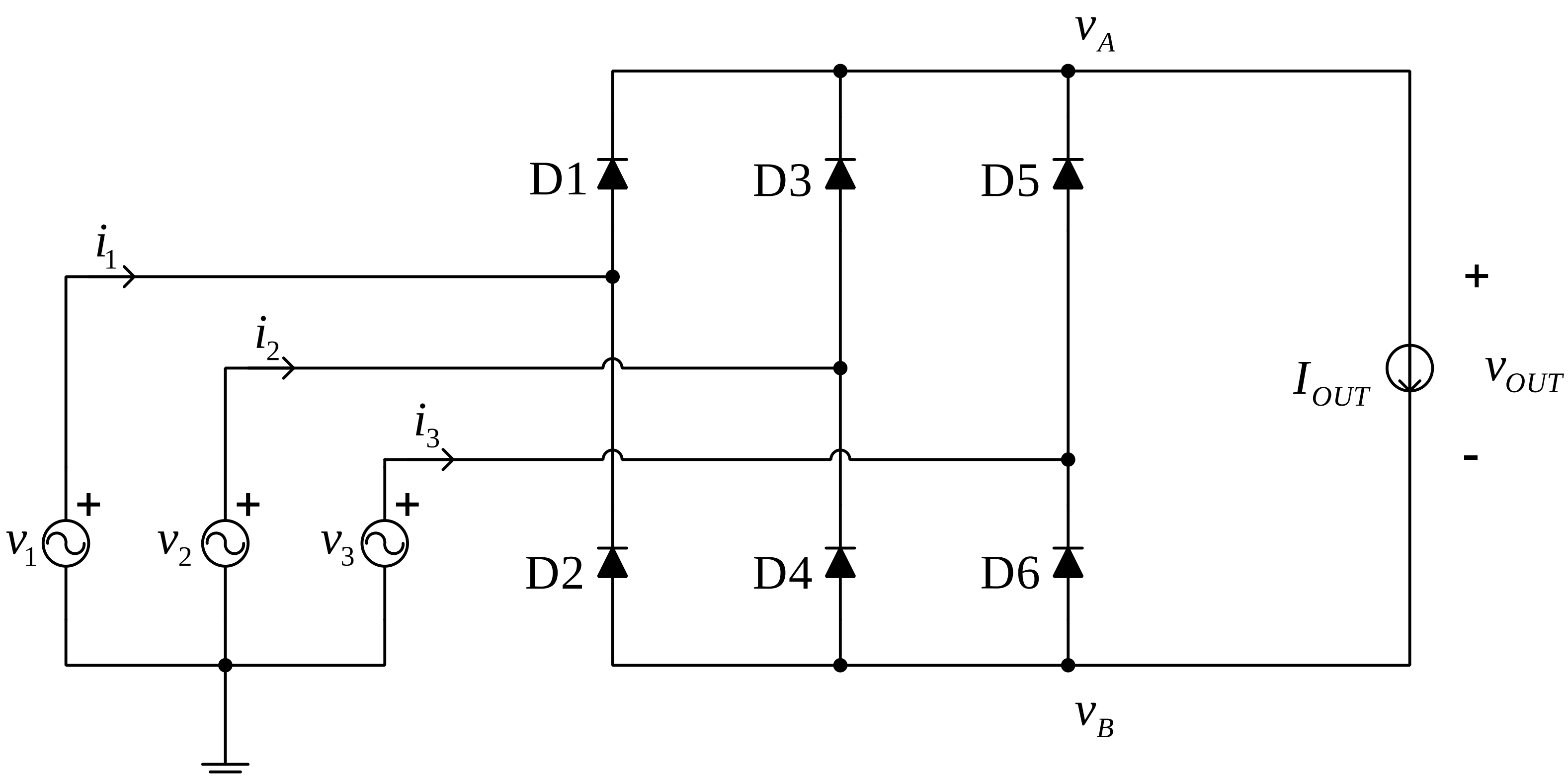
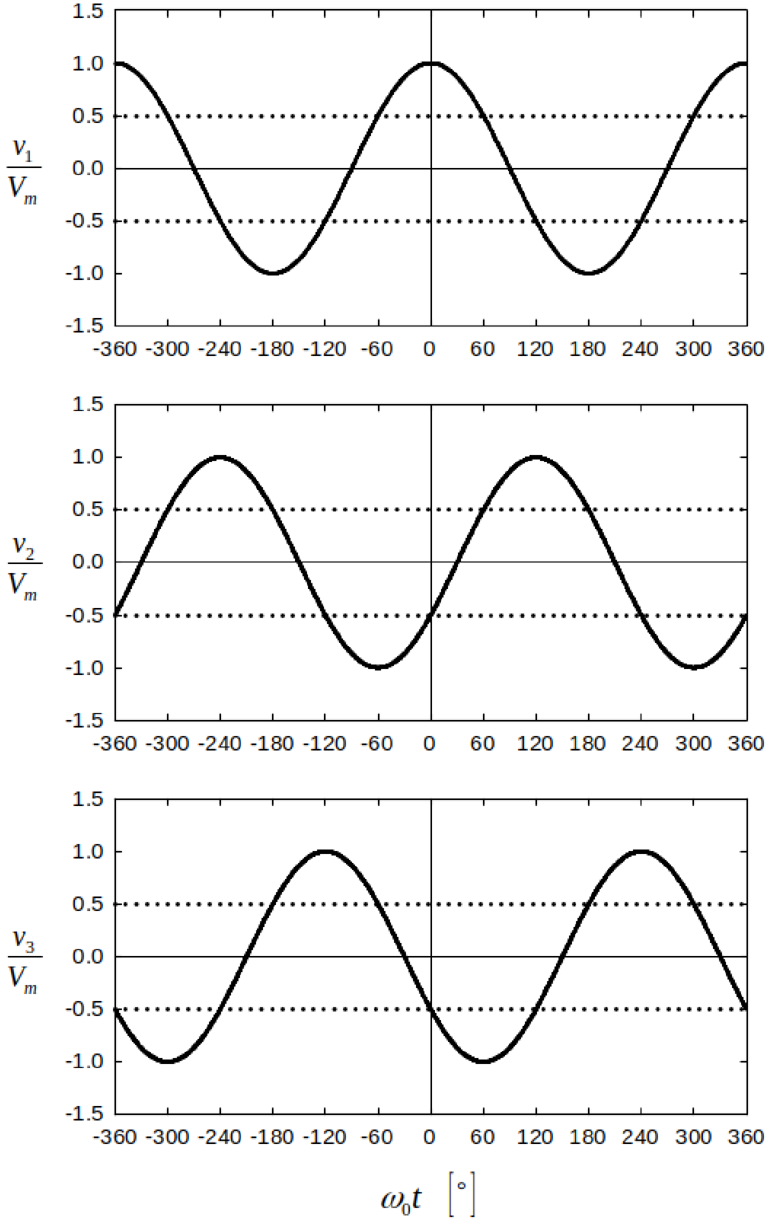
Current Injection Methods In Three-Phase Rectifiers

Predrag Pejović

University of Belgrade



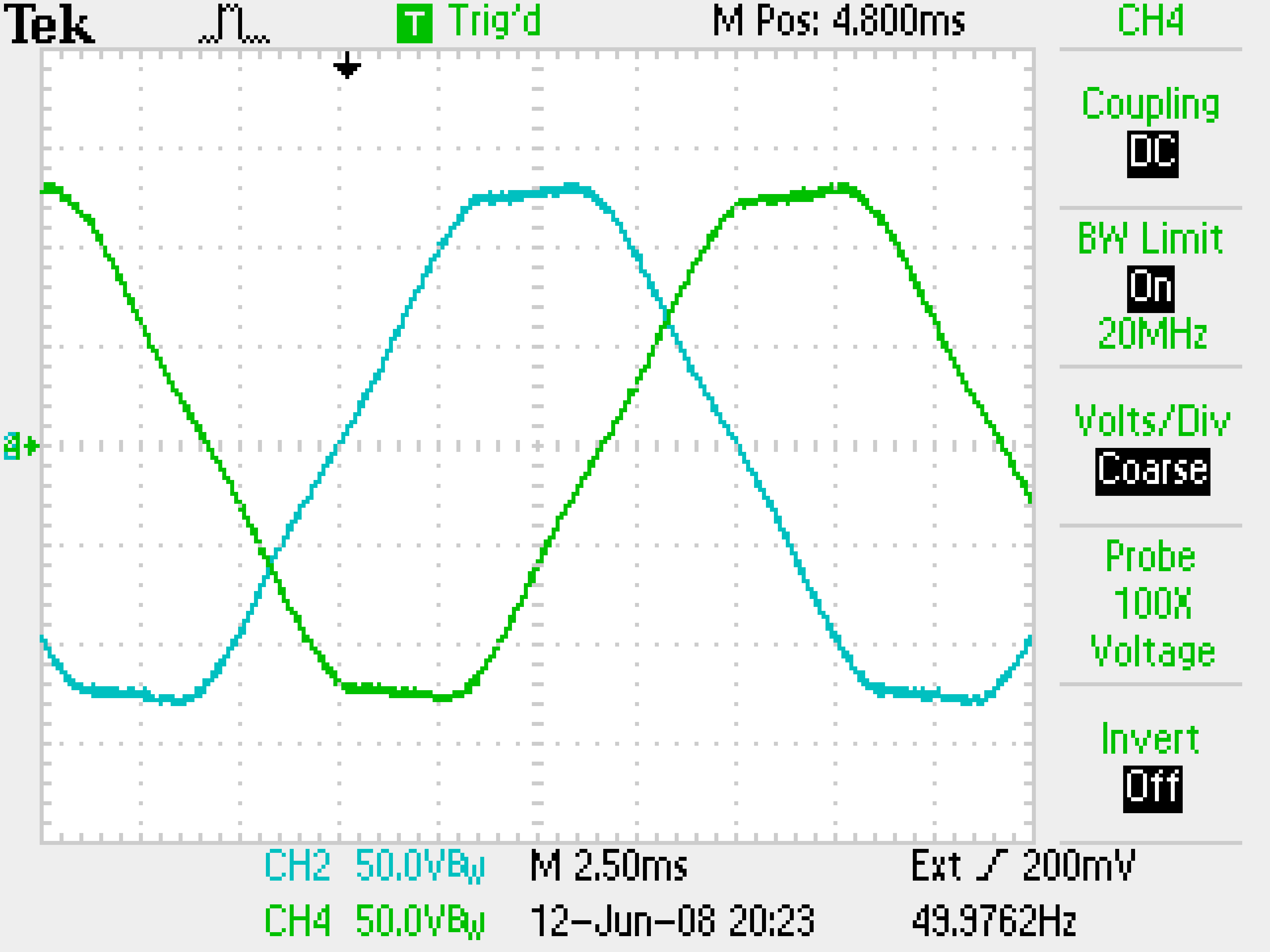
v_A
D1
D3
D5
i_1
+
i_2
I_{OUT}
v_{OUT}
i_3
-
+
+
+
v_1
v_2
v_3
D2
D4
D6
v_B

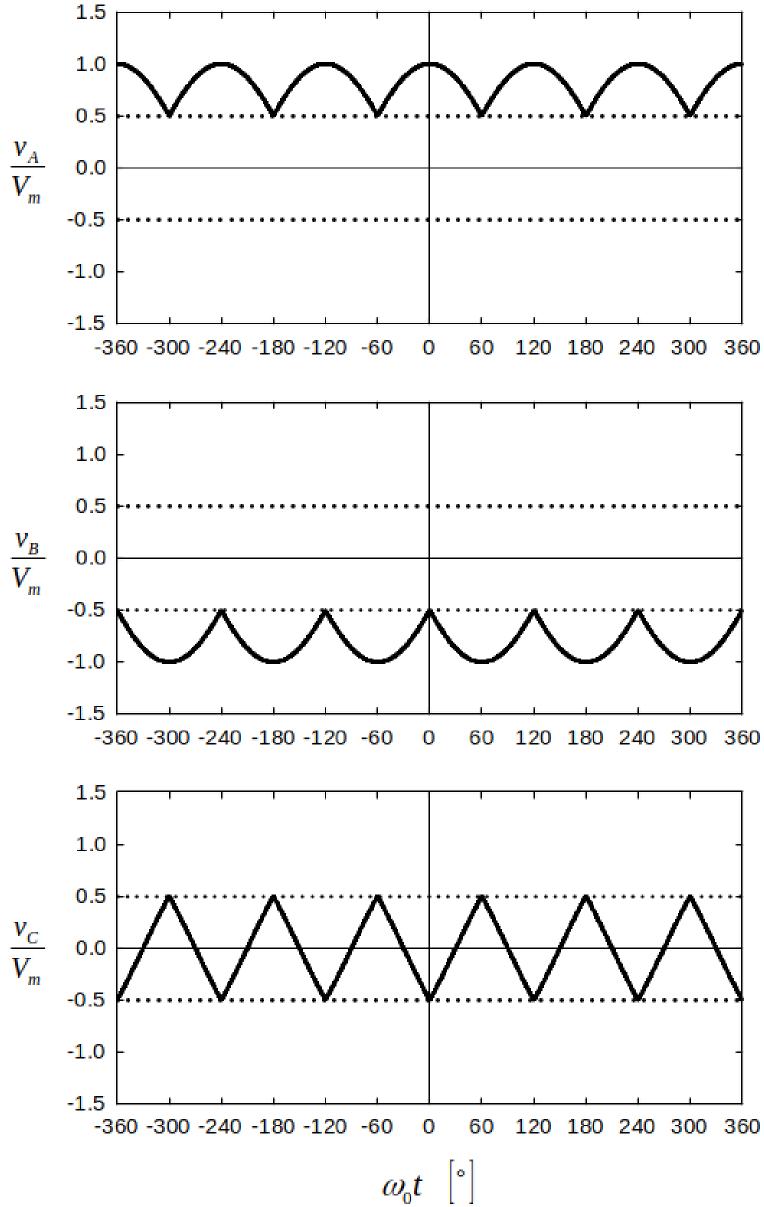


$$v_1 = V_m \cos(\omega_0 t)$$

$$v_2 = V_m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$v_3 = V_m \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$


Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH2 50.0V
CH4 50.0V
M 2.50ms
12-Jun-08 20:23
Ext 200mV
49.9762Hz



$$v_A = \max(v_1, v_2, v_3)$$

$$v_A = \frac{3\sqrt{3}}{\pi} V_m \left(\frac{1}{2} + \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{9n^2-1} \cos(3n\omega_0 t) \right)$$

$$v_B = \min(v_1, v_2, v_3)$$

$$v_B = \frac{3\sqrt{3}}{\pi} V_m \left(-\frac{1}{2} + \sum_{n=1}^{+\infty} \frac{1}{9n^2-1} \cos(3n\omega_0 t) \right)$$

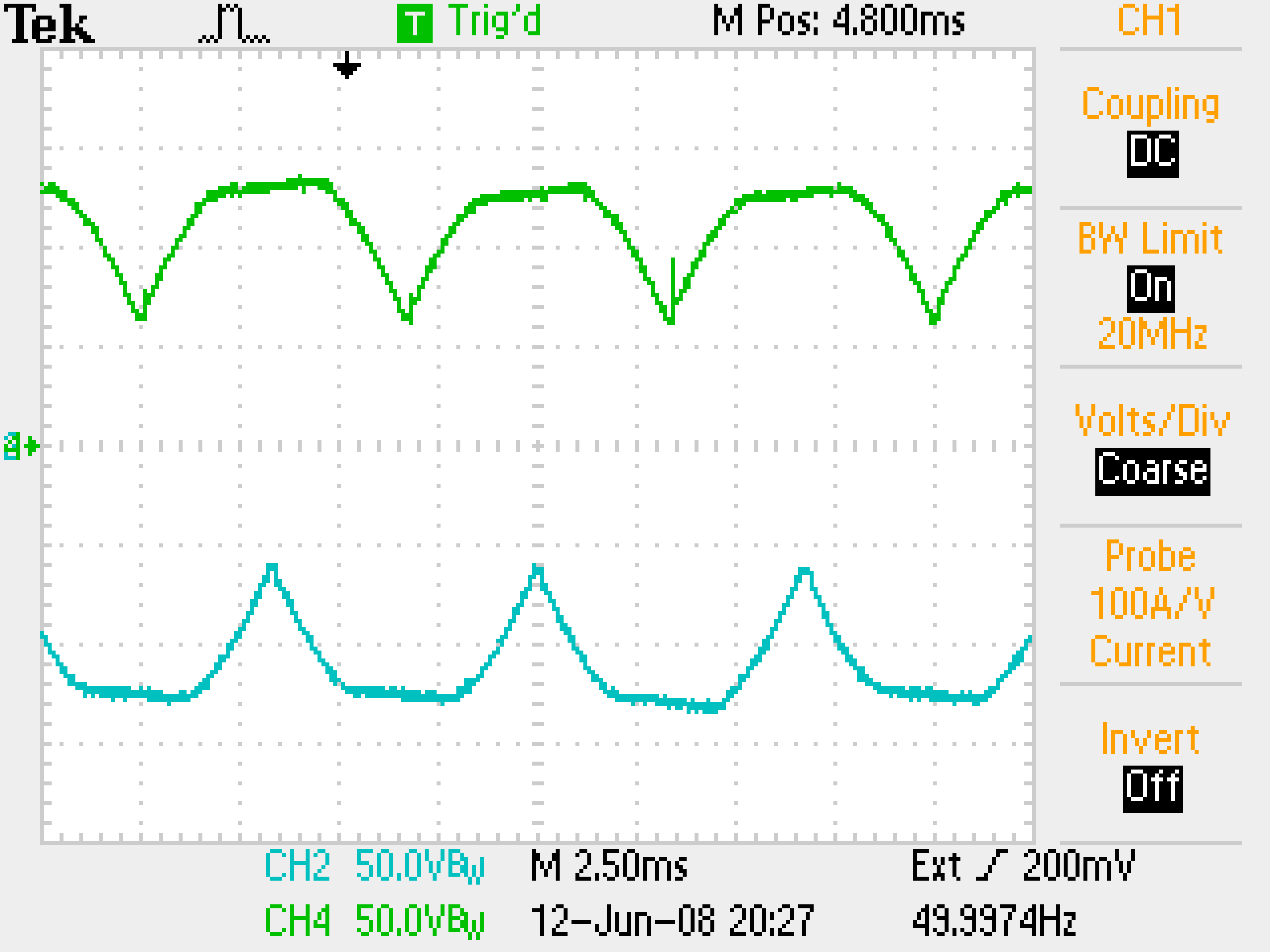
$$v_1 + v_2 + v_3 = 0$$

$$v_C = -v_A - v_B$$

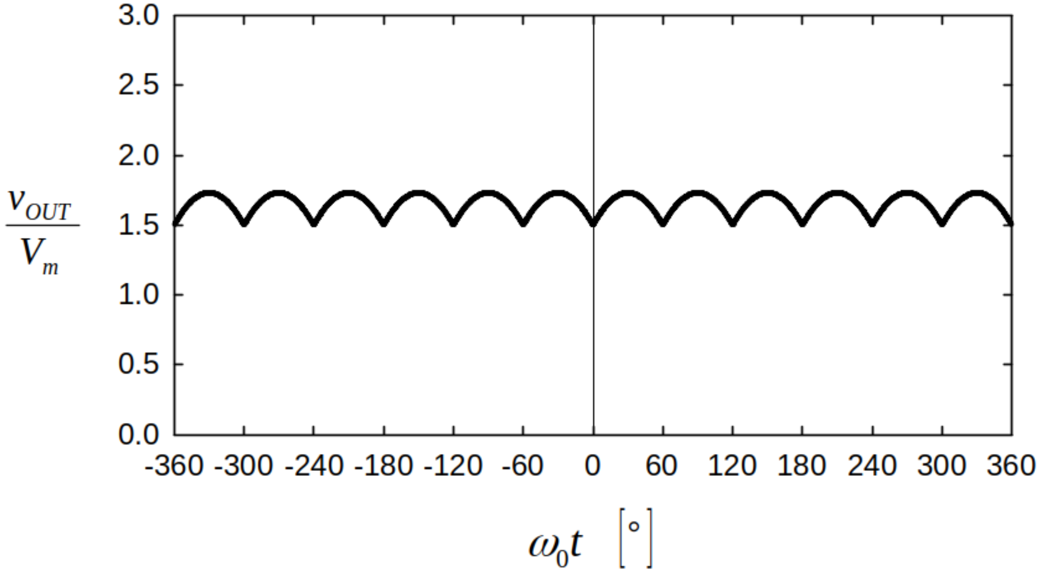
$$v_C = -\frac{3\sqrt{3}}{\pi} V_m \sum_{k=1}^{+\infty} \frac{2}{(6k-3)^2-1} \cos((6k-3)\omega_0 t)$$

$$v_{AV} = \frac{1}{2}(v_A + v_B) = -\frac{1}{2} v_C$$

$$v_{AV} = \frac{3\sqrt{3}}{\pi} V_m \sum_{k=1}^{+\infty} \frac{1}{(6k-3)^2-1} \cos((6k-3)\omega_0 t)$$



Tek
Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100A/V
Current
Invert
Off
CH2 50.0VBW
CH4 50.0VBW
M 2.50ms
12-Jun-08 20:27
Ext 200mV
49.9974Hz

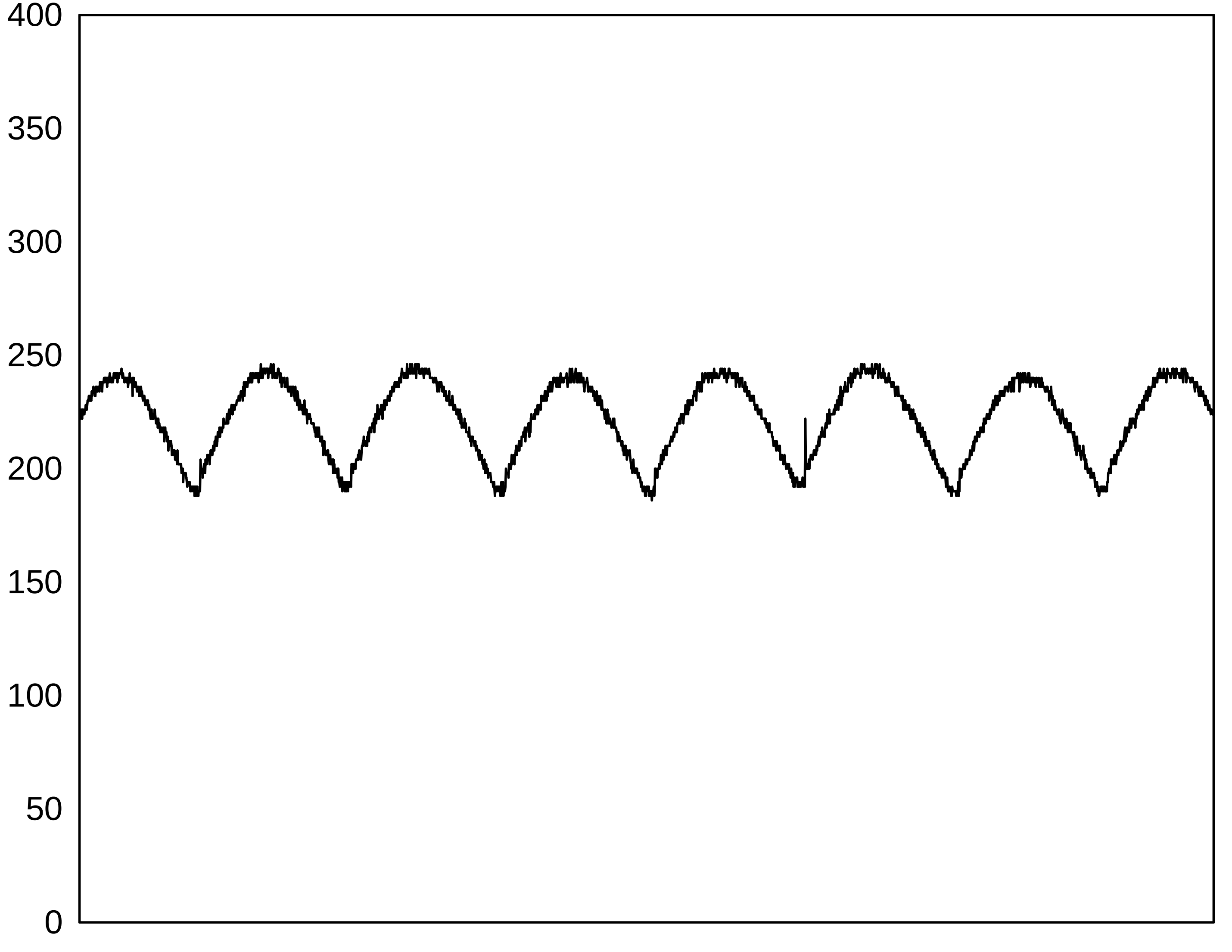


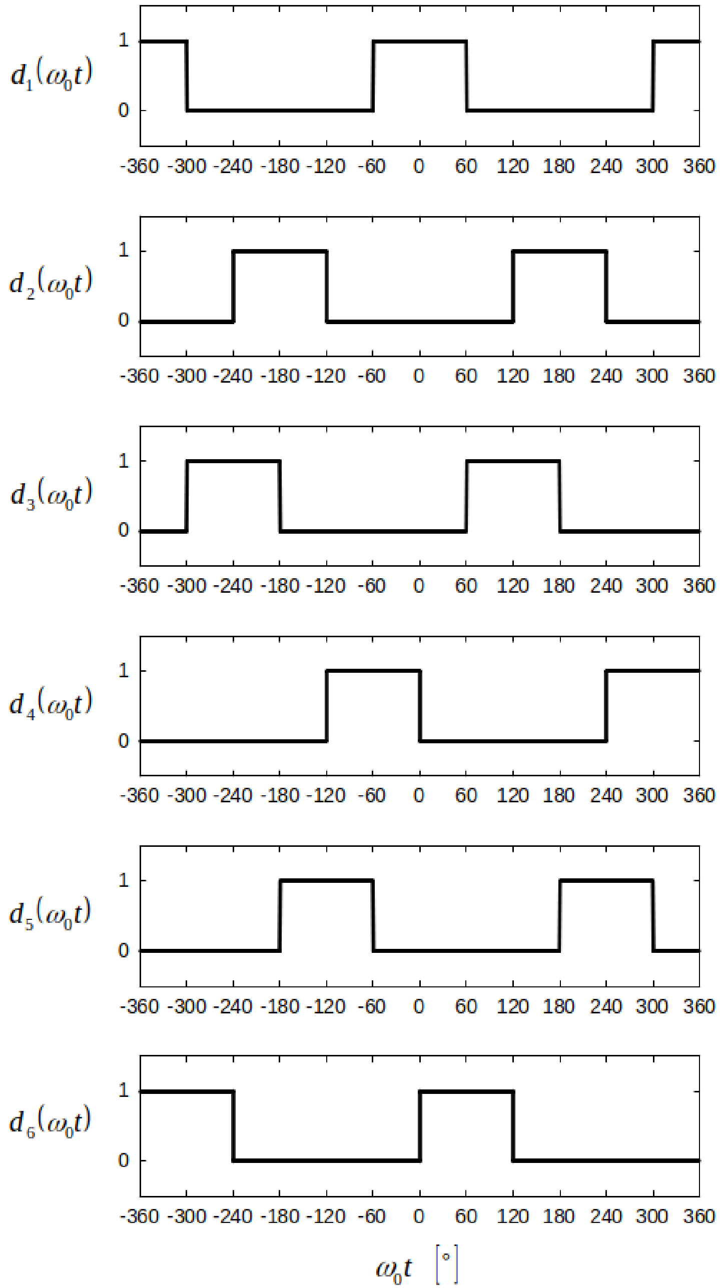
$$v_{OUT} = v_A - v_B$$

$$v_{OUT} = \frac{3\sqrt{3}}{\pi} V_m \left(1 - \sum_{k=1}^{+\infty} \frac{2}{36k^2 - 1} \cos(6k\omega_0 t) \right)$$

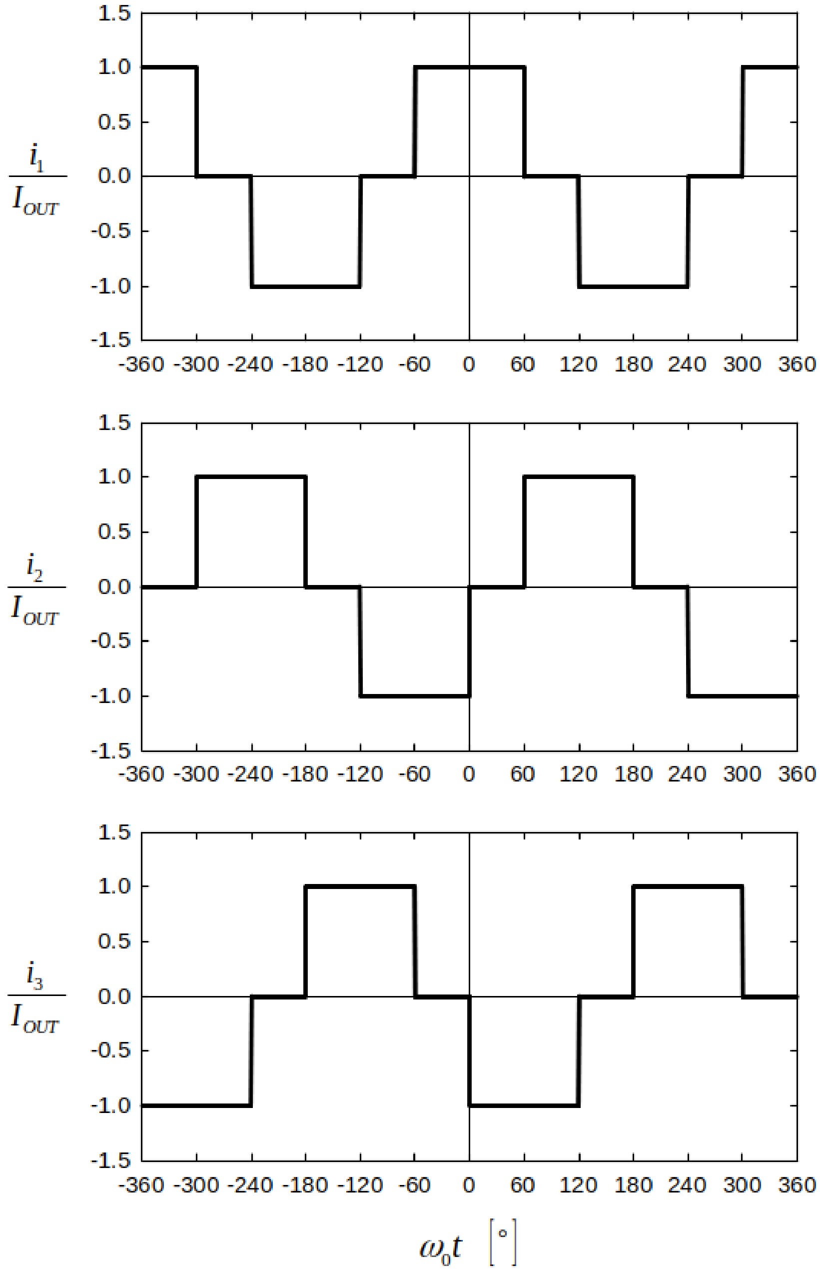
$$V_{OUT} = \frac{3\sqrt{3}}{\pi} V_m \approx 1.65 V_m \approx 2.34 V_{P\ RMS}$$

$$\hat{v}_{OUT} = -\frac{3\sqrt{3}}{\pi} V_m \sum_{k=1}^{+\infty} \frac{2}{36k^2 - 1} \cos(6k\omega_0 t)$$


400
350
300
250
200
150
100
50
0



$$i_p = I_{OUT}\left(d_{2p-1}(\omega_0 t) - d_{2p}(\omega_0 t)\right)$$

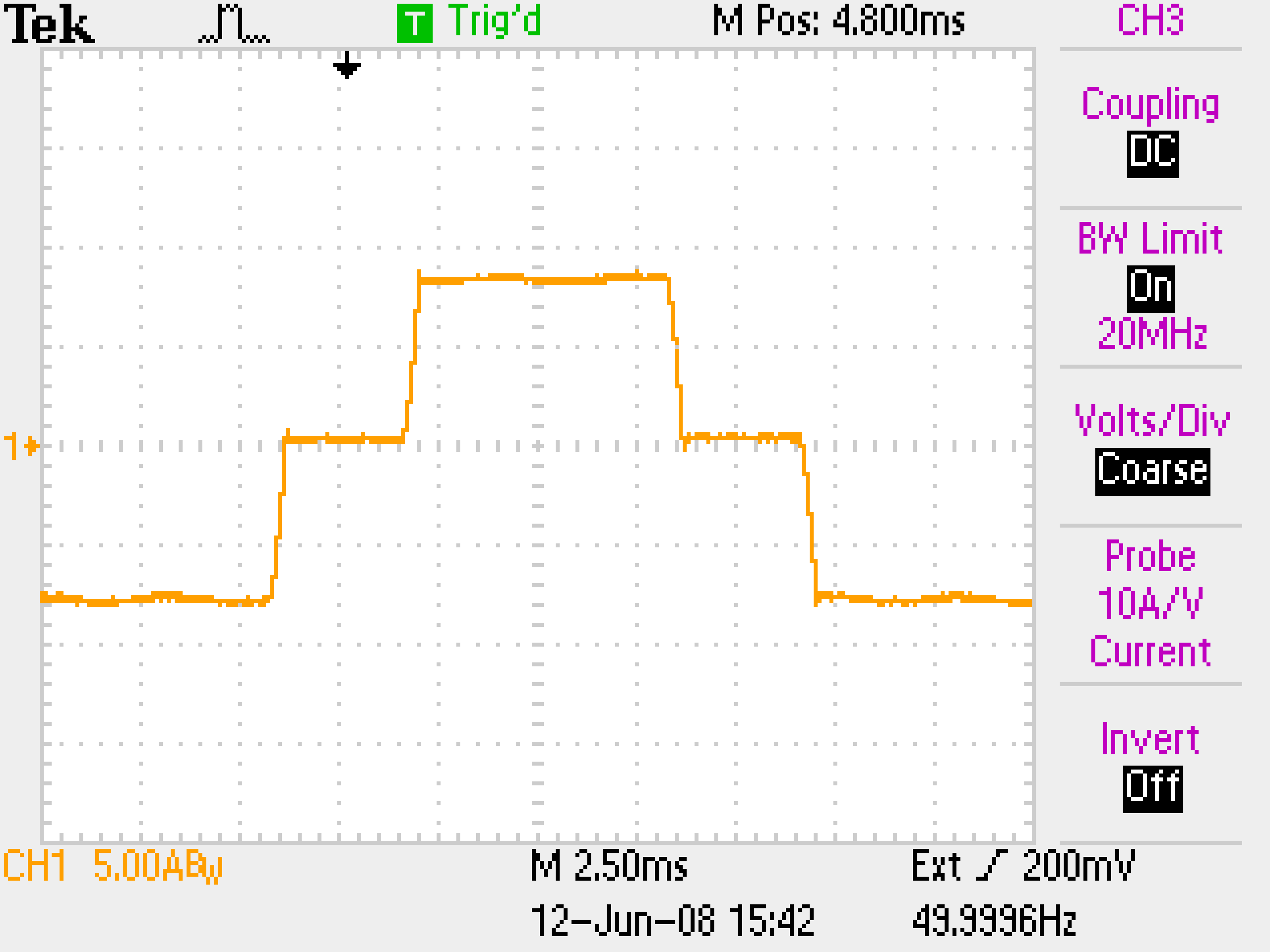


$$PF = \frac{P_{IN}}{S_{IN}} = \frac{3}{\pi} = 0.9549$$

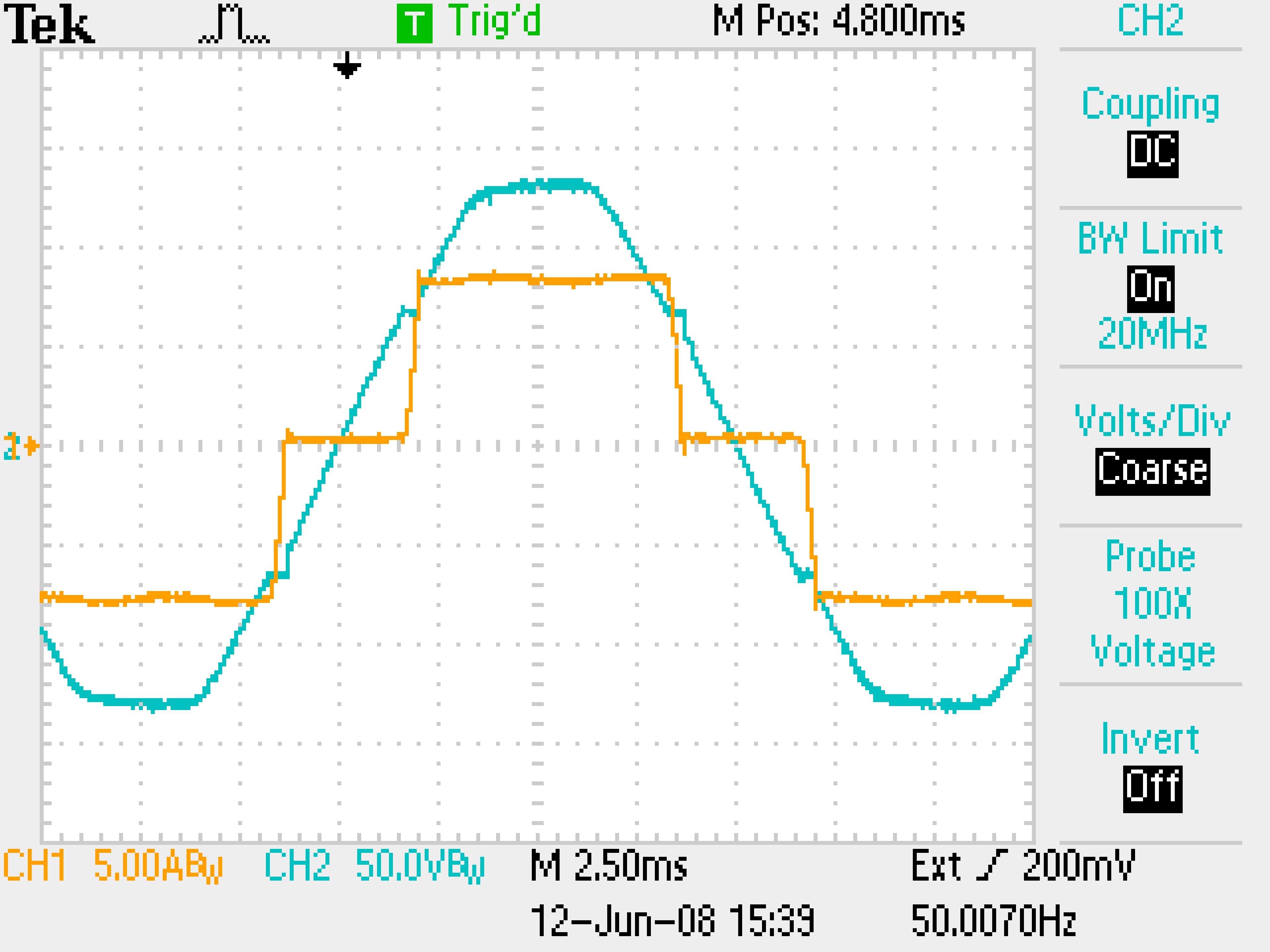
$$\mathrm{DPF} = \cos\phi_1 = 1$$

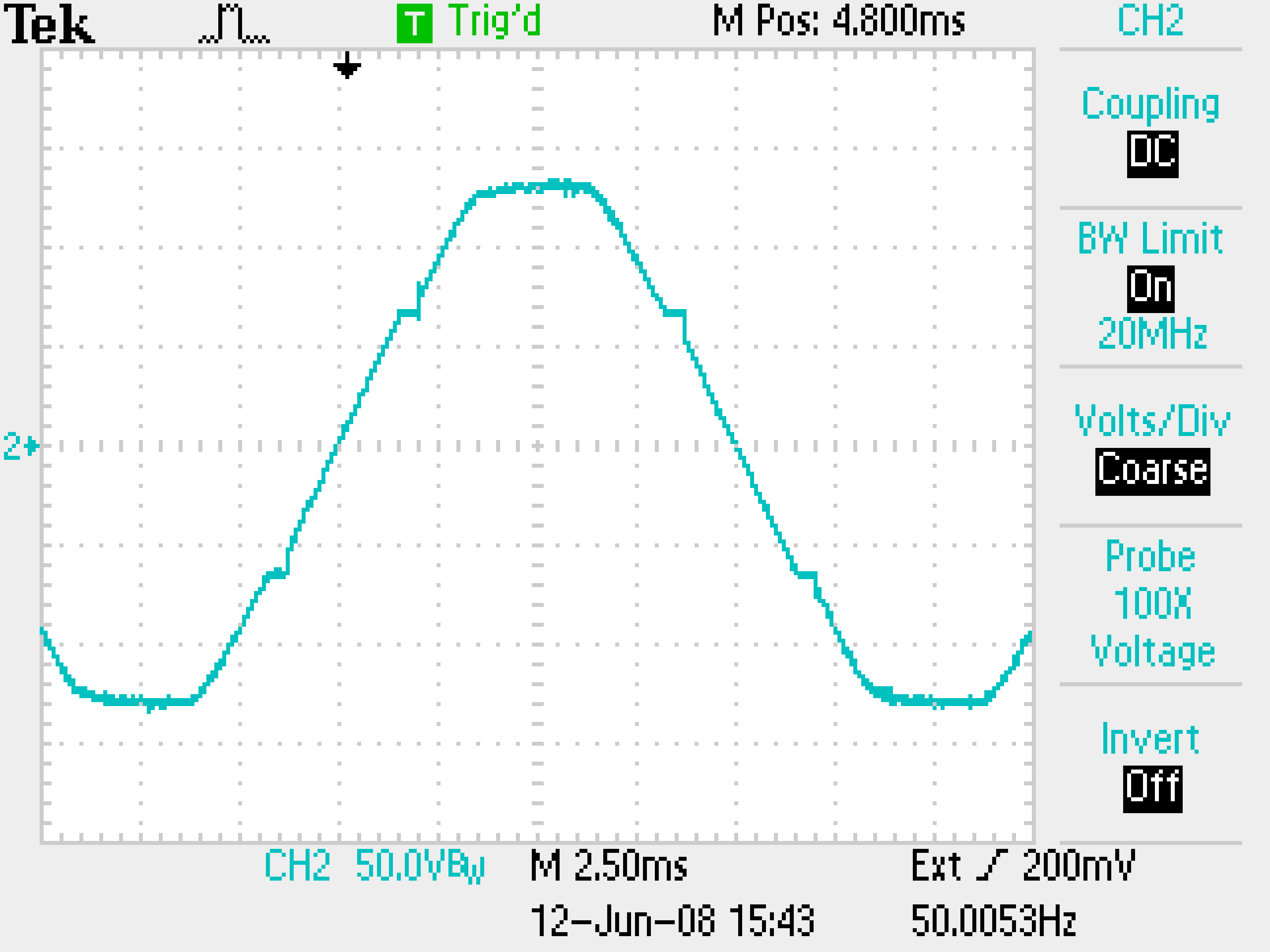
$$THD = \frac{\sqrt{I_{RMS}^2 - I_{1\,RMS}^2}}{I_{1\,RMS}}$$

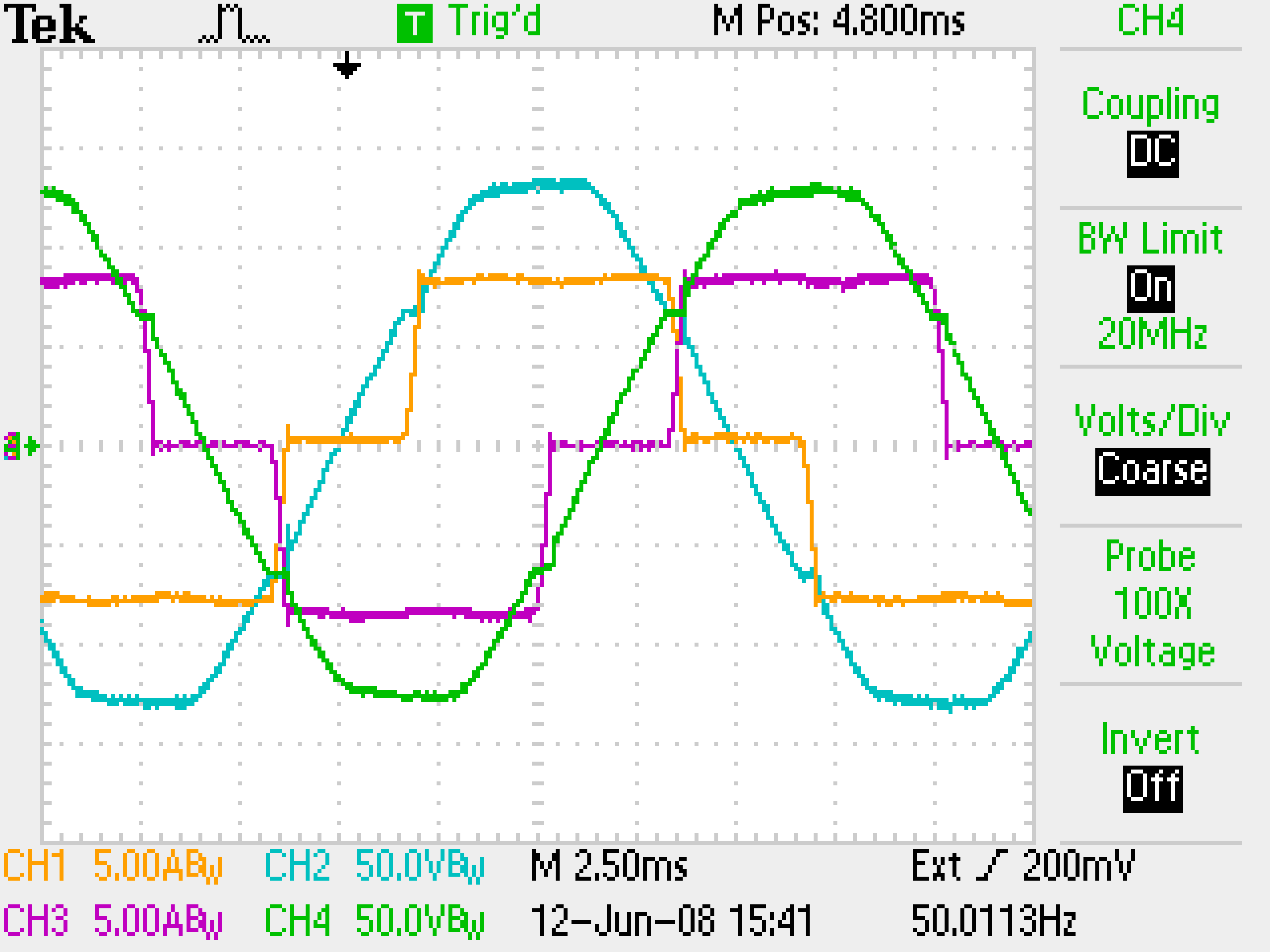
$$THD = \frac{1}{3}\sqrt{\pi^2 - 9} = 31.08\,\%$$

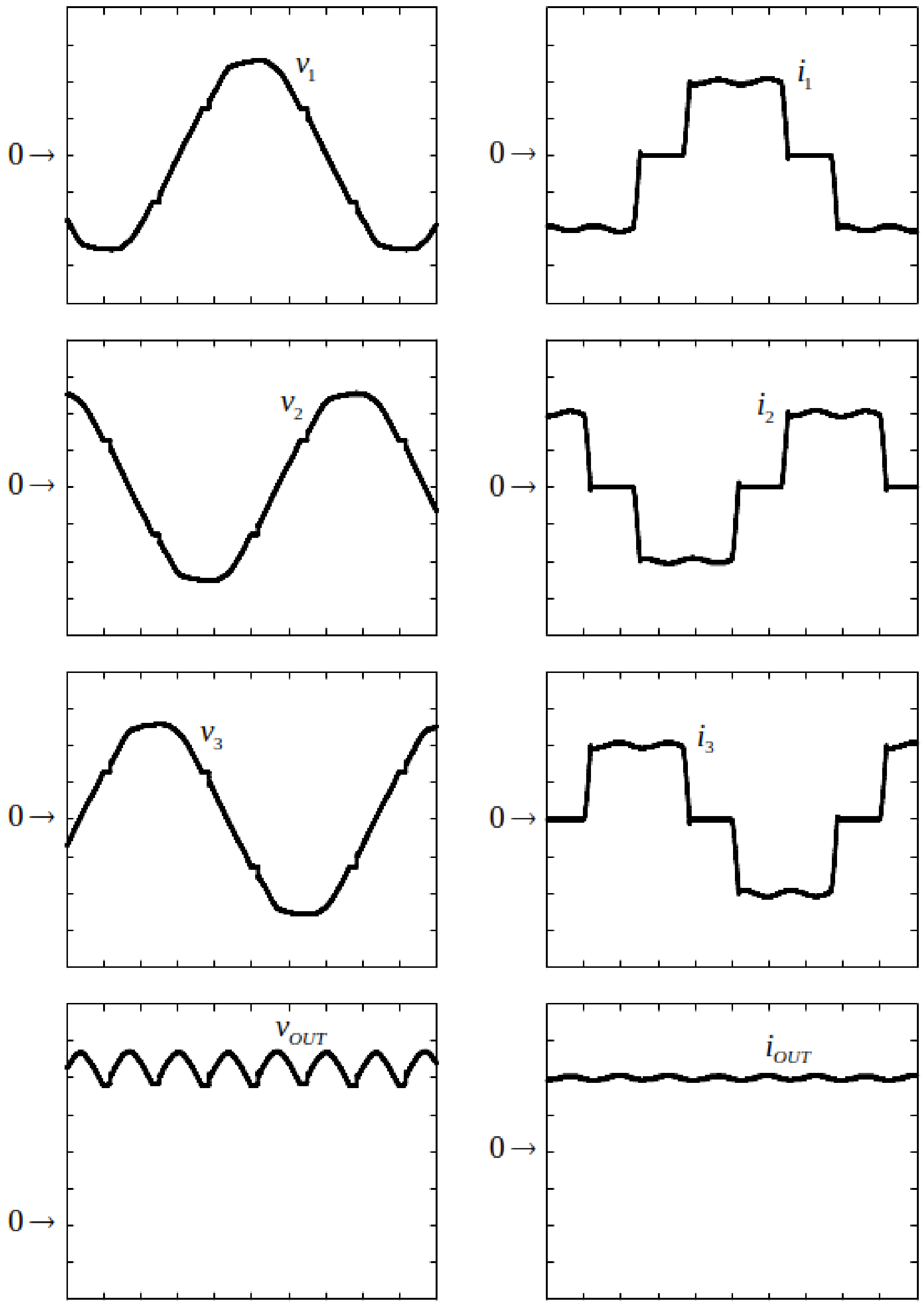


Tek
Trig'd
M Pos: 4.800ms
CH3
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
10A/V
Current
Invert
Off
1
CH1 5.00AΩ
M 2.50ms
Ext 200mV
12-Jun-08 15:42
49.9996Hz

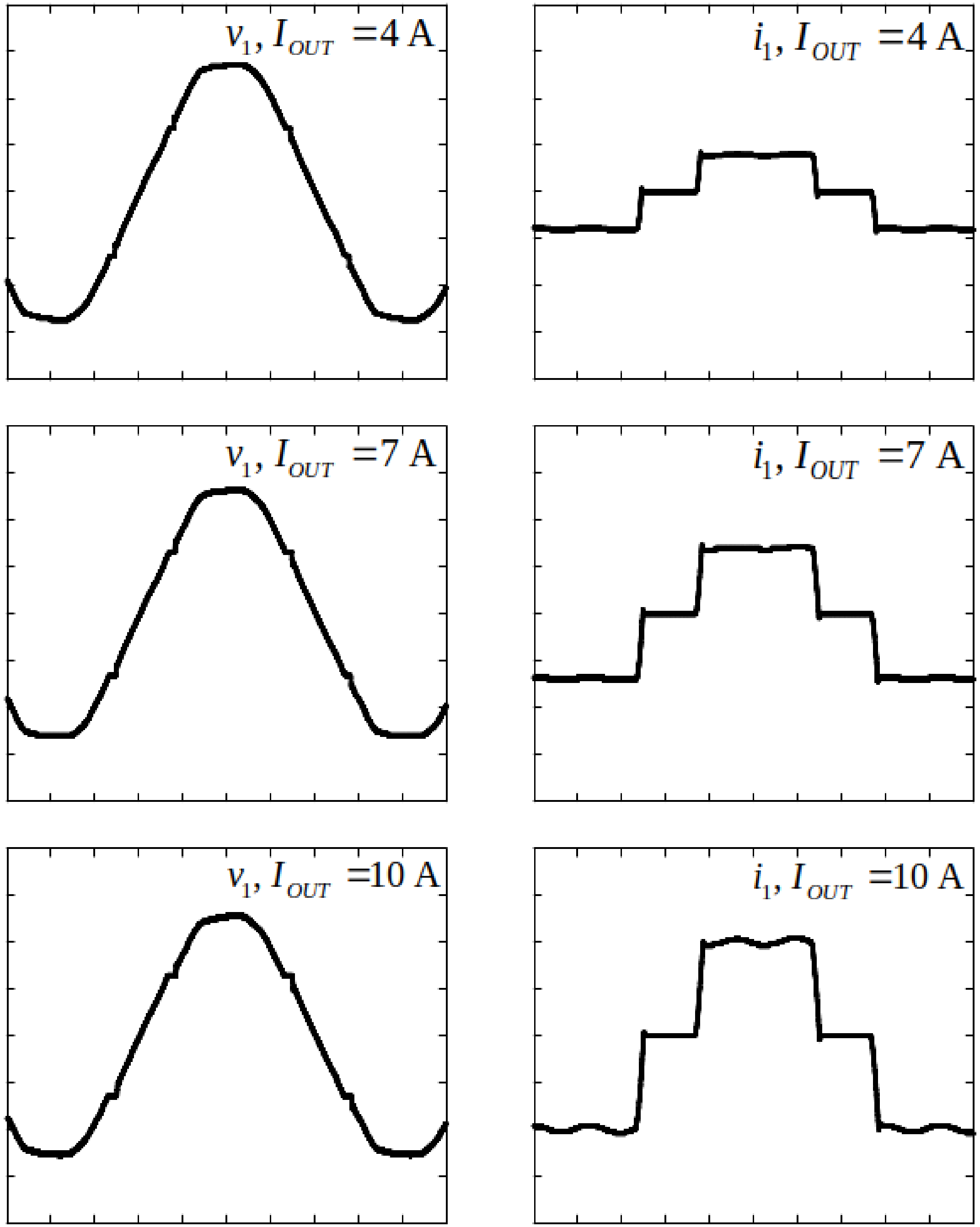

Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 15:39
50.0070Hz


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
2
CH2 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 15:43
50.0053Hz


Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00AΩ
CH2 50.0VΩ
M 2.50ms
Ext ∫ 200mV
CH3 5.00AΩ
CH4 50.0VΩ
12-Jun-08 15:41
50.0113Hz

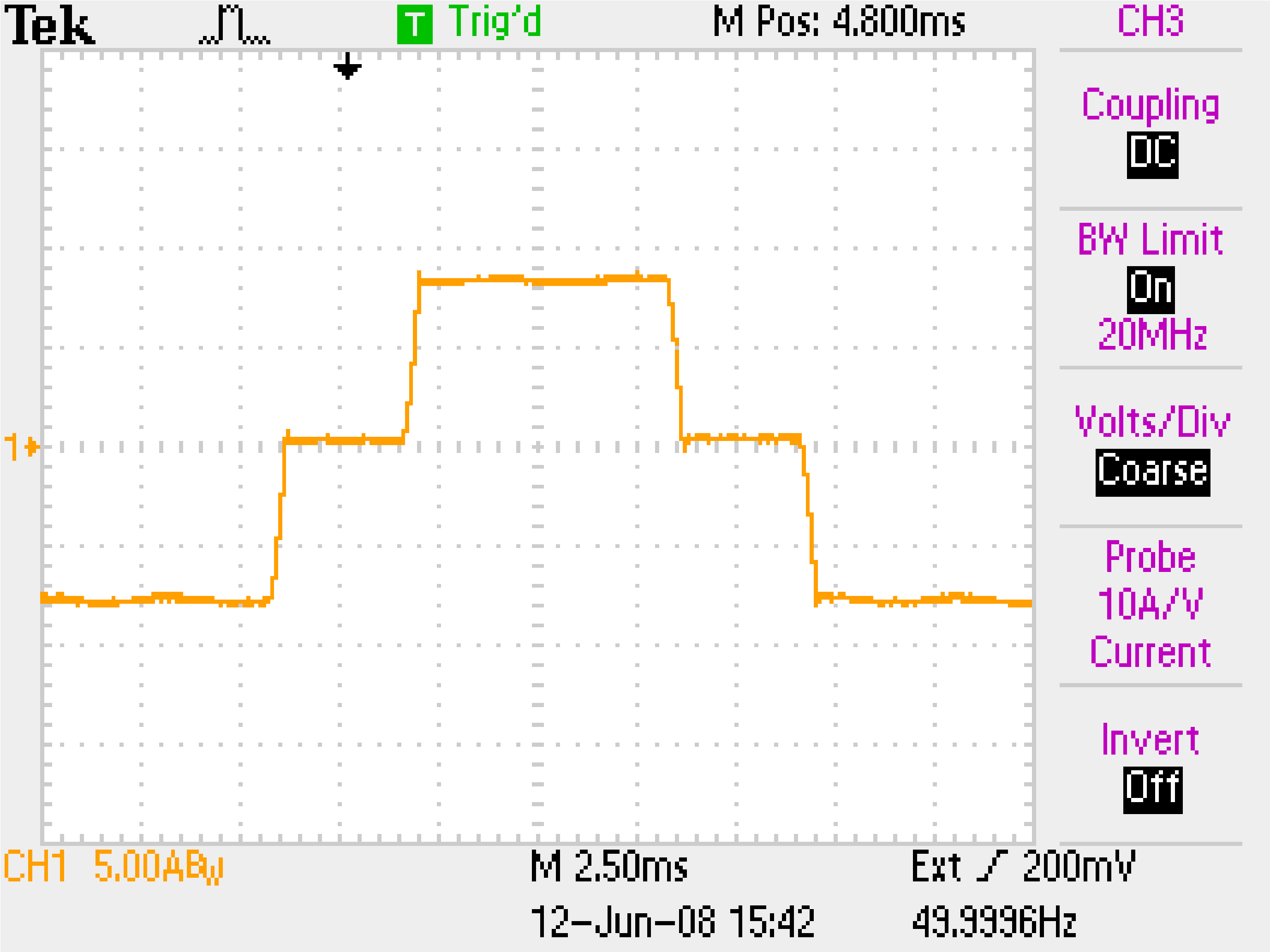


v_1
i_1
v_2
i_2
v_3
i_3
v_{OUT}
i_{OUT}
0 →



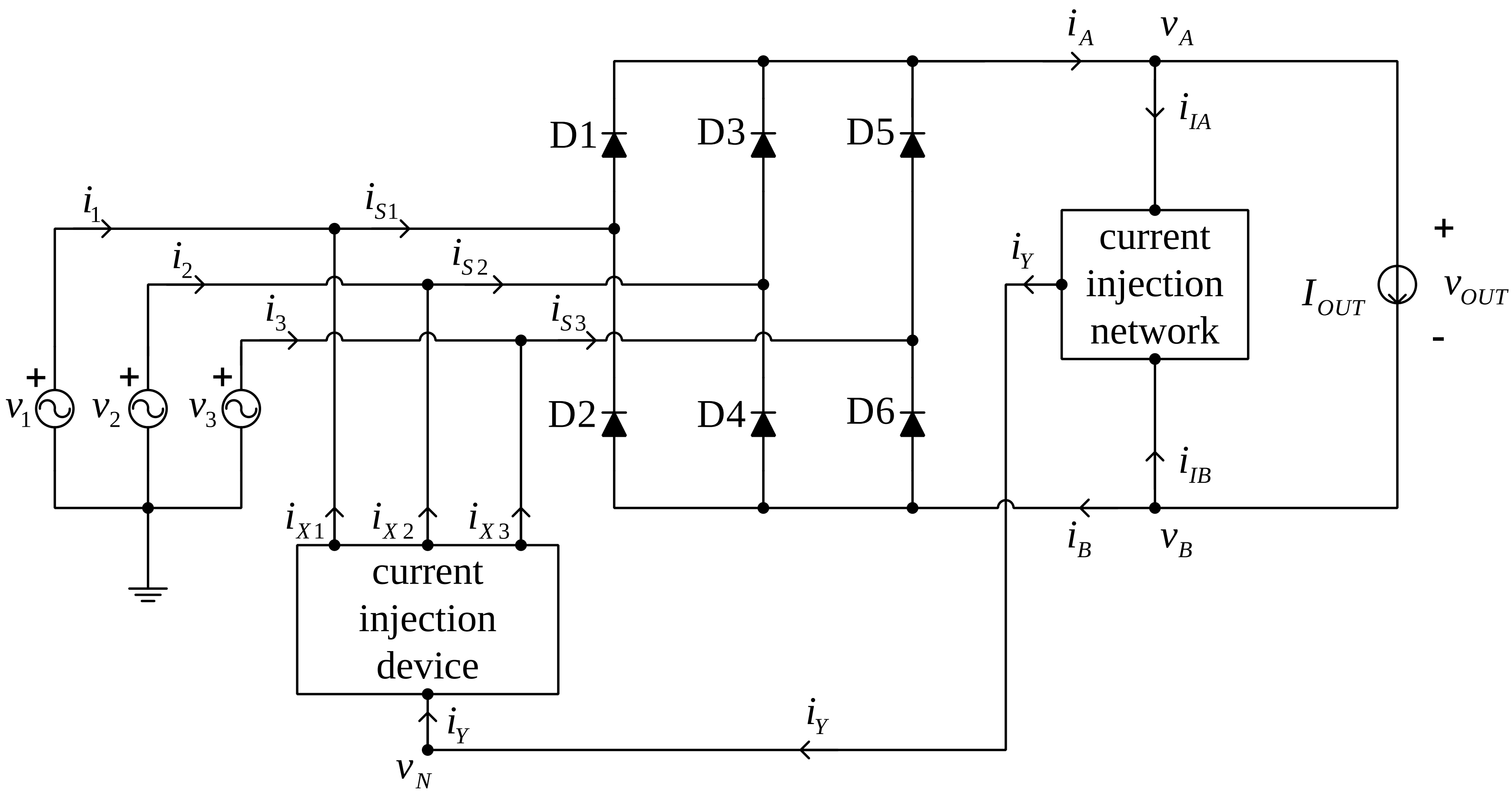
v_1, I_{OUT} =4 A
i_1, I_{OUT} =4 A
v_1, I_{OUT} =7 A
i_1, I_{OUT} =7 A
v_1, I_{OUT} =10 A
i_1, I_{OUT} =10 A

I_{OUT}	THD(v_{IN})	THD(i_{IN})	P_{OUT}	S_{IN}	PF
4 A	3.17 %	29.47 %	909.8 W	946.8 VA	0.9610
7 A	3.34 %	28.65 %	1544.3 W	1607.8 VA	0.9605
10 A	3.42 %	27.94 %	2121.0 W	2212.1 VA	0.9588



How to patch the gaps?

The Method of Current Injection



$$i_1(\omega_0 t) = d_1(\omega_0 t)\, i_A(\omega_0 t) - d_2(\omega_0 t)\, i_B(\omega_0 t) - i_{X1}(\omega_0 t)$$

$$i_2(\omega_0 t) = d_3(\omega_0 t)\, i_A(\omega_0 t) - d_4(\omega_0 t)\, i_B(\omega_0 t) - i_{X2}(\omega_0 t)$$

$$i_3(\omega_0 t) = d_5(\omega_0 t)\, i_A(\omega_0 t) - d_6(\omega_0 t)\, i_B(\omega_0 t) - i_{X3}(\omega_0 t)$$

$$i_1(\omega_0 t) = i_2\left(\omega_0 t - \frac{2\pi}{3}\right) = i_3\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$d_1(\omega_0 t) = d_3\left(\omega_0 t - \frac{2\pi}{3}\right) = d_5\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$d_2(\omega_0 t) = d_4\left(\omega_0 t - \frac{2\pi}{3}\right) = d_6\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$i_A(\omega_0 t) = i_A\left(\omega_0 t - \frac{2\pi}{3}\right) = i_A\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$i_B(\omega_0 t) = i_B\left(\omega_0 t - \frac{2\pi}{3}\right) = i_B\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$i_{X1}(\omega_0 t) = i_{X2}\left(\omega_0 t - \frac{2\pi}{3}\right) = i_{X3}\left(\omega_0 t - \frac{4\pi}{3}\right)$$

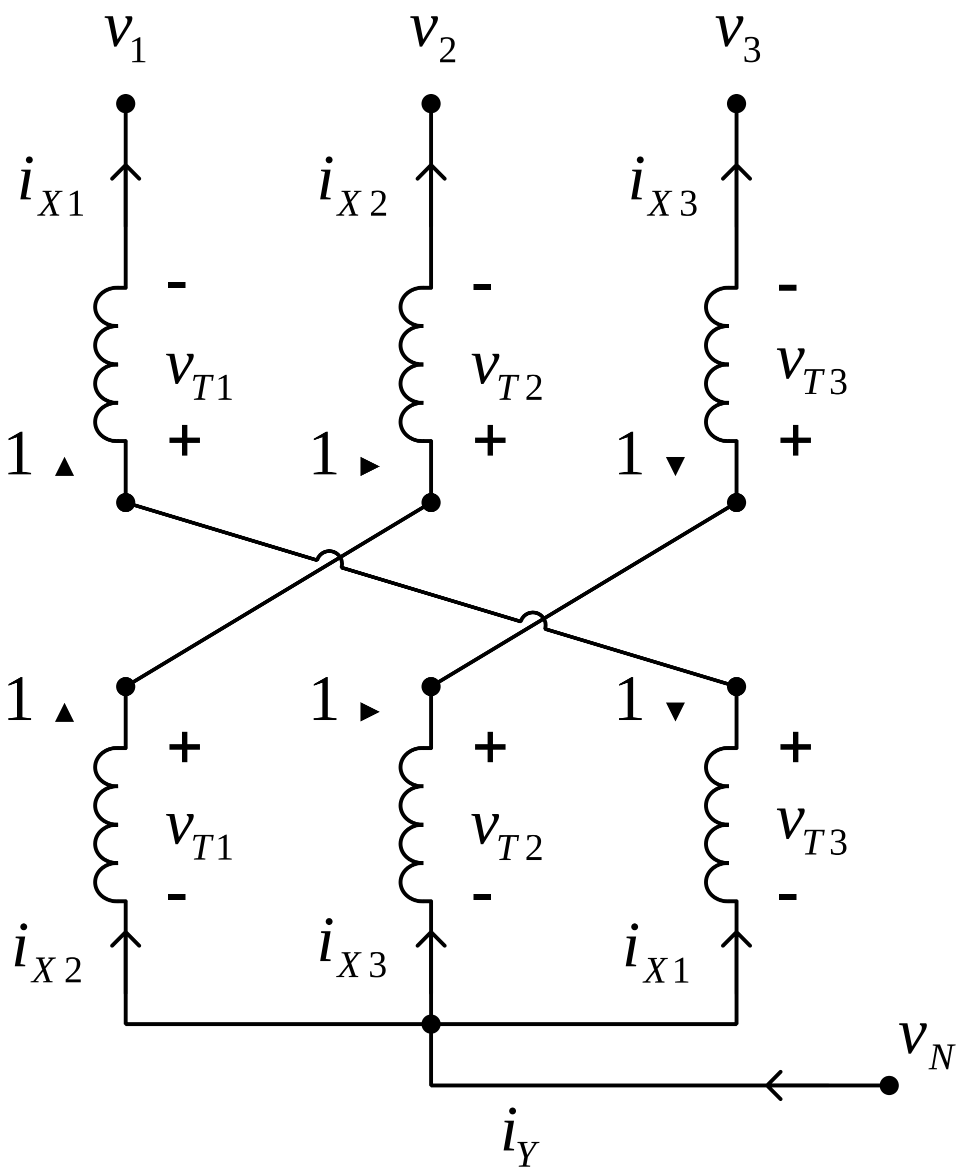
$$i_{X1}(\omega_0 t) = i_{X2}(\omega_0 t) = i_{X3}(\omega_0 t) = i_X(\omega_0 t)$$

$$i_X(\omega_0 t) = i_X\left(\omega_0 t - \frac{2\pi}{3}\right) = i_X\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$i_X(\omega_0 t) = \frac{1}{3} i_Y(\omega_0 t)$$

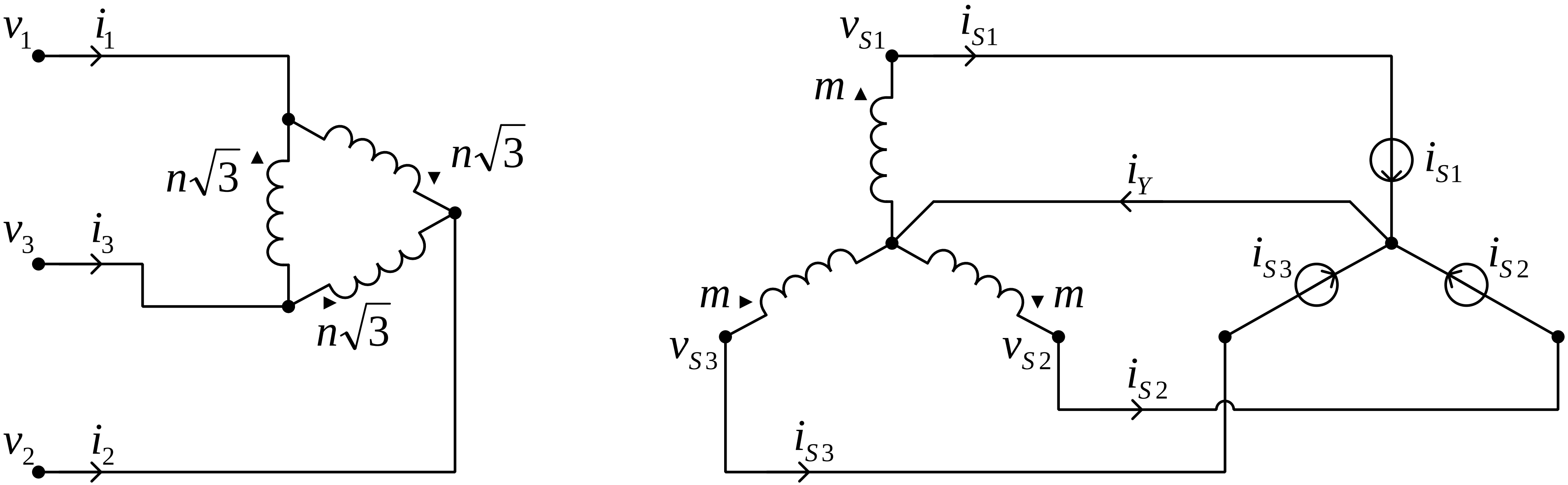
$$i_Y(\omega_0 t) = i_A(\omega_0 t) - i_B(\omega_0 t)$$

Current Injection Devices

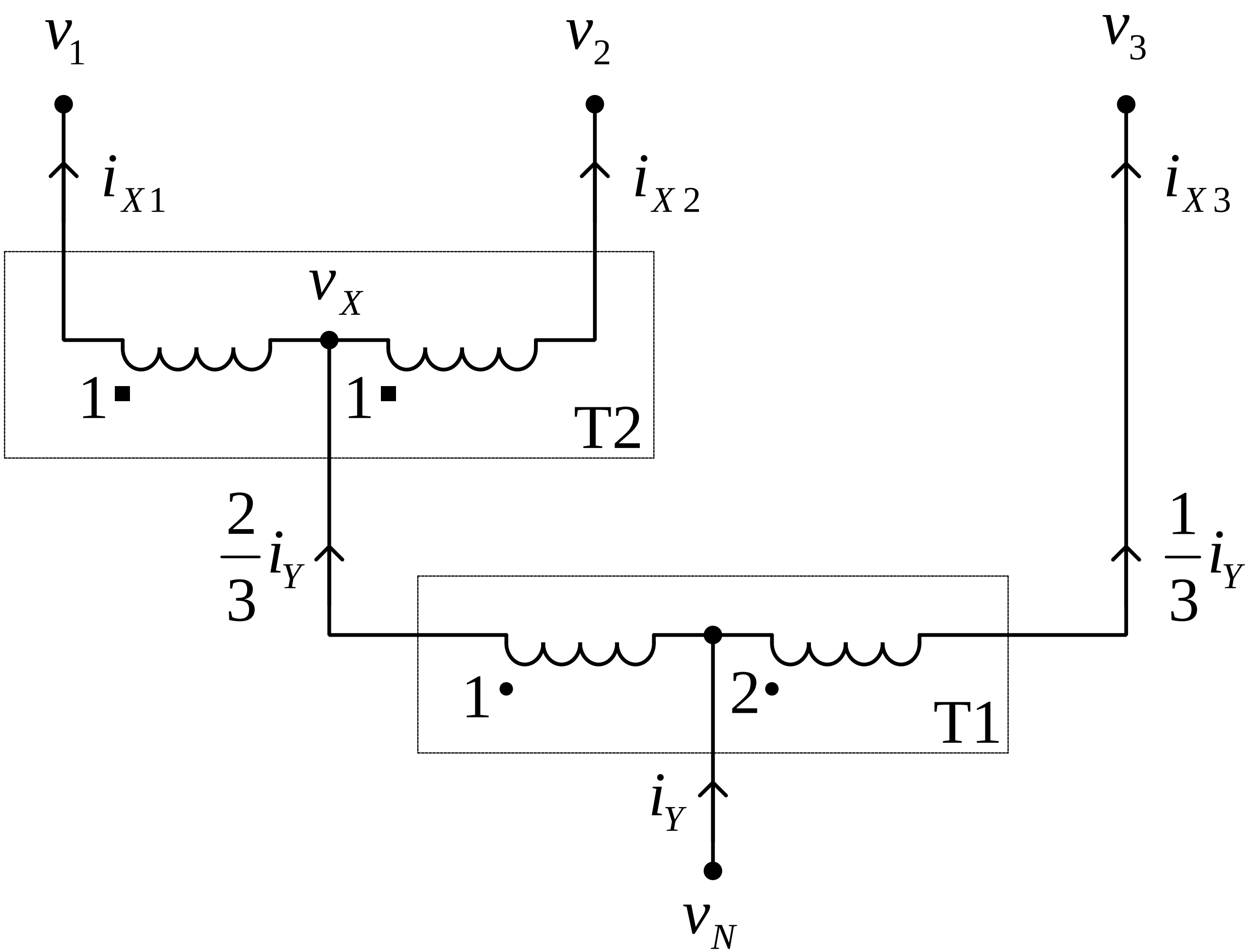


$$i_{X1} = i_{X2} = i_{X3} = i_X = \frac{1}{3} i_Y$$

$$v_N = \frac{1}{3}\left(v_1 + v_2 + v_3\right) = 0$$

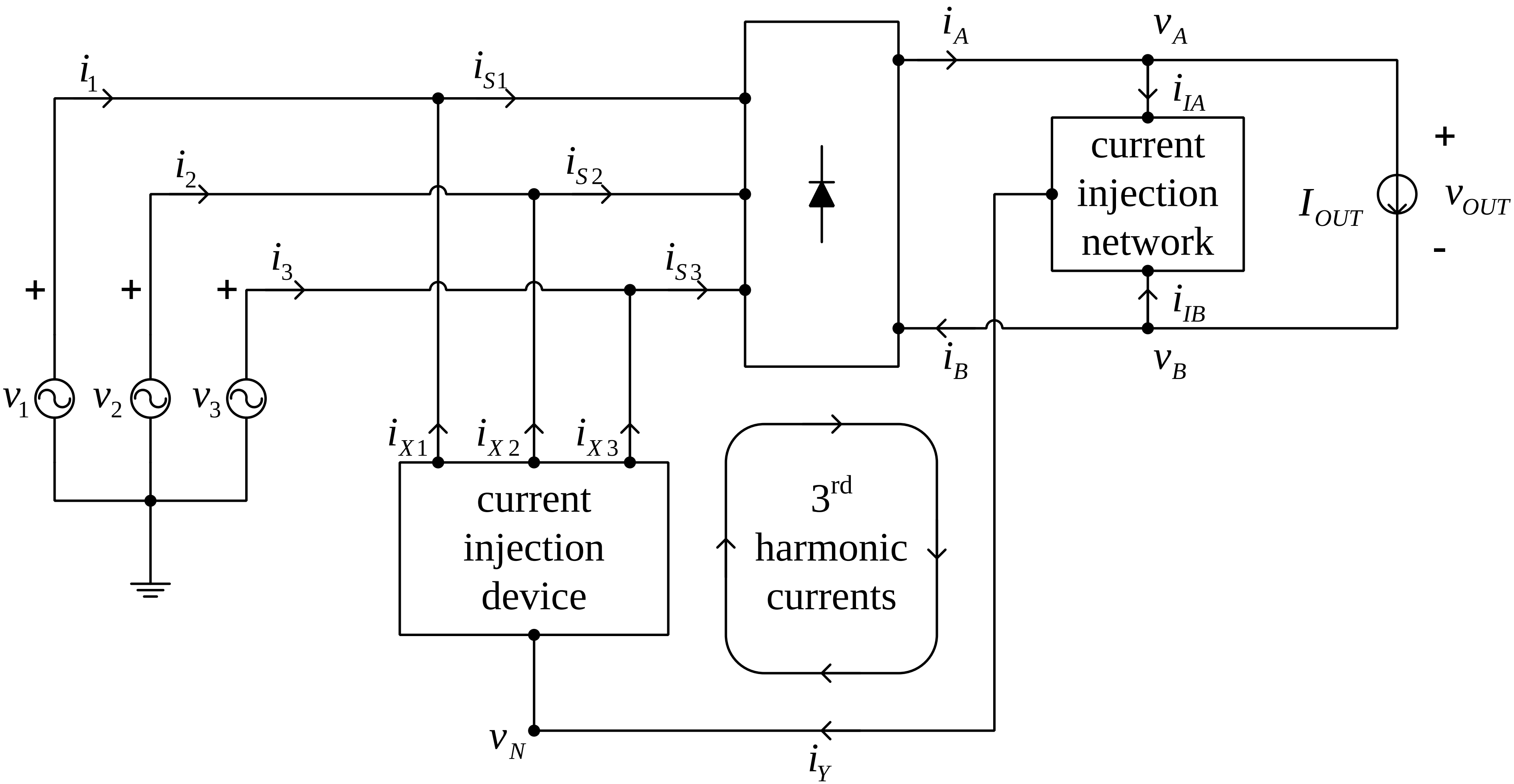


v_1
i_1
v_3
i_3
v_2
i_2
$n\sqrt{3}$
$n\sqrt{3}$
$n\sqrt{3}$
v_{S1}
i_{S1}
m
i_{S1}
i_Y
m
m
i_{S3}
i_{S2}
v_{S3}
v_{S2}
i_{S2}
i_{S3}

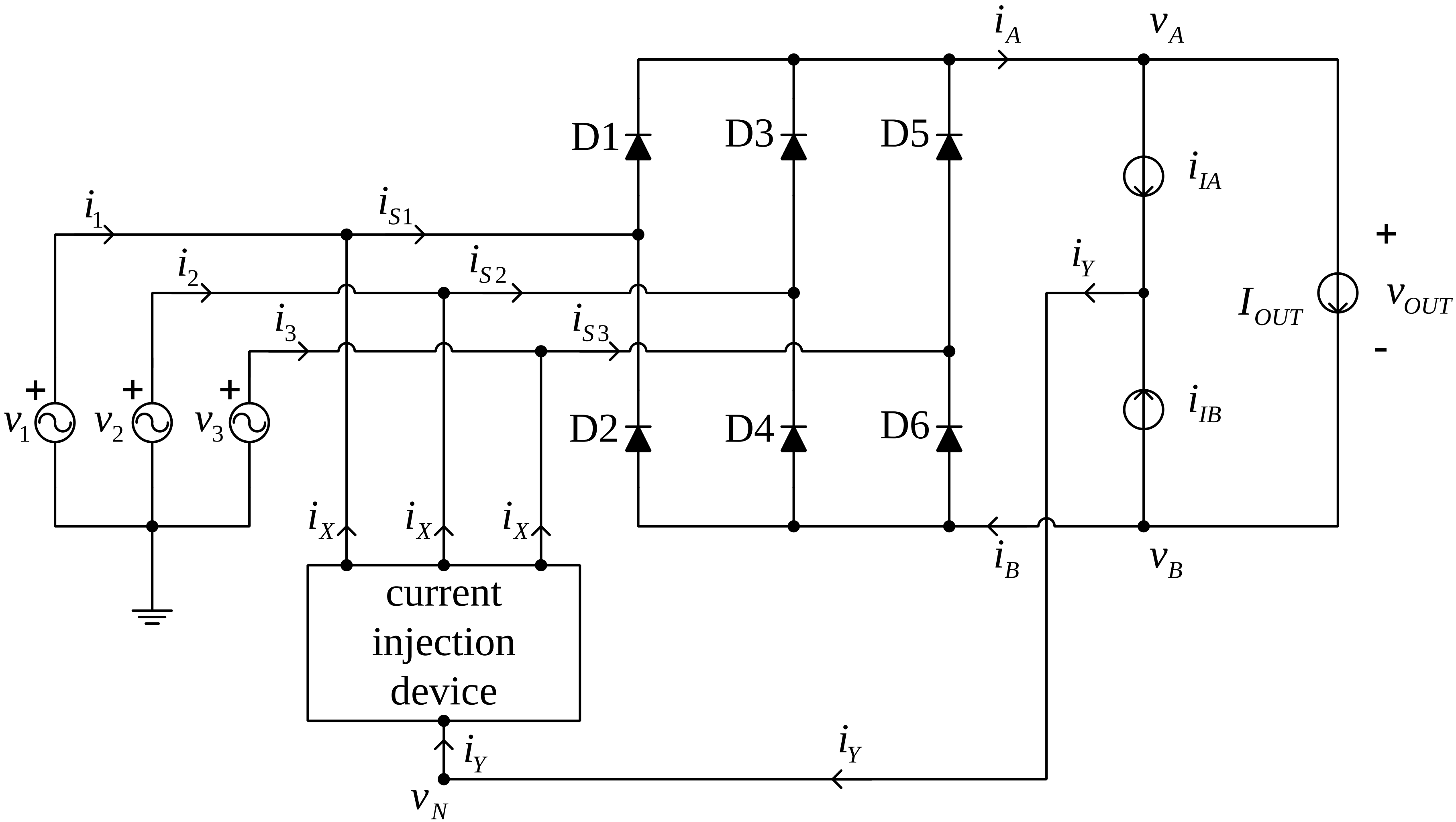


v_1
v_2
v_3
i_{X1}
i_{X2}
i_{X3}
v_X
1
1
T2
$\frac{2}{3}i_Y$
$\frac{1}{3}i_Y$
1
2
T1
i_Y
v_N

A Common Misconception



Third-Harmonic Current Injection



$$i_{IA}=i_{IB}=kI_{OUT}\cos\left(3\omega_0 t-\phi\right)$$

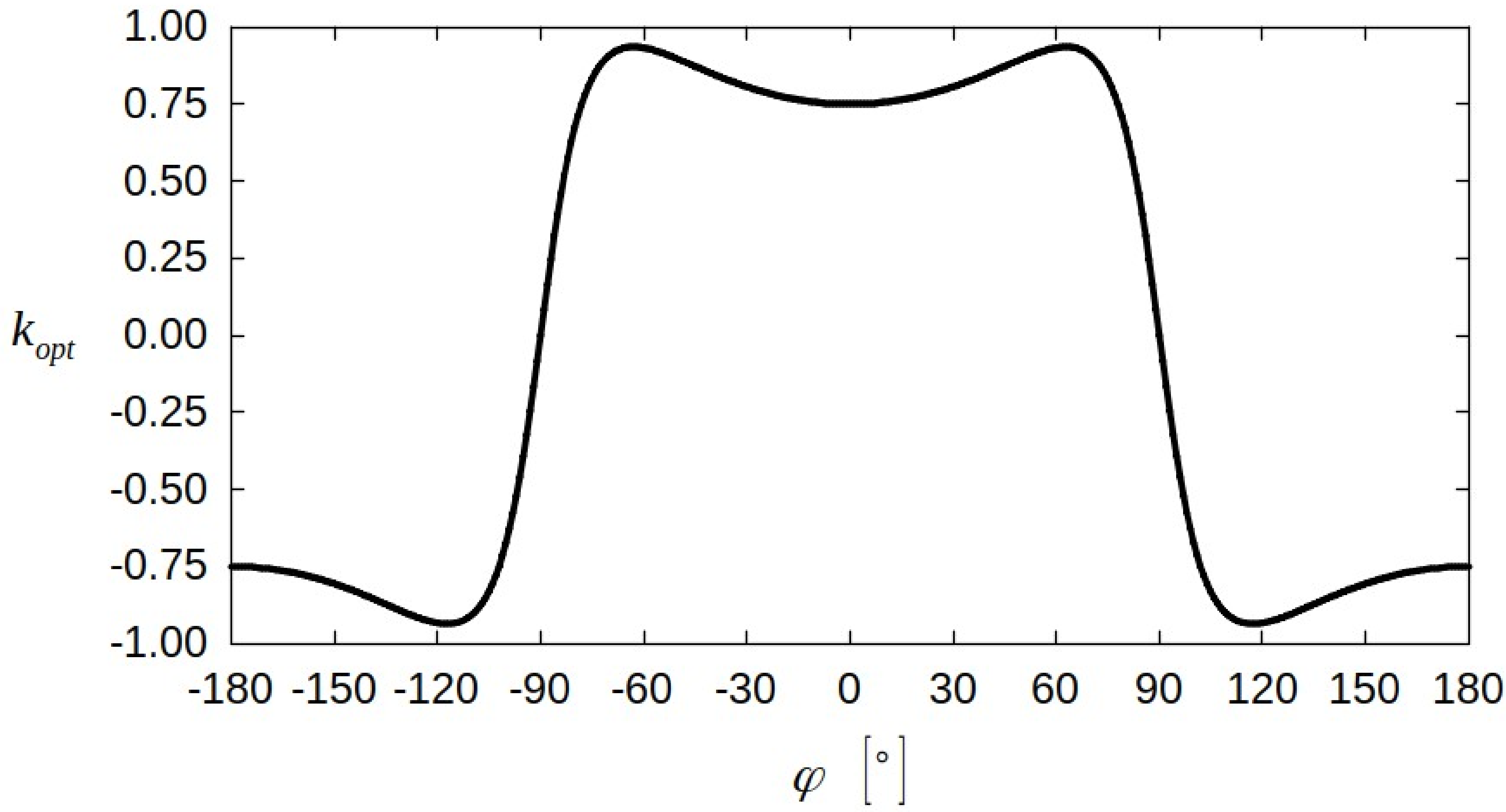
$$\frac{i_1}{I_{OUT}}=\begin{cases} 1+\frac{1}{3}k\cos\left(3\omega_0 t-\phi\right) & \text{for } -60°<\omega_0 t<60° \\ -\frac{2}{3}k\cos\left(3\omega_0 t-\phi\right) & \text{for } 60°<\left|\omega_0 t\right|<120° \\ -1+\frac{1}{3}k\cos\left(3\omega_0 t-\phi\right) & \text{for } 120°<\omega_0 t<240° \end{cases},$$

$$THD\left(k,\phi\right)=\frac{\sqrt{I_{RMS}^2-I_{1RMS}^2}}{I_{1RMS}}$$

$$\frac{\partial\, THD\left(k,\phi\right)}{\partial\, k}=0$$

$$k_{opt}\left(\phi\right)=\frac{48\cos\phi}{\sqrt{576\left(\cos\phi\right)^4+624\left(\cos\phi\right)^2+25}+24\left(\cos\phi\right)^2+5}$$

$$THD\left(k_{opt}\left(\phi\right),\phi\right)=\frac{\sqrt{6\left(4\pi^2-45\right)\left(\cos\phi\right)^2+5\pi^2-45+\left(\pi^2-9\right)\sqrt{576\left(\cos\phi\right)^4+624\left(\cos\phi\right)^2+25}}}{3\sqrt{\sqrt{576\left(\cos\phi\right)^4+624\left(\cos\phi\right)^2+25}+30\left(\cos\phi\right)^2+5}}$$



1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
k_{opt}
-180
-150
-120
-90
-60
-30
0
30
60
90
120
150
180
φ [°]

$$k_{OPT} = k_{opt}(0) = \frac{3}{4}$$

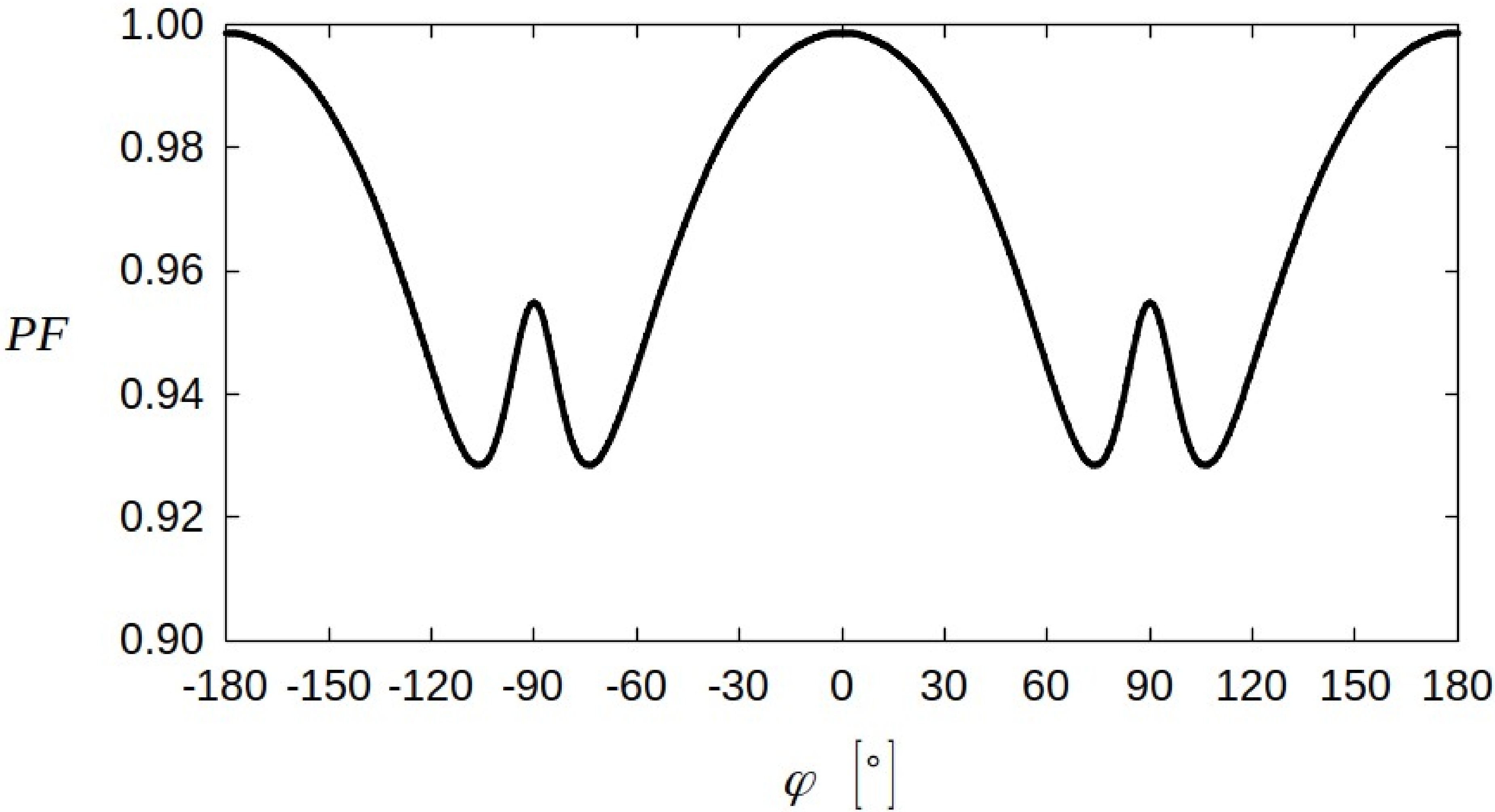
$$THD_{min} = THD\left(\frac{3}{4}, 0\right) = \sqrt{\frac{32\pi^2}{315} - 1} \approx 5.12\%$$

$$THD(0, 0) = \frac{1}{3}\sqrt{\pi^2 - 9} \approx 31.08\%$$

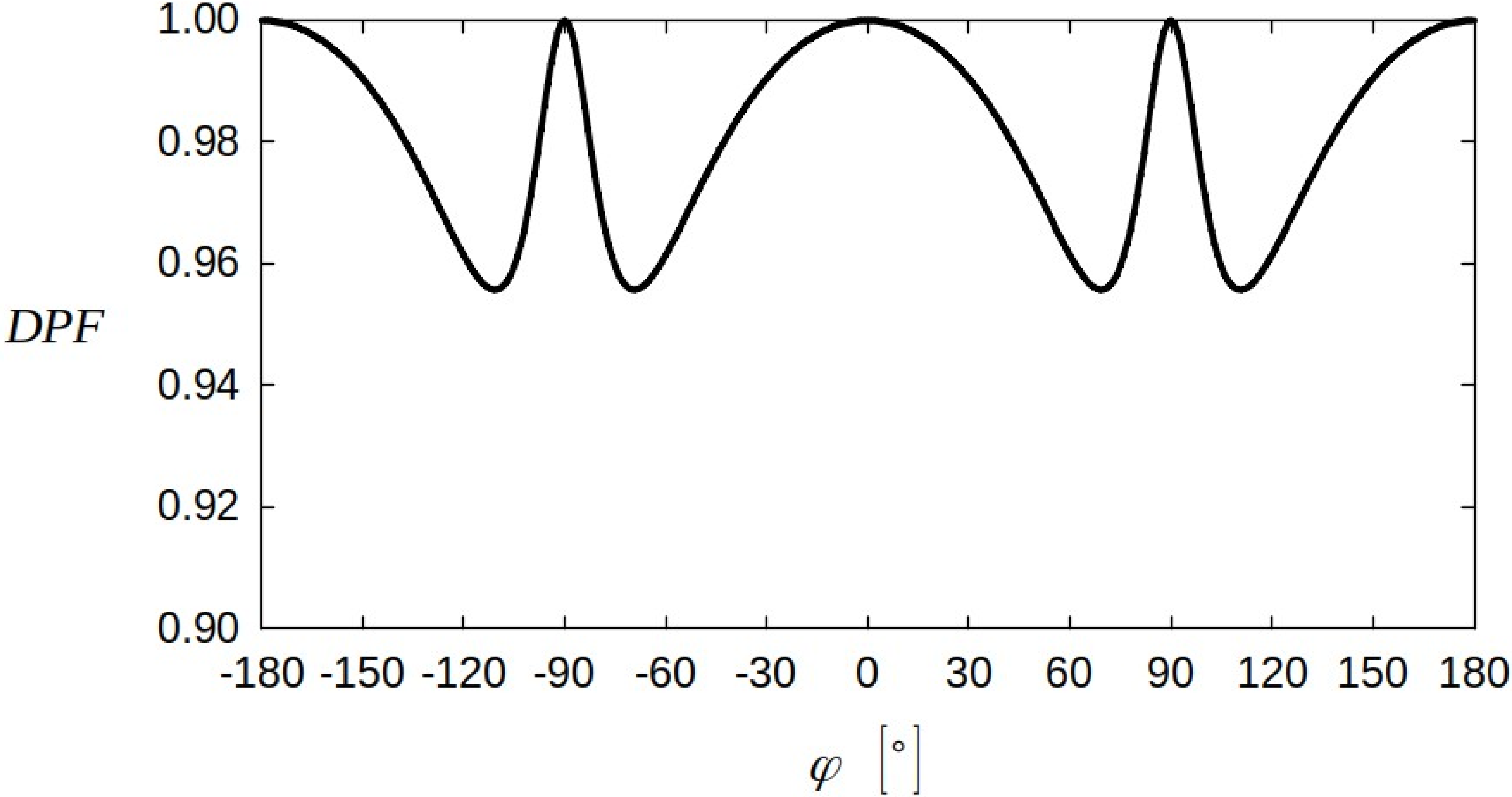
$$P_{INJ} = \frac{3\sqrt{3}}{8\pi} V_m I_{OUT} k \cos\phi$$

$$P_{OUT} = V_{OUT} I_{OUT} = \frac{3\sqrt{3}}{\pi} V_m I_{OUT}$$

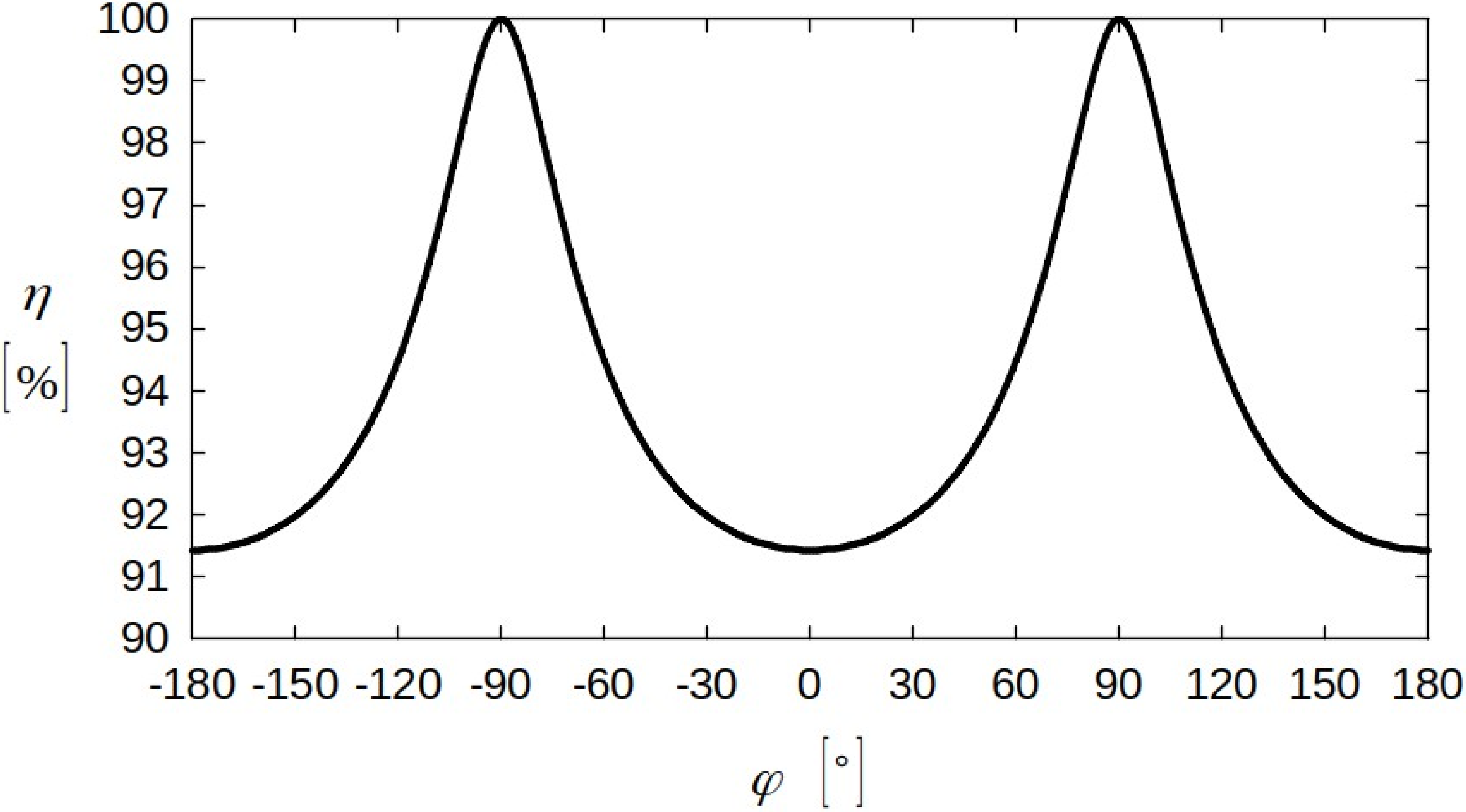
$$\eta(k, \phi) = \frac{8}{8 + k\cos\phi}$$



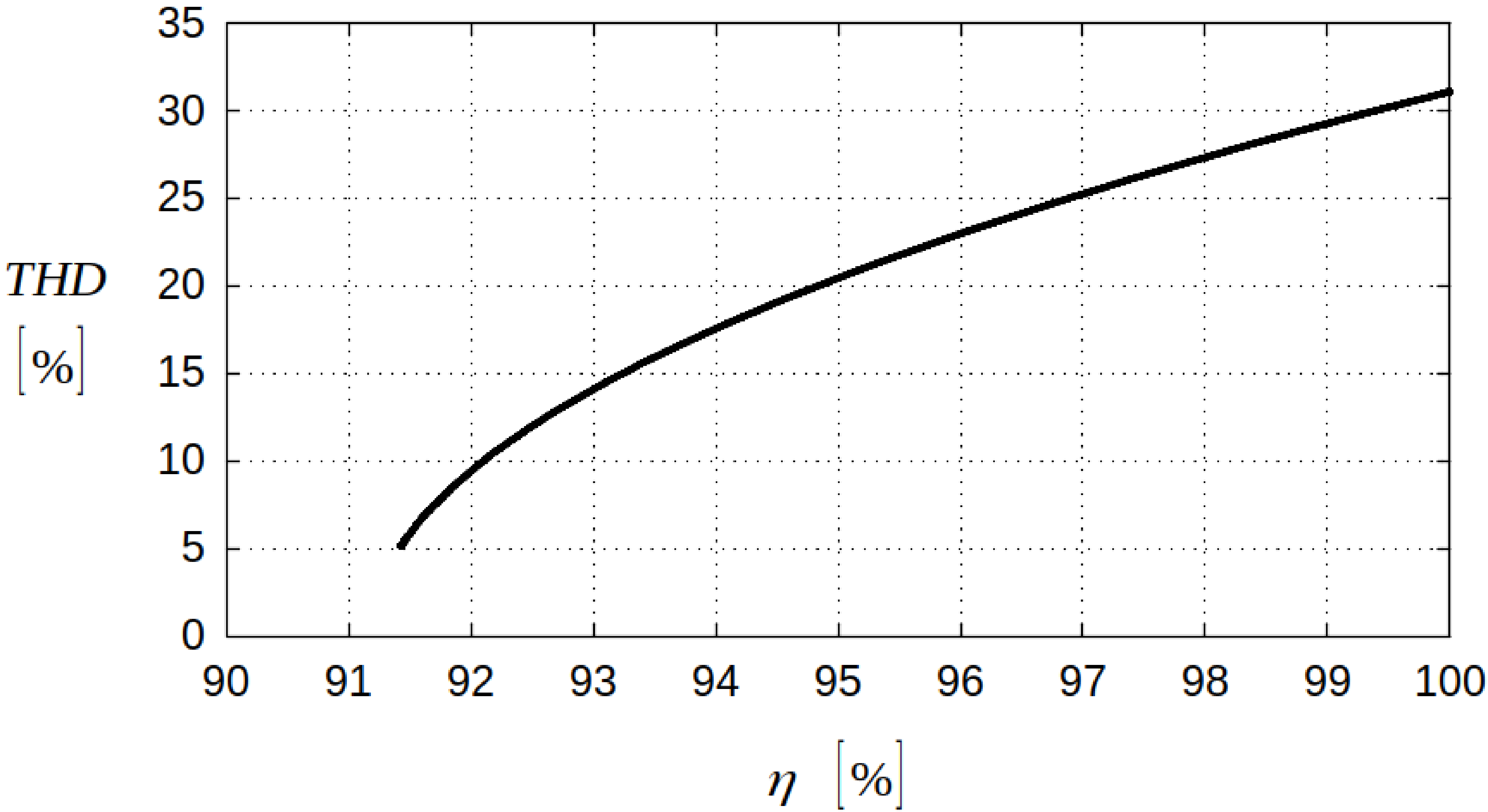
1.00
0.98
0.96
0.94
0.92
0.90
PF
-180 -150 -120 -90 -60 -30 0 30 60 90 120 150 180
φ [°]



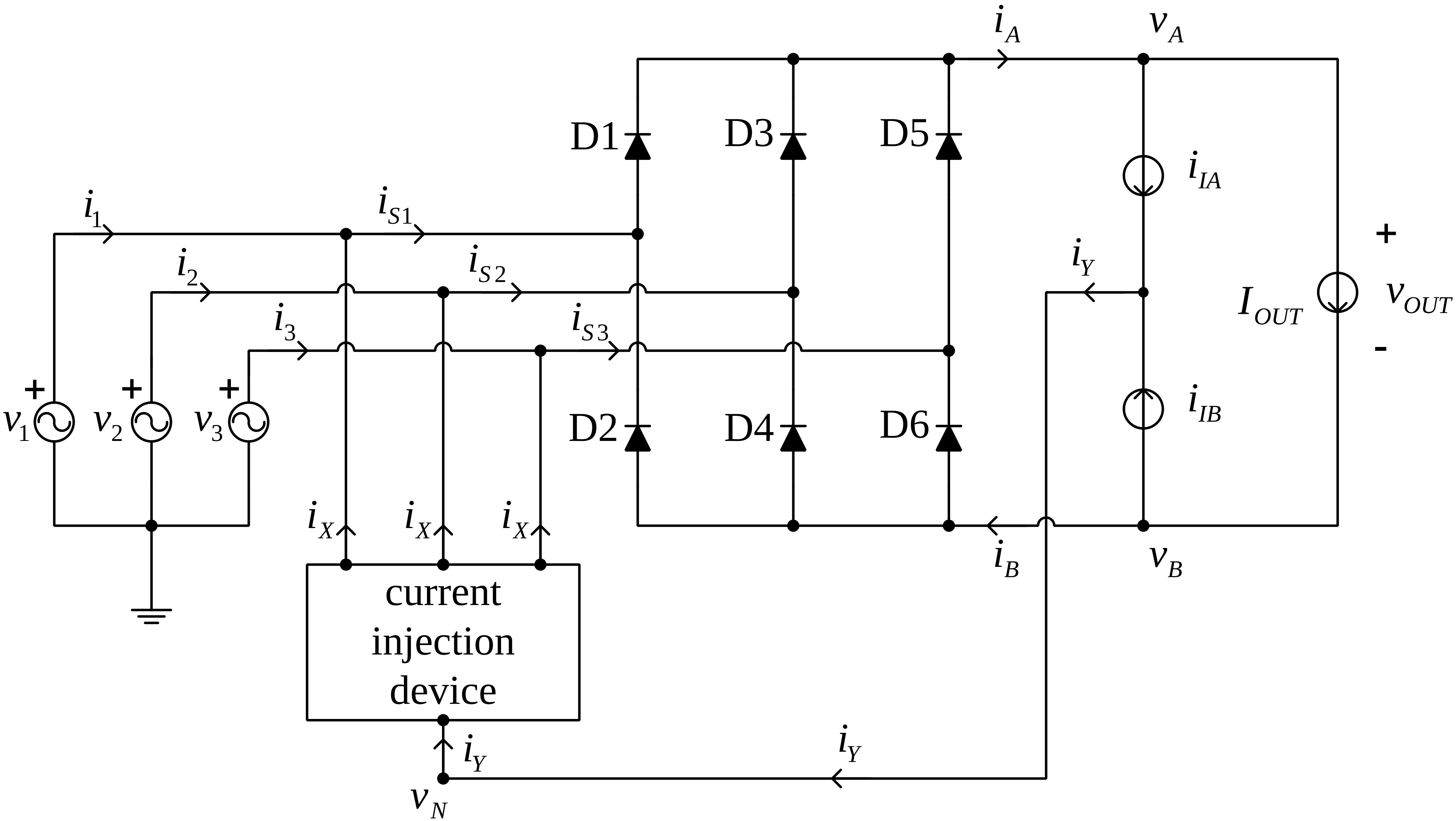
DPF
1.00
0.98
0.96
0.94
0.92
0.90
-180
-150
-120
-90
-60
-30
0
30
60
90
120
150
180
φ [°]



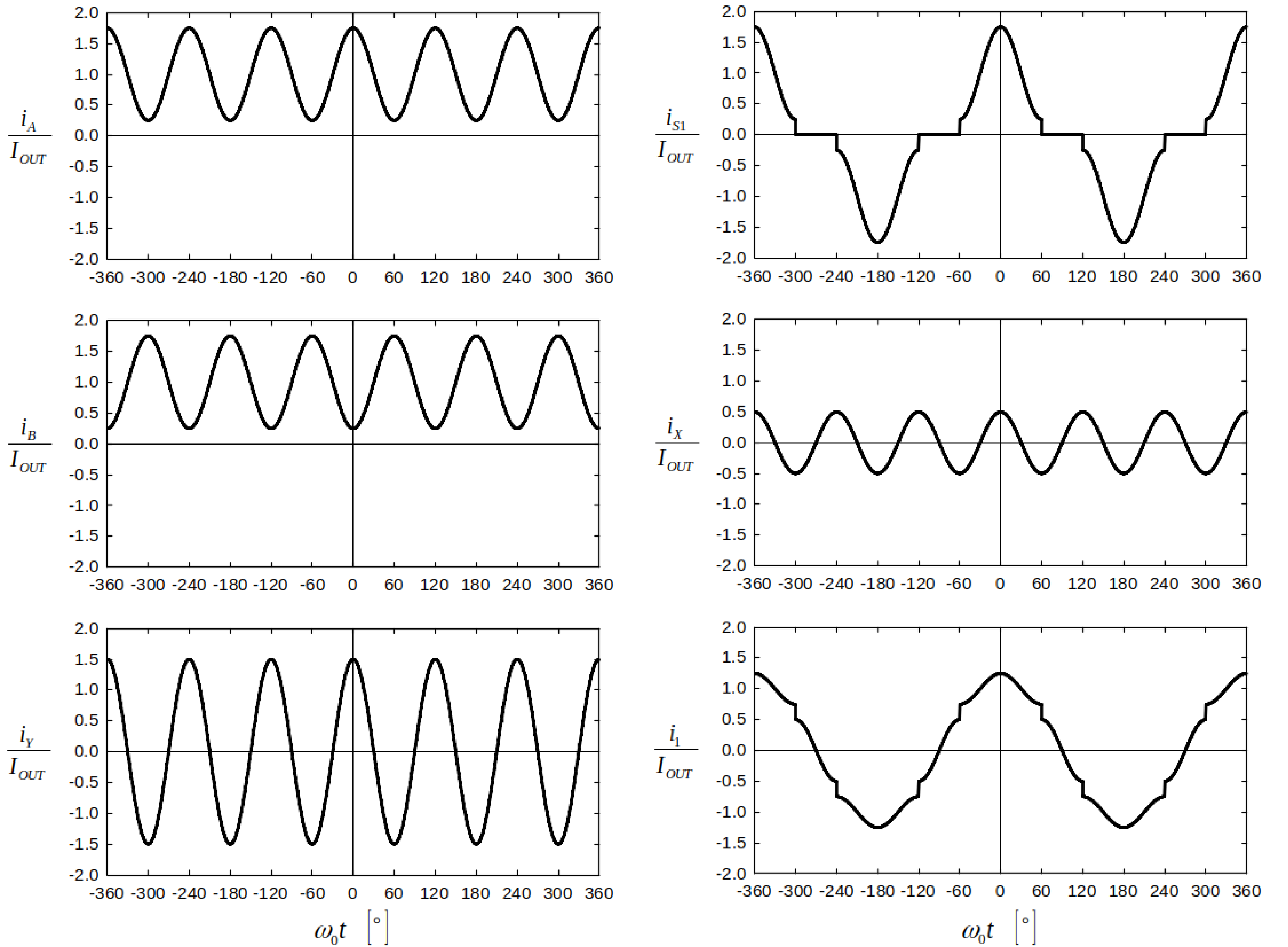
100
99
98
97
96
95
94
93
92
91
90
η
[%]
-180
-150
-120
-90
-60
-30
0
30
60
90
120
150
180
φ [°]



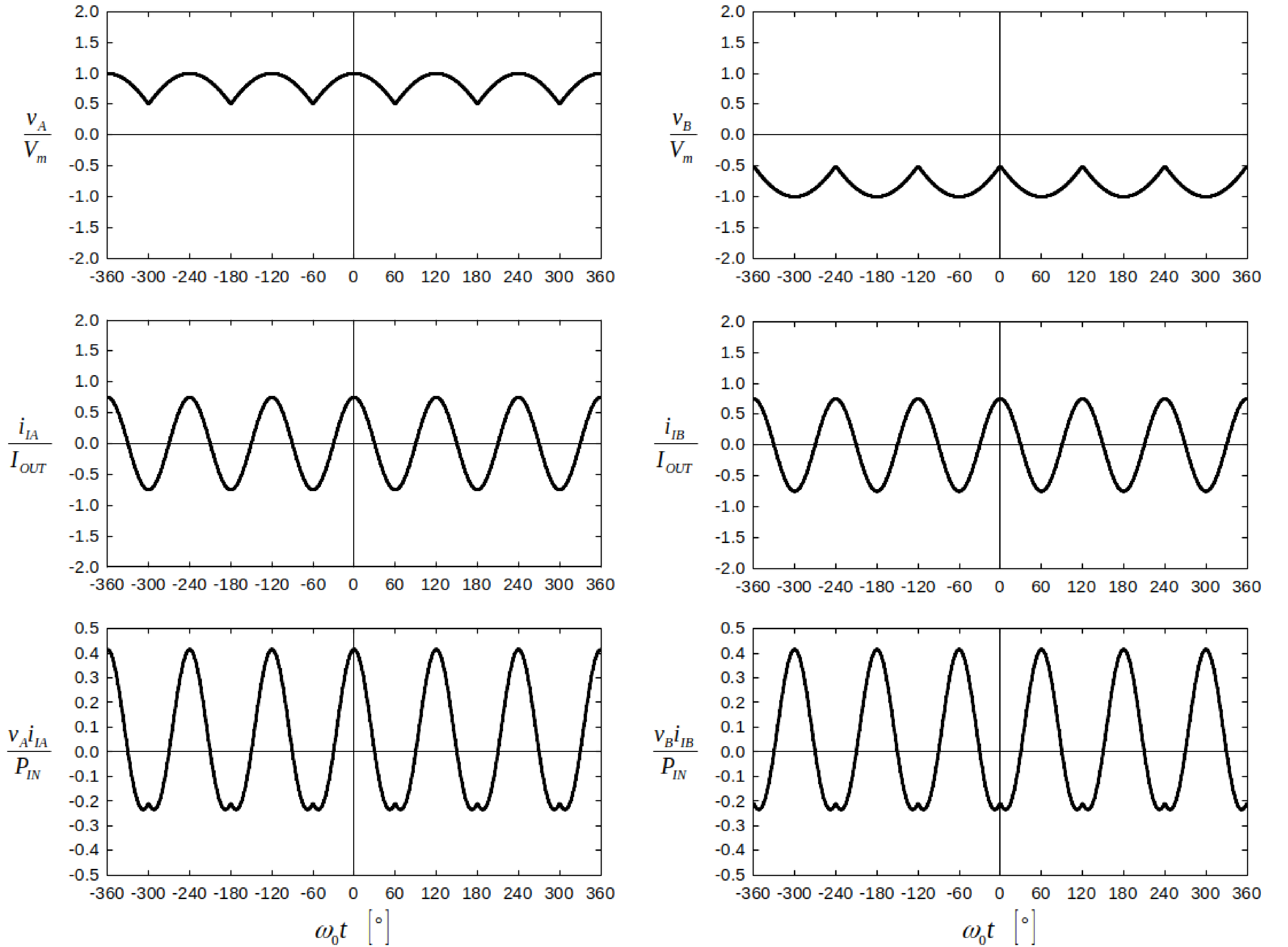
THD [%]
35
30
25
20
15
10
5
0
90
91
92
93
94
95
96
97
98
99
100
η [%]



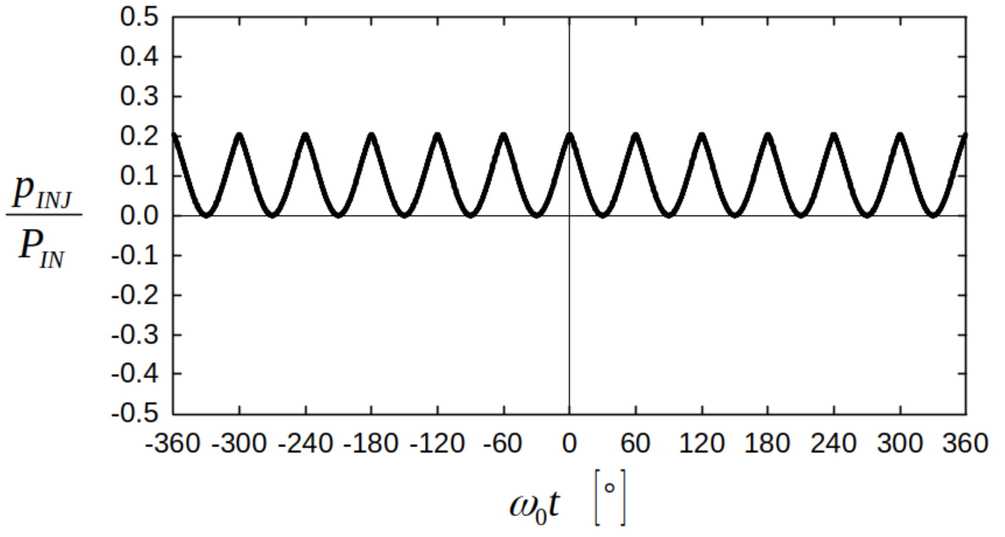
i_A
v_A
D1
D3
D5
i_IA
i_1
i_S1
i_2
i_S2
i_Y
+
I_OUT
v_OUT
-
i_3
i_S3
+
+
+
v_1
v_2
v_3
D2
D4
D6
i_IB
i_X
i_X
i_X
i_B
v_B
current
injection
device
i_Y
i_Y
v_N



i_A / I_{OUT}
i_B / I_{OUT}
i_Y / I_{OUT}
i_{S1} / I_{OUT}
i_X / I_{OUT}
i_1 / I_{OUT}
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]

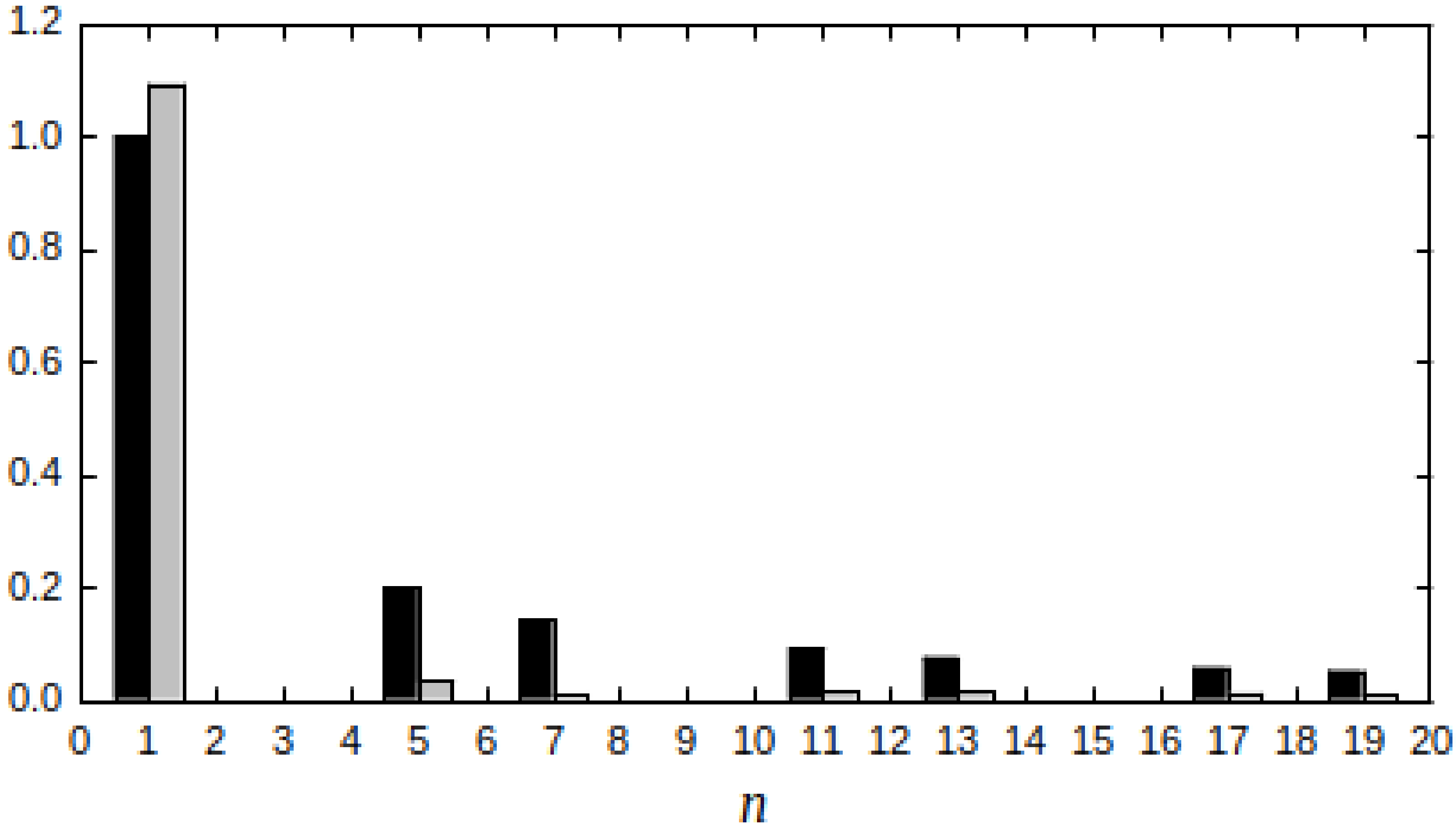


$\frac{v_A}{V_m}$
$\frac{v_B}{V_m}$
$\frac{i_{IA}}{I_{OUT}}$
$\frac{i_{IB}}{I_{OUT}}$
$\frac{v_A i_{IA}}{P_{IN}}$
$\frac{v_B i_{IB}}{P_{IN}}$
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]



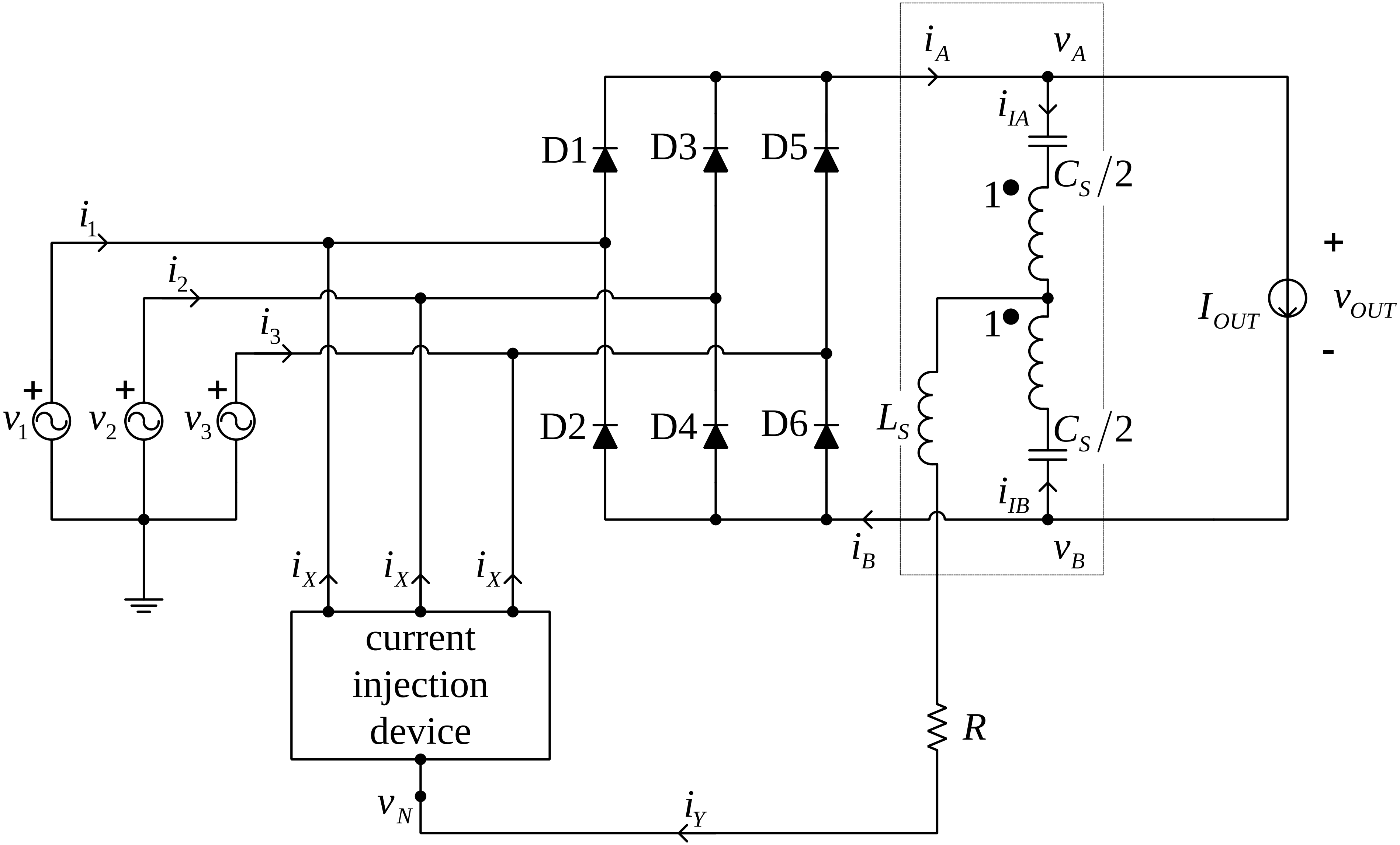
$$P_{INJ\max}=\frac{9\sqrt{3}}{32\pi}V_m I_{OUT}=\frac{3}{32}P_{OUT}=\frac{3}{35}P_{IN}\approx 8.57\,\%\;P_{IN}$$

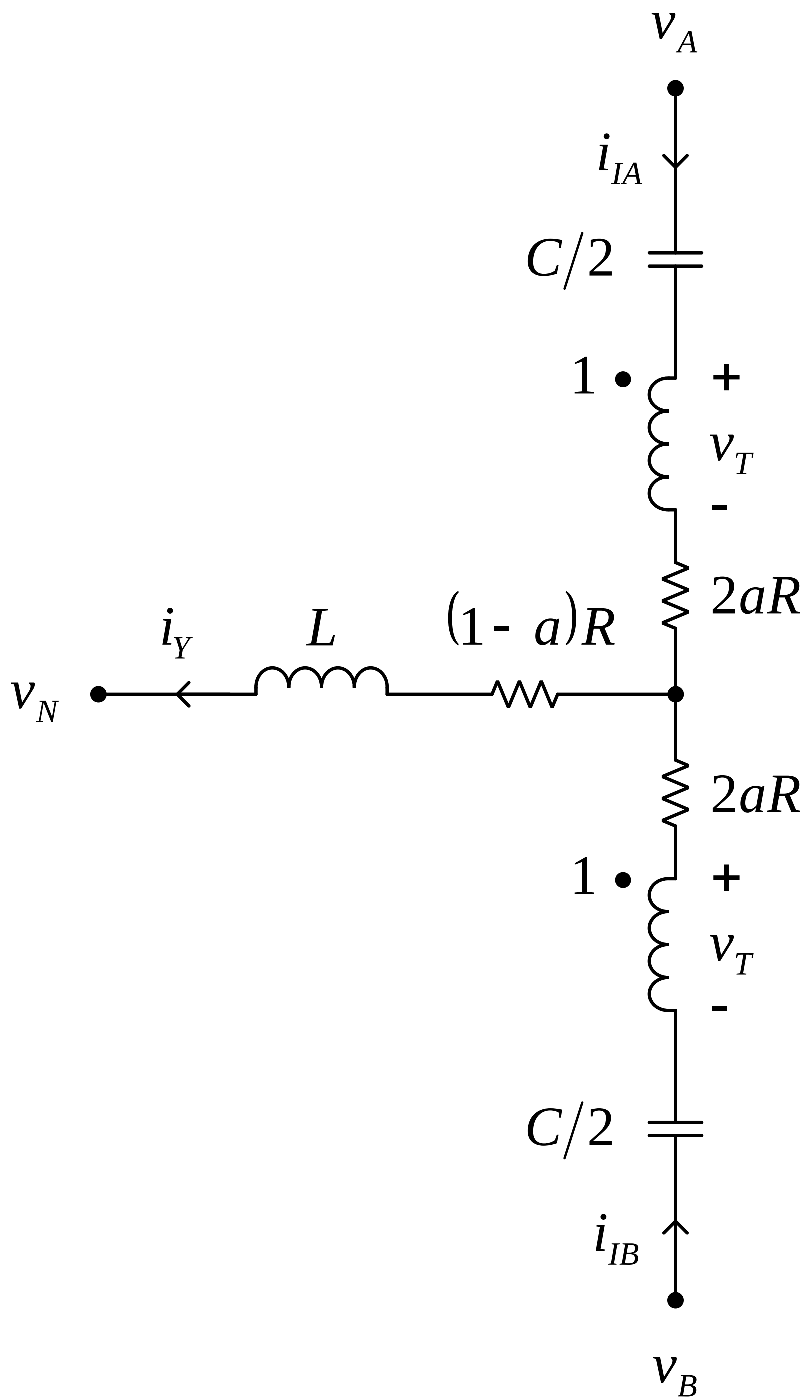
$$\eta_{\min}=\frac{32}{35}=91.43\,\%$$



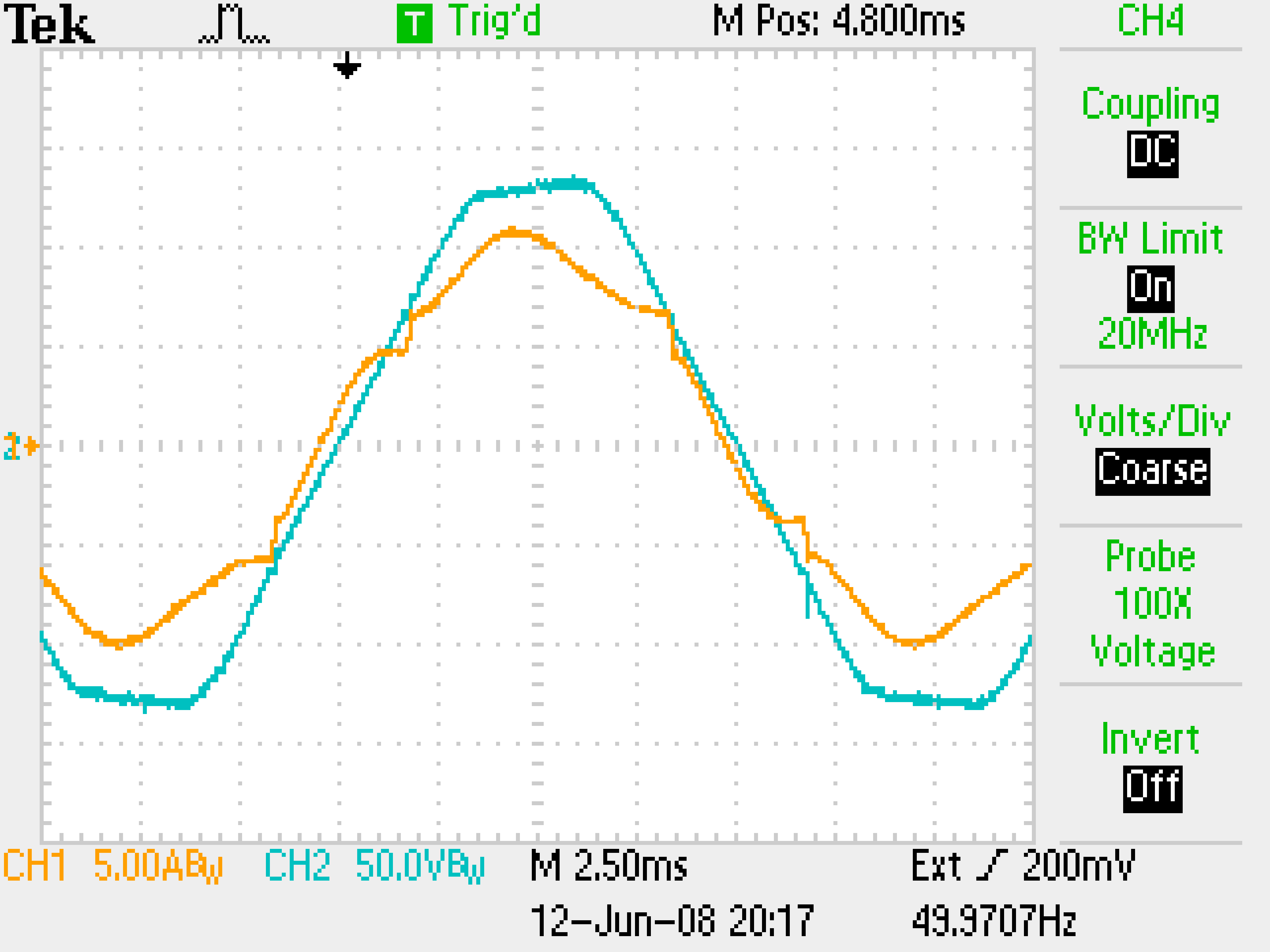
Normalized amplitude spectra of the input currents; black bars: current injection is not applied; gray bars: the optimal third-harmonic current injection

Current Injection Networks

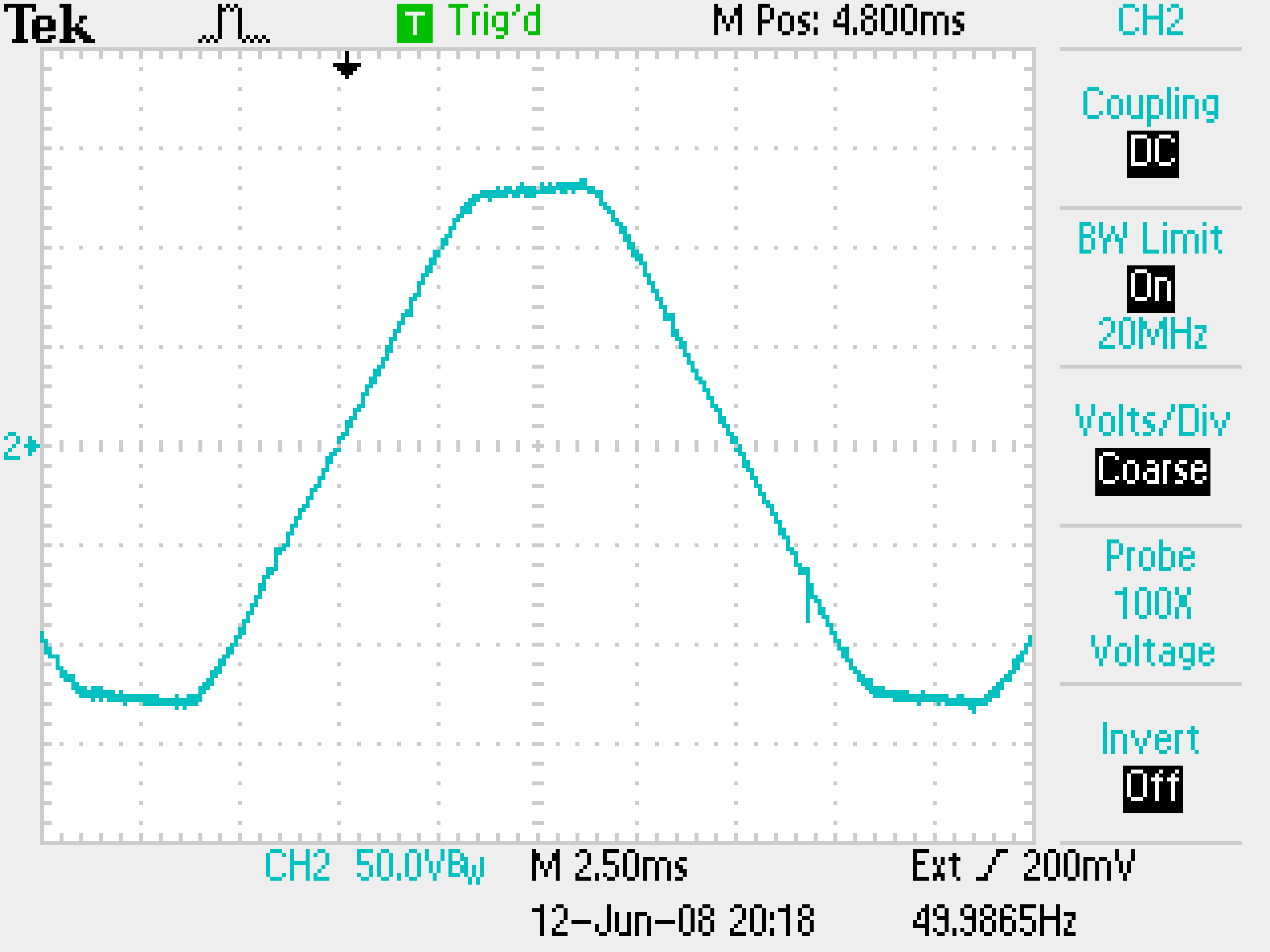


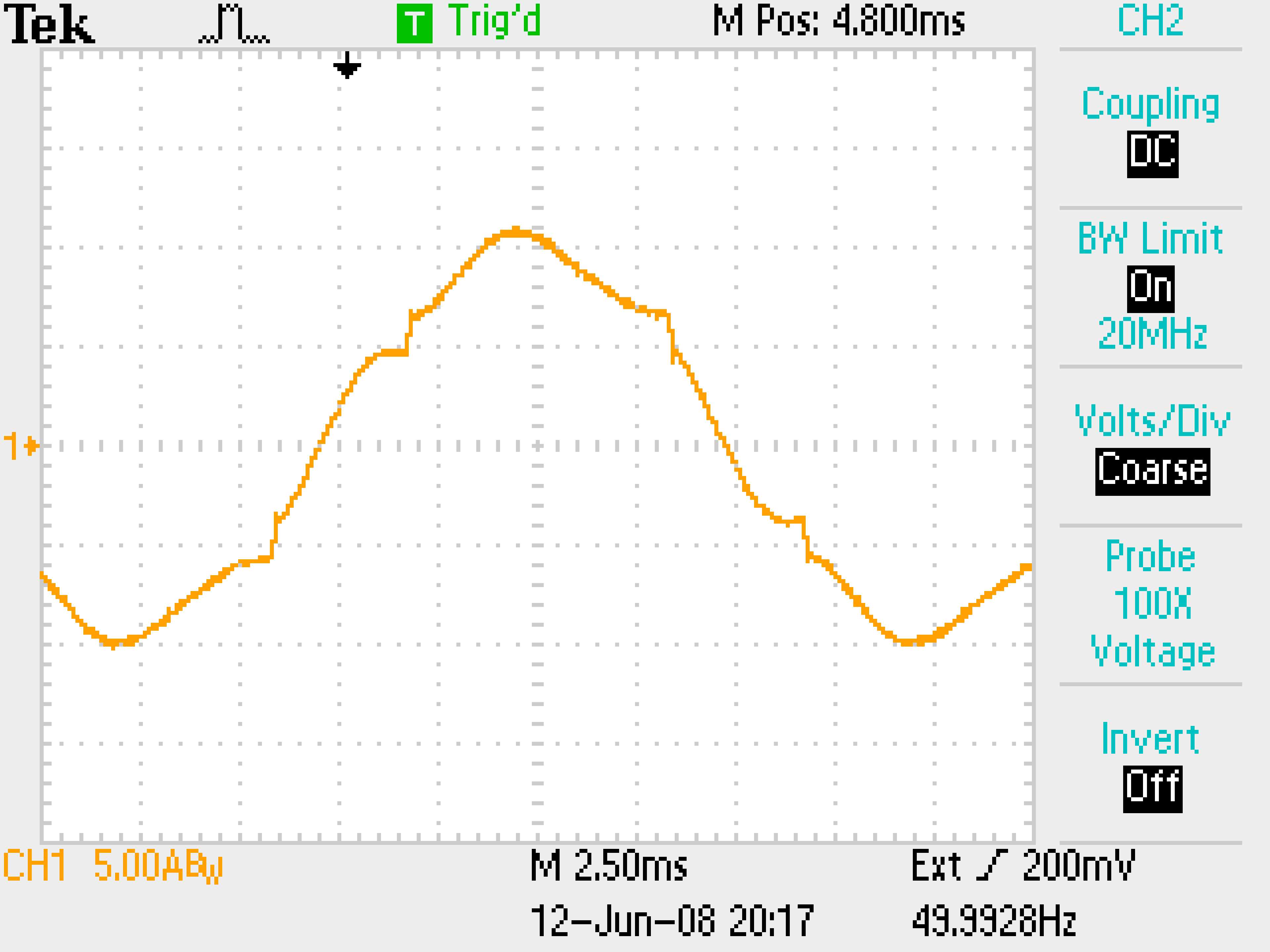


vA
iIA
C/2
1
+
vT
-
2aR
iY
L
(1- a)R
vN
2aR
1
+
vT
-
C/2
iIB
vB

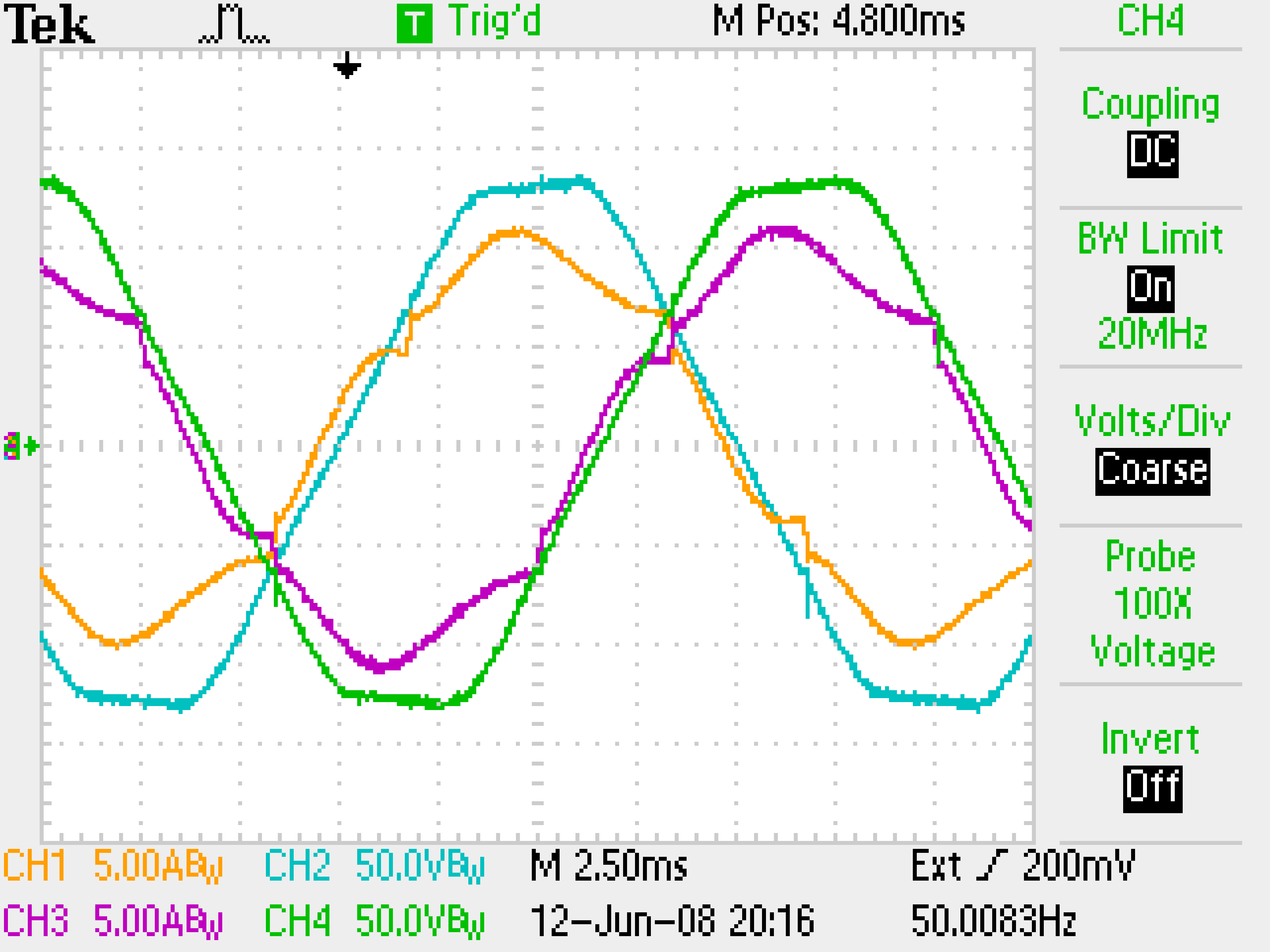


Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00AΩ
CH2 50.0VΩ
M 2.50ms
Ext 200mV
12-Jun-08 20:17
49.9707Hz


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
2
CH2 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 20:18
49.9865Hz



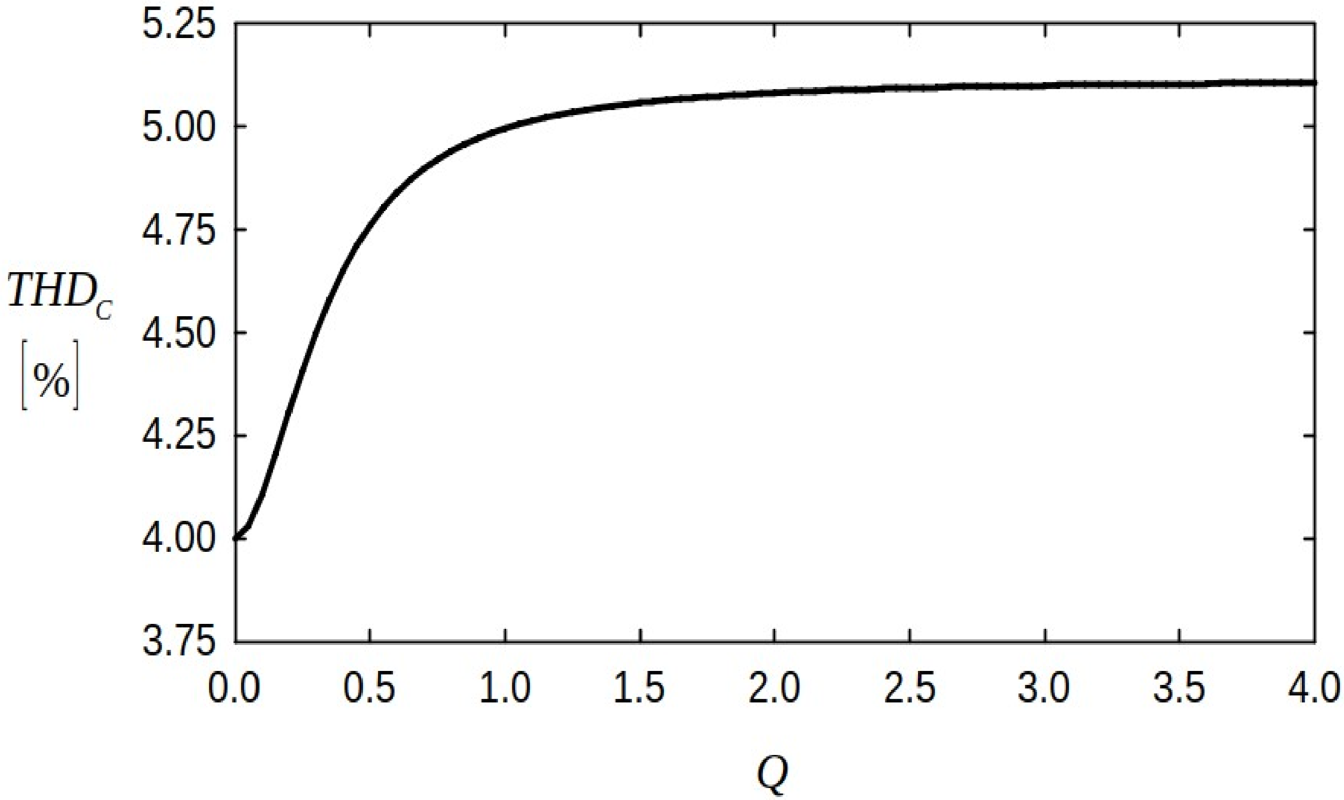
Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00AB
M 2.50ms
Ext 200mV
12-Jun-08 20:17
49.9928Hz

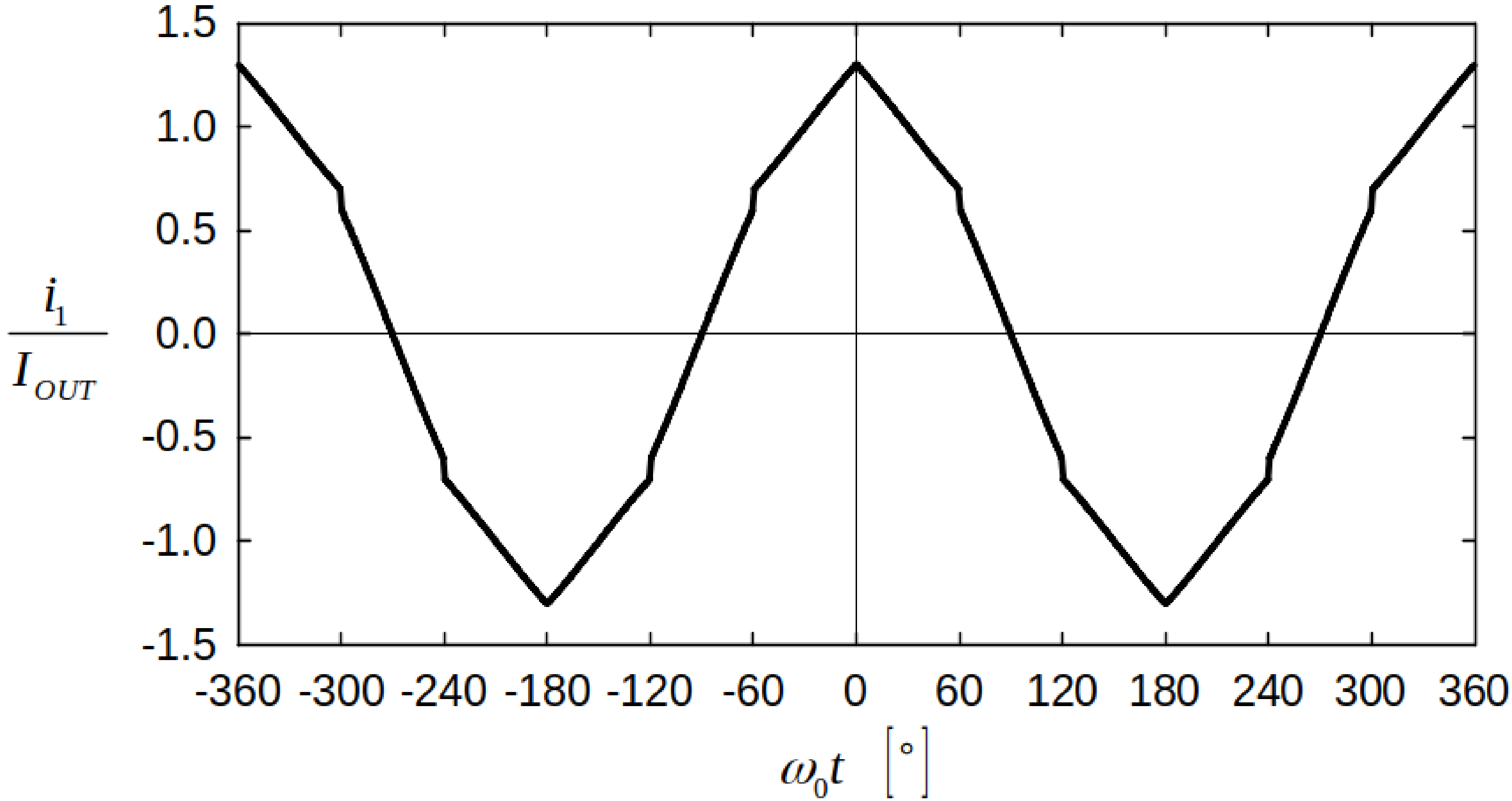


Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
CH3 5.00A
CH4 50.0V
12-Jun-08 20:16
50.0083Hz

Dependence on the Q-factor?

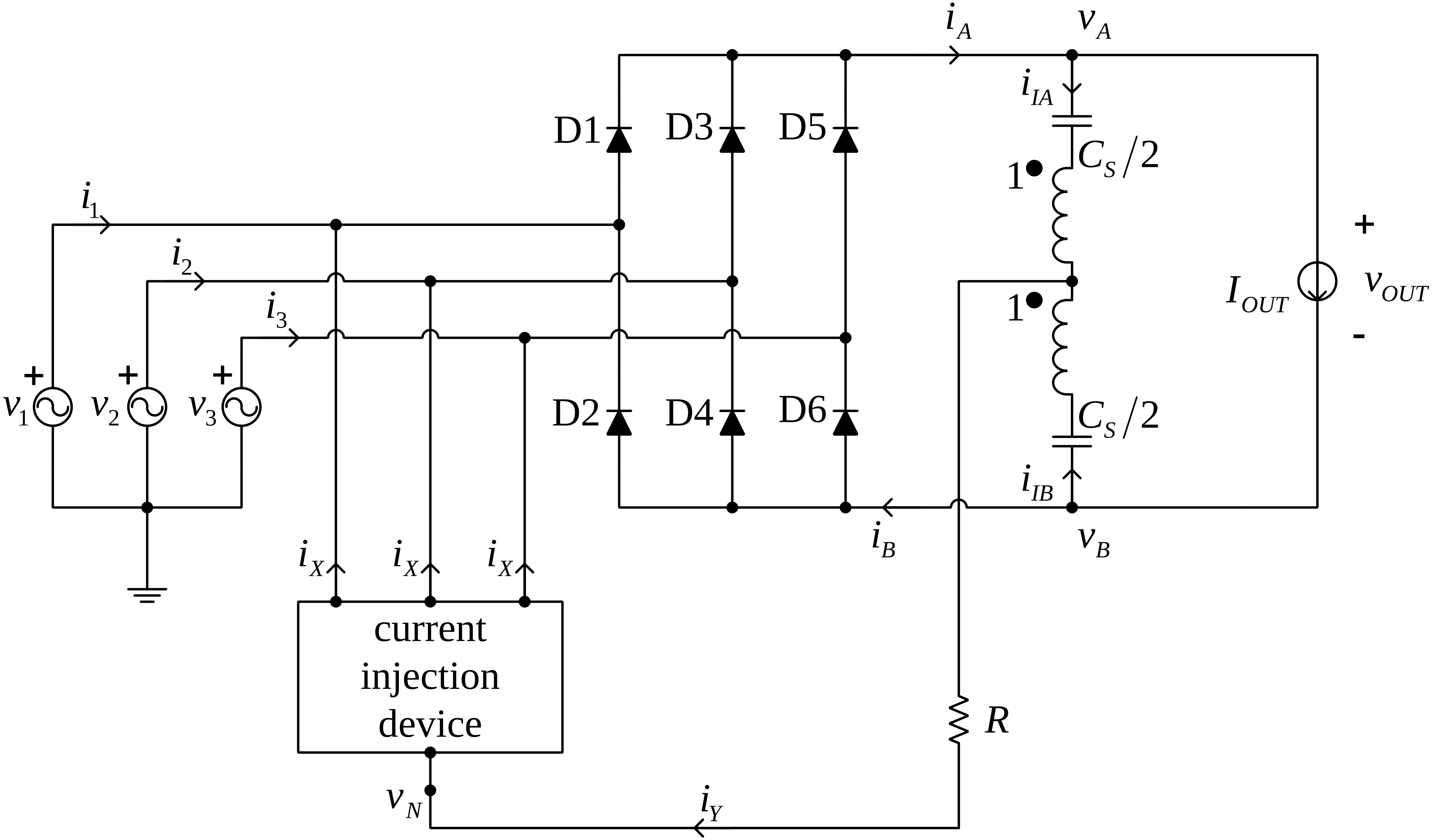
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$





1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
$\frac{i_1}{I_{OUT}}$
-360
-300
-240
-180
-120
-60
0
60
120
180
240
300
360
$\omega_0 t$ [°]

Current Injection Based Rectifier for Q --> 0



$$\sigma = \frac{V_m}{I_{OUT} R}$$

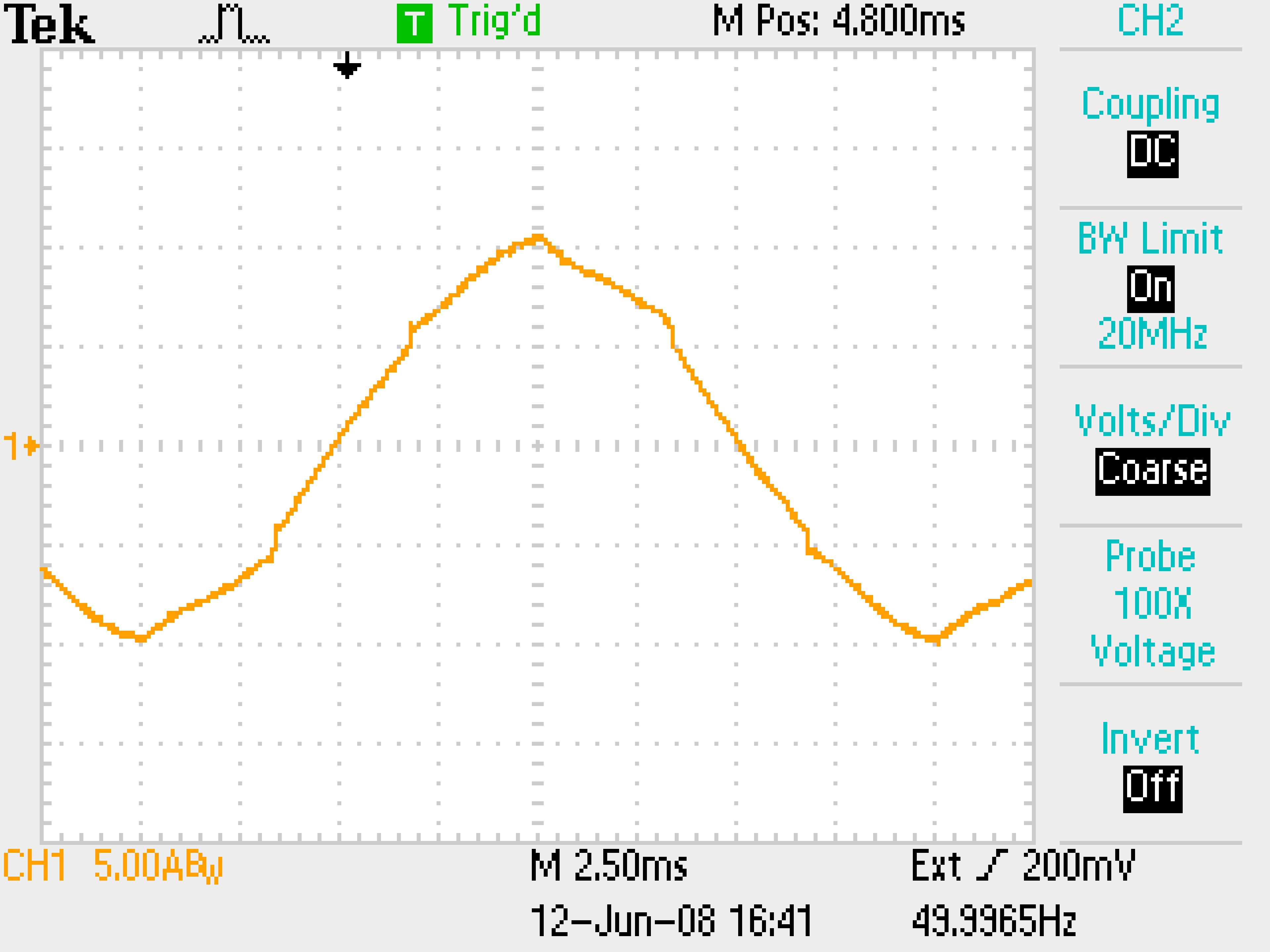
$$THD(\sigma) = \frac{\sqrt{(4\pi^2 - 27)\sigma^2 + 96(9 - 2\sqrt{3}\pi)\sigma + 768(\pi^2 - 9)}}{(2\pi - 3\sqrt{3})\sigma + 48\sqrt{3}}$$

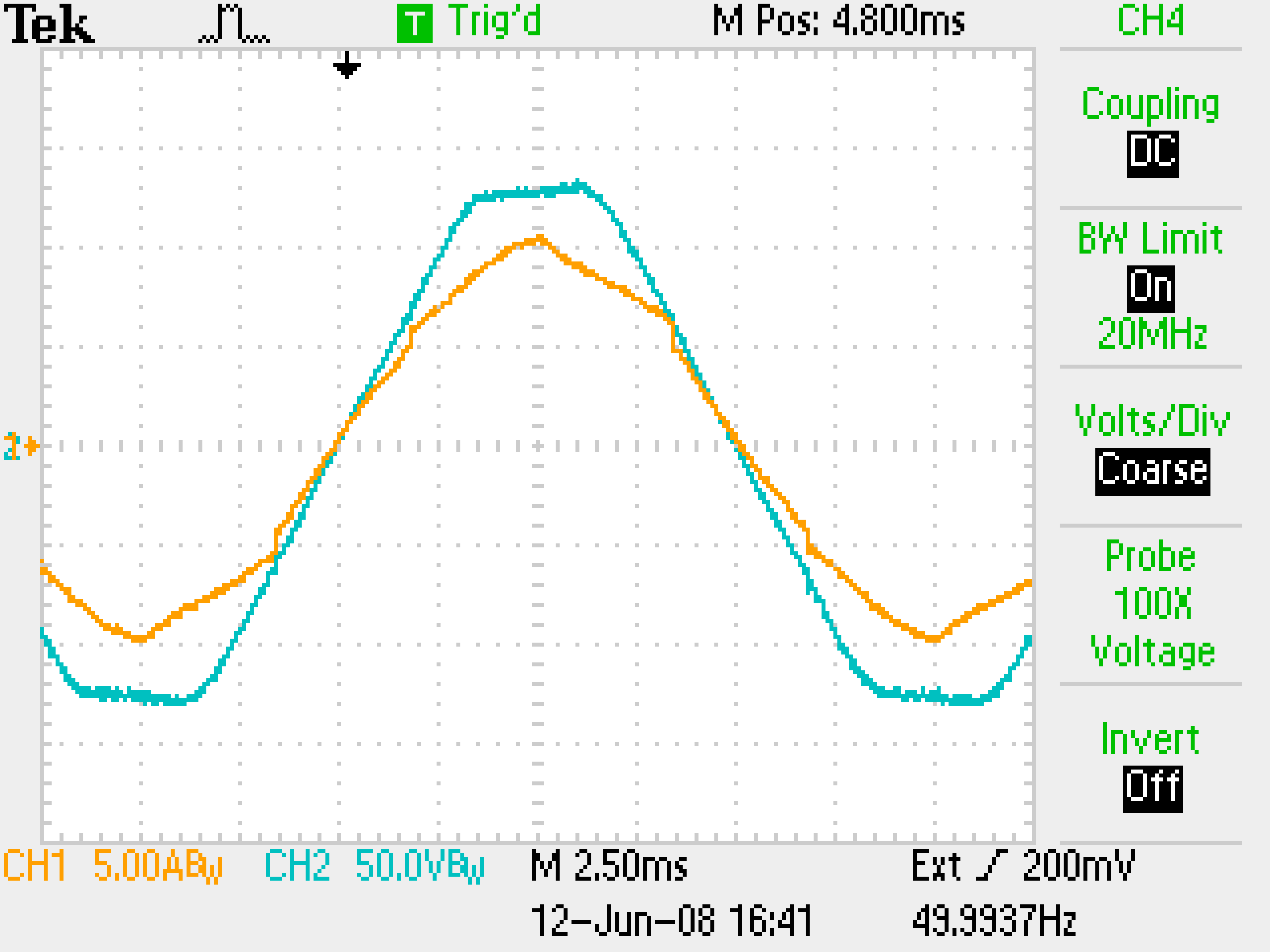
$$\frac{d(THD(\sigma))}{d\sigma} = 0$$

$$\sigma = \frac{4\pi}{\sqrt{3}}$$

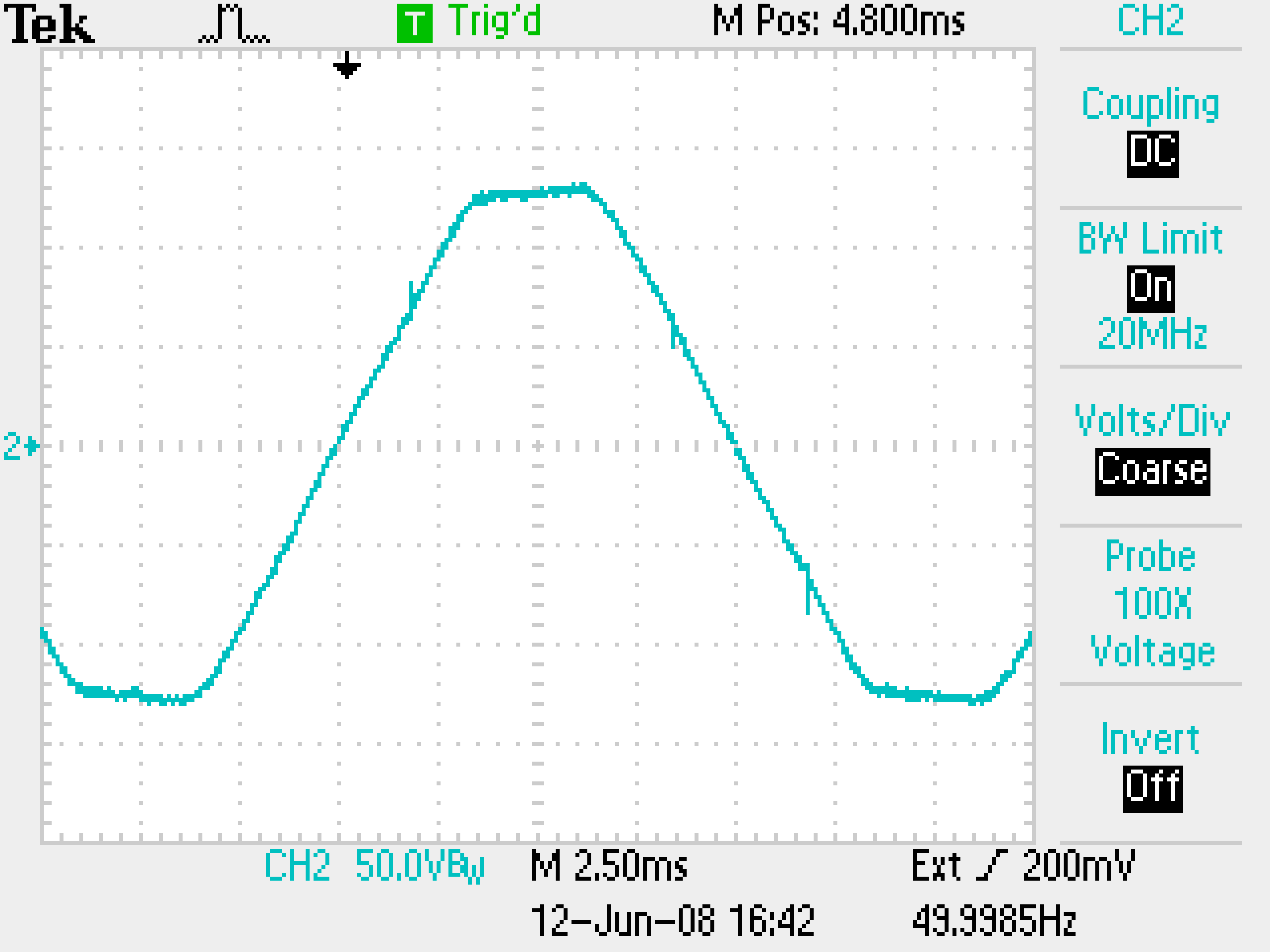
$$THD_{min} = THD\left(\frac{4\pi}{\sqrt{3}}\right) = \frac{\sqrt{4\pi^4 - 27\pi^2 + 216\sqrt{3}\pi - 1296}}{2\pi^2 - 3\sqrt{3}\pi + 36} \approx 4.01\%$$

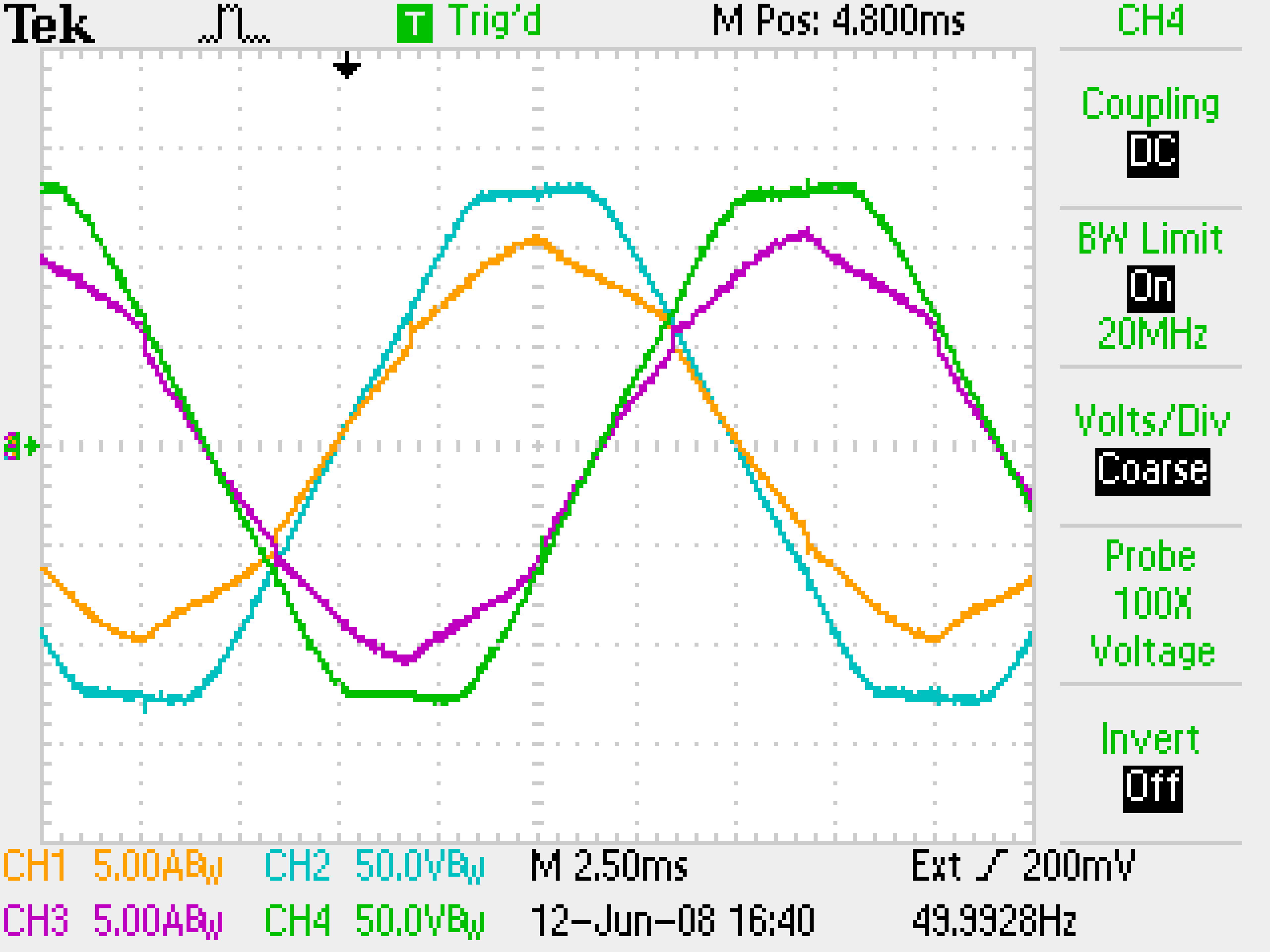
$$P_{INJ} = \frac{\pi(2\pi - 3\sqrt{3})}{36 + \pi(2\pi - 3\sqrt{3})} P_{IN} \approx 8.66\%\ P_{IN}$$


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00ABw
M 2.50ms
Ext 200mV
12-Jun-08 16:41
49.9965Hz

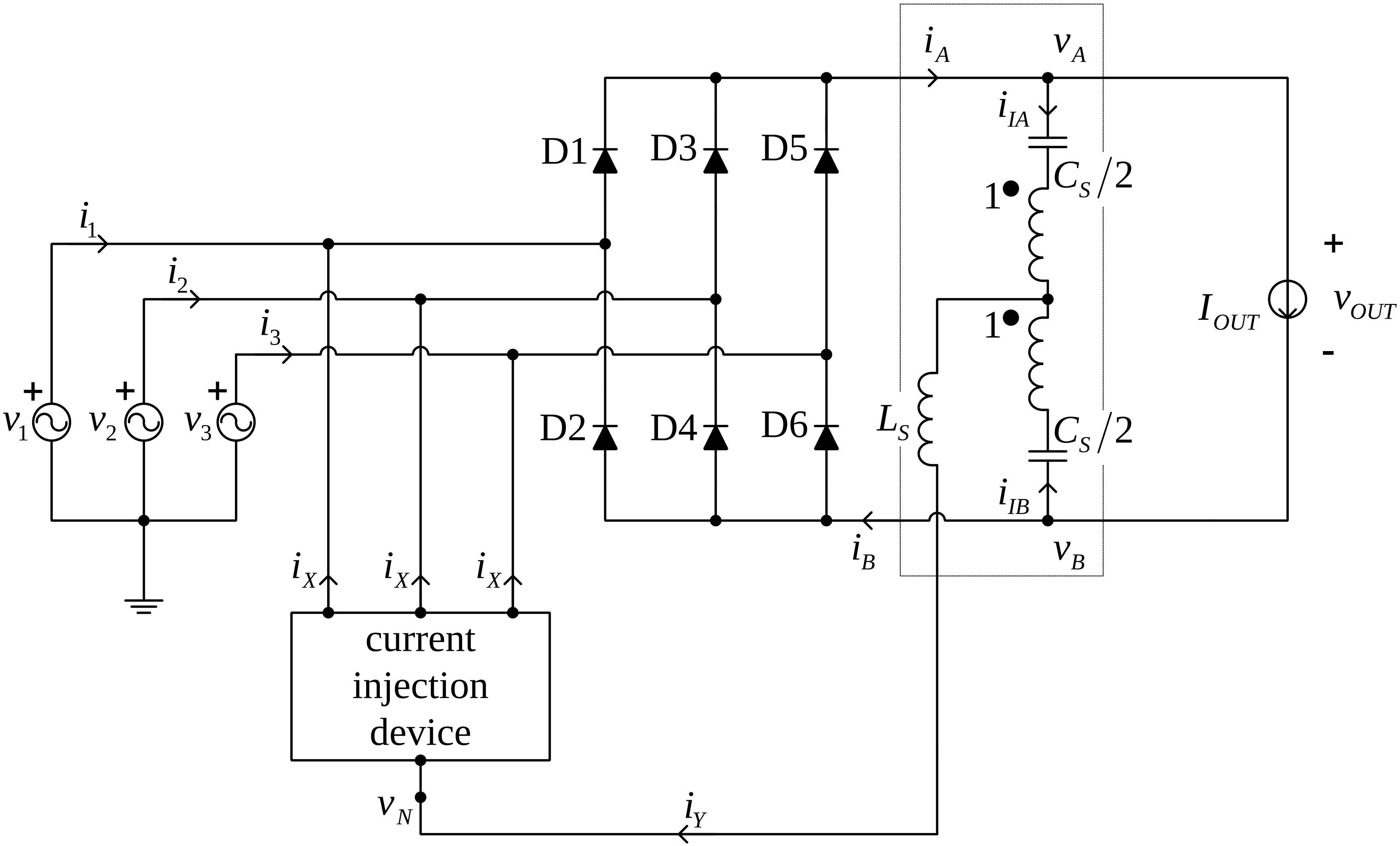


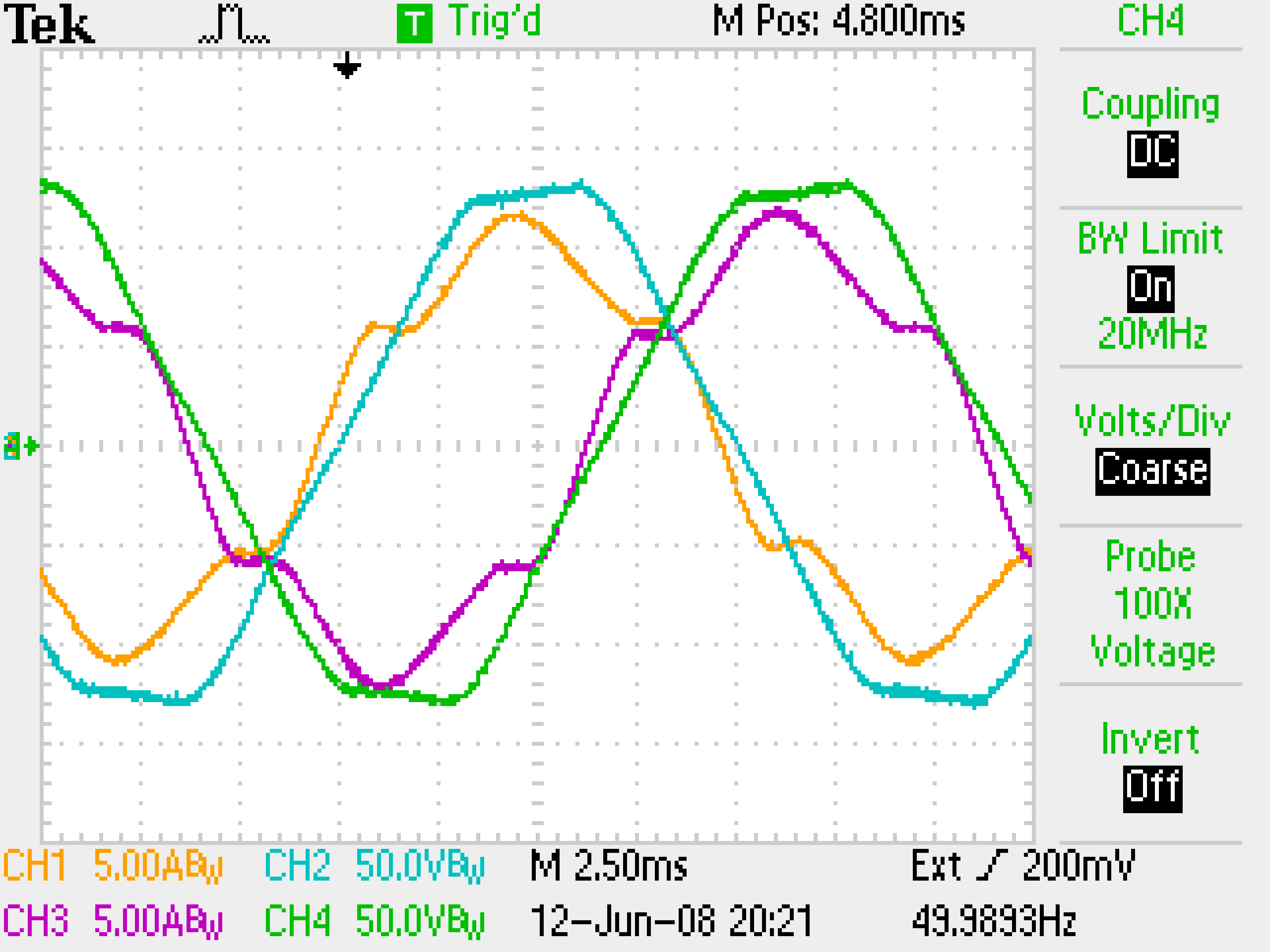
Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 16:41
49.9937Hz


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
2
CH2 50.0V
M 2.50ms
Ext 200mV
12-Jun-08 16:42
49.9985Hz


Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
CH3 5.00A
CH4 50.0V
12-Jun-08 16:40
49.9928Hz

The Discontinuous Conduction Mode




Tek
Trig'd
M Pos: 4.800ms
CH4
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A
CH2 50.0V
M 2.50ms
Ext 200mV
CH3 5.00A
CH4 50.0V
12-Jun-08 20:21
49.9893Hz

The Optimal Current Injection

$$i_1+i_2+i_3=0$$

$$i_X=\frac{1}{3}i_Y=\frac{1}{3}\left(i_A-i_B\right)$$

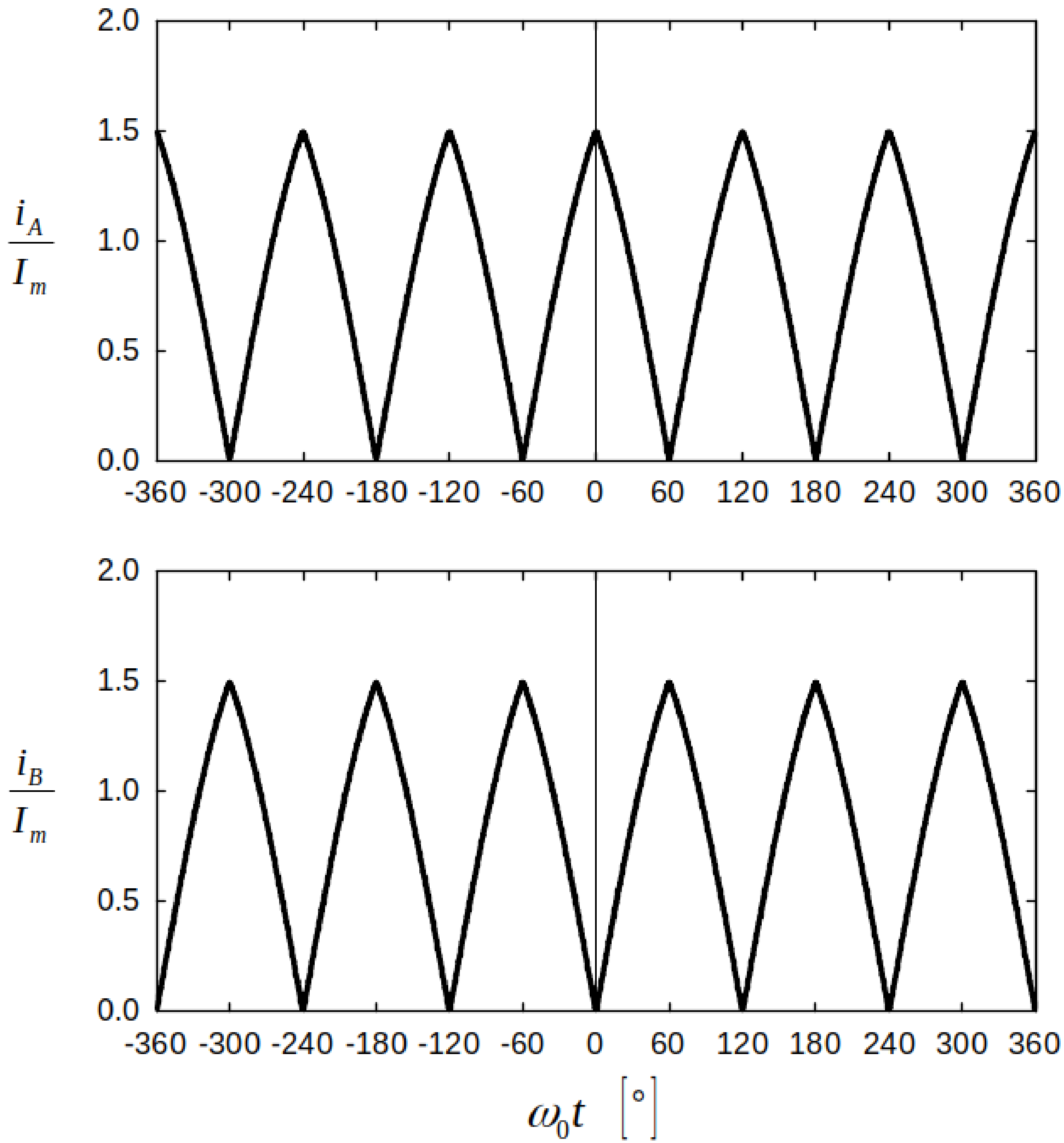
$$i_1=d_1 i_A-d_2 i_B-\frac{1}{3}\left(i_A-i_B\right)$$

$$i_2=d_3 i_A-d_4 i_B-\frac{1}{3}\left(i_A-i_B\right)$$

$$i_3=d_5 i_A-d_6 i_B-\frac{1}{3}\left(i_A-i_B\right)$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}=\begin{bmatrix} d_1-\frac{1}{3} & -d_2+\frac{1}{3} \\ d_3-\frac{1}{3} & -d_4+\frac{1}{3} \\ d_5-\frac{1}{3} & -d_6+\frac{1}{3} \end{bmatrix}\begin{bmatrix} i_A \\ i_B \end{bmatrix}$$

$$\begin{bmatrix} i_A \\ i_B \end{bmatrix}=\begin{bmatrix} d_1-\frac{1}{3} & -d_2+\frac{1}{3} \\ d_3-\frac{1}{3} & -d_4+\frac{1}{3} \end{bmatrix}^{-1}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



2.0
1.5
1.0
0.5
0.0
$\frac{i_A}{I_m}$
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
2.0
1.5
1.0
0.5
0.0
$\frac{i_B}{I_m}$
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]

$$i_{IA}=\frac{3\sqrt{3}}{\pi}I_m\sum_{n=1}^{+\infty}\frac{1-2(-1)^n}{9n^2-1}\cos(3n\omega_0 t)$$

$$1-2(-1)^n=\begin{cases}3, & \text{for odd } n\\ -1, & \text{for even } n\end{cases},$$

$$i_{IB}=\frac{3\sqrt{3}}{\pi}I_m\sum_{n=1}^{+\infty}\frac{2-(-1)^n}{9n^2-1}\cos(3n\omega_0 t)$$

$$2-(-1)^n=\begin{cases}3, & \text{for odd } n\\ 1, & \text{for even } n\end{cases}.$$

$$i_{IA}=i_{ODD}+i_{EVEN}$$

$$i_{IB}=i_{ODD}-i_{EVEN}$$

$$i_{ODD}=\frac{3\sqrt{3}}{\pi}I_m\sum_{k=1}^{+\infty}\frac{3}{(6k-3)^2-1}\cos((6k-3)\,\omega_0 t)$$

$$i_{EVEN}=-\frac{3\sqrt{3}}{\pi}I_m\sum_{k=1}^{+\infty}\frac{1}{36k^2-1}\cos(6k\omega_0 t)$$

$$v_{AV}=\frac{3\sqrt{3}}{\pi}V_m\sum_{k=1}^{+\infty}\frac{1}{(6k-3)^2-1}\cos((6k-3)\,\omega_0 t)$$

$$\hat{v}_{OUT}=-\frac{3\sqrt{3}}{\pi}V_m\sum_{k=1}^{+\infty}\frac{2}{36k^2-1}\cos(6k\omega_0 t)$$

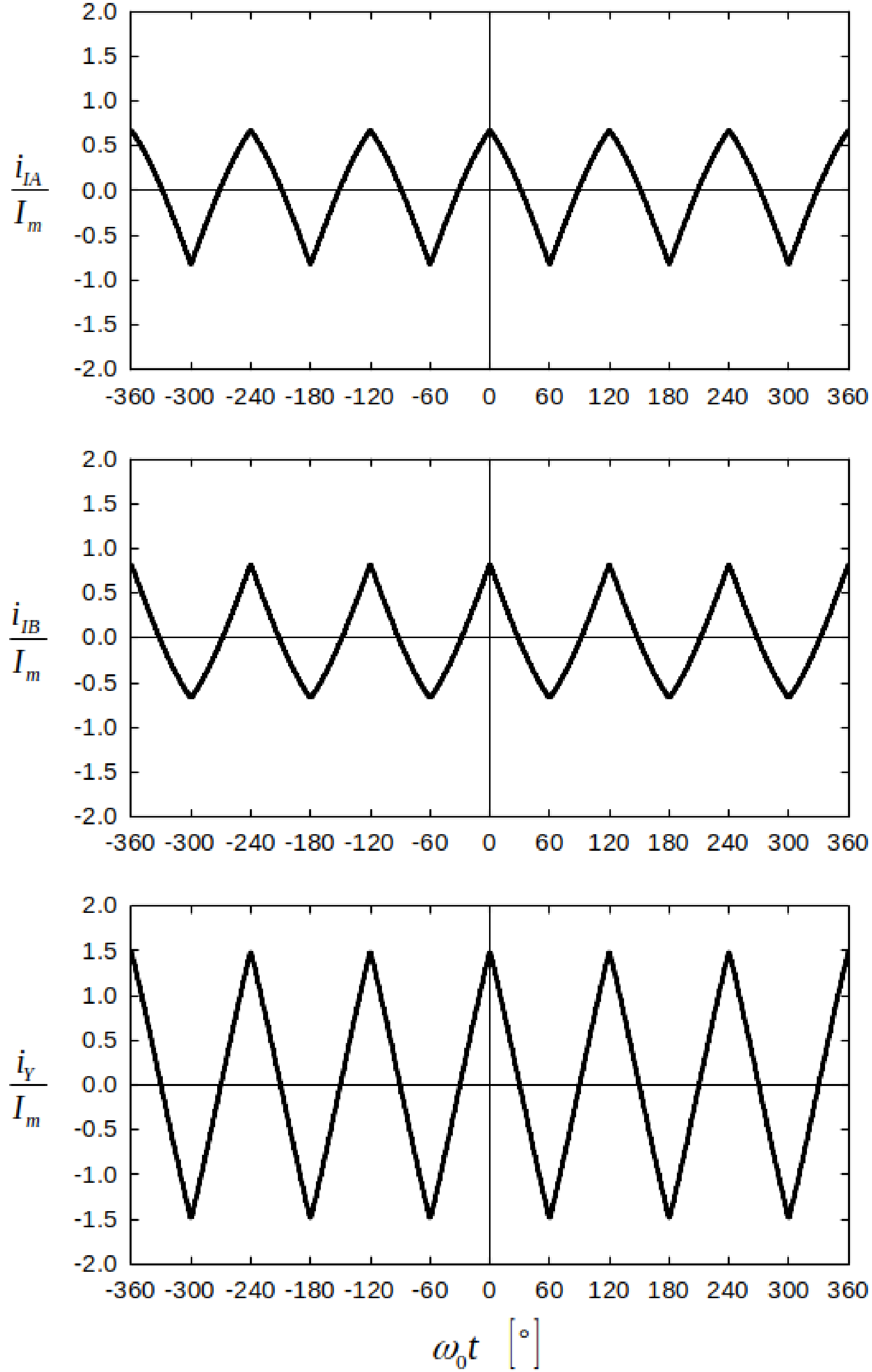
$$\frac{v_{AV}}{i_{ODD}}=\frac{1}{3}R_E$$

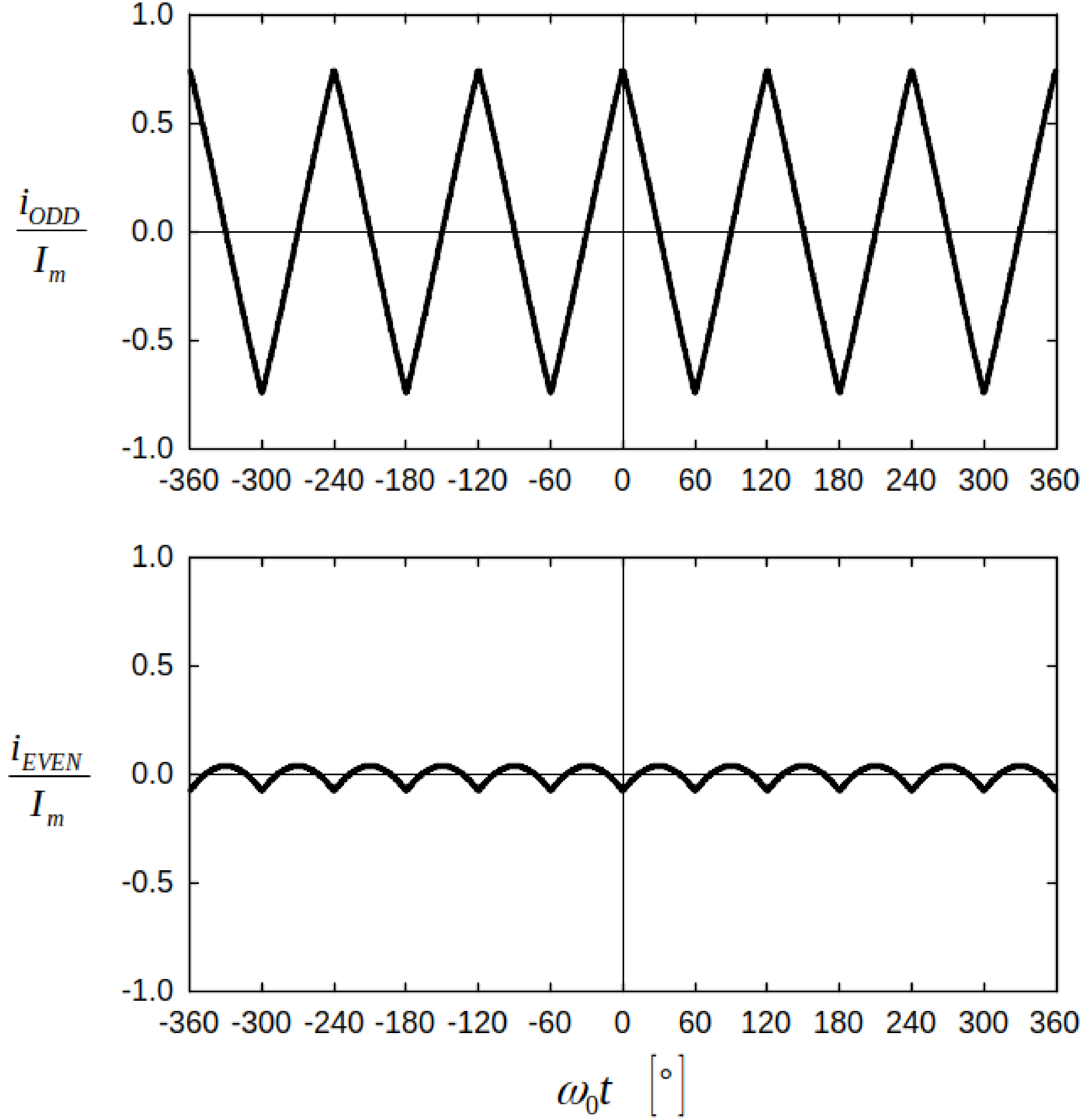
$$\frac{\hat{v}_{OUT}}{i_{EVEN}}=2R_E$$

$$i_Y=i_{IA}+i_{IB}=2i_{ODD}$$

$$\frac{v_{AV}}{i_Y}=\frac{1}{6}R_E$$

$$R_E=\frac{V_m}{I_m}$$


2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
$\frac{i_{IA}}{I_m}$
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\frac{i_{IB}}{I_m}$
$\frac{i_Y}{I_m}$
$\omega_0 t$ [°]

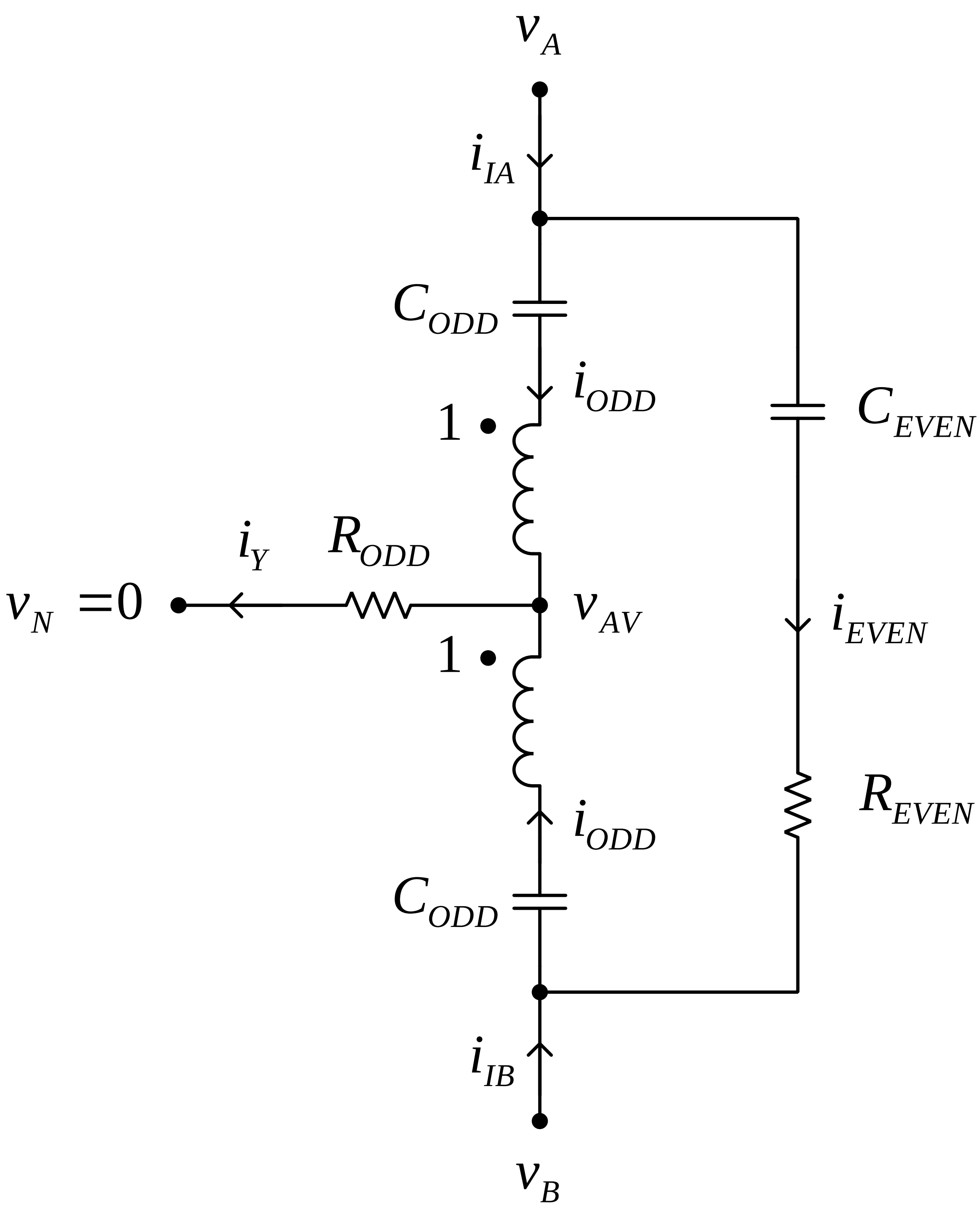


1.0
0.5
0.0
-0.5
-1.0
$\frac{i_{ODD}}{I_m}$
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\frac{i_{EVEN}}{I_m}$
$\omega_0 t \quad [^\circ]$

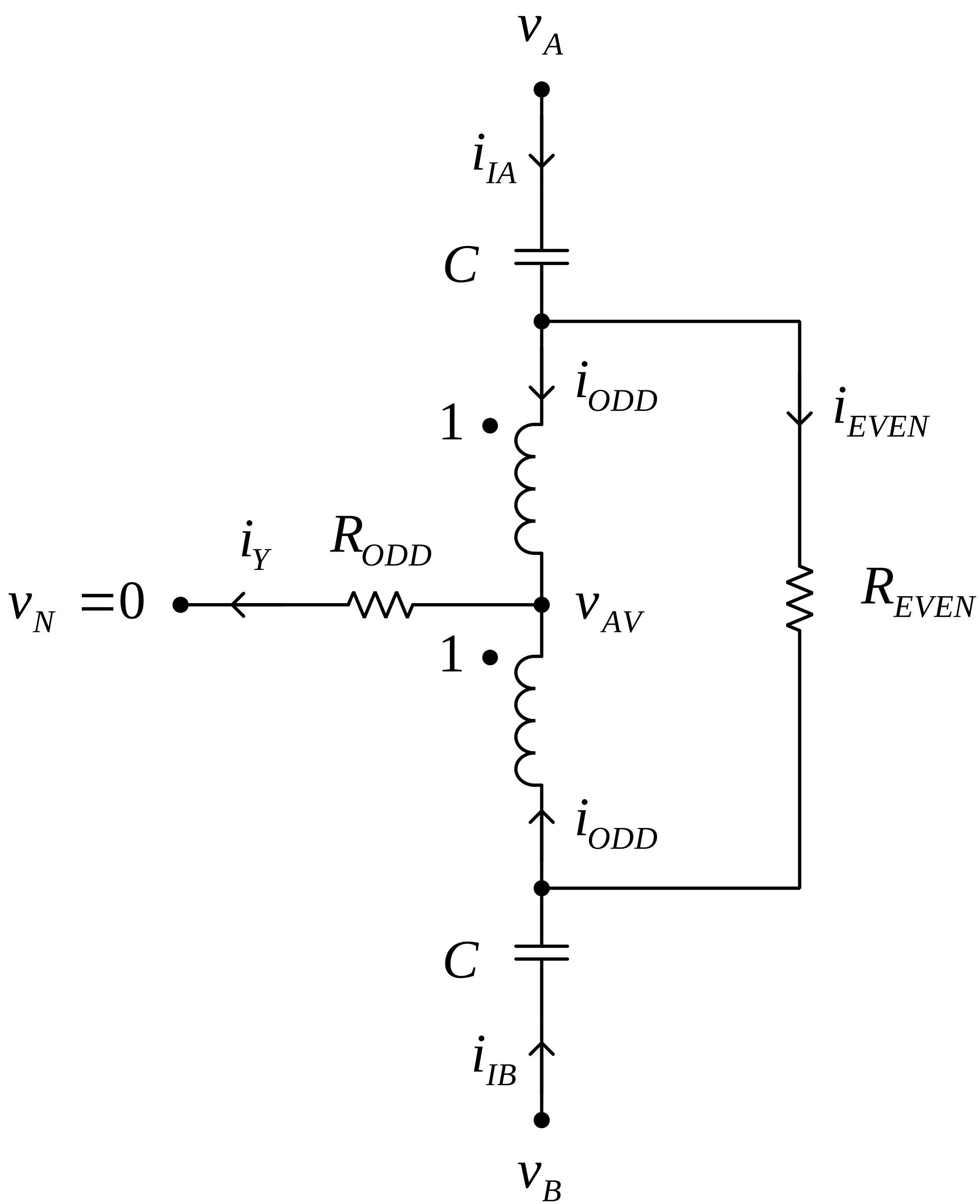
$$P_{INJ} = P_{IN} - P_{OUT} = \left(1 - \left(\frac{3}{\pi}\right)^2\right) P_{IN} = 8.81\% \ P_{IN}$$

$$\eta = \frac{P_{OUT}}{P_{IN}} = \left(\frac{3}{\pi}\right)^2 \approx 91.19\%$$

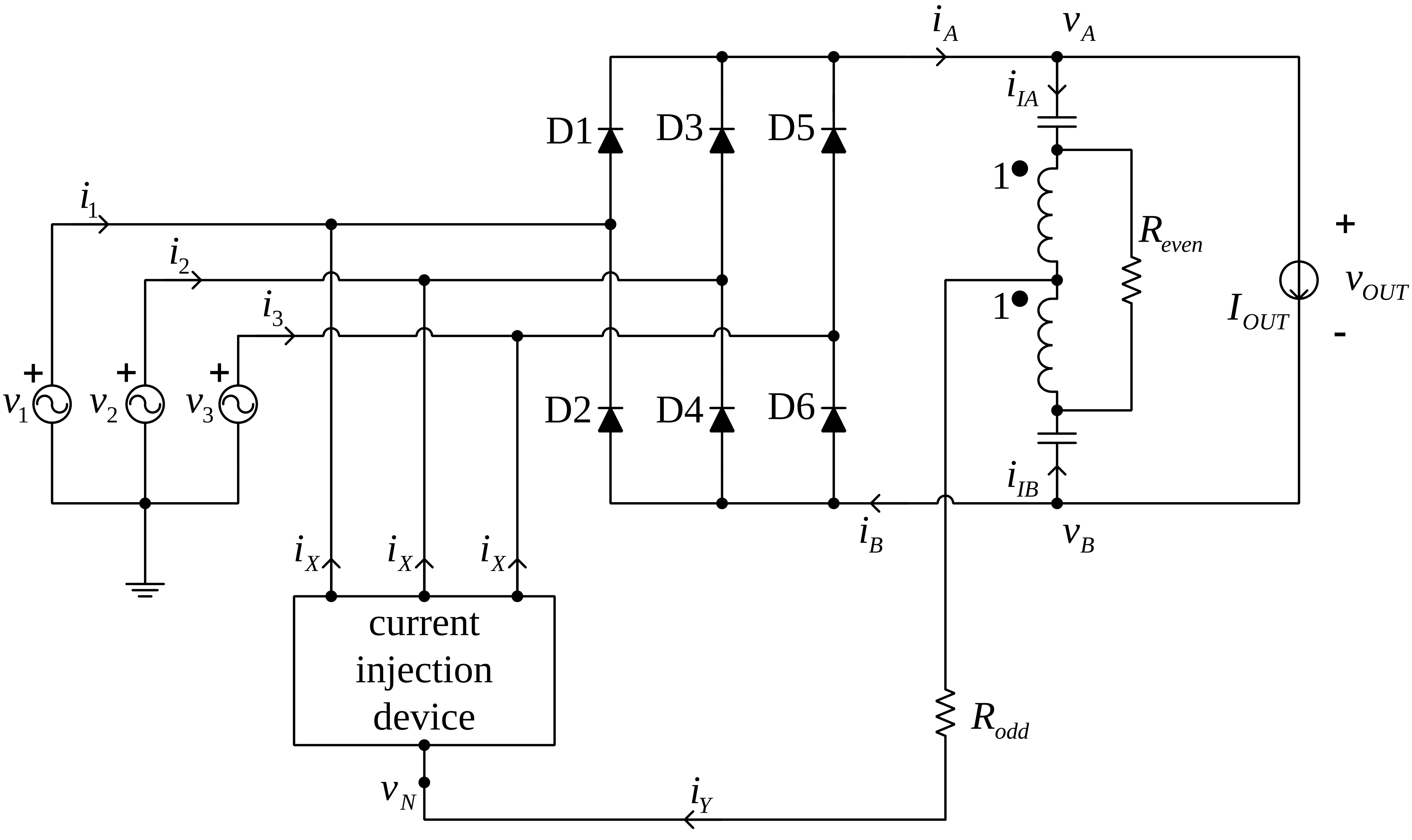
$$\Delta\eta = \frac{32}{35} - \left(\frac{3}{\pi}\right)^2 = 0.24\%$$



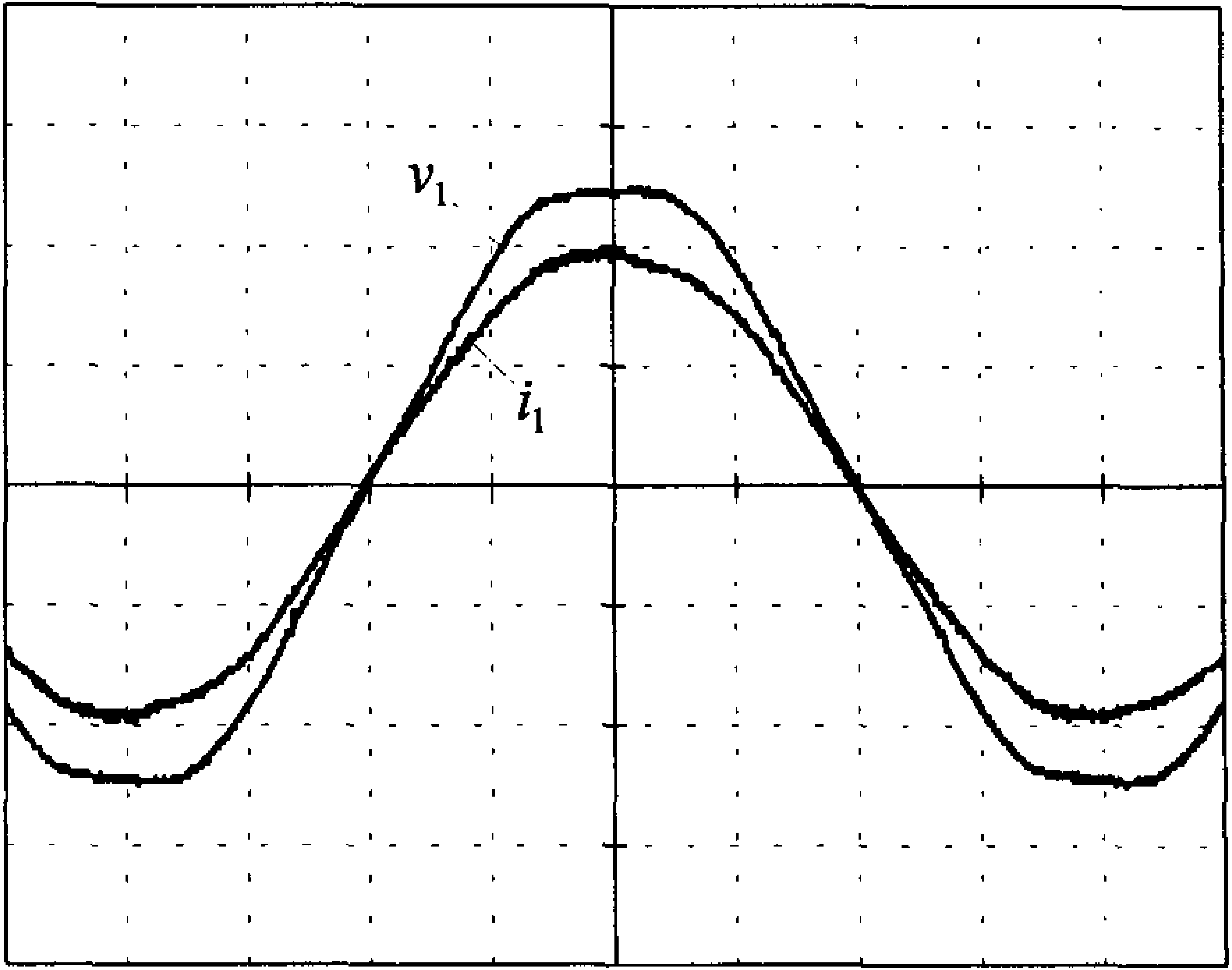
v_A
i_{IA}
C_{ODD}
i_{ODD}
C_{EVEN}
1
i_Y
R_{ODD}
$v_N = 0$
v_{AV}
i_{EVEN}
1
R_{EVEN}
i_{ODD}
C_{ODD}
i_{IB}
v_B



v_A
i_{IA}
C
i_{ODD}
i_{EVEN}
1
i_Y
R_{ODD}
$v_N = 0$
v_{AV}
R_{EVEN}
1
i_{ODD}
C
i_{IB}
v_B



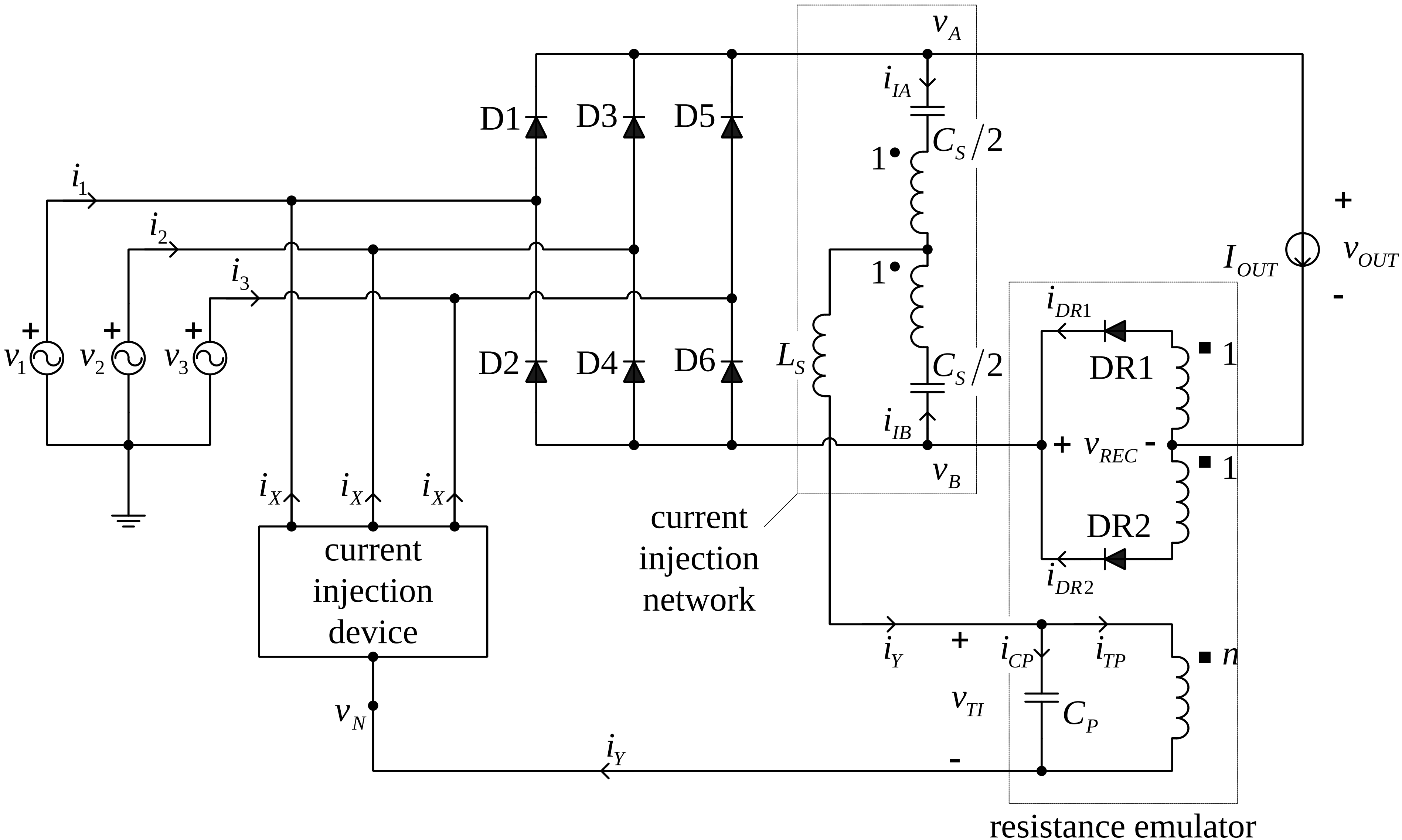
i_A
v_A
i_{IA}
D1
D3
D5
1
R_{even}
+
i_1
i_2
1
v_{OUT}
I_{OUT}
i_3
-
+
+
+
v_1
v_2
v_3
D2
D4
D6
i_{IB}
i_B
v_B
i_X
i_X
i_X
current injection device
R_{odd}
v_N
i_Y

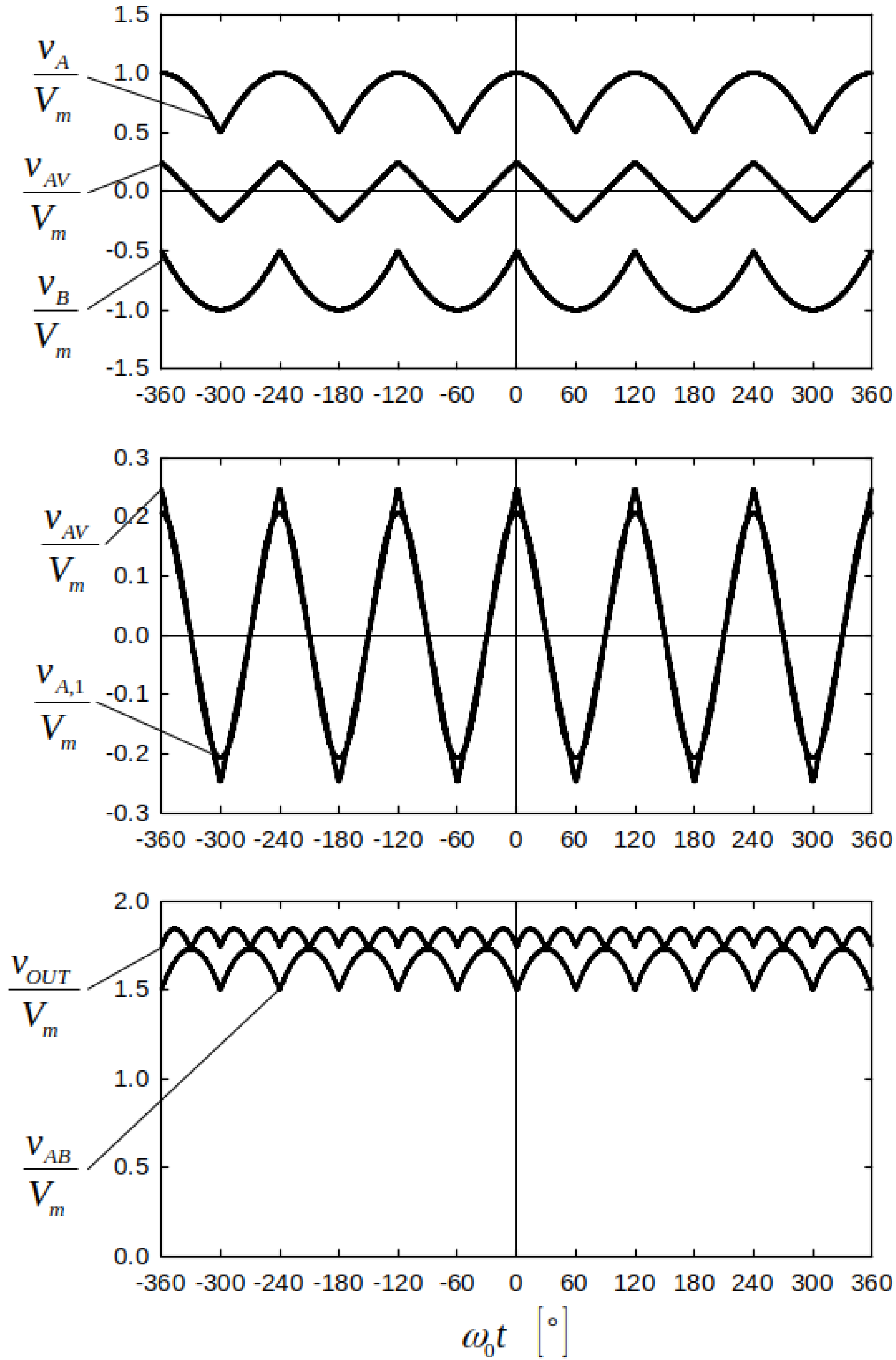


v_1
i_1

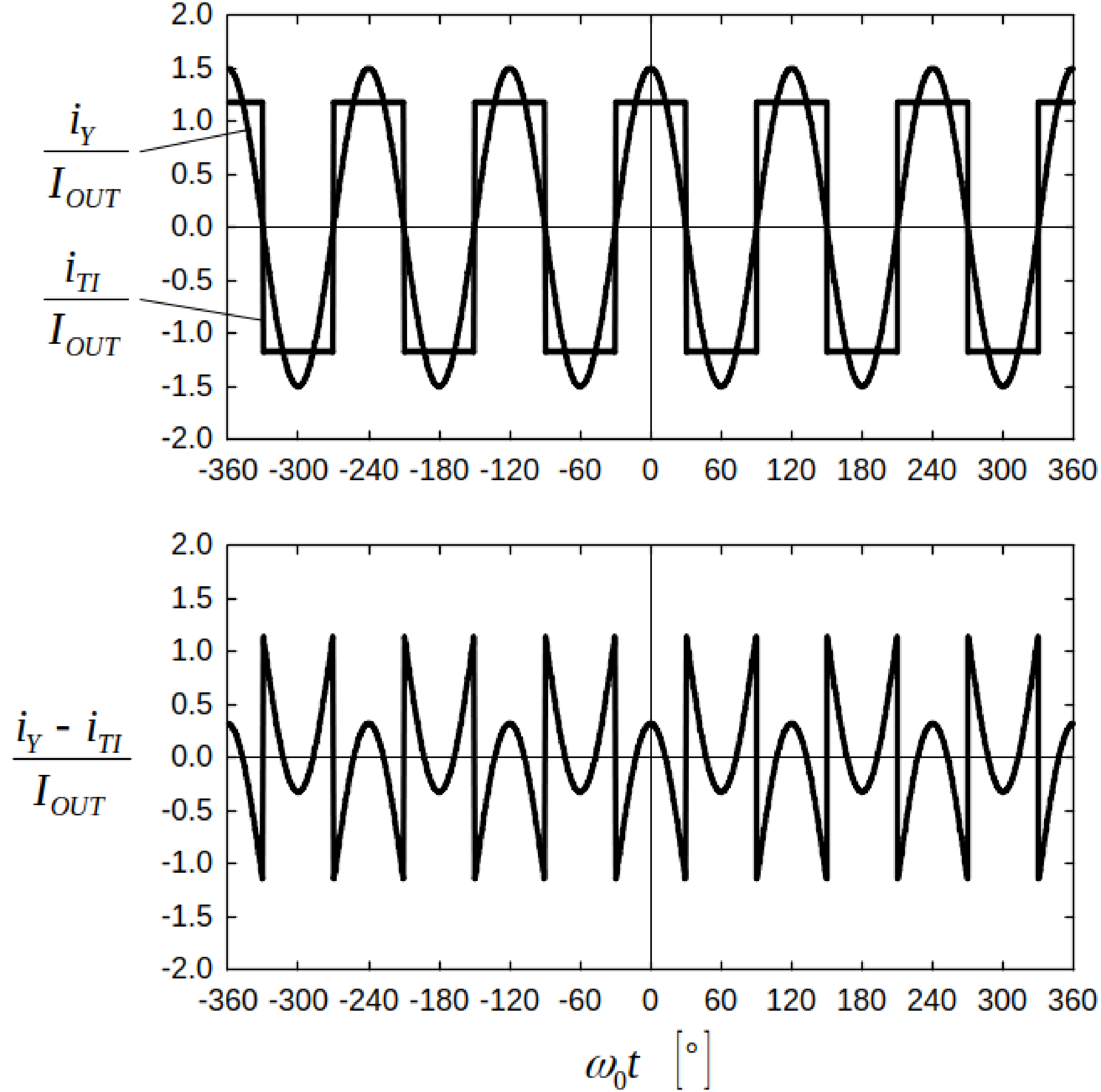
Phase, k	1	2	3
$THD(v_k)$	2.99%	2.66%	3.17%
$THD(i_k)$	2.58%	2.67%	2.57%
PF_k	0.9994	0.9995	0.9991

Recovery of the Current Injection Network Power Applying A Current-Loaded Resistance Emulator





v_A/V_m
v_{AV}/V_m
v_B/V_m
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
v_{AV}/V_m
$v_{A,1}/V_m$
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
v_{OUT}/V_m
v_{AB}/V_m
2.0
1.5
1.0
0.5
0.0
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]



$\frac{i_Y}{I_{OUT}}$
$\frac{i_{TI}}{I_{OUT}}$
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\frac{i_Y - i_{TI}}{I_{OUT}}$
$\omega_0 t$ [°]

$$R_X = \frac{V_m}{I_{OUT\,\max}}$$

$$C_S = \frac{\pi}{3\sqrt{3}} \frac{1}{\omega_0 R_X}$$

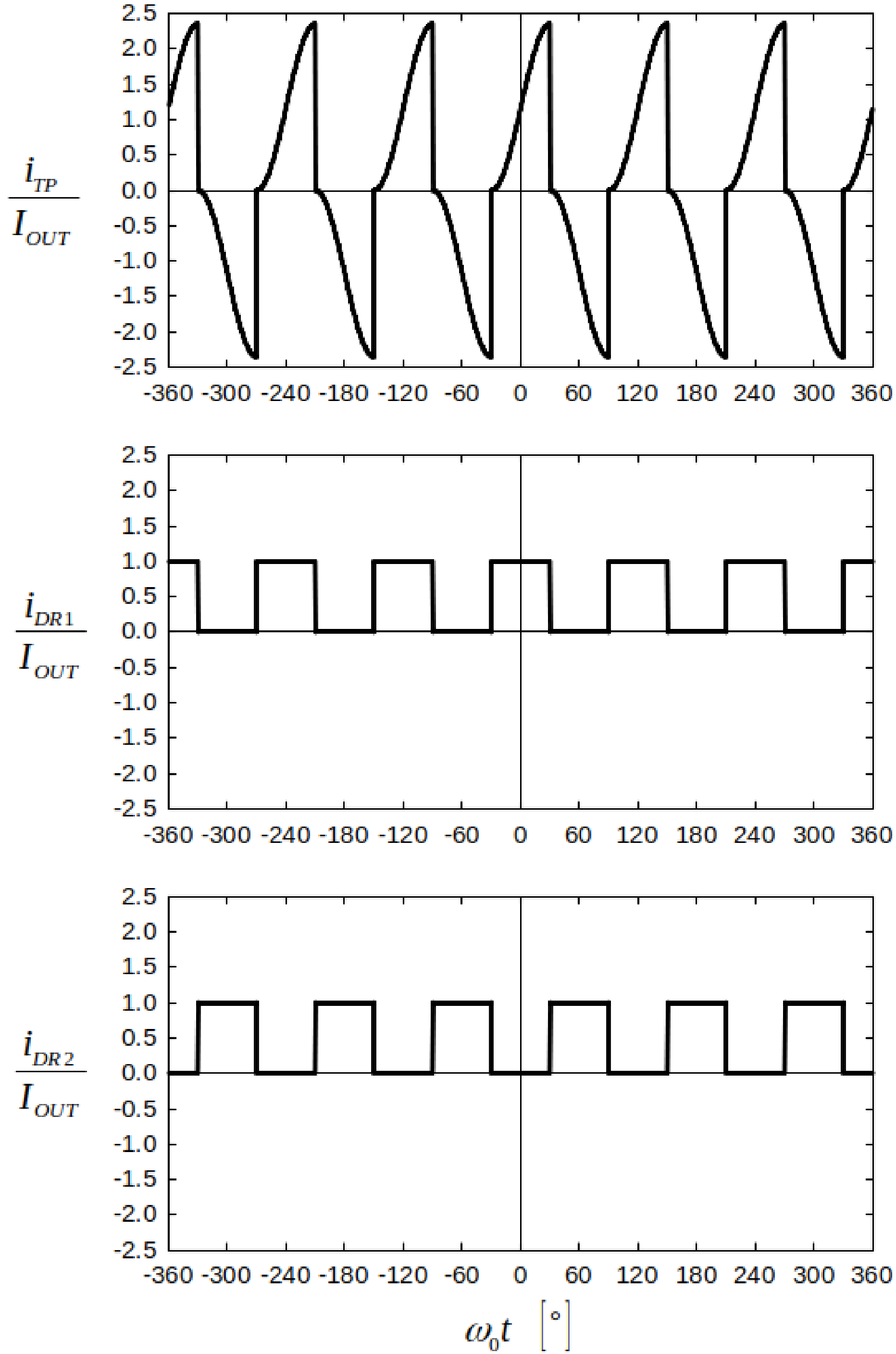
$$L_S = \frac{1}{\pi\sqrt{3}} \frac{R_X}{\omega_0}$$

$$S_L = \frac{2}{35} P_{OUT} \approx 5.71\% \ P_{OUT}$$

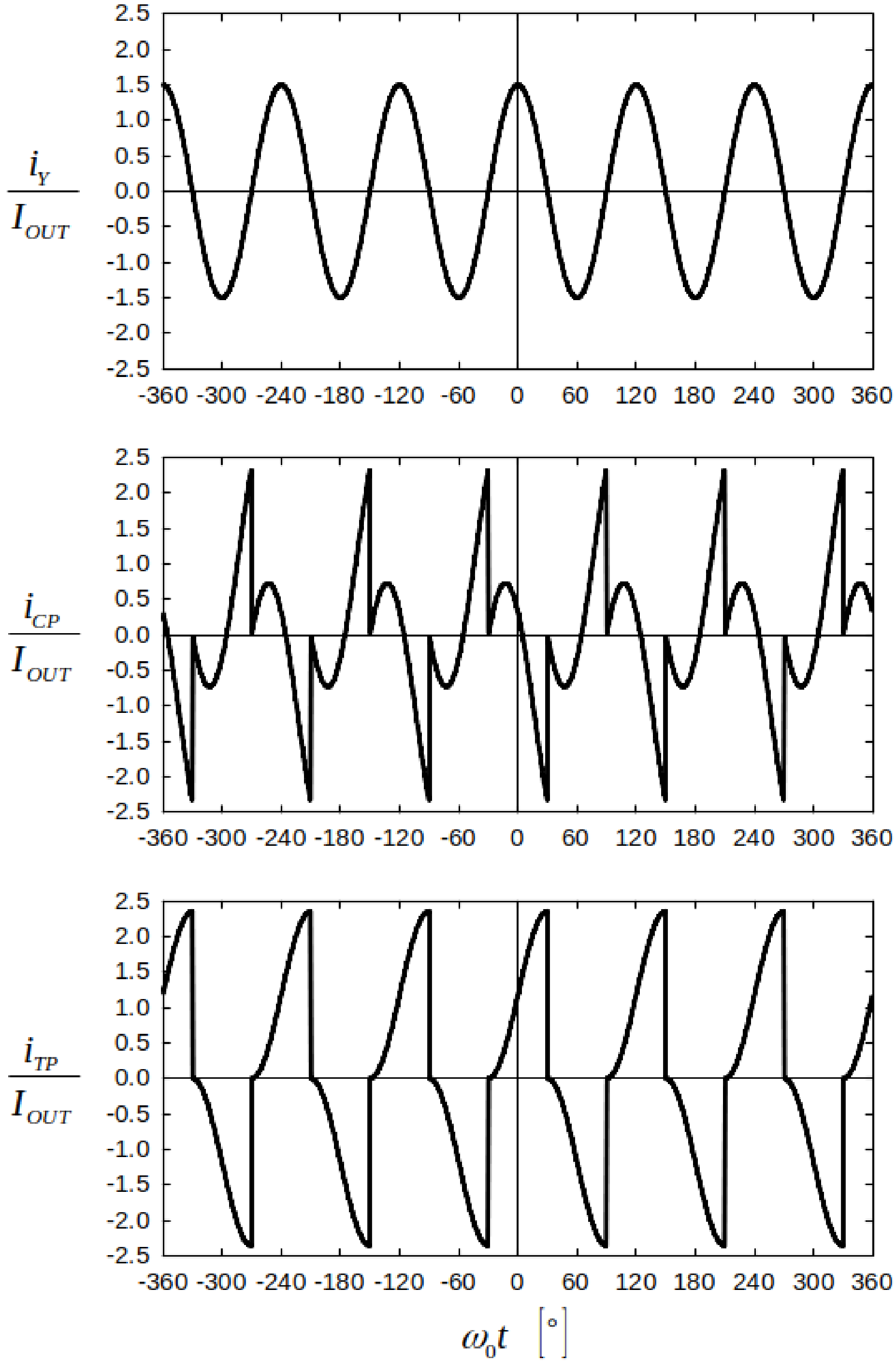
$$n = \frac{8}{3\pi}$$

$$S_{TRE} = \frac{\pi}{140}\left(1 + \frac{\sqrt{3}}{2}\right) P_{OUT} \approx 4.19\% \ P_{OUT}$$

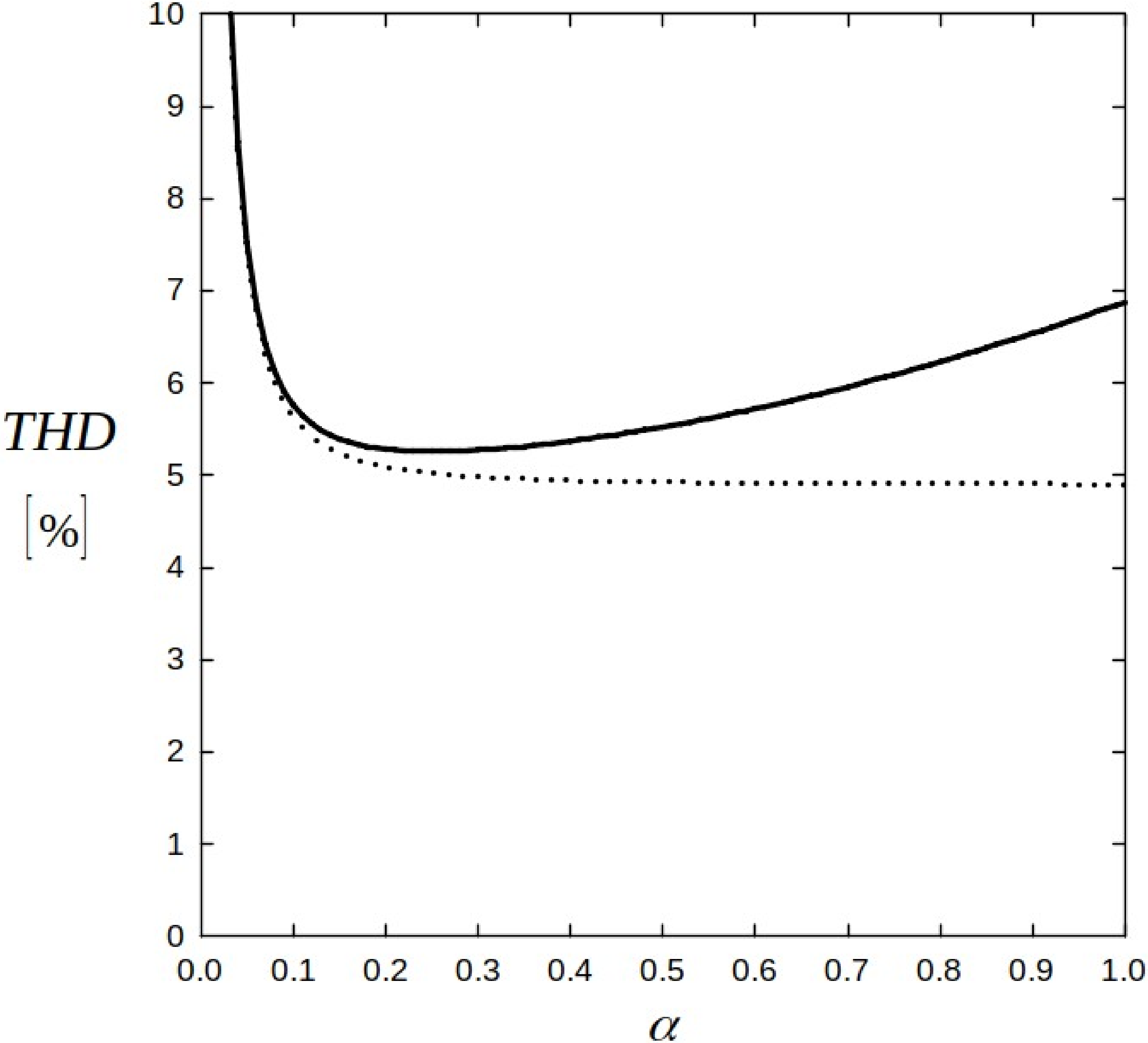
$$S_{T\,1:1} = \frac{2}{35}\left(\sqrt{\pi^2 - 9} - 3\arccos\frac{3}{\pi}\right) P_{OUT} = 0.16\% \ P_{OUT}$$



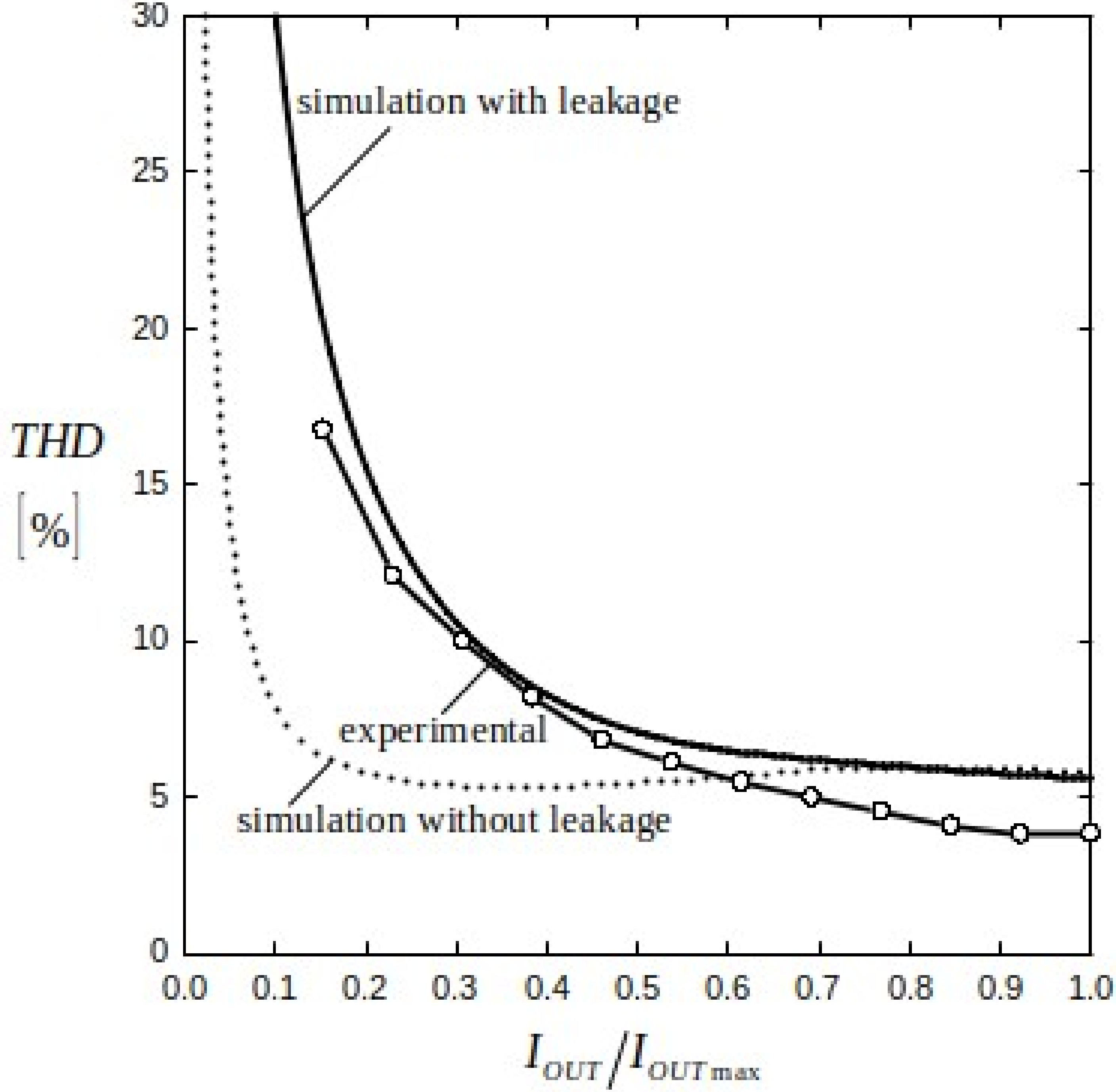
$\frac{i_{TP}}{I_{OUT}}$
$\frac{i_{DR1}}{I_{OUT}}$
$\frac{i_{DR2}}{I_{OUT}}$
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]



$\frac{i_Y}{I_{OUT}}$
$\frac{i_{CP}}{I_{OUT}}$
$\frac{i_{TP}}{I_{OUT}}$
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]

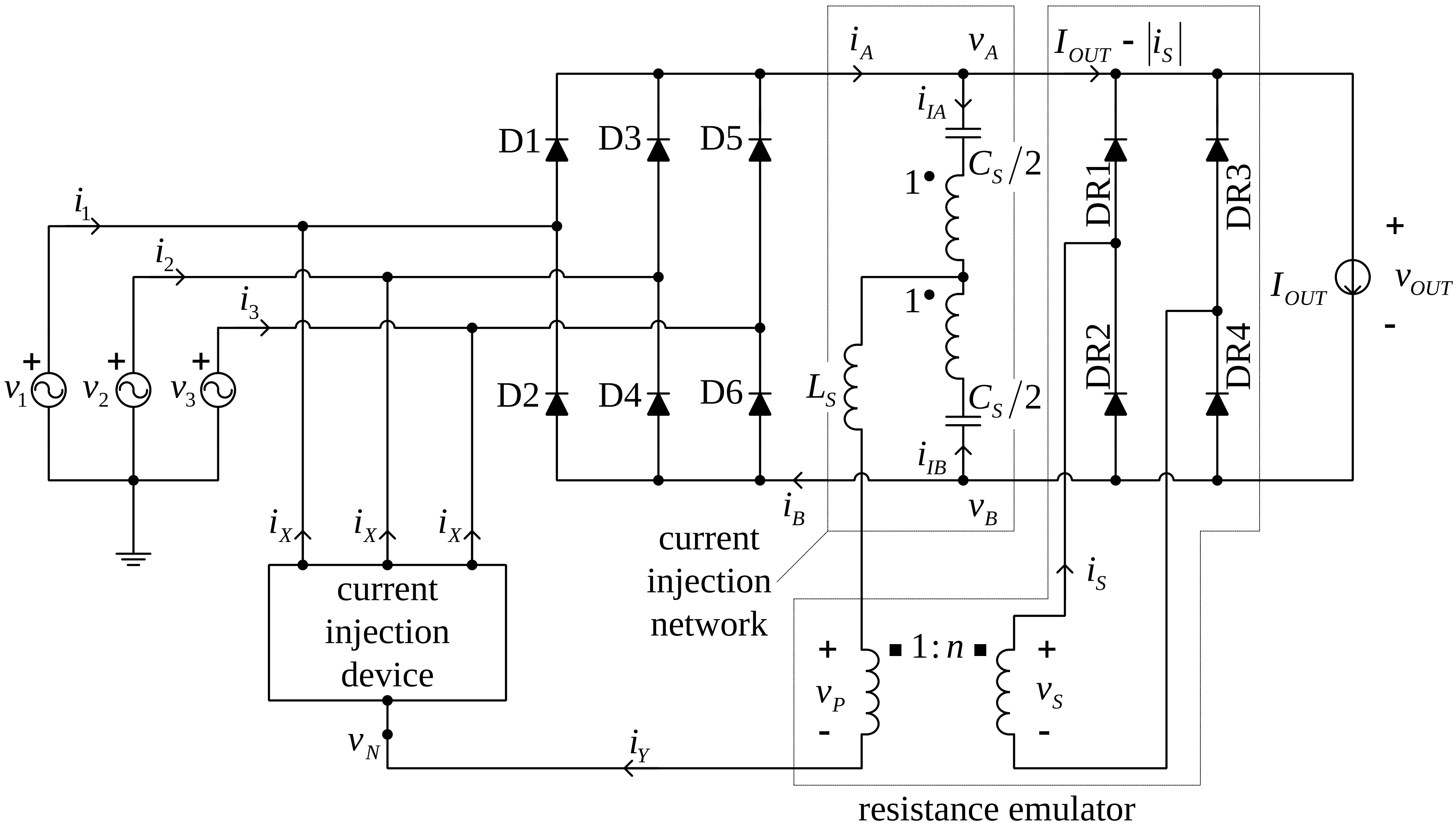


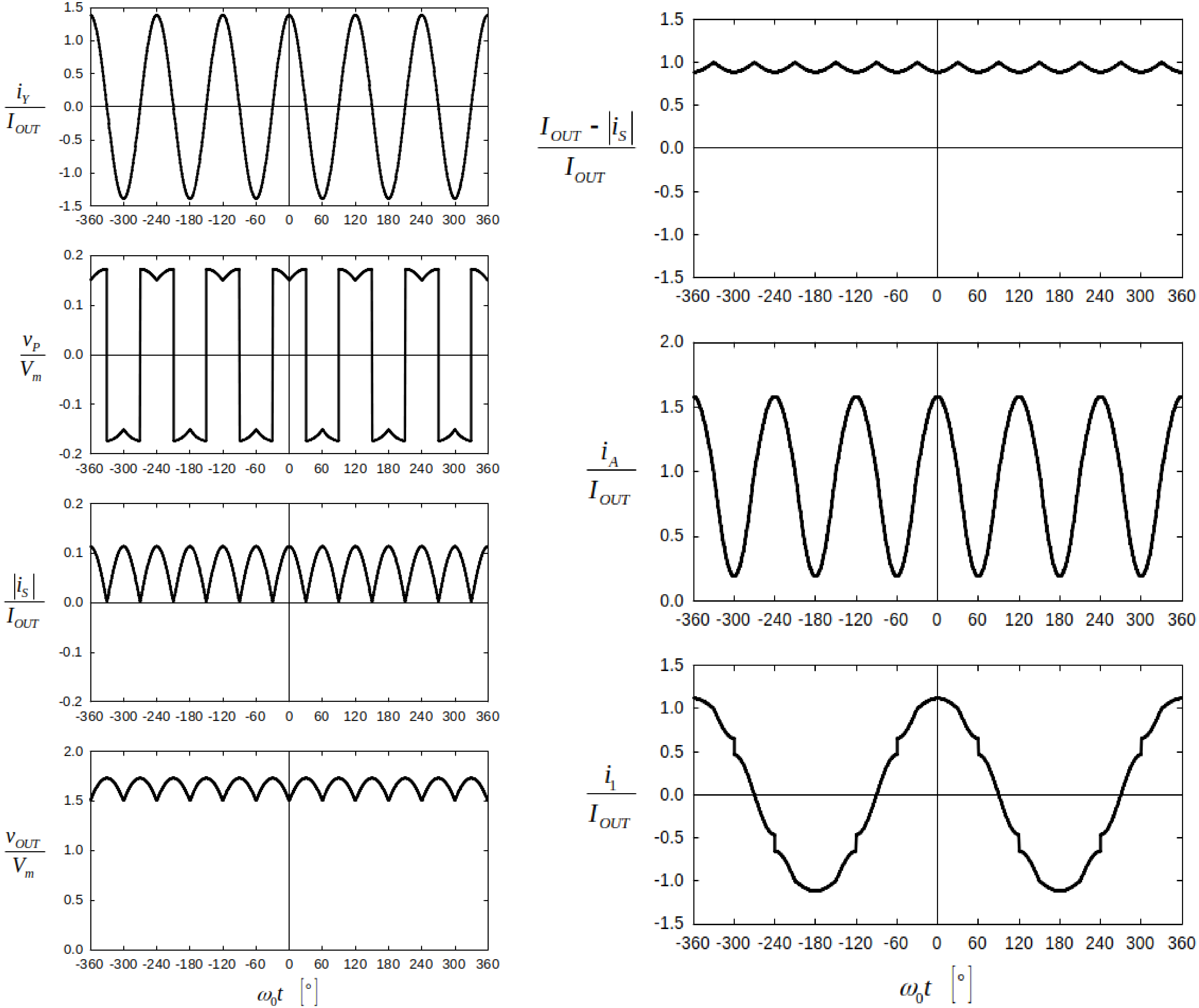
$$\alpha = \frac{I_{OUT}}{I_{OUT\ \max}}$$



THD
[%]
simulation with leakage
experimental
simulation without leakage
0
5
10
15
20
25
30
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$I_{OUT}/I_{OUT\max}$

Recovery of the Current Injection Network Power Applying A Voltage-Loaded Resistance Emulator





$\frac{i_Y}{I_{OUT}}$
$\frac{v_P}{V_m}$
$\frac{|i_S|}{I_{OUT}}$
$\frac{v_{OUT}}{V_m}$
$\frac{I_{OUT} - |i_S|}{I_{OUT}}$
$\frac{i_A}{I_{OUT}}$
$\frac{i_1}{I_{OUT}}$
-360 -300 -240 -180 -120 -60 0 60 120 180 240 300 360
$\omega_0 t$ [°]

Analysis . . .

$$i_Y = kI_{OUT}\cos(3\omega_0 t)$$

$$THD(k,n) = \frac{\sqrt{3}\sqrt{k^2\left(n^2\left(32\pi^2 - 27\right) + 540n + 12\left(32\pi^2 - 225\right) - 96kn\left(9n + 2\left(16\pi - 45\right)\right) + 768n^2\left(\pi^2 - 9\right)\right)}}{9\left(k\left(n - 10\right) + 16n\right)}$$

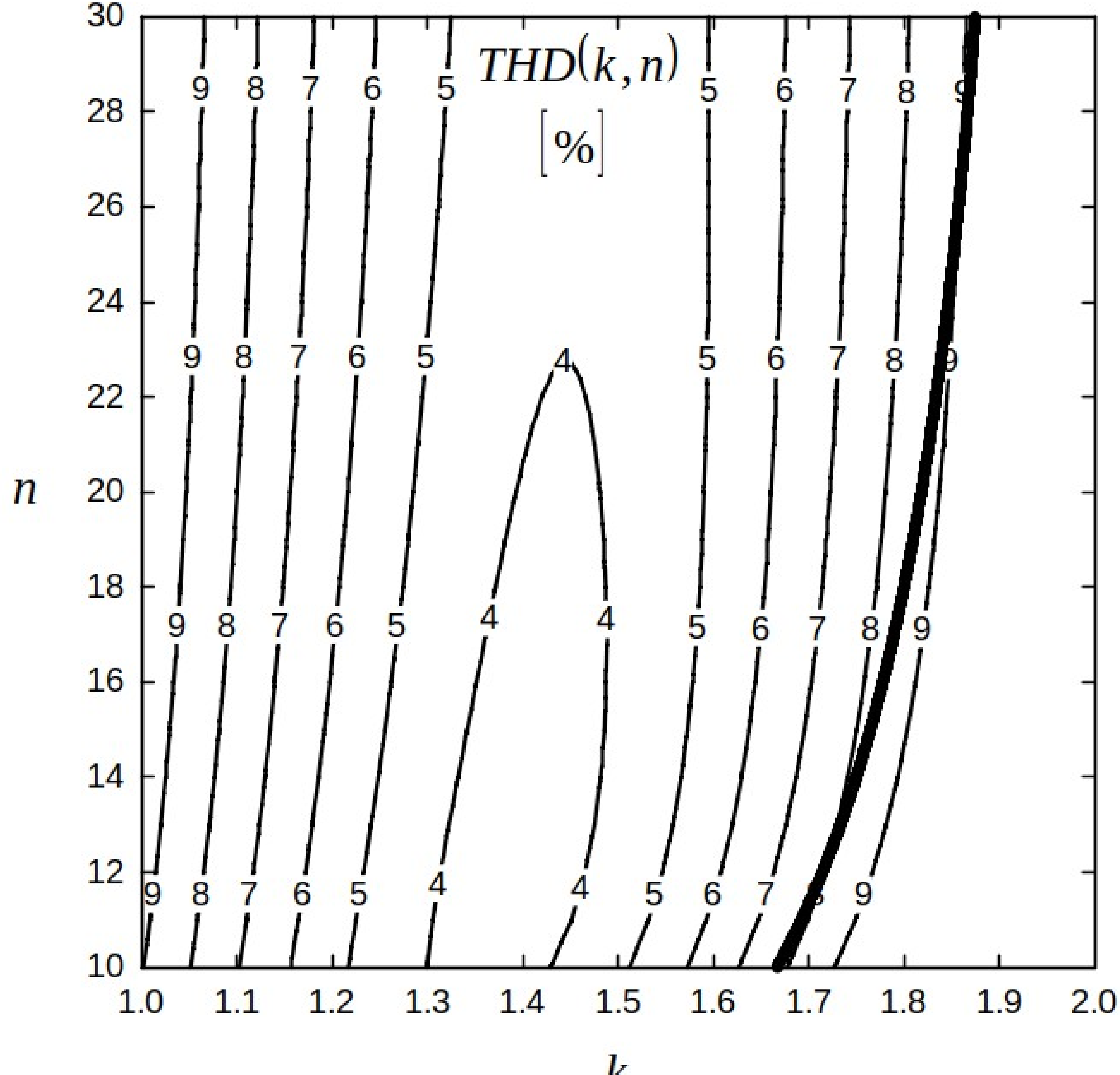
$$kI_{OUT} = \frac{3\sqrt{3}}{8\pi}\left(1 - \frac{10}{n}\right)\frac{V_m}{R}$$

$$n = 10$$

$$\frac{d\,THD(k,10)}{d\,k} = 0$$

$$k = \frac{30}{7\pi} \approx 1.36$$

$$THD = \frac{1}{3}\sqrt{\pi^2 - \frac{69}{7}} \approx 3.72\%$$



THD(k, n) [%]
n
k
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
10
12
14
16
18
20
22
24
26
28
30

$$\frac{\partial\, THD\,(k,n)}{\partial\, k}=0$$

$$k_{opt}(n)=\frac{3n\,(\pi n-2(5\pi-16))}{2\,(\pi n^2+3n+6(2\pi-5))}$$

$$\frac{d\, THD\,(k_{opt}(n),n)}{d\, n}=0$$

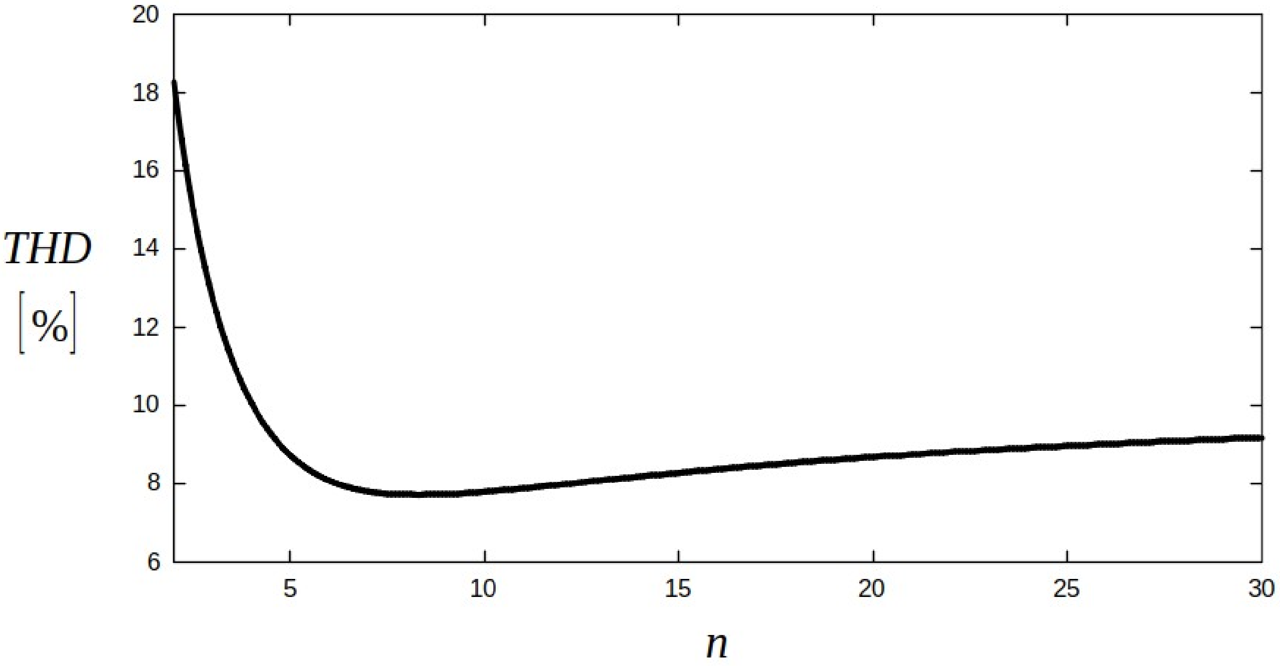
$$k_{OPT}=\frac{3(\pi^2-8)}{\pi(2\pi-5)}\approx 1.39$$

$$n_{OPT}=\frac{6(\pi^2-8)}{\pi(16-5\pi)}\approx 12.23$$

$$THD_{MIN}=THD\,(k_{OPT},n_{OPT})=\frac{1}{3}\sqrt{\frac{8\pi^4-199\pi^2+360\pi+54}{15\pi^2-40\pi-6}}\approx 3.64\,\%$$

$$R_{OPT}=\frac{\pi(2\pi-5)(7\pi^2-20\pi-6)}{18(\pi^2-8)^2}\,\frac{V_{OUT}}{I_{OUT}}\approx 1.64\,\%\;\frac{V_{OUT}}{I_{OUT}}$$

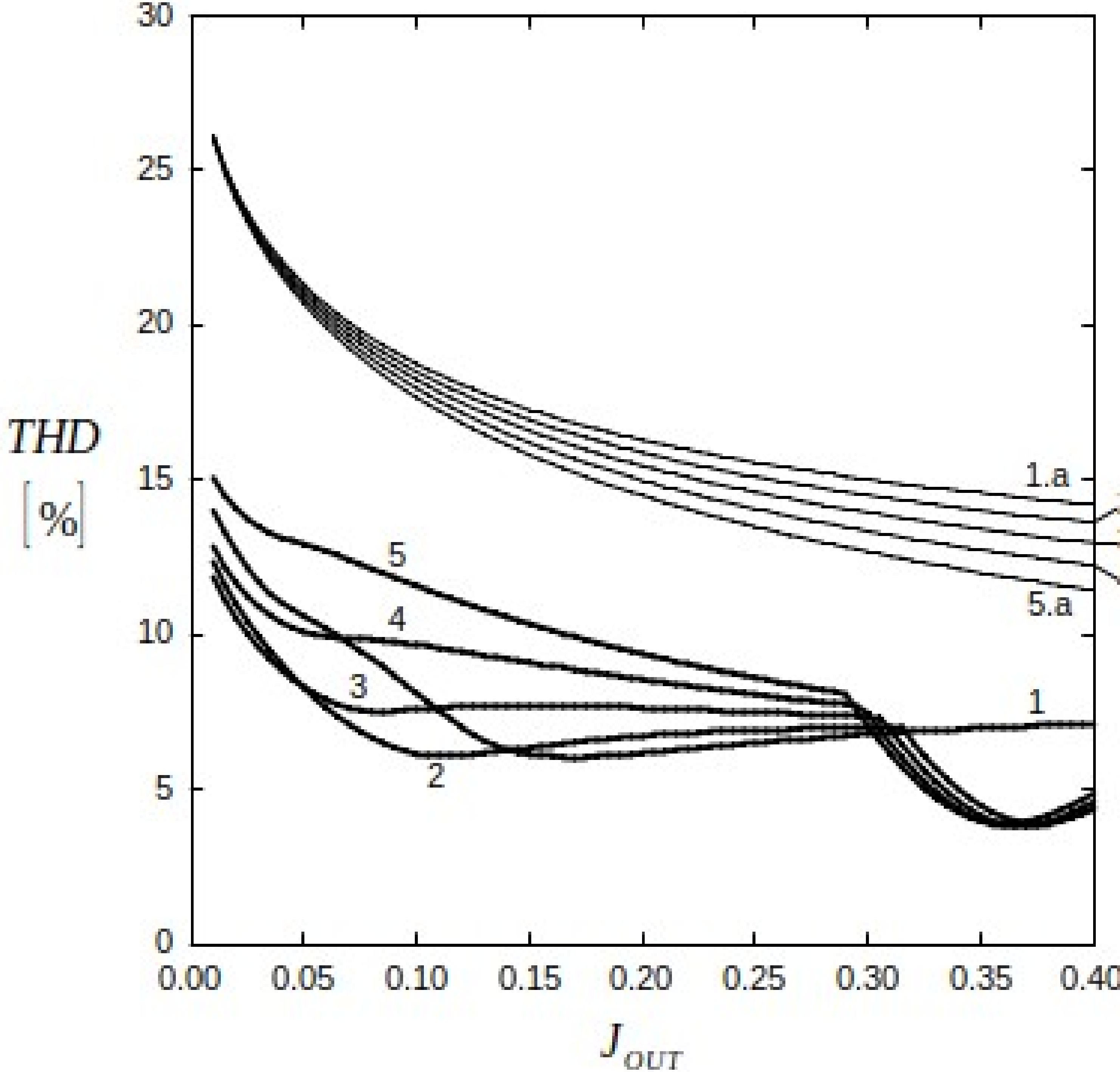
$$P_R=\frac{7\pi^2-20\pi-6}{4\pi(2\pi-5)}\,P_{OUT}\approx 1.58\,\%\;P_{OUT}$$



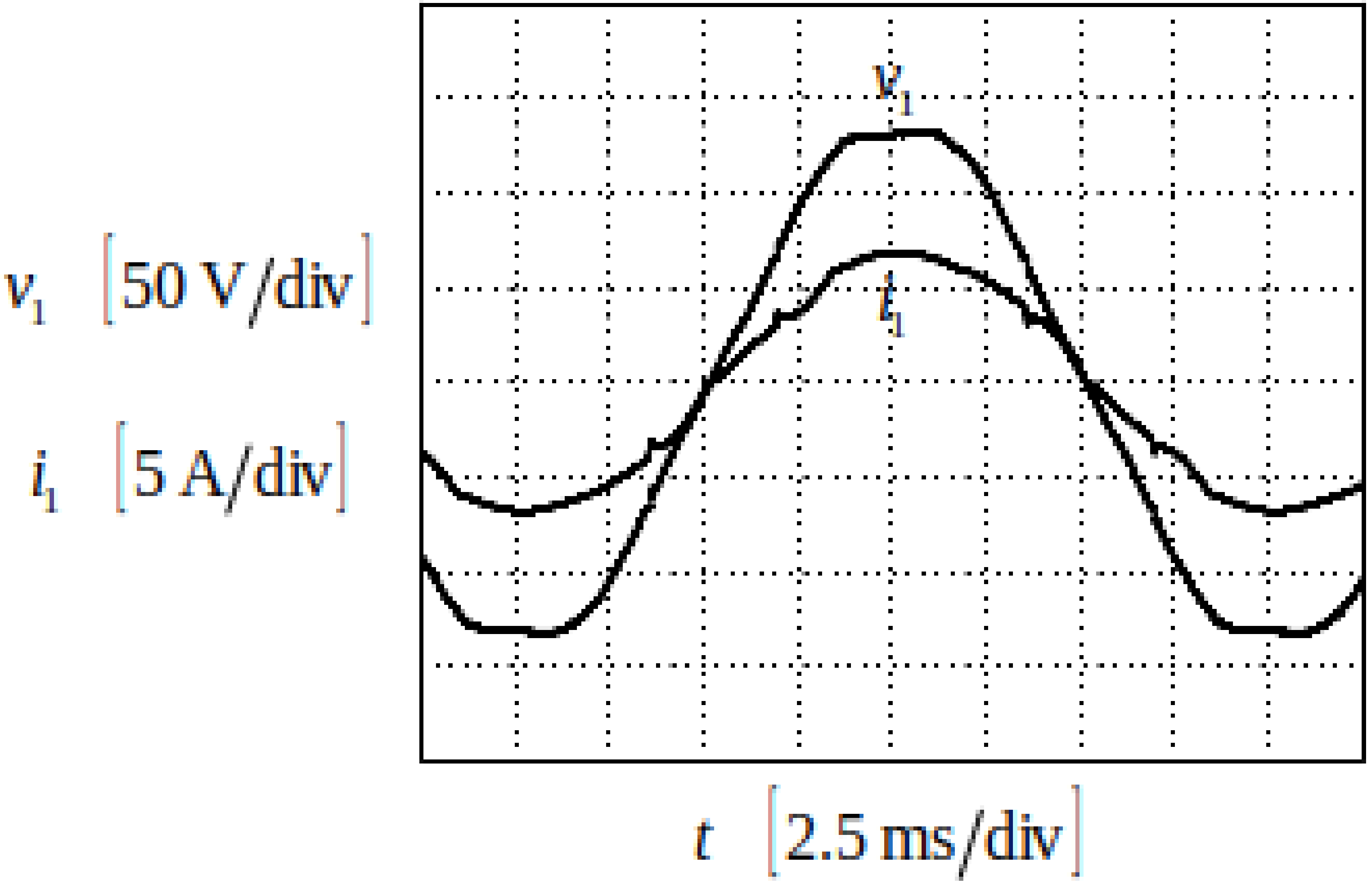
$$THD_{DCM}\left(n_{DCM\ OPT}\right)=\frac{\sqrt{936\pi^4-3424\pi^3-16377\pi^2+89640\pi-104976}}{9\left(13\pi-36\right)}\approx 7.71\%$$

$$THD_{DCM}\left(10\right)=\frac{1}{9}\sqrt{\frac{61\pi^2-36\pi-486}{6}}\approx 7.79\%$$

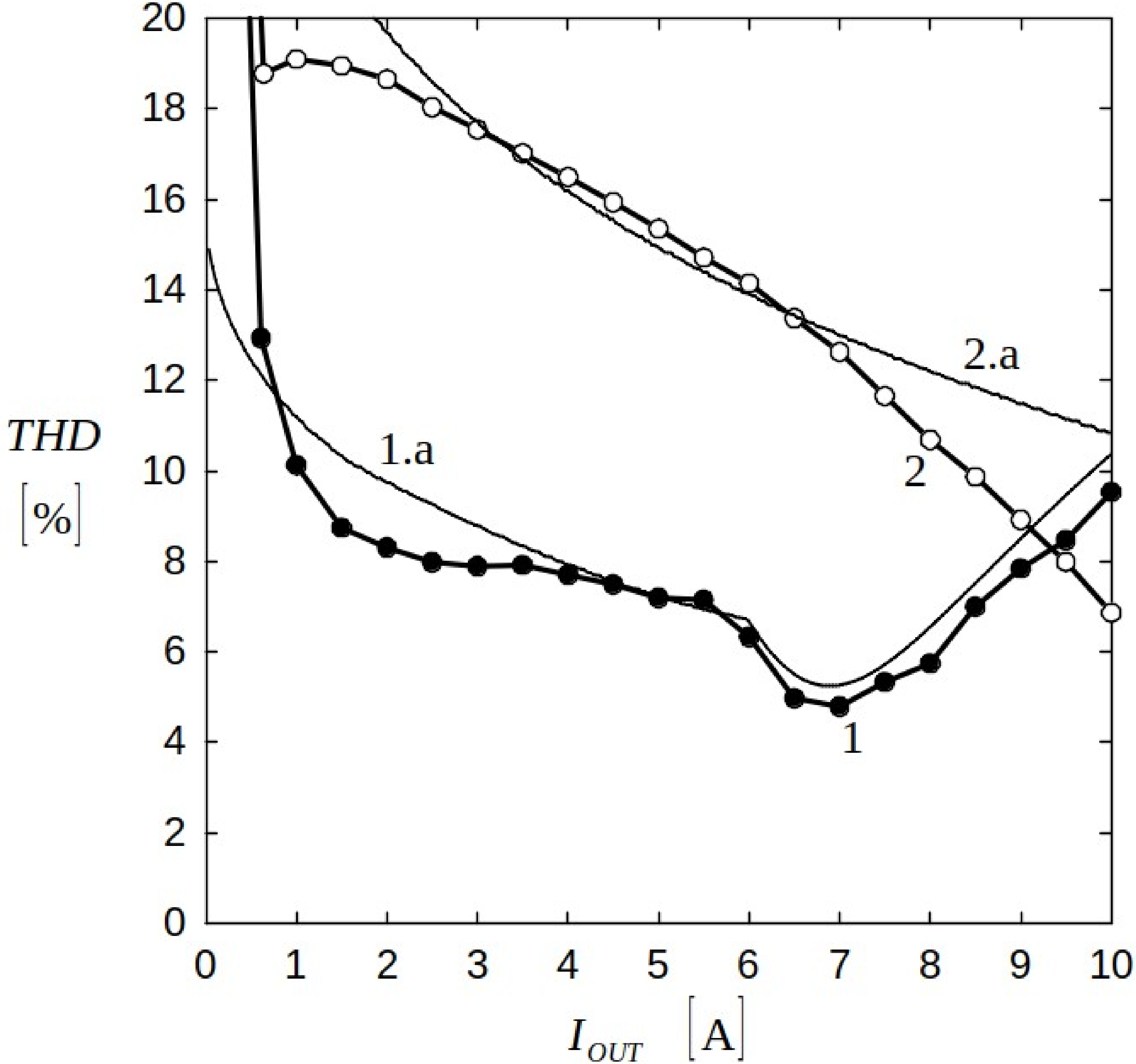
$$\lim_{n\to\infty}\left(THD_{DCM}\left(n\right)\right)=\frac{1}{27}\sqrt{\frac{224\pi^2-2187}{3}}\approx 10.43\%$$



THD
[%]
J_OUT
30
25
20
15
10
5
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
1.a
2.a
3.a
4.a
5.a
1
2
3
4
5

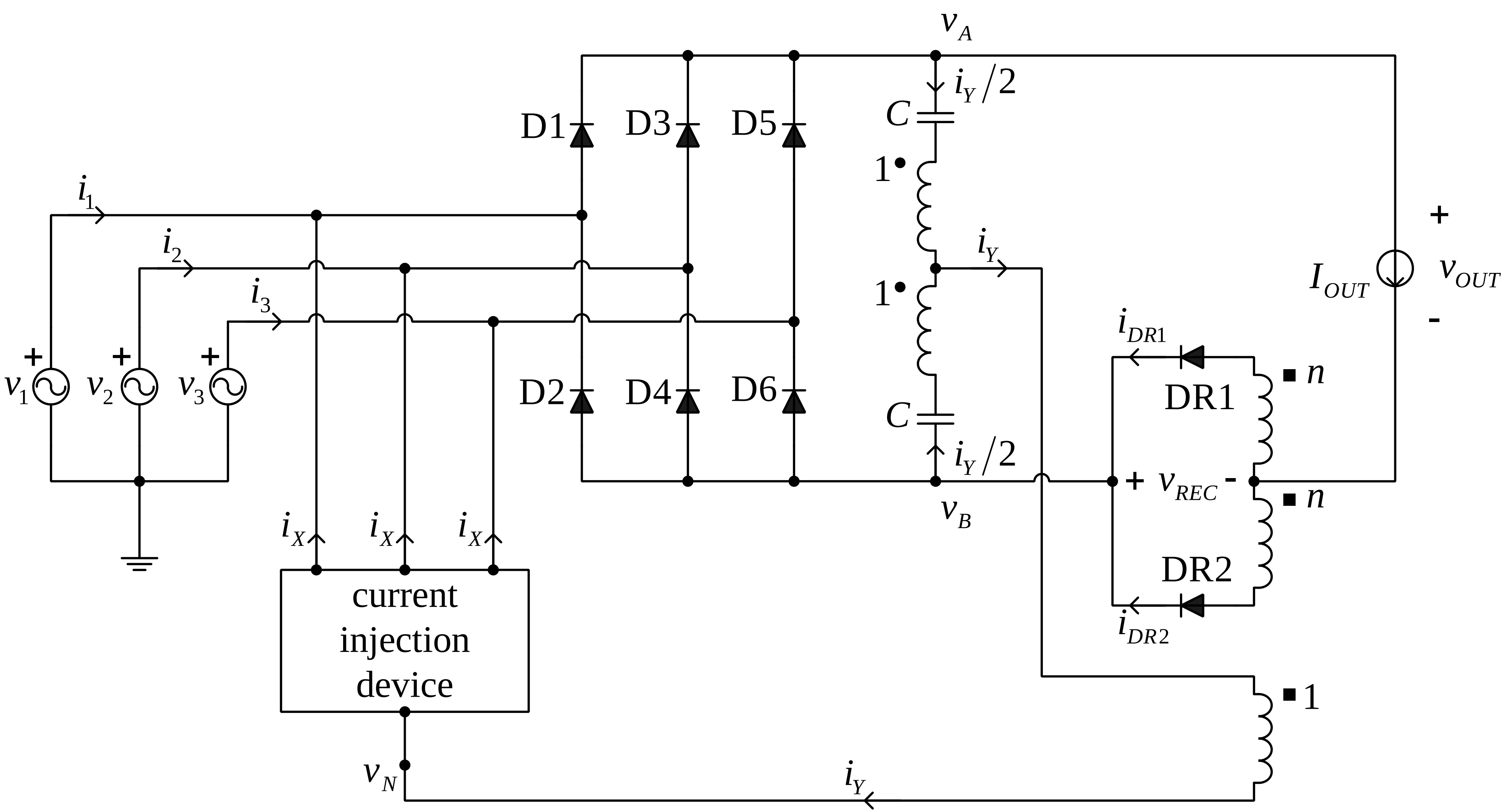


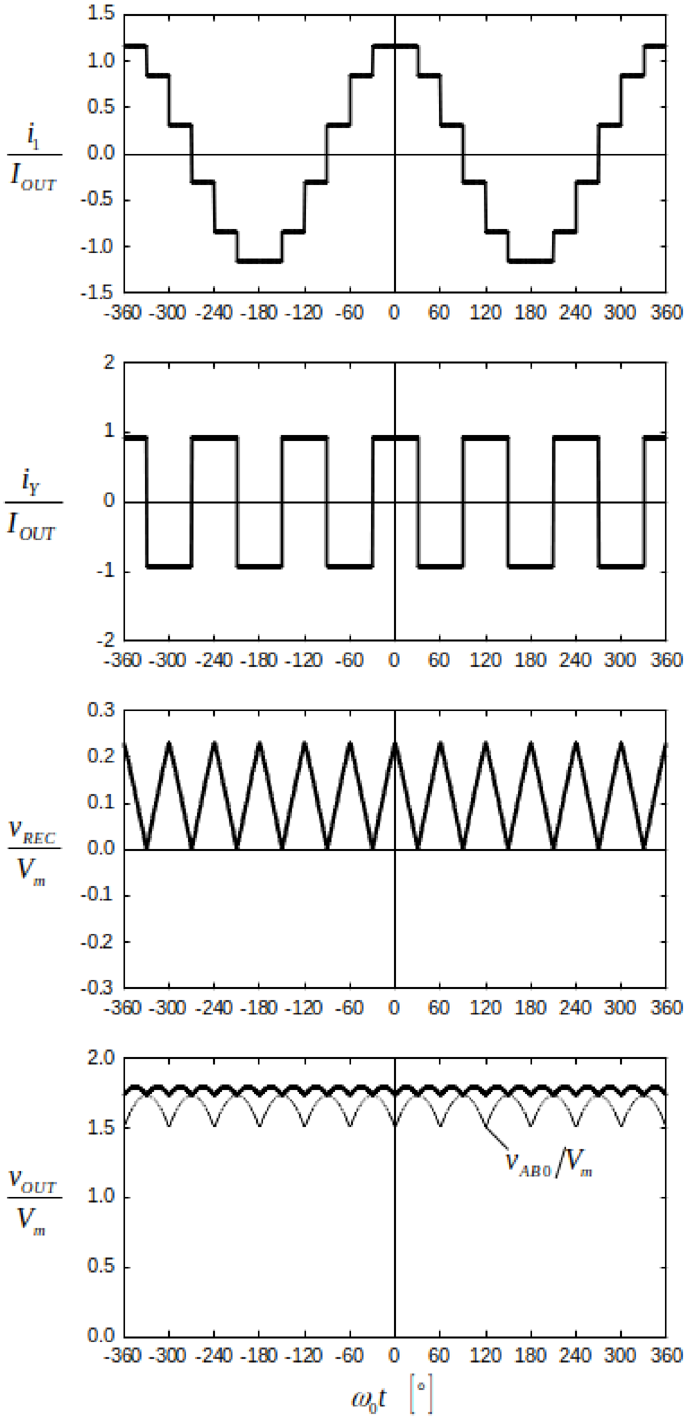
v_1 [50 V/div]
i_1 [5 A/div]
v_1
i_1
t [2.5 ms/div]



20
18
16
14
12
10
8
6
4
2
0
THD
[%]
0
1
2
3
4
5
6
7
8
9
10
I_{OUT} [A]
1.a
2.a
1
2

Square-Wave Current Injection Rectifier that Applies Current-Loaded Resistance Emulator





$$n_{OPT} = 4\sqrt{3} - 6 \approx 0.9282$$

$$THD_{min} = THD\left(n_{OPT}\right) = \frac{\sqrt{\left(2+\sqrt{3}\right)\pi^2 - 36}}{6} \approx 15.22\,\%$$

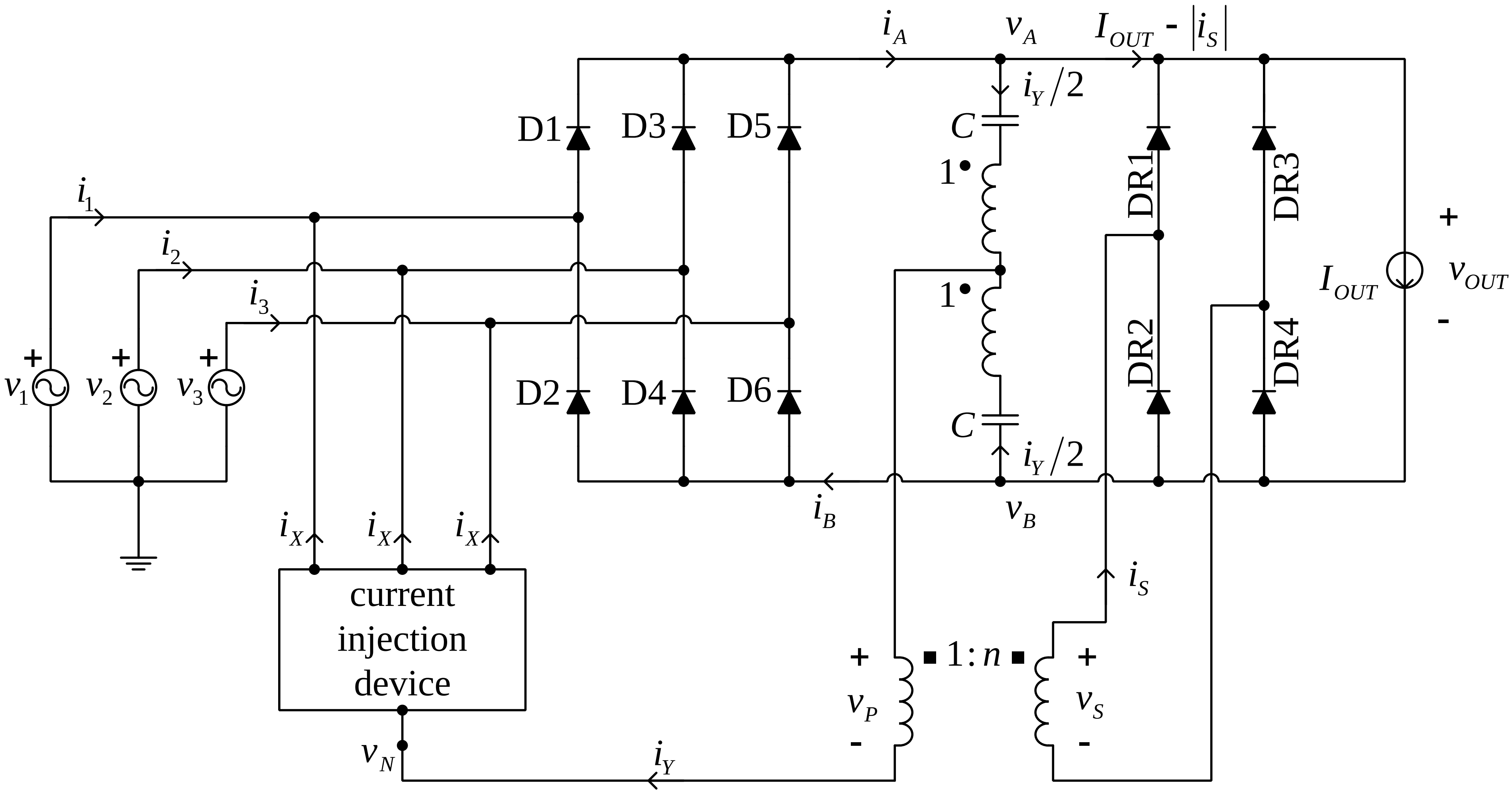
$$v_{OUT} = V_m \frac{24\sqrt{3} - 36}{\pi}\left(1 - 2\sum_{k=1}^{+\infty} \frac{1}{144k^2 - 1}\cos\left(12k\omega_0 t\right)\right)$$

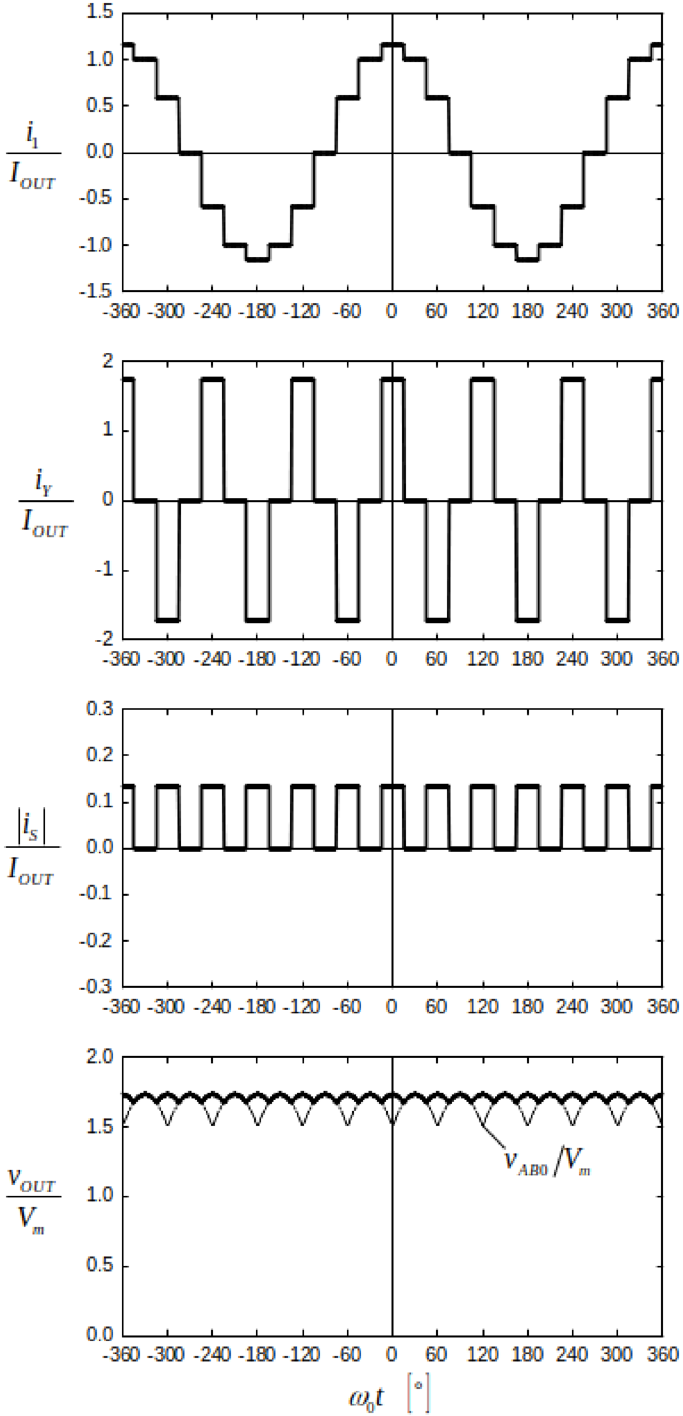
$$\frac{P_{INJ}}{P_{IN}} = \frac{2 - \sqrt{3}}{4} \approx 6.70\,\%$$

$$S_{T\ 1:1} = \frac{\sqrt{6}}{48}\left(\sqrt{\pi^2 - 9} - 3\arccos\left(\frac{3}{\pi}\right)\right)P_{OUT} \approx 0.145\,\%\ P_{OUT}$$

$$S_{T\ n:n:1} = \frac{\left(2 - \sqrt{3}\right)\left(1 + \sqrt{2}\right)\pi}{48\sqrt{2}} P_{OUT} \approx 2.99\,\%\ P_{OUT}$$

Square-Wave Current Injection Rectifier that Applies Voltage-Loaded Resistance Emulator





$$n_{OPT}=6+4\sqrt{3}\approx 12.93$$

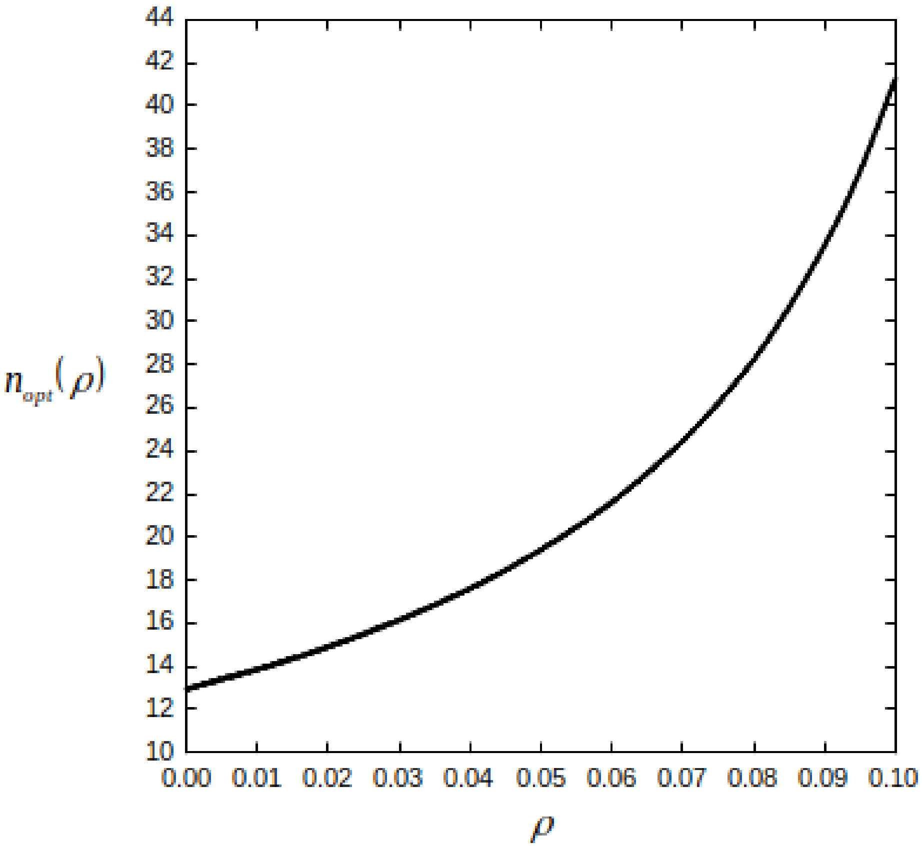
$$THD_{min}=THD\left(n_{OPT}\right)=\frac{\sqrt{\left(2+\sqrt{3}\right)\pi^2-36}}{6}\approx 15.22\%$$

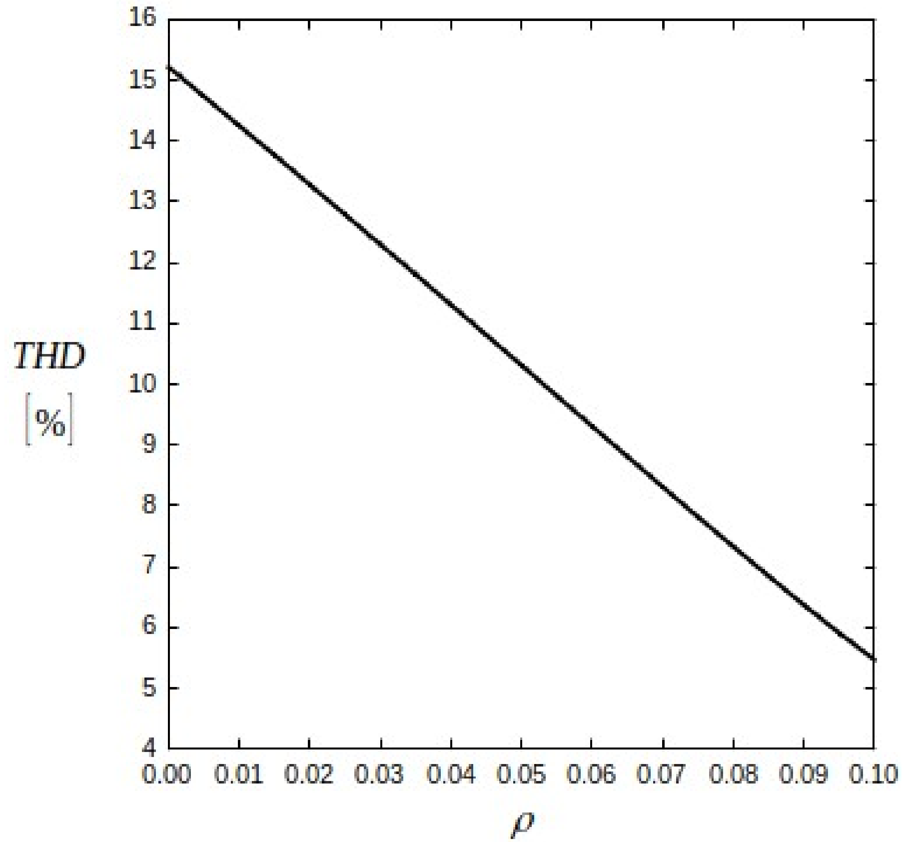
$$S_{T\,1:1}=\left(\frac{\sqrt{3}}{8}\arcsin\frac{3\sqrt{2}\left(\sqrt{3}-1\right)}{\pi}+\frac{3+\sqrt{3}}{48\sqrt{2}}\sqrt{\pi^2+36\sqrt{3}-72}-\frac{2\pi\sqrt{3}}{32}\right)P_{OUT}$$

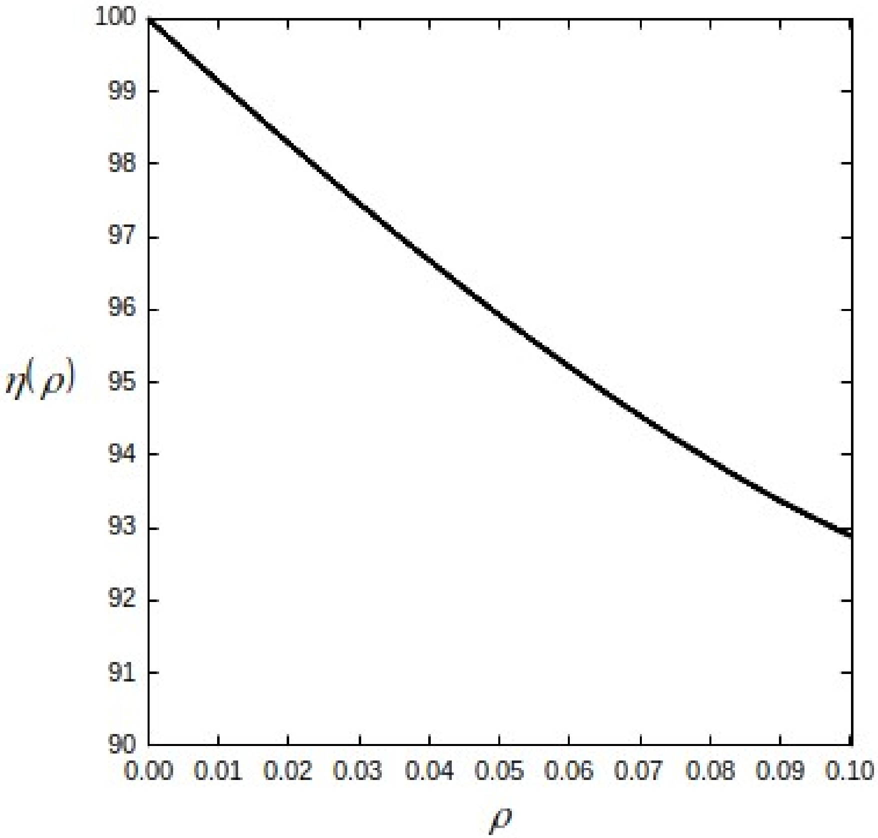
$$S_{T\,1:1}\approx 0.025\%\ P_{OUT}$$

$$S_{T\,1:n}=\frac{\pi\sqrt{2}}{48}\left(1+\sqrt{3}-\sqrt{6}\right)P_{OUT}\approx 2.62\%\ P_{OUT}$$

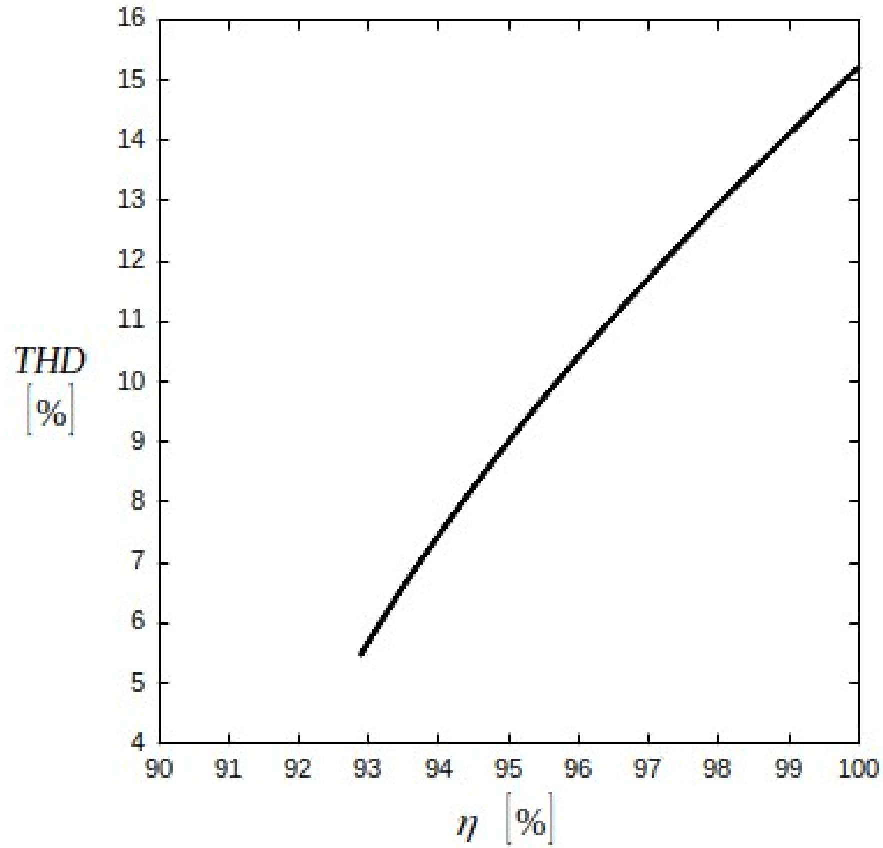
$$\rho = \frac{RI_{OUT}}{V_m}$$



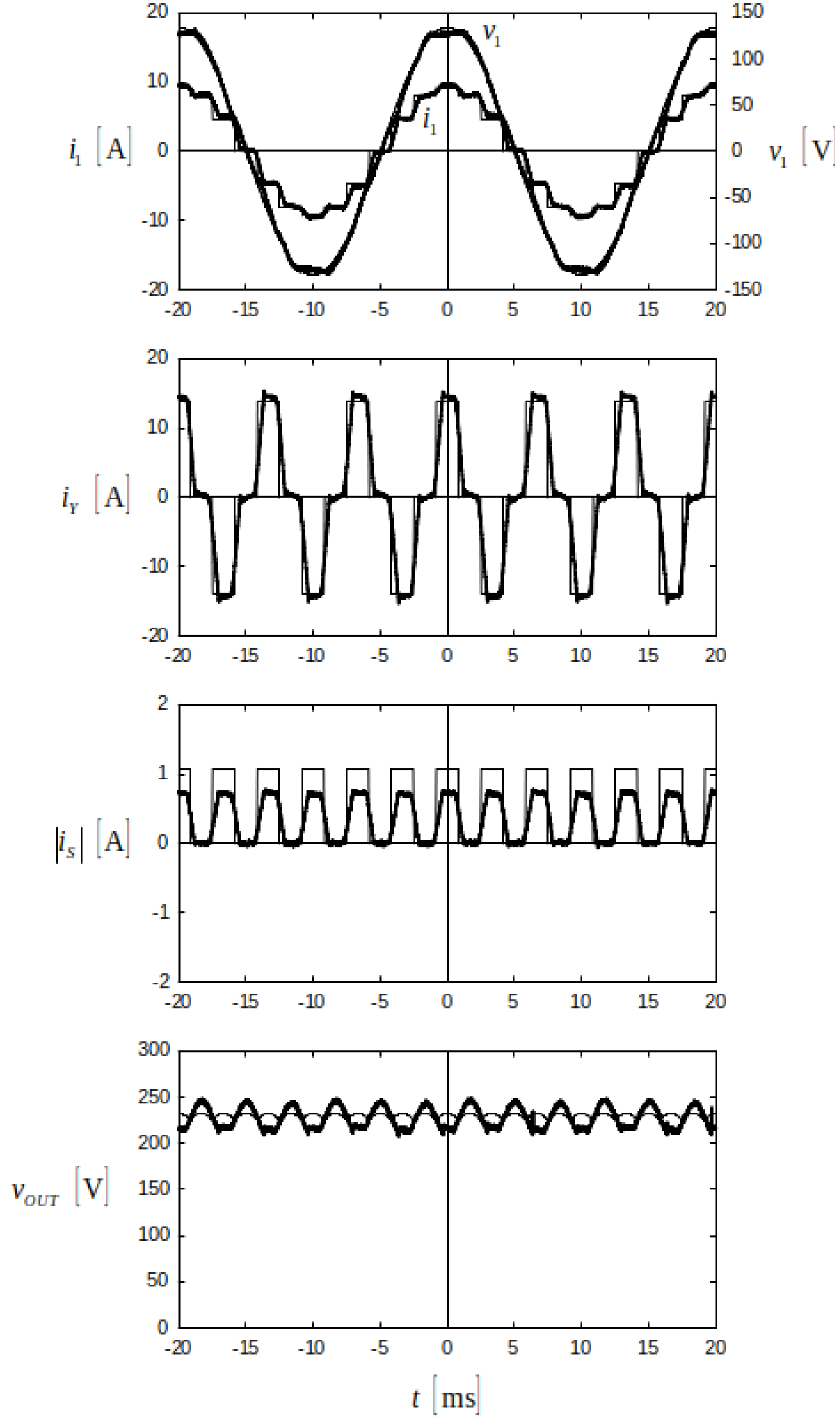




$\eta(\rho)$
100
99
98
97
96
95
94
93
92
91
90
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
ρ

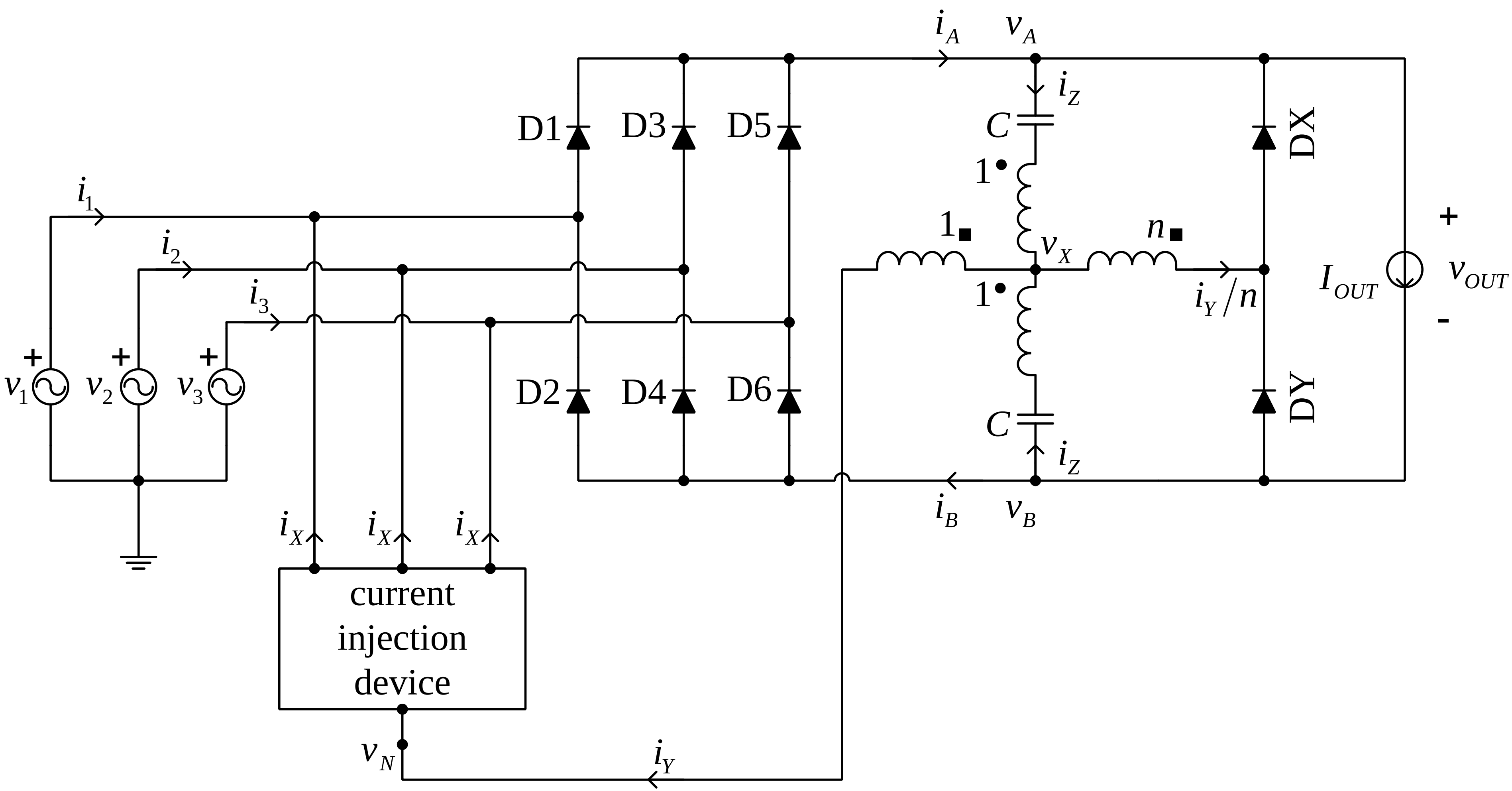


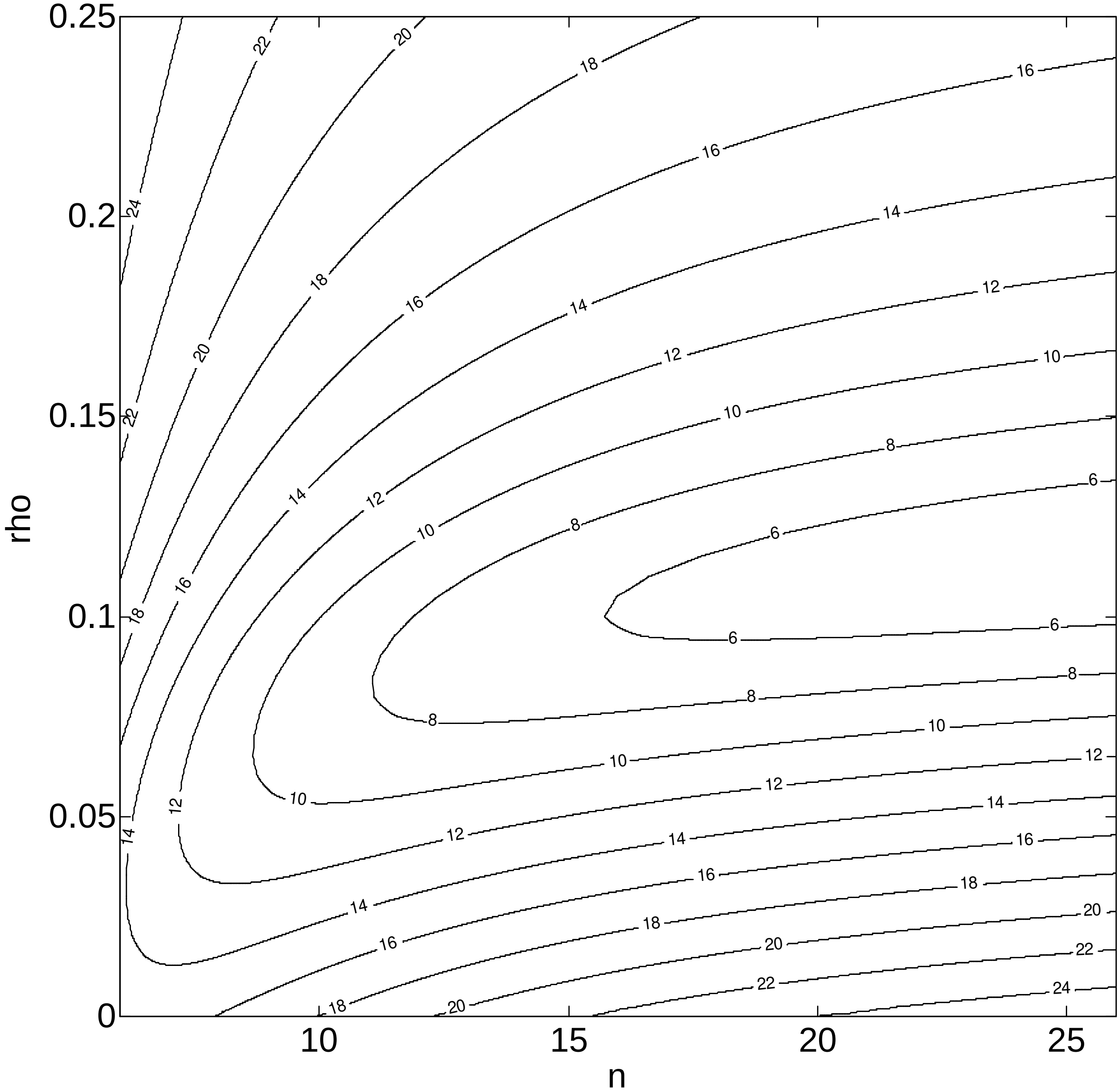
THD $[\%]$
16
15
14
13
12
11
10
9
8
7
6
5
4
90
91
92
93
94
95
96
97
98
99
100
η $[\%]$



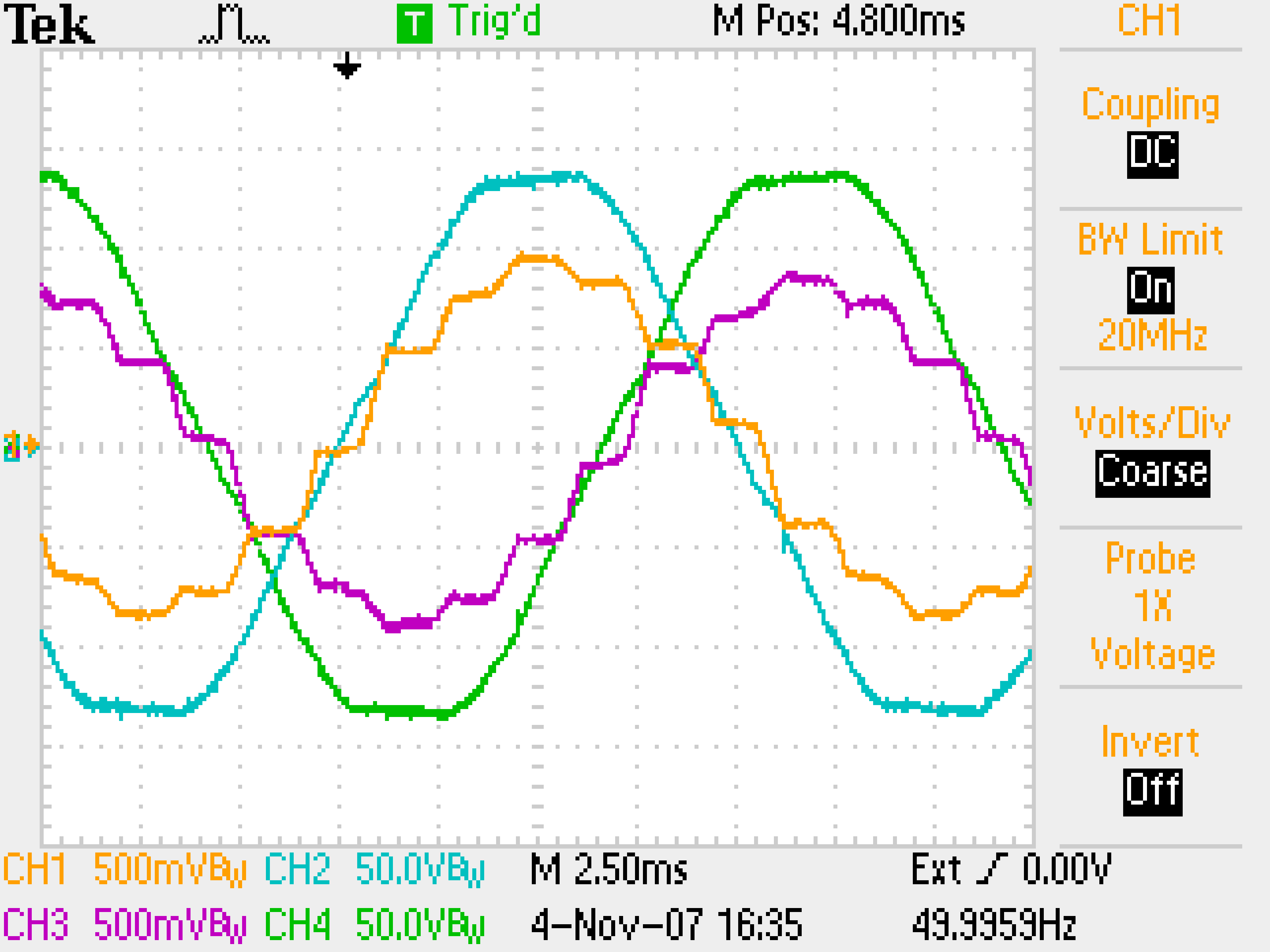
i_1 [A]
v_1 [V]
v_1
i_1
i_Y [A]
$|i_S|$ [A]
v_{OUT} [V]
t [ms]

Really New Results: PESC 2008, paper #1: voltage-loaded resistance emulator

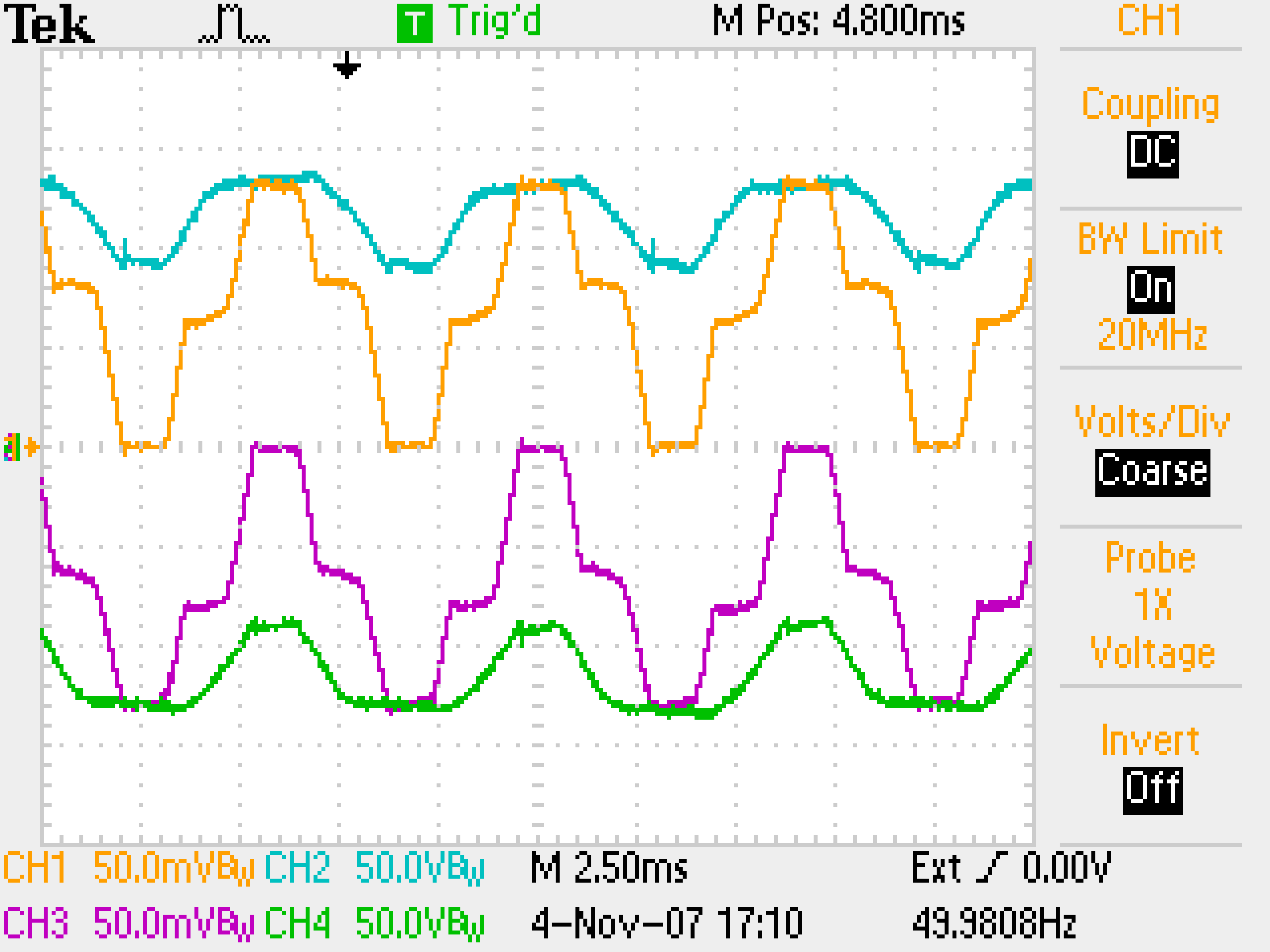


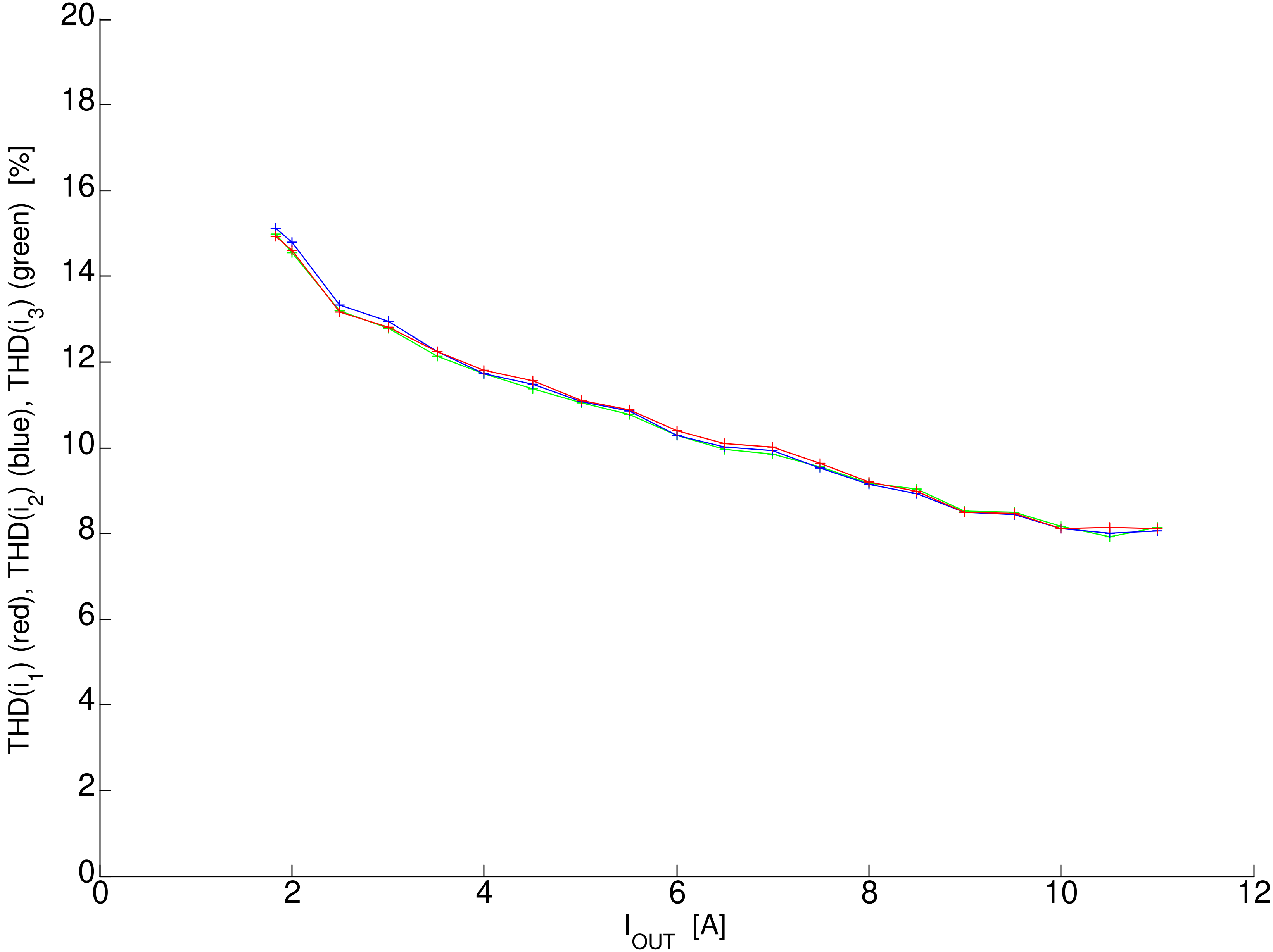


Dependence of the THD on *n* and *ρ*



Tek
Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
1X
Voltage
Invert
Off
CH1 500mV BW
CH2 50.0V BW
M 2.50ms
Ext 0.00V
CH3 500mV BW
CH4 50.0V BW
4-Nov-07 16:35
49.9959Hz


Tek
Trig'd
M Pos: 4.800ms
CH1
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
1X
Voltage
Invert
Off
CH1 50.0mV BW CH2 50.0V BW
M 2.50ms
Ext 0.00V
CH3 50.0mV BW CH4 50.0V BW
4-Nov-07 17:10
49.9808Hz

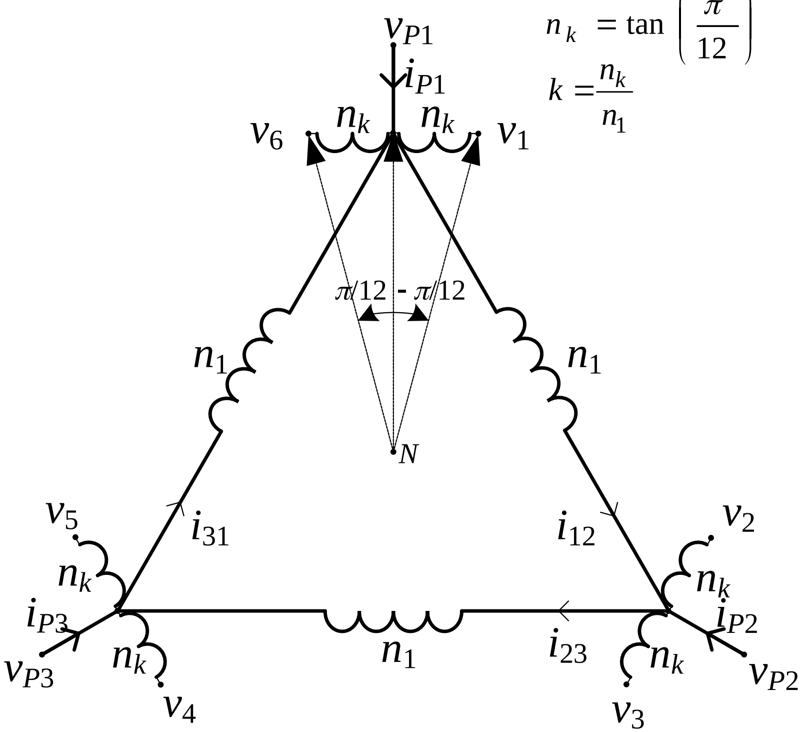

THD(i1) (red), THD(i2) (blue), THD(i3) (green) [%]
0
2
4
6
8
10
12
14
16
18
20
0
2
4
6
8
10
12
IOUT [A]

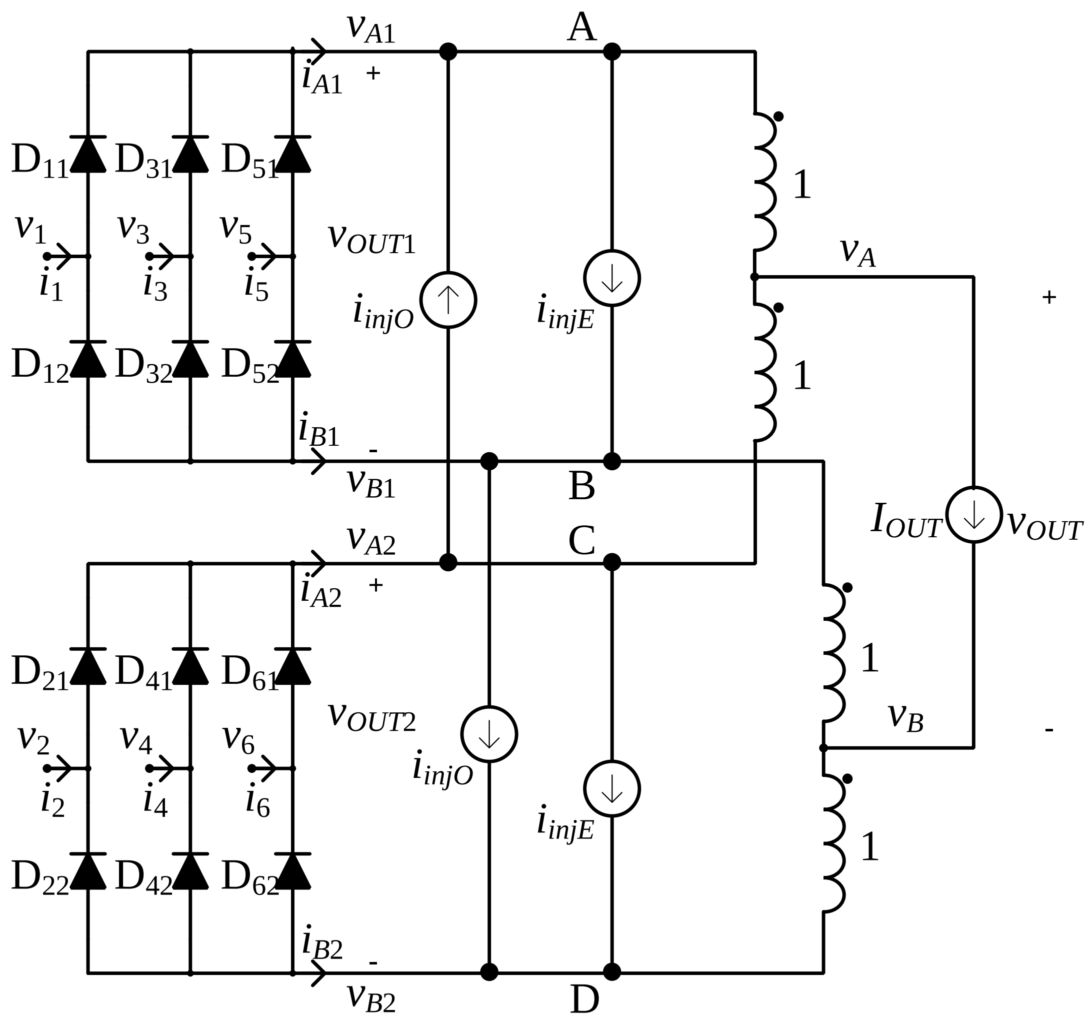
Really New Results: PESC 2008, paper #2: Current injection in multipulse rectifiers

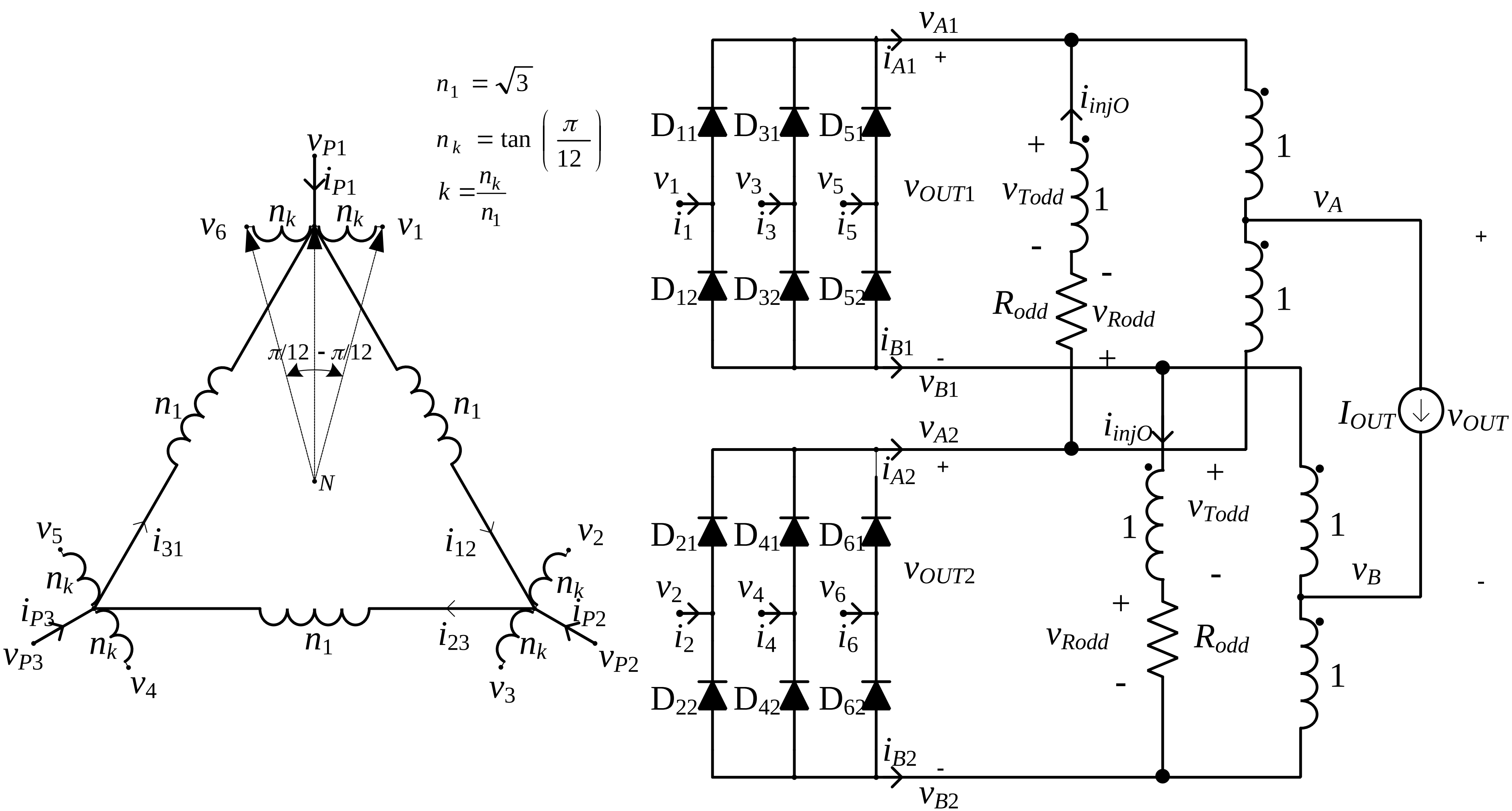
$$n_1 = \sqrt{3}$$

$$n_k = \tan\left(\frac{\pi}{12}\right)$$

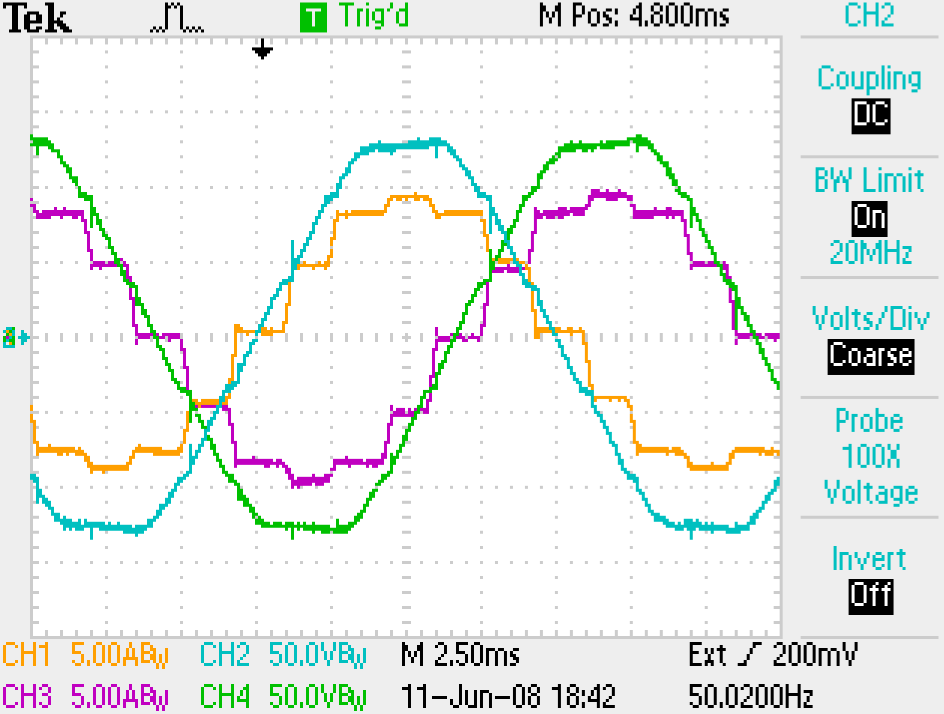
$$k = \frac{n_k}{n_1}$$

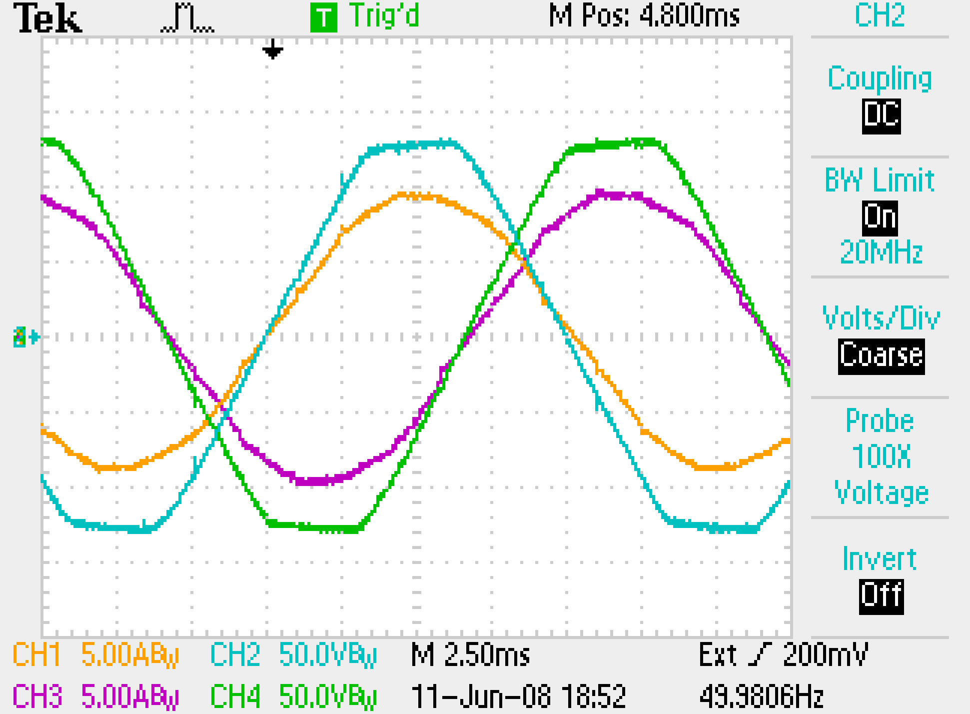


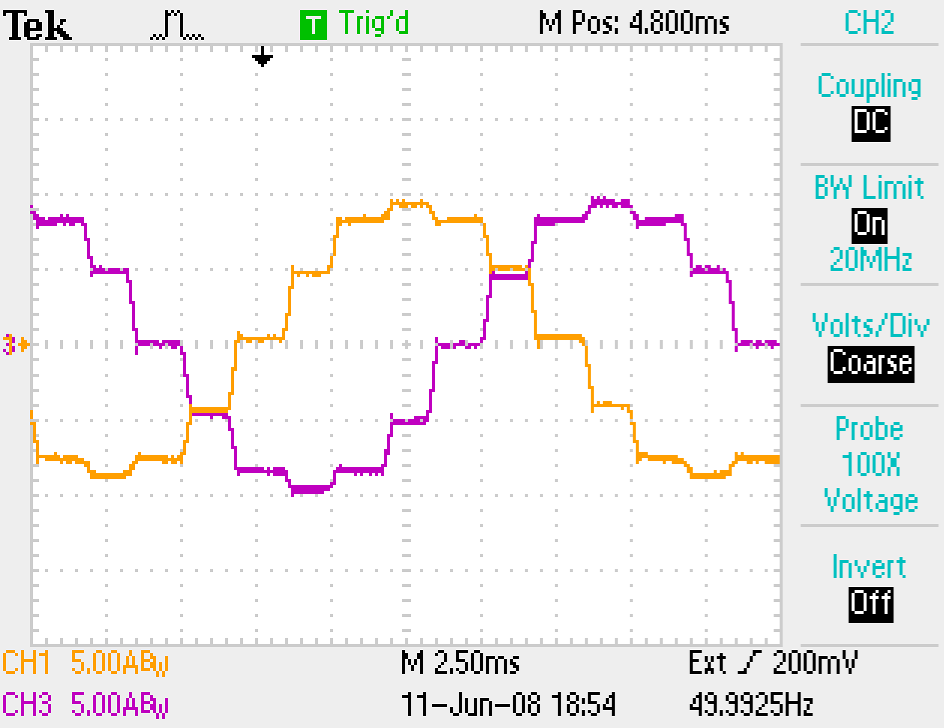


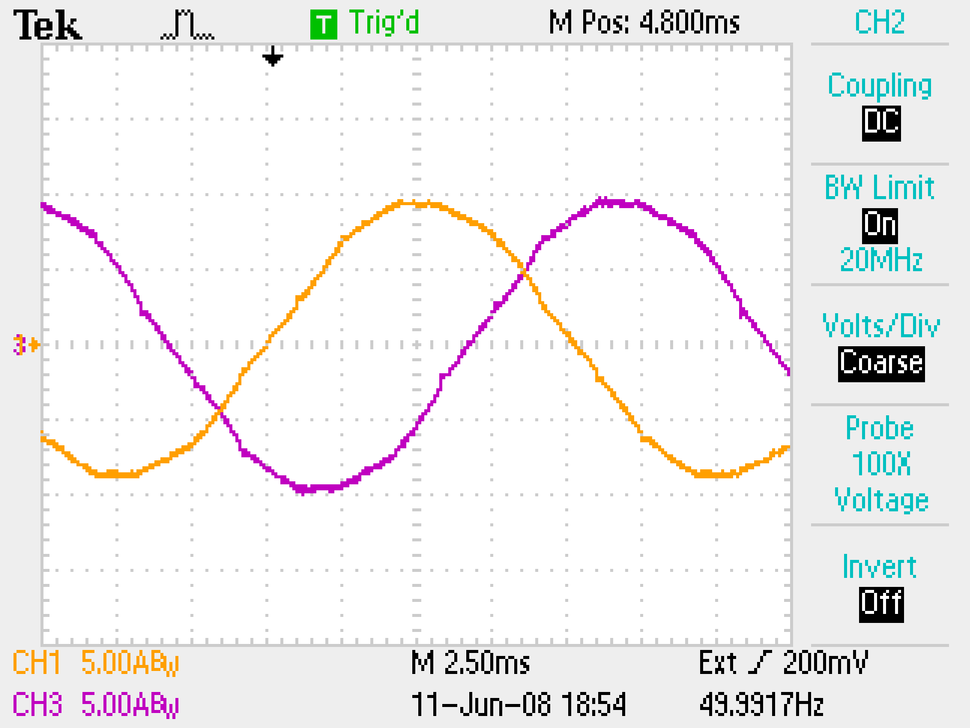


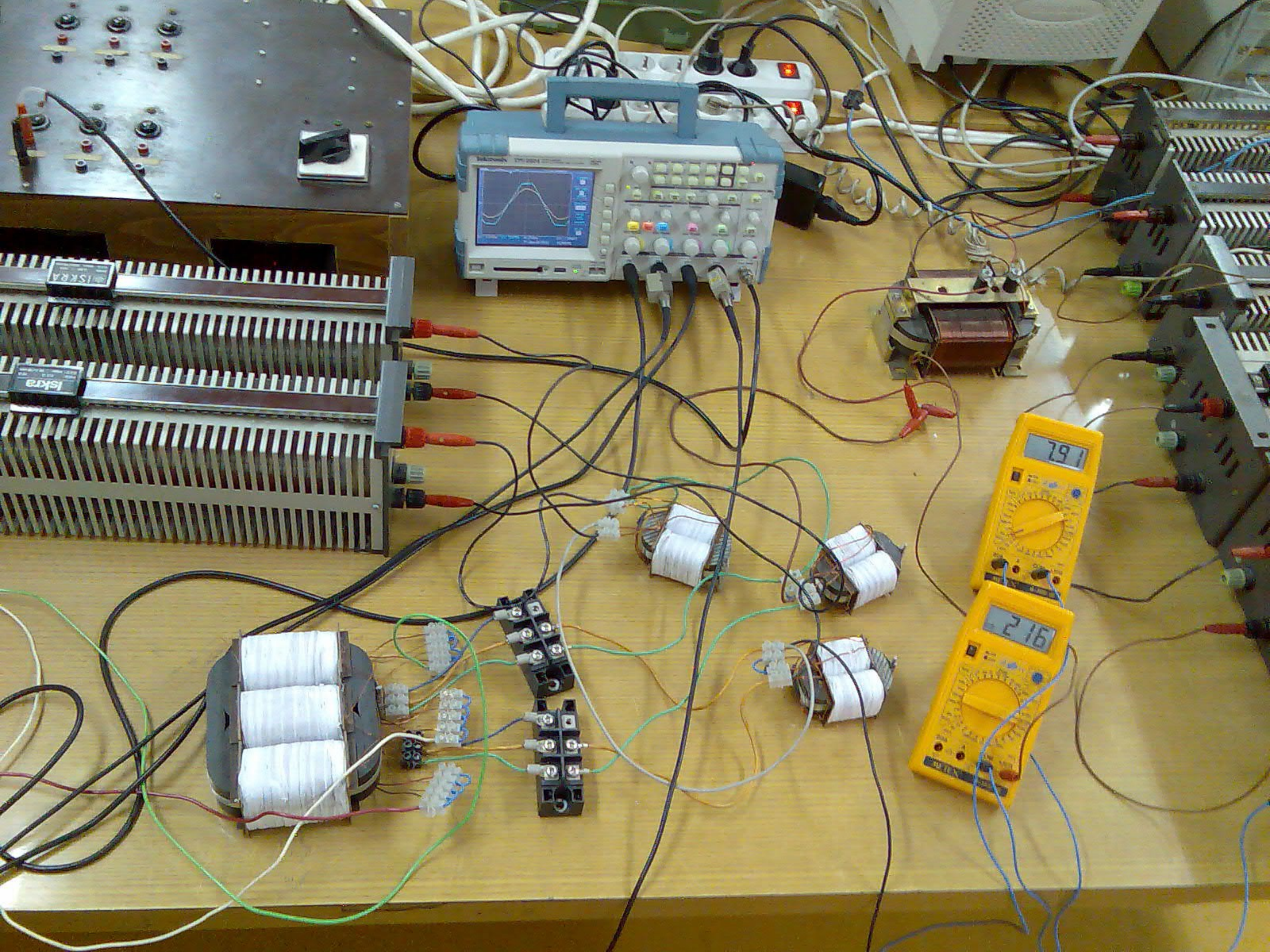
n1 = √3
nk = tan (π/12)
k = nk/n1
vP1
iP1
v6
nk
nk
v1
π/12
π/12
n1
n1
N
v5
i31
i12
v2
nk
nk
iP3
iP2
vP3
nk
n1
i23
nk
vP2
v4
v3
vA1
iA1
+
iinjO
D11
D31
D51
+
1
1
v1
v3
v5
vOUT1
vTodd
vA
i1
i3
i5
-
-
+
D12
D32
D52
1
Rodd
vRodd
iB1
-
+
vB1
IOUT
vOUT
iinjO
vA2
iA2
+
+
vTodd
D21
D41
D61
1
1
vOUT2
-
vB
-
v2
v4
v6
+
i2
i4
i6
vRodd
Rodd
-
1
D22
D42
D62
iB2
-
vB2


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A/div CH2 50.0V/div M 2.50ms Ext 200mV
CH3 5.00A/div CH4 50.0V/div 11-Jun-08 18:42 50.0200Hz


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A/div CH2 50.0V/div M 2.50ms Ext 200mV
CH3 5.00A/div CH4 50.0V/div 11-Jun-08 18:52 49.9806Hz


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A/div M 2.50ms Ext 200mV
CH3 5.00A/div 11-Jun-08 18:54 49.9925Hz


Tek
Trig'd
M Pos: 4.800ms
CH2
Coupling
DC
BW Limit
On
20MHz
Volts/Div
Coarse
Probe
100X
Voltage
Invert
Off
CH1 5.00A/div M 2.50ms Ext 200mV
CH3 5.00A/div 11-Jun-08 18:54 49.9917Hz


ISKRA
Iskra

Finally: Conclusions

- Current injection methods to improve THD and PF in three-phase rectifiers
- Three methods: third-harmonic, optimal, square-wave
- Several circuits that implement the method
- Resistance emulation
- The list of circuits exhausted?
- Which solution is the best?
- Is there a best solution?