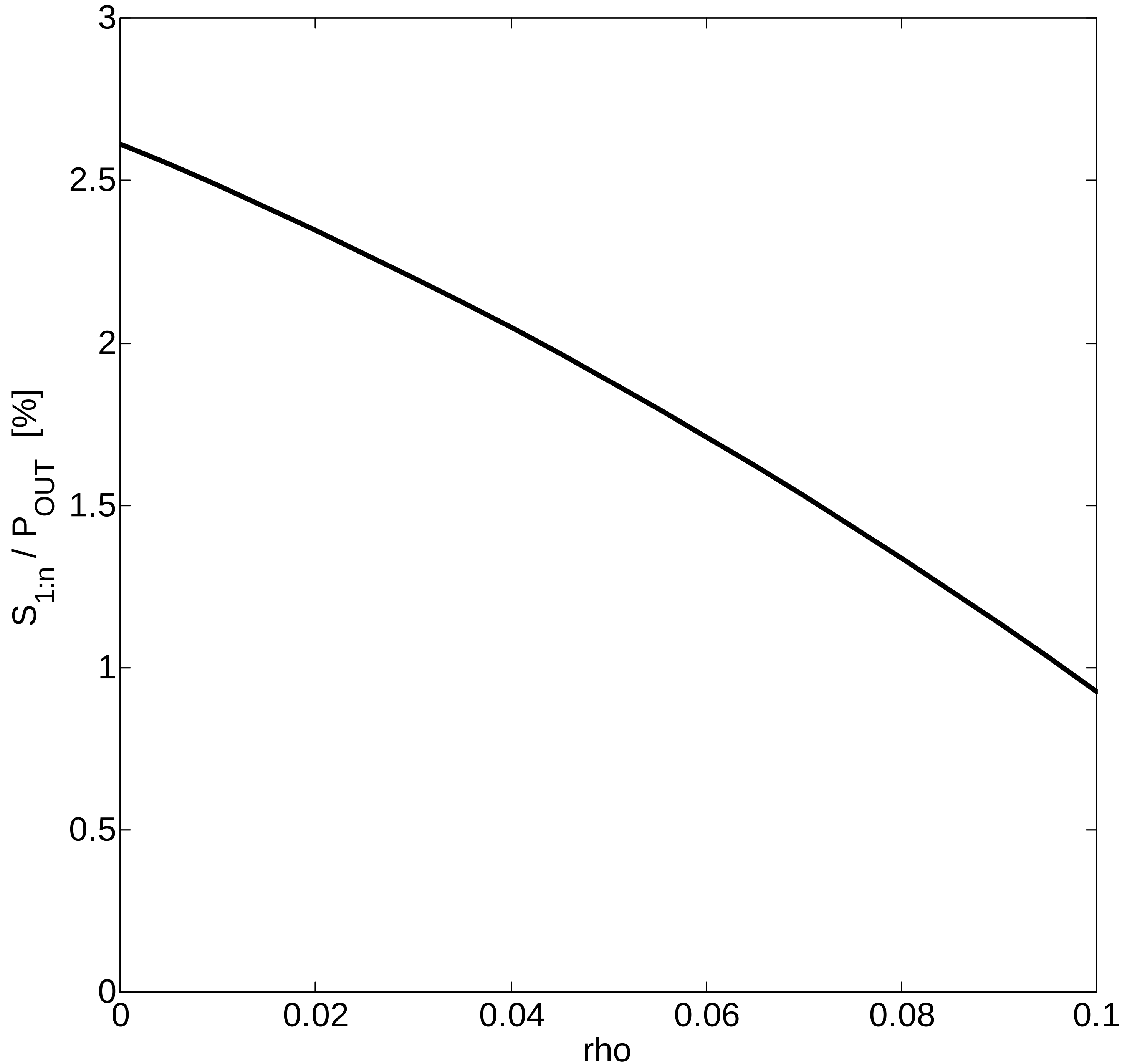


25
20
15
10
5
0
n_OPT
0
0.02
0.04
0.06
0.08
0.1
rho

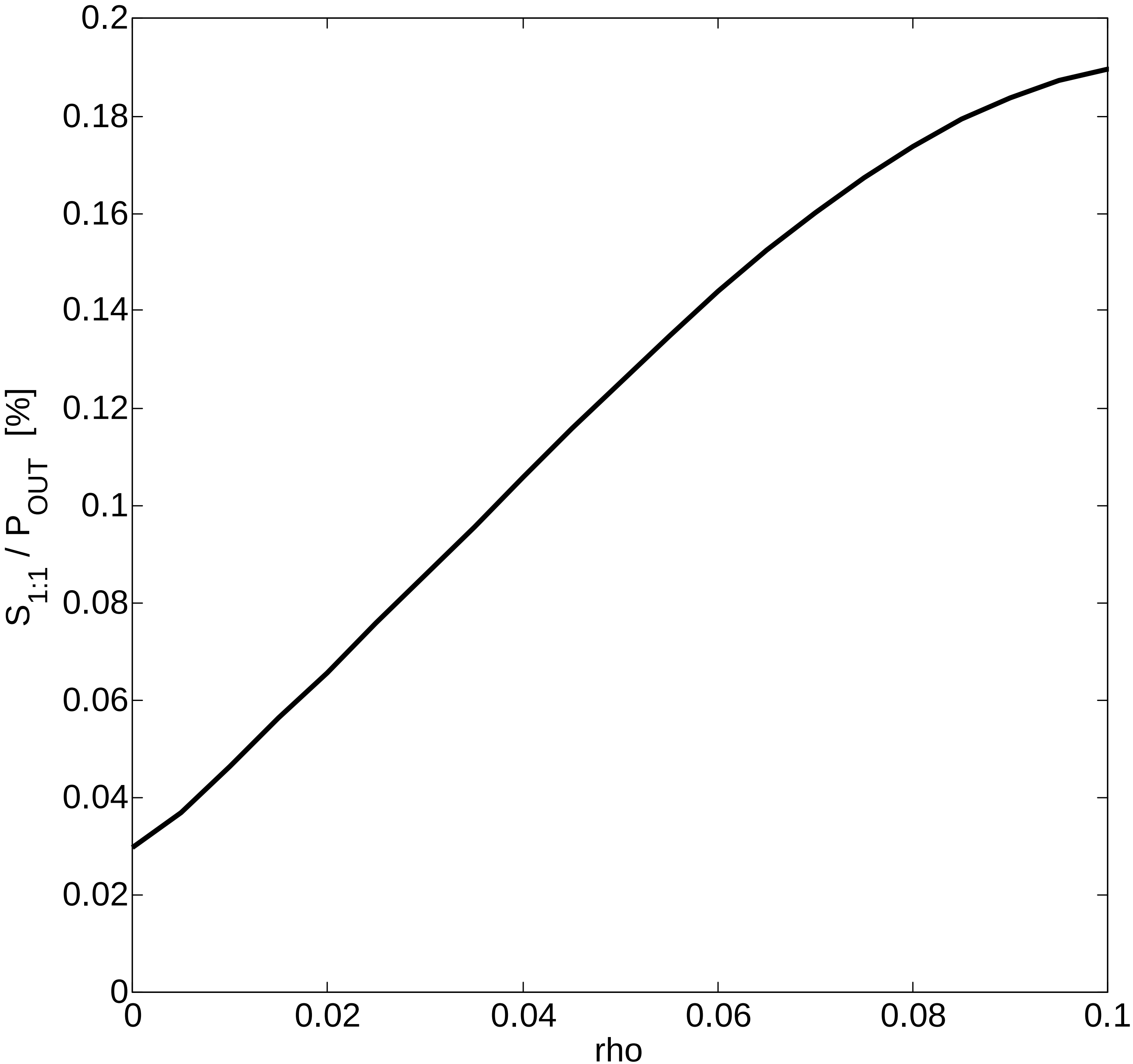

efficiency [%]
100
99
98
97
96
95
94
93
92
91
90
0
0.02
0.04
0.06
0.08
0.1
rho



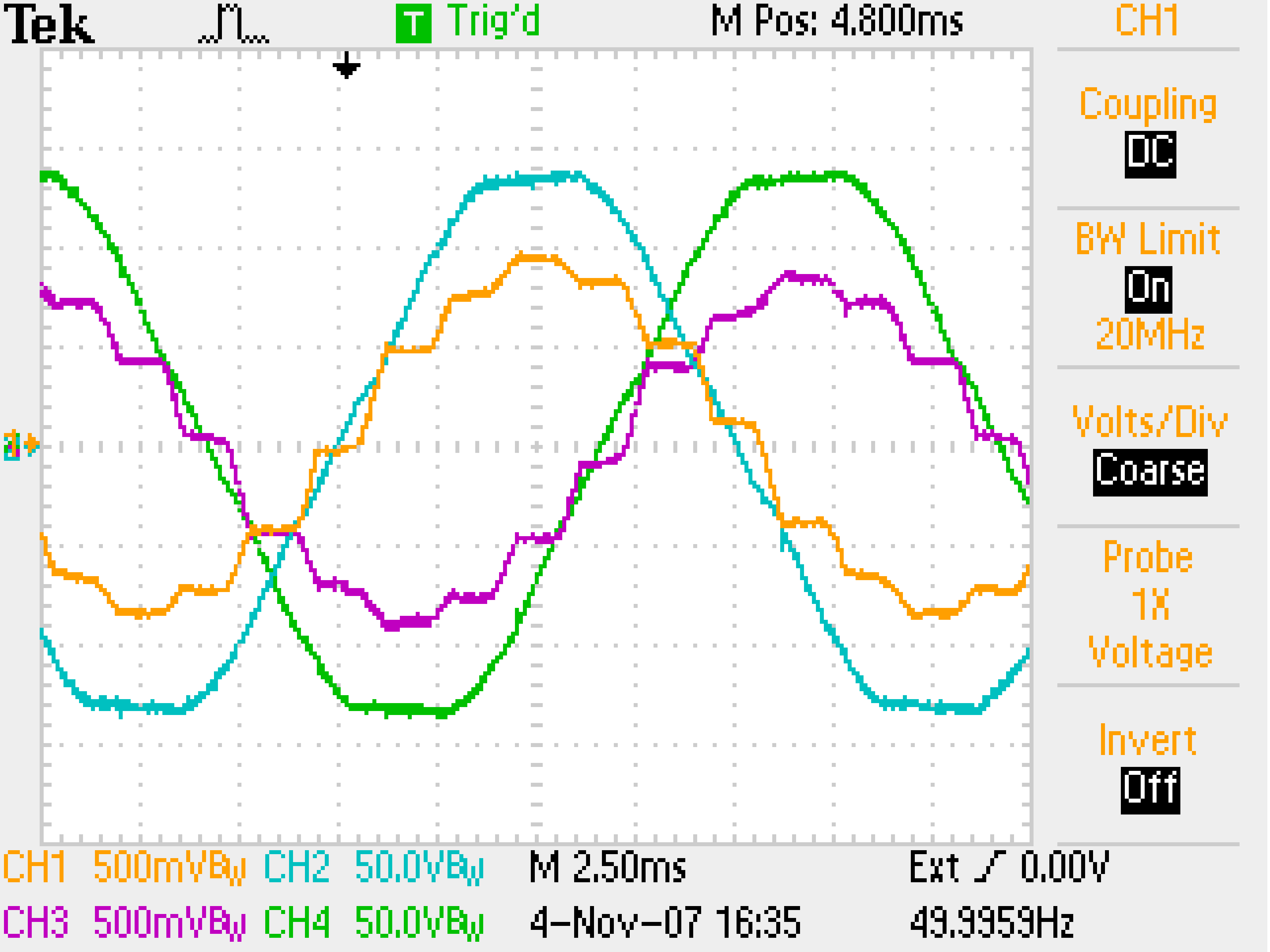
efficiency [%]
100
99
98
97
96
95
94
93
92
91
90
0
2
4
6
8
10
12
14
16
THD



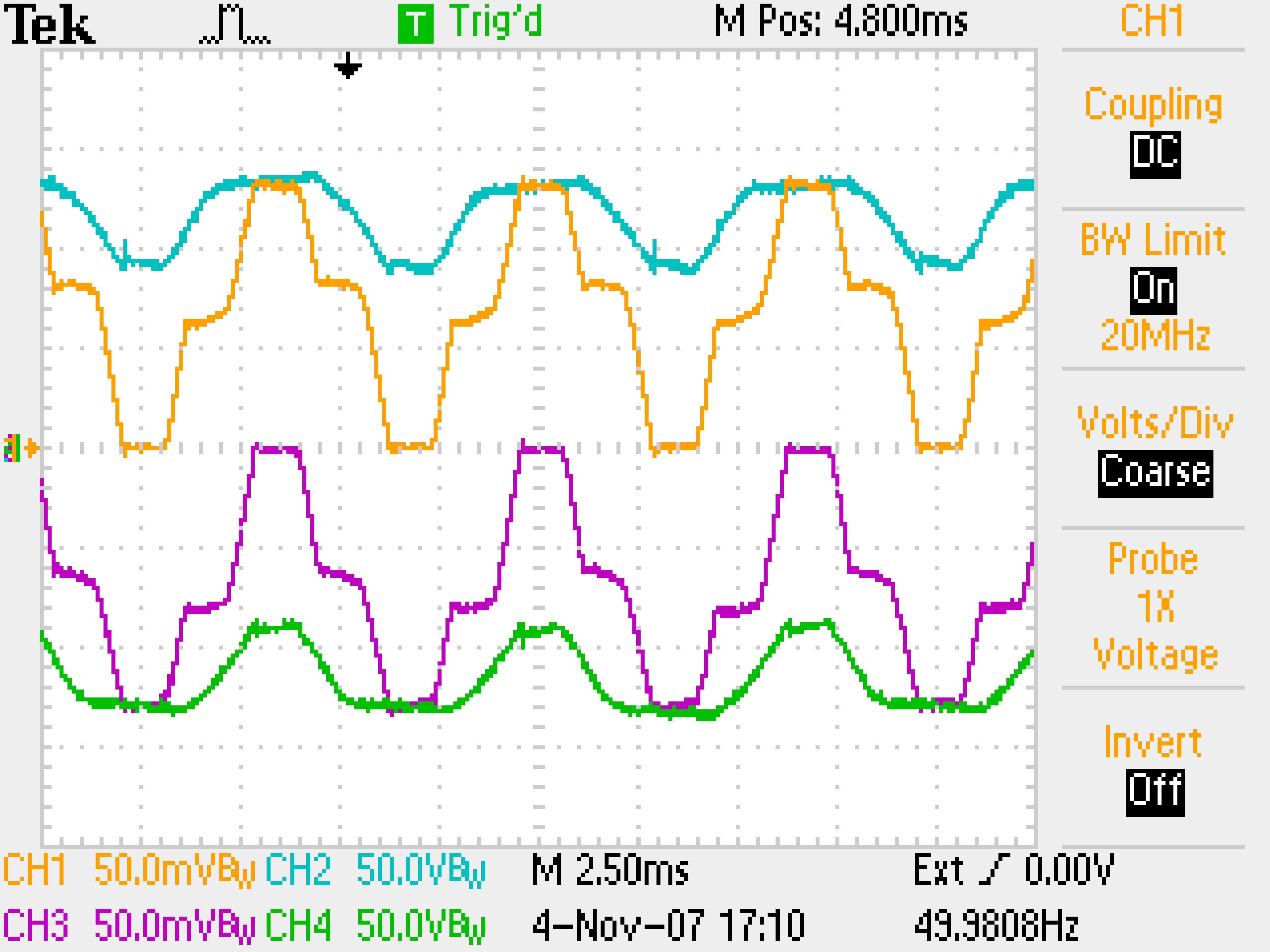
3
2.5
2
1.5
1
0.5
0
0
0.02
0.04
0.06
0.08
0.1
rho
$S_{1:n}$ / $P_{OUT}$ [%]



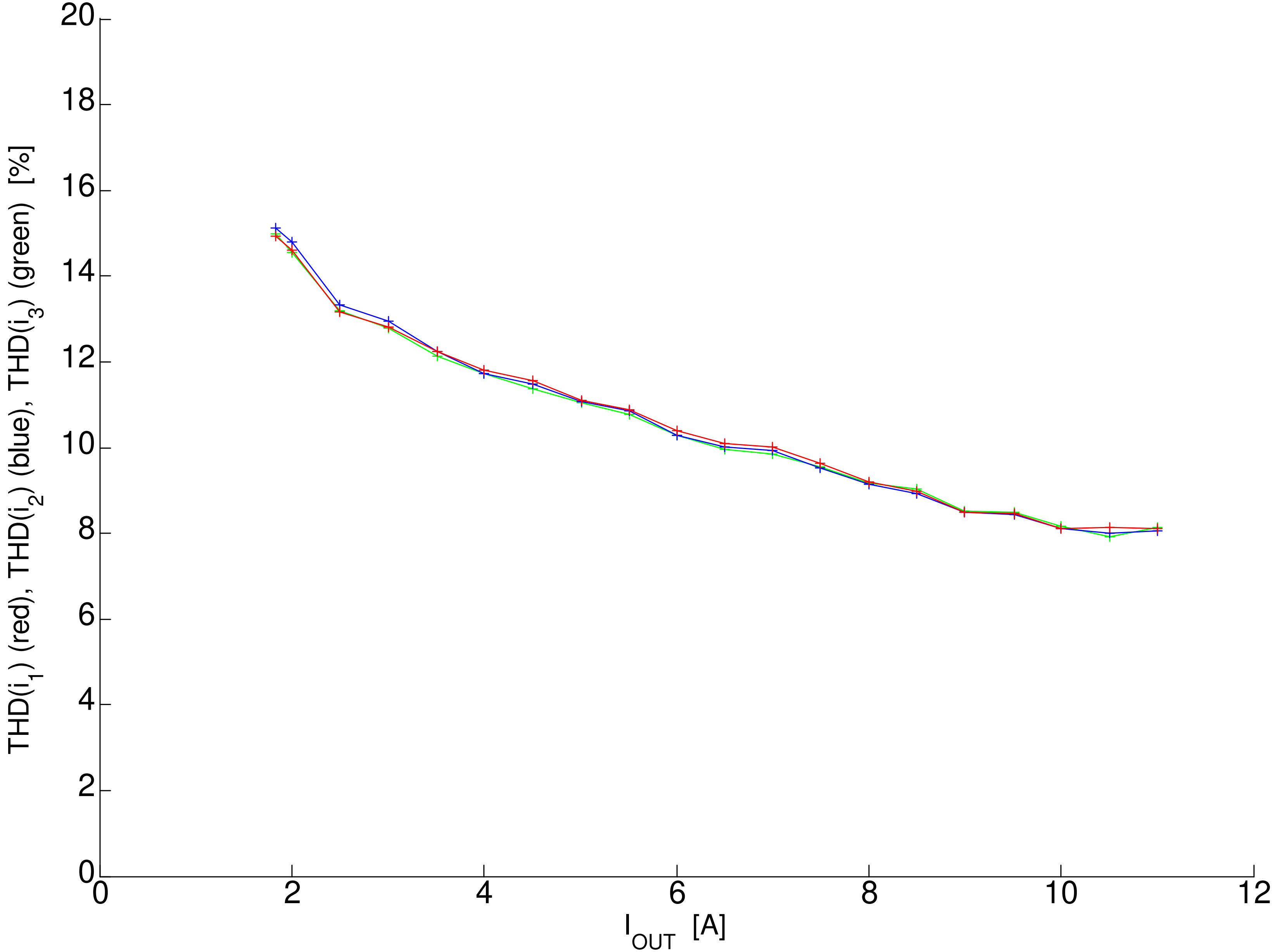
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.02
0.04
0.06
0.08
0.1
rho
S1:1 / POUT [%]



Tek
Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
1X
Voltage
Invert
Off
CH1 500mV
CH2 50.0V
M 2.50ms
Ext 0.00V
CH3 500mV
CH4 50.0V
4-Nov-07 16:35
49.9959Hz

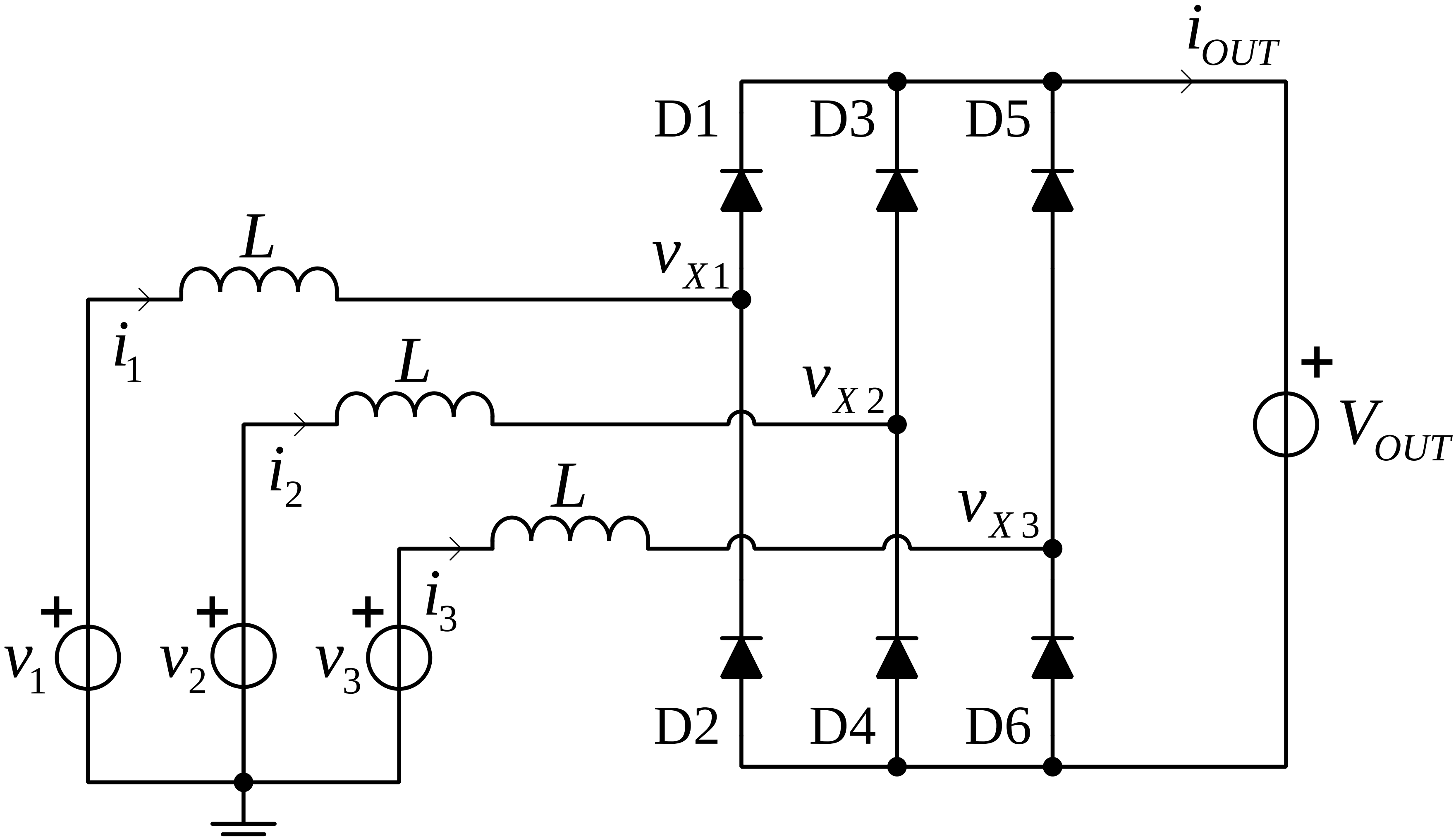


Tek
Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
1X
Voltage
Invert
Off
CH1 50.0mV BW CH2 50.0V BW
M 2.50ms
Ext 0.00V
CH3 50.0mV BW CH4 50.0V BW
4-Nov-07 17:10
49.9808Hz

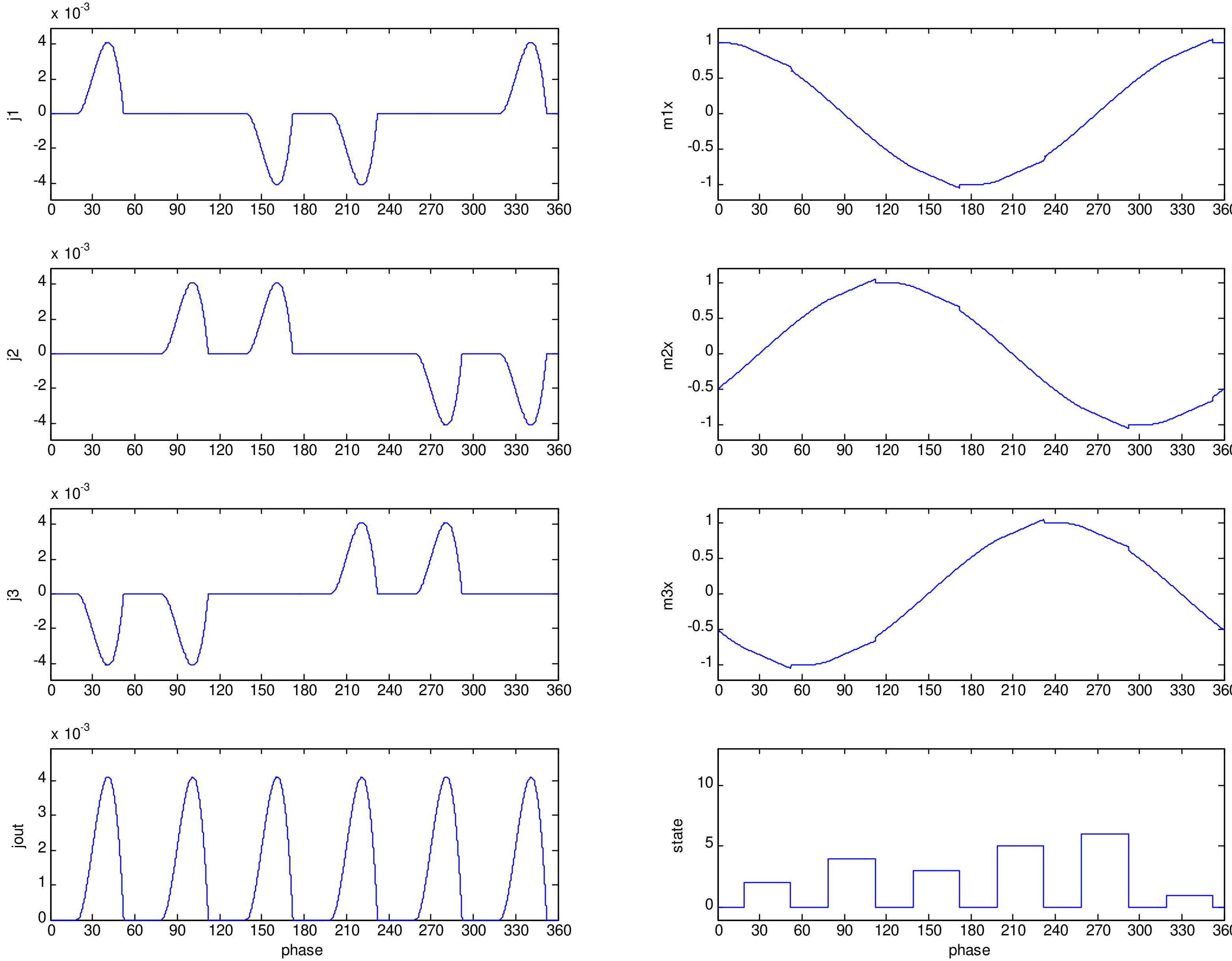


THD(i1) (red), THD(i2) (blue), THD(i3) (green) [%]
I_OUT [A]
0
2
4
6
8
10
12
14
16
18
20

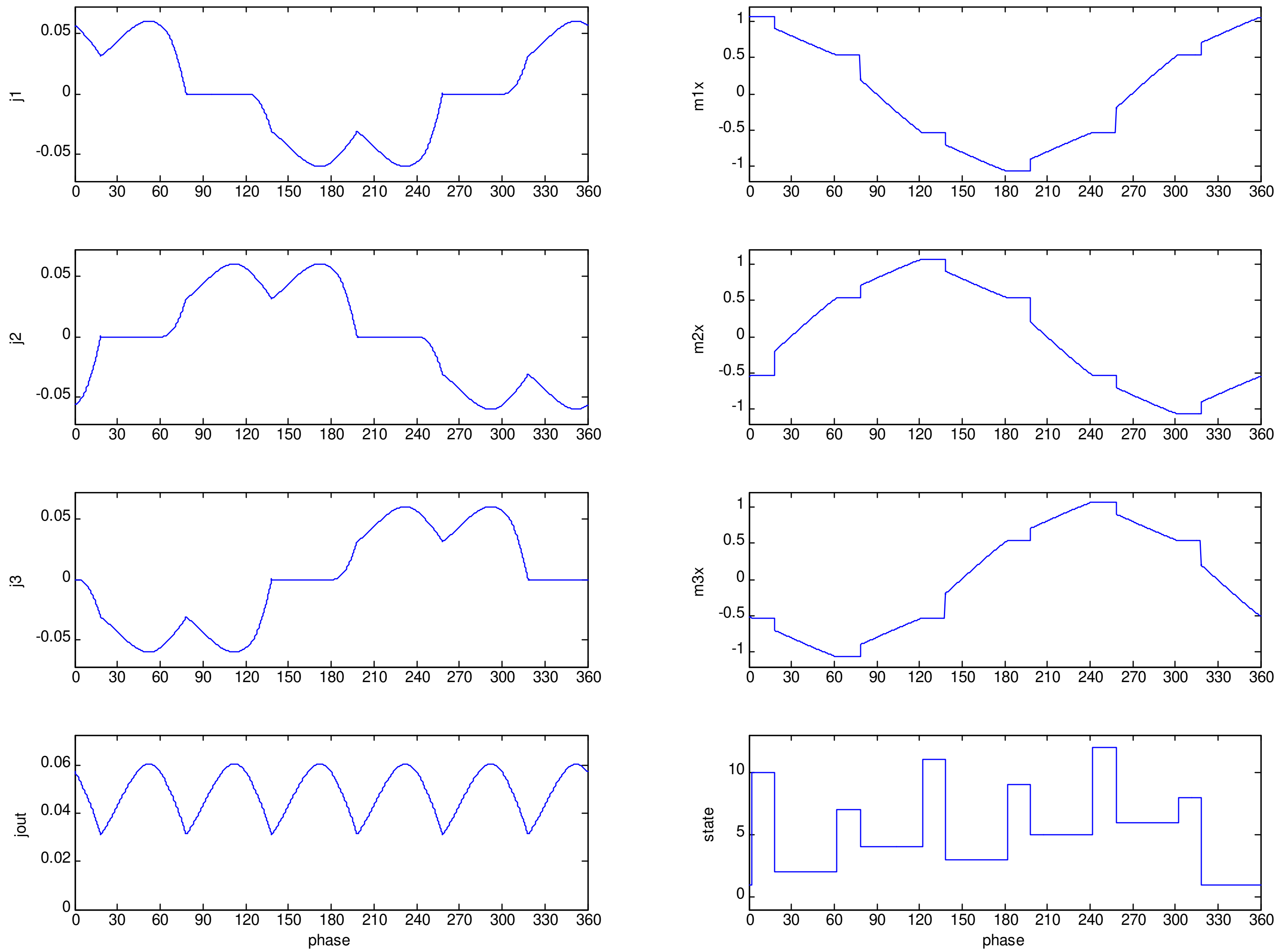
an application in education:



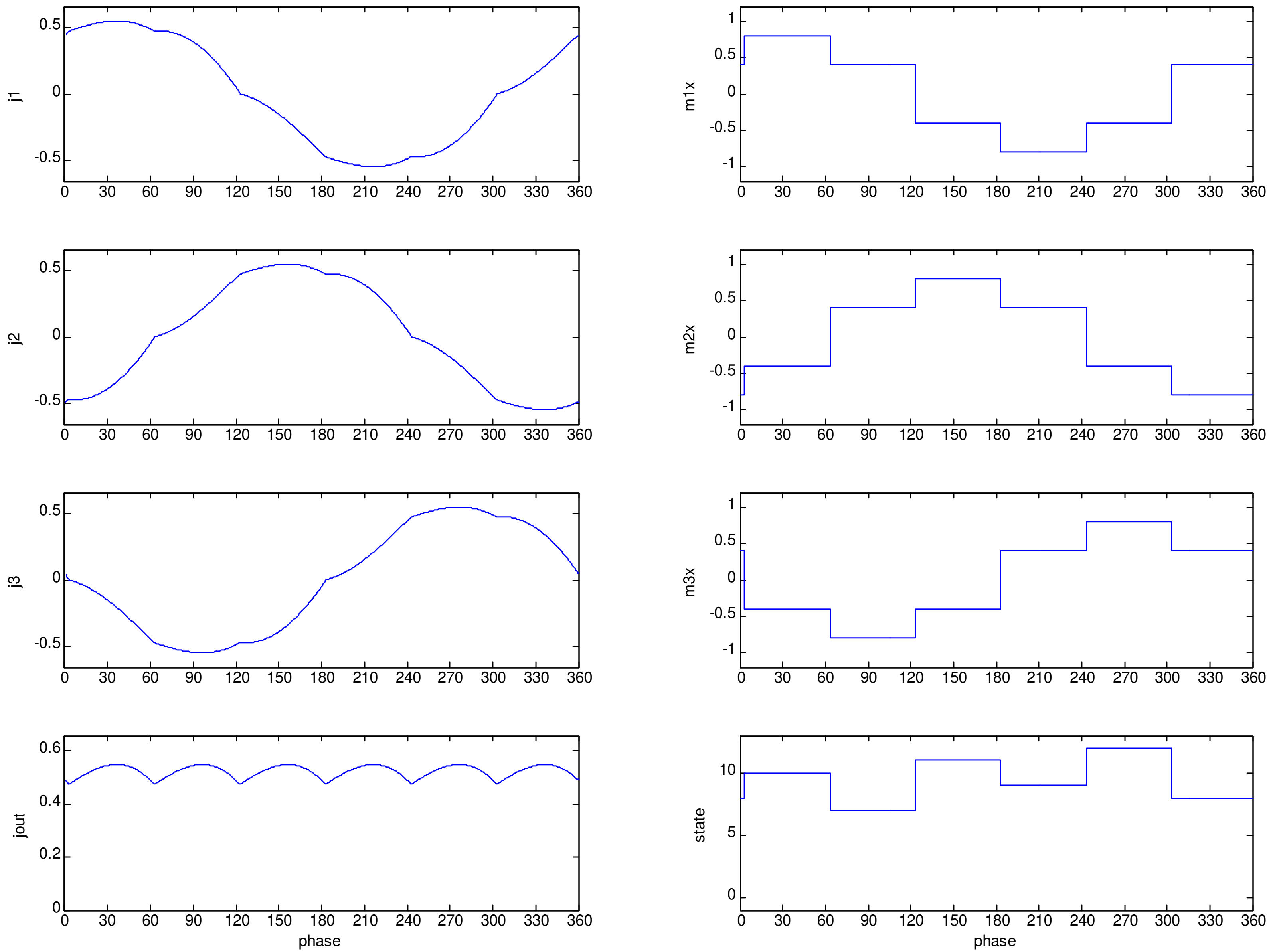

13 states, state transition rules, 2[nd] order circuit, . . .

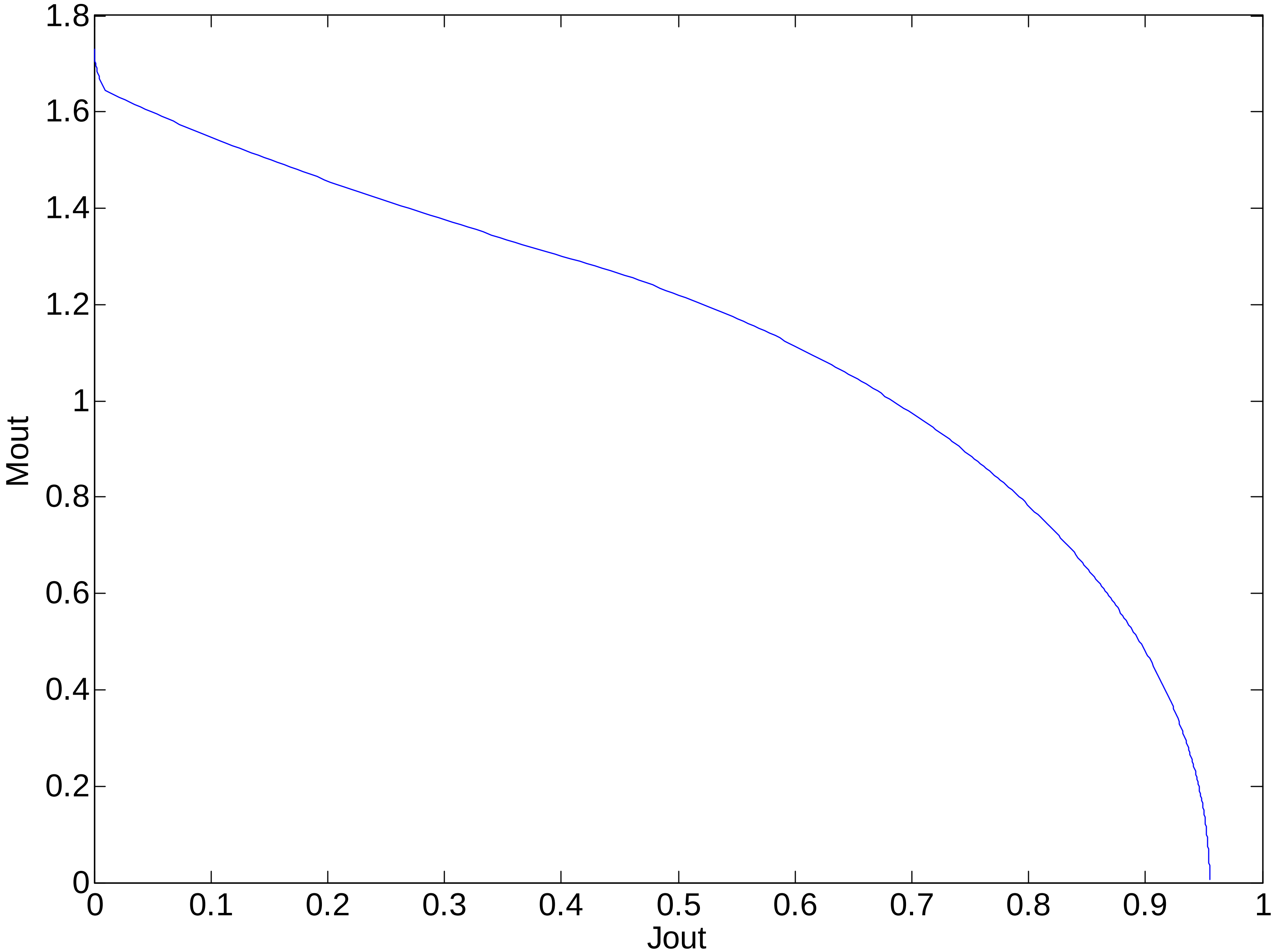


x 10^-3
j1
m1x
j2
m2x
j3
m3x
jout
state
phase
phase
0 30 60 90 120 150 180 210 240 270 300 330 360

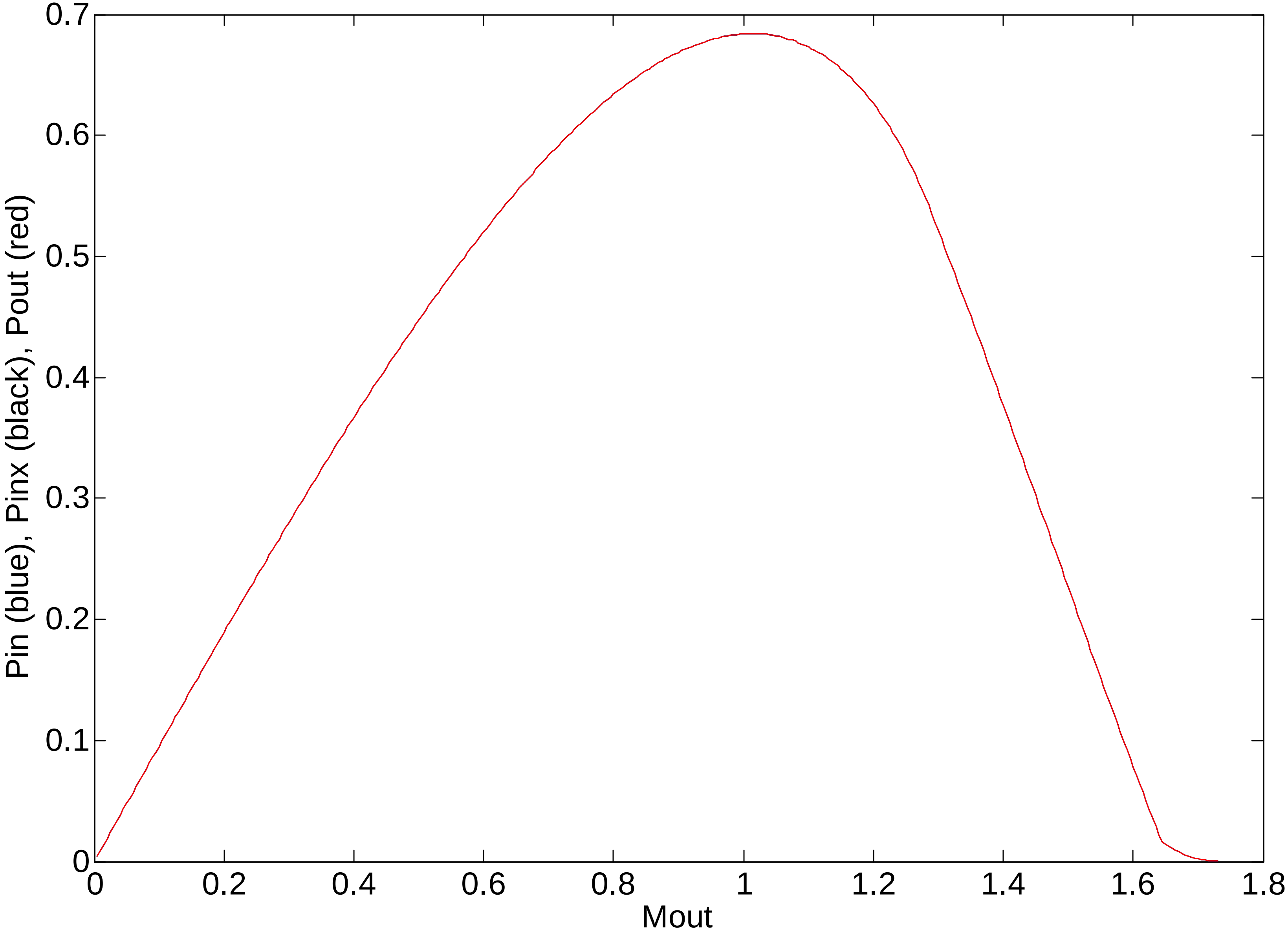


j1
m1x
j2
m2x
j3
m3x
jout
state
phase
phase

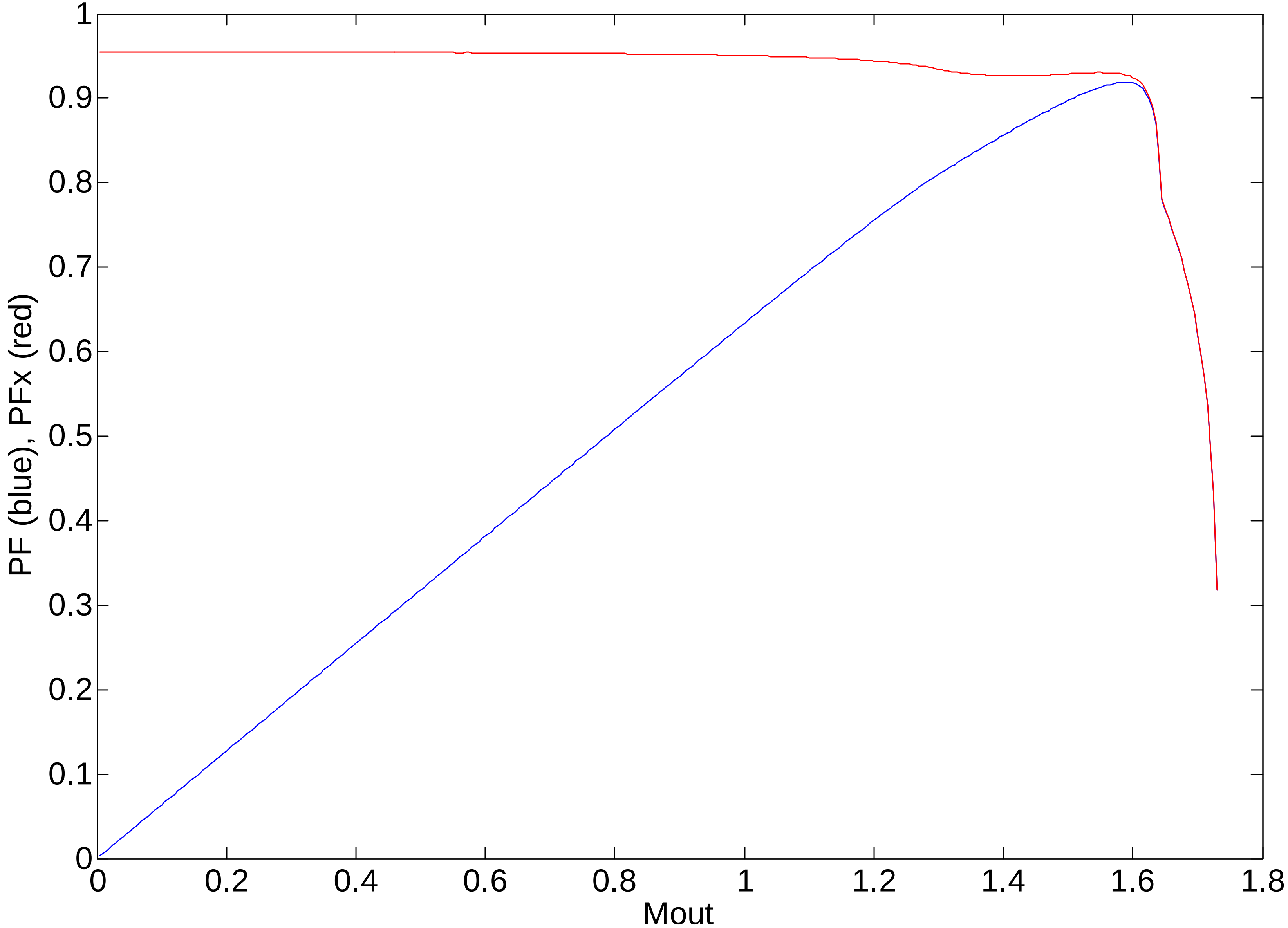

j1
j2
j3
jout
m1x
m2x
m3x
state
phase
phase

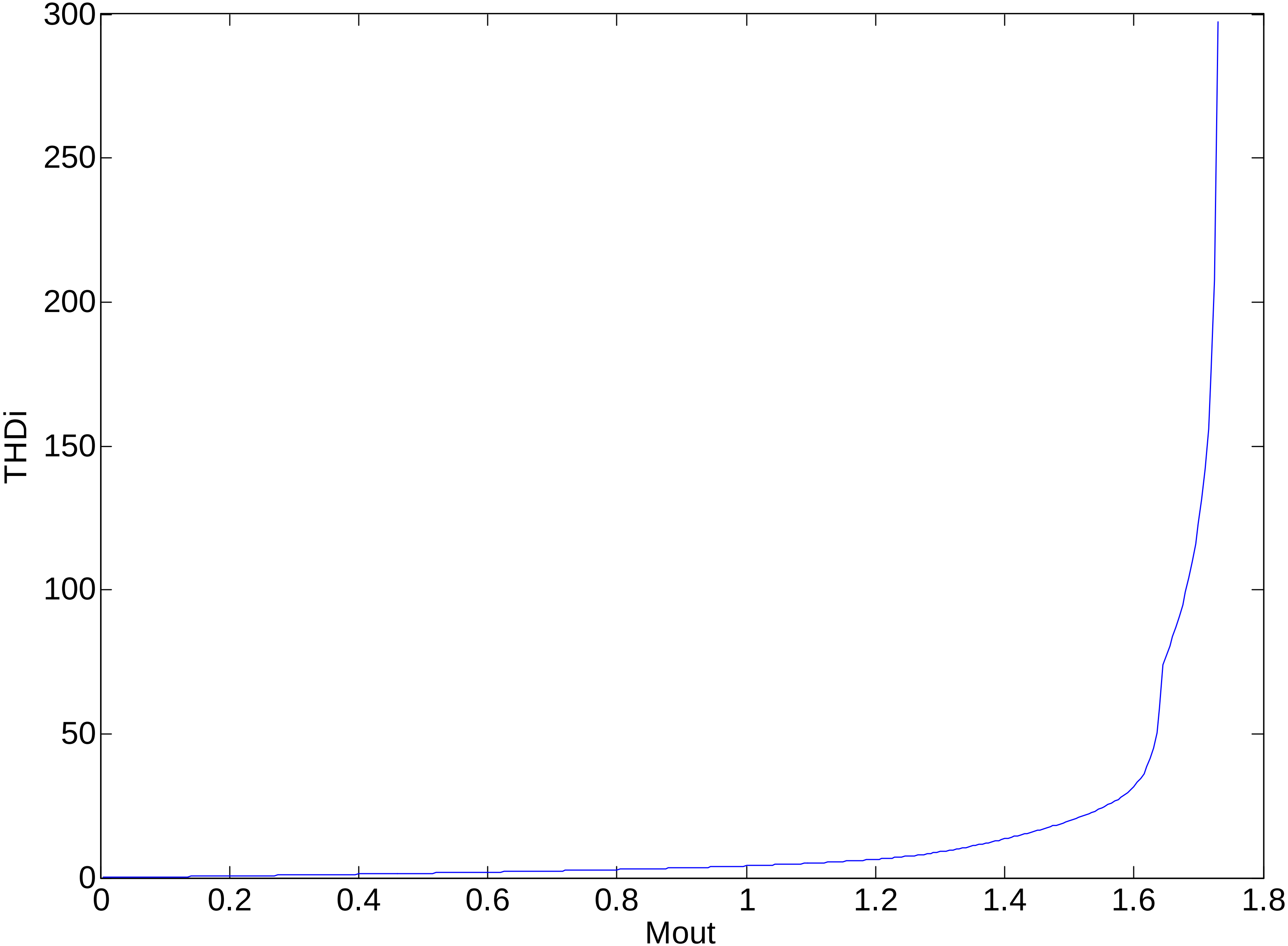


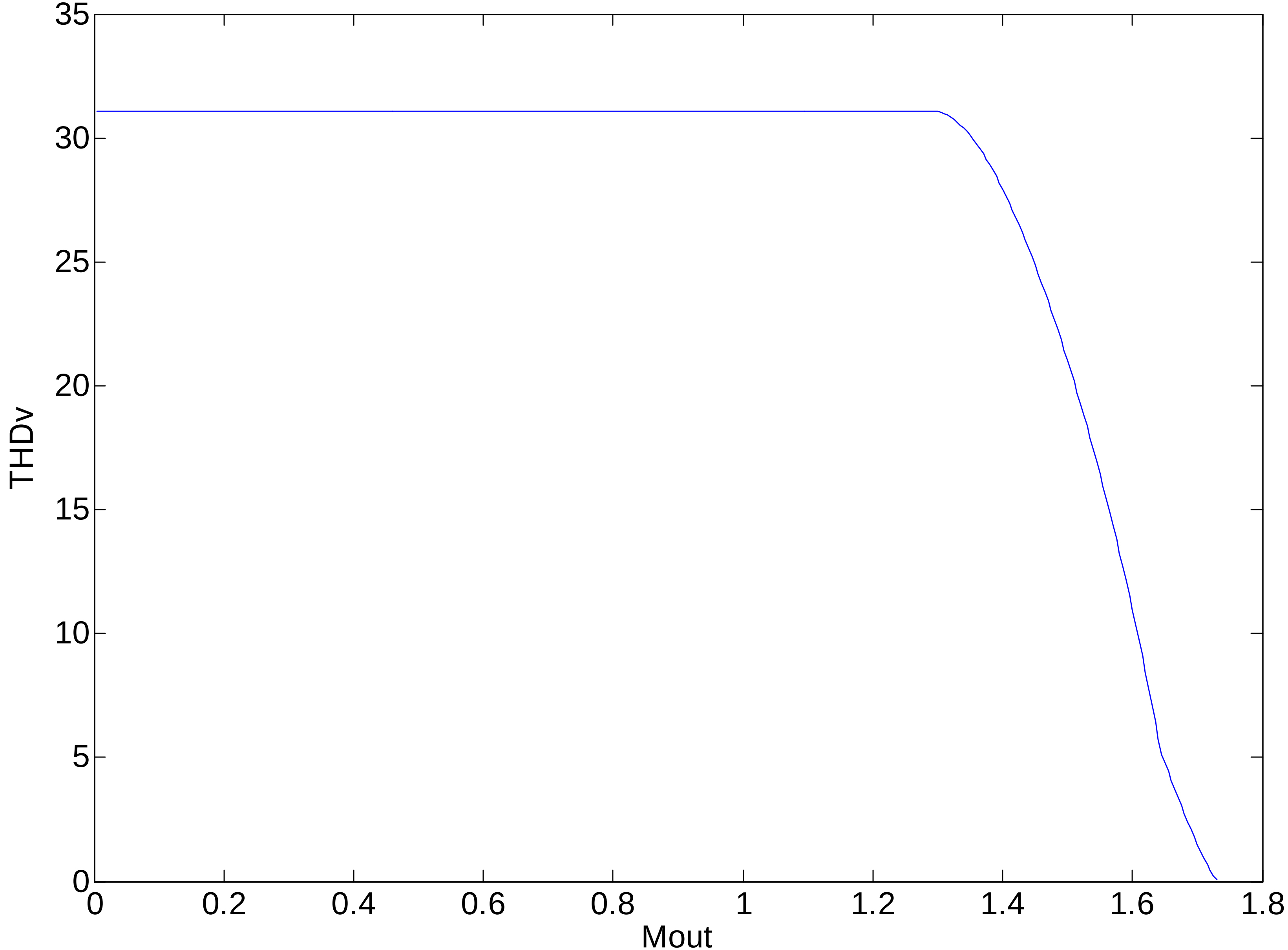
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Mout
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Jout

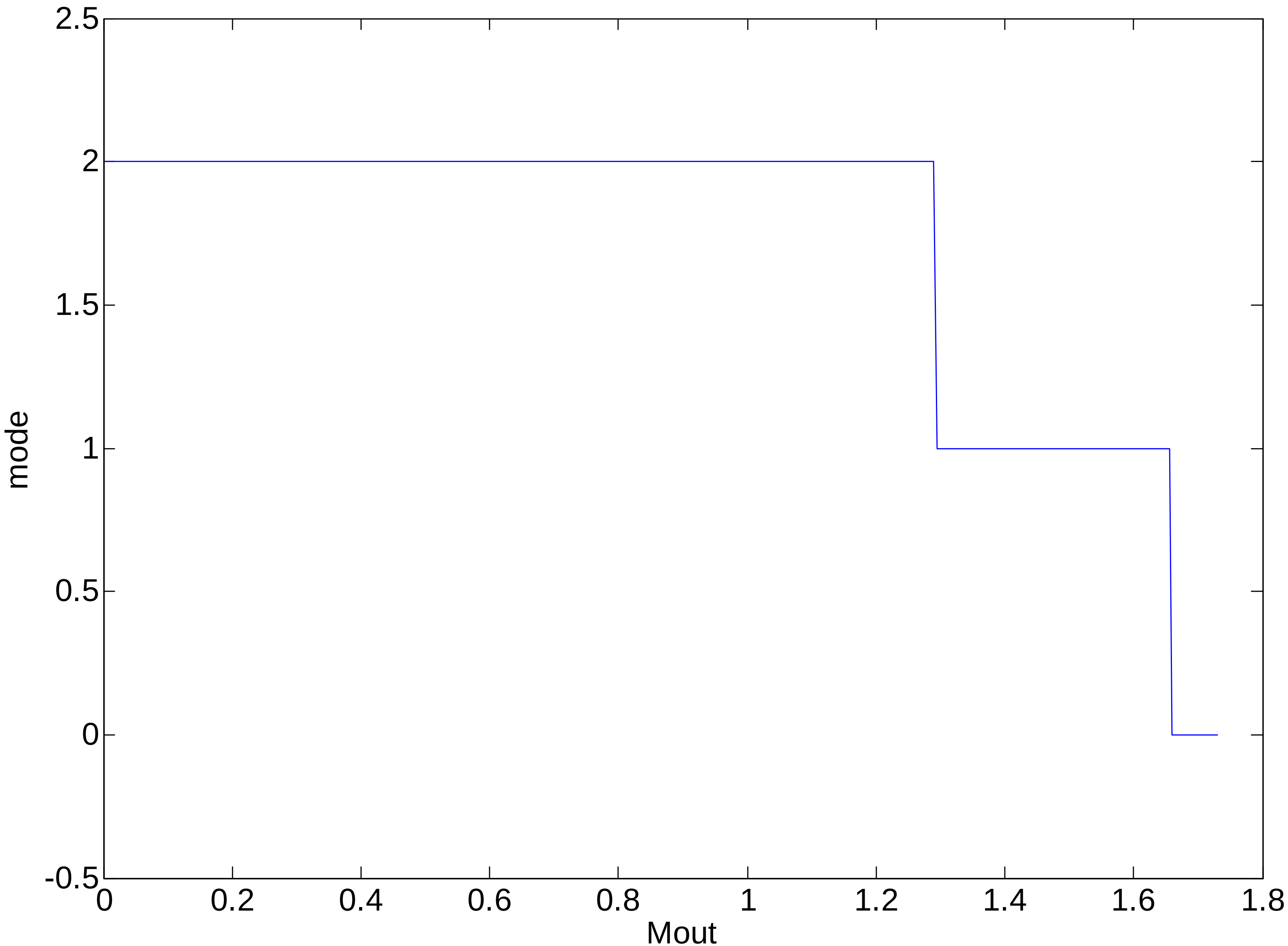


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Pin (blue), Pinx (black), Pout (red)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Mout


PF (blue), PFx (red)
Mout
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


300
250
200
150
100
50
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
THDi
Mout


THDv
Mout
0
5
10
15
20
25
30
35
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


2.5
2
1.5
1
0.5
0
-0.5
mode
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Mout

## Conclusions

- How to make simulation results general enough?
- Normalization
- Analytical preparation
- Numerical simulation
- Diagrams
- Steady state acceleration algorithm
- Problems with dynamic (algebraic) degeneration
- Analysis of dynamic circuits as resistive circuits
- Application of simulation in education
- Analytical solutions versus numerical solutions?