On $(1, 2)^*$ -Fuzzy Soft b -Continuity in Fuzzy Soft Bitopological Spaces

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Abstract

The purpose of this paper is to introduce the concepts of $(1, 2)^*$ -fuzzy soft b -continuous maps, $(1, 2)^*$ -fuzzy soft b -irresolute maps and the relations with other weak forms of fuzzy soft continuous maps in fuzzy soft bitopological spaces. Also we introduce and investigate the basic properties of $(1, 2)^*$ -fuzzy soft b -irresolutemap and $(1, 2)^*$ -fuzzy soft b -open (closed) maps.

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1 Introduction

In the year 1965, Zadeh [30], introduced the concept of fuzzy set theory and its applications that can be found in many branches of mathematical and engineering sciences including management science, control engineering, computer science, artificial intelligence (see, [6], [7]).

In the year 1999, Russian researcher Molodtsov [16], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences (see, [17], [29]. In 2003, Maji et al. [15], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, Chen [5], presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory.

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In 1963, Kelly [11], first initiated the concept of bitopological spaces and other authors have contributed to development and construction some of properties of such spaces (see, [12], [20]) as a generalizations of which are in general topology.

In 2014, Ittanagi [8], introduced and studied the concept of soft bitopological spaces and other authors have contributed to development and construction some properties of such spaces (see, [9], [10], [21], [22]).

The notion of soft bitopological space was introduced using different soft topologies on an initial universe set. On the other hand, the mixed type of soft set theory was given using different soft topologies (see, [28], [1], [2], [18]).

In 2015, Mukherjee1 and Park [19] first introduced the notion of fuzzy soft bitopological space and they introduced the notions of $\tau_1\tau_2$ -fuzzy soft open (closed) sets, $\tau_1\tau_2$ -fuzzy soft interior (resp. closure and studied some of their basic properties. Also, my works ([25], [26], [27]) were extension and continuation of studying in this trend by characterizing a new type of fuzzy soft sets and introducing some separation axioms in fuzzy soft bitopological spaces. The purpose of this paper is to introduce the concepts of $(1, 2)^*$ -fuzzy soft b -continuous maps, $(1, 2)^*$ -fuzzy soft b -irresolute maps and the relations with other weak forms of fuzzy soft continuous maps in fuzzy soft bitopological spaces. Also we introduce and investigate the basic properties of $(1, 2)^*$ -fuzzy soft b -irresolute map and $(1, 2)^*$ -fuzzy soft b -open (closed) maps.

2 Preliminaries

In this section we are going to present the basic definitions and results of fuzzy soft set and fuzzy soft bitopological space which will be a central role in our paper.

Throughout our discussion, X refers to an initial universe, E the set of all parameters for X and $P(X)$ denotes the power set of X .

Definition 2.1 [30] A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \rightarrow [0, 1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for every x in X . Let I^X denotes the family of all fuzzy sets on X .

A member A in I^X is contained in a member B of I^X denoted $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for every $x \in X$ (see [30]).

Let $A, B \in I^X$, we have the following properties on fuzzy sets (see [30]).

- (1) Equality: $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in X$,
- (2) Intersection: $C = A \wedge B \in I^X$ by $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,
- (3) Union: $D = A \vee B \in I^X$ by $\mu_D(x) = \max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$,
- (4) Complement: $E = A^c \in I^X$ by $\mu_E(x) = 1 - \mu_A(x)$ for all $x \in X$

Definition 2.2 [30] The empty fuzzy set on X denoted by $\tilde{0}$ is a function which maps each $x \in X$ to 0. That is, $\tilde{0}(x) = 0$ for all $x \in X$.

A universal fuzzy set denoted by $\tilde{1}$ is a function which maps each $x \in X$ to 1. That is, $\tilde{1}(x) = 1$ for all $x \in X$.

Definition 2.3 [16] Let $A \subseteq E$. A pair (F, A) is called a soft set over X if F is a mapping $F : A \rightarrow P(X)$.

Definition 2.4 [14] Let $A \subseteq E$. A pair (f, A) , denoted by f_A , is called a fuzzy soft set over X , where f is a mapping given by $f : A \rightarrow I^X$ defined by $f_A(e) = \mu_{f_A}^e$ where

$$\mu_{f_A}^e = \begin{cases} \tilde{0}, & \text{if } e \notin A; \\ \text{otherwise,} & \text{if } e \in A. \end{cases}$$

$\widetilde{(X, E)}$ denotes the family of all fuzzy soft sets over (X, E) .

Definition 2.5 [15] A fuzzy soft set $f_A \in \widetilde{(X, E)}$ is said to be:

(a) NULL fuzzy soft set, denoted by $\tilde{\varphi}$, if for all $e \in A$, $f_A(e) = \tilde{0}$.

(b) absolute fuzzy soft set, denoted by $\tilde{E}$, if for all $e \in E$, $f_A(e) = \tilde{1}$.

Definition 2.6 [24] The complement of a fuzzy soft set f_A , denoted by f_A^c where $f_A^c :$

$E \rightarrow I^X$ is a mapping given by $\mu_{f_A^c}^e = \tilde{1} - \mu_{f_A}^e$, for all $e \in E$ and where $\tilde{1}(x) = 1$, for all $x \in X$. Clearly $(f_A^c)^c = f_A$.

Definition 2.7 [24] Let $f_A, g_B \in \widetilde{(X, E)}$. f_A is fuzzy soft subset of g_B , denoted by $f_A \tilde{\subseteq} g_B$, if $A \subseteq B$ and $\mu_{f_A}^e \leq \mu_{g_B}^e$ for all $e \in A$, i.e. $\mu_{f_A}^e(x) \leq \mu_{g_B}^e(x)$ for all $x \in X$ and for all $e \in A$.

Definition 2.8 [24] Let $f_A, g_B \in \widetilde{(X, E)}$. The union of f_A and g_B is also a fuzzy soft set h_C , where $C = A \cup B$ and for all $e \in C$, $h_C(e) = \mu_{h_C}^e = \mu_{f_A}^e \vee \mu_{g_B}^e$. Here we write $h_C = f_A \tilde{\cup} g_B$.

Definition 2.9 [24] Let $f_A, g_B \in \widetilde{(X, E)}$. The intersection of f_A and g_B is also a fuzzy soft set d_C , where $C = A \cap B$ and for all $e \in C$, $d_C(e) = \mu_{d_C}^e = \mu_{f_A}^e \wedge \mu_{g_B}^e$. Here we write

$$d_C = f_A \tilde{\cap} g_B.$$

Definition 2.10 [13] The fuzzy soft set $f_A \tilde{\in} (\widetilde{X, E})$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu_A^e(x) = \alpha (0 < \alpha \leq 1)$ and $\mu_A^e(y) = 0$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e or f_e .

Definition 2.11 [13] The fuzzy soft point f_e is said to be belonging to the fuzzy soft set (g, A) , denoted by $f_e \tilde{\in} (g, A)$, if for the element $e \in A$, $\alpha \leq \mu_{gA}^e(x)$, (0

Definition 2.12 [4] Let f_A be fuzzy soft set over X . The two fuzzy soft points $f_{e_1}, f_{e_2} \tilde{\in} f_A$ are said to be equal if $\mu_{f_{e_1}}(x) = \mu_{f_{e_2}}(x)$ for all $x \in X$. Thus $f_{e_1} \neq f_{e_2}$ if and only $\mu_{f_{e_1}}(x) \neq \mu_{f_{e_2}}(x)$ for all $x \in X$.

Definition 2.13 [24] A fuzzy soft topology τ over (X, E) is a family of fuzzy soft sets over (X, E) satisfying the following properties:

- (i) $\tilde{0}_E, \tilde{1}_E \in \tau$
- (ii) if $f_A, g_B \in \tau$, then $f_A \tilde{\cap} g_B \in \tau$,
- (iii) if $f_{A_\alpha} \in \tau$ for all $\alpha \in \Delta$ an index set, then $\bigcup_{\alpha \in \Delta} f_{A_\alpha} \in \tau$.

Definition 2.14 [19] If τ is a fuzzy soft topology on (X, E) the triple (X, E, τ) is said to be a fuzzy soft topological space. Also each member of τ is called a fuzzy soft open set in (X, E, τ) .

The complement of a fuzzy soft open set is a fuzzy soft closed set.

Definition 2.15 [19] Let (X, E, τ_1) and (X, E, τ_2) be the two different fuzzy soft topologies on (X, E) . Then (X, E, τ_1, τ_2) is called a fuzzy soft bitopological space on which no separation axioms are assumed unless explicitly stated.

The members of $\tau_i (i = 1, 2)$ are called $\tau_i (i = 1, 2)$ -fuzzy soft open sets and the complement of $\tau_i (i = 1, 2)$ -fuzzy soft open sets are called $\tau_i (i = 1, 2)$ -fuzzy soft closed sets.

Definition 2.16 [19] A fuzzy soft set $f_E \tilde{\in} (\widetilde{X, E})$ is called $\tau_1 \tau_2$ -fuzzy soft open set if $f_E = g_E \tilde{\cup} h_E$ such that $g_E \tilde{\in} \tau_1$ and $h_E \tilde{\in} \tau_2$.

The complement of $\tau_1 \tau_2$ -fuzzy soft open set is called $\tau_1 \tau_2$ -fuzzy soft closed set.

The family of all $\tau_1 \tau_2$ -fuzzy soft open (closed) sets in (X, E, τ_1, τ_2) is denoted by $\tau_1 \tau_2 FSO(X, \tau_1, \tau_2)_E$ ($\tau_1 \tau_2 FSC(X, \tau_1, \tau_2)_E$), respectively.

Remark 2.17 [19] $\tau_i (i = 1, 2)$ -fuzzy soft open is $\tau_1 \tau_2$ -fuzzy soft open but the converse is not true.

Theorem 2.18 [19] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space, then $\tau = \tau_1 \tilde{\cap} \tau_2$ is a fuzzy soft topological space over (X, E) .

Remark 2.19 [19] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space, then $\tau = \tau_1 \tilde{\cap} \tau_2$ is

not a fuzzy soft topological space over (X, E) .

Definition 2.20 [19] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then the $\tau_i (i = 1, 2)$ -fuzzy soft closure of f_E , denoted by $\tau_i cl(f_E)$, is the intersection of all $\tau_i (i = 1, 2)$ -fuzzy soft closed supersets of f_E .

Clearly, $\tau_i cl(f_E)$ is the smallest $\tau_i (i = 1, 2)$ -fuzzy soft closed set over (X, E) which contains f_E .

Definition 2.21 [19] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then the $\tau_1 \tau_2$ -fuzzy soft closure of f_E , denoted by $\tau_1 \tau_2 cl(f_E)$, is the intersection of all $\tau_1 \tau_2$ -fuzzy soft closed supersets of f_E .

Clearly, $\tau_1 \tau_2 cl(f_E)$ is the smallest $\tau_1 \tau_2$ -fuzzy soft closed set over (X, E) which contains f_E .

Remark 2.22 [19] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then $\tau_1 \tau_2 cl(f_E) \subseteq \tau_i cl(f_E) (i = 1, 2)$.

Definition 2.23 [19] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then the $\tau_i (i = 1, 2)$ -fuzzy soft interior of f_E , denoted by $\tau_i int(f_E)$, is the union of all $\tau_i (i = 1, 2)$ -fuzzy soft open subsets of f_E .

Clearly, $\tau_i int(f_E)$ is the largest $\tau_i (i = 1, 2)$ -fuzzy soft open set over (X, E) which contained in f_E .

Definition 2.24 [19] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then the $\tau_1\tau_2$ -fuzzy soft interior of f_E , denoted by $\tau_1\tau_2\text{int}(f_E)$, is the union of all $\tau_1\tau_2$ -fuzzy soft open subsets of f_E .

Clearly, $\tau_1\tau_2\text{cl}(f_E)$ is the largest $\tau_1\tau_2$ -fuzzy soft open set over (X, E) which contained in f_E .

Remark 2.25 [19] If (X, E, τ_1, τ_2) is a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then $\tau_i\text{int}(f_E) (i = 1, 2) \subseteq \tau_1\tau_2\text{int}(f_E)$.

Definition 2.26 [27] Let (X, E, τ_1, τ_2) be a soft bitopological space. Then, the family of all $\tau_1\tau_2$ -fuzzy soft open sets is a supra fuzzy soft topology on (X, E) . This supra fuzzy soft topology, will denoted by τ_{12} , i.e., $\tau_{12} = \tau_1\tau_2\text{FSO}(X, \tau_1, \tau_2)_E = \{g_E = g_{1E} \cup g_{2E} : g_{iE} \in \tau_i, i = 1, 2\}$ and the triple (X, E, τ_{12}) is the supra fuzzy soft topological space associated to the fuzzy soft bitopological space (X, E, τ_1, τ_2) .

Definition 2.27 [25] A fuzzy soft set f_E in a fuzzy soft bitopological space (X, E, τ_1, τ_2) is called

(i) $(1, 2)^*$ -fuzzy soft pre open set if $f_E \subseteq \tau_1\tau_2\text{int}(\tau_1\tau_2\text{cl}(f_E))$ and $(1, 2)^*$ -fuzzy soft pre closed set if $\tau_1\tau_2\text{cl}(\tau_1\tau_2\text{int}(f_E)) \subseteq f_E$.

(ii) $(1, 2)^*$ -fuzzy soft semi open set if $f_E \subseteq \tau_1\tau_2\text{cl}(\tau_1\tau_2\text{int}(f_E))$ and $(1, 2)^*$ -fuzzy soft semi closed set if $\tau_1\tau_2\text{int}(\tau_1\tau_2\text{cl}(f_E)) \subseteq f_E$.

(iii) $(1, 2)^*$ -fuzzy soft β -open set if $f_E \subseteq \tau_1\tau_2\text{cl}(\tau_1\tau_2\text{int}(\tau_1\tau_2\text{cl}(f_E)))$ and $(1, 2)^*$ -fuzzy soft β -closed set if $\tau_1\tau_2\text{int}(\tau_1\tau_2\text{cl}(\tau_1\tau_2\text{int}(f_E))) \subseteq f_E$.

Definition 2.28 [25] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$. Then f_E is called $(1, 2)^*$ -fuzzy soft b-open set (briefly, $(1, 2)^*$ -fsb-open) if $f_E \subseteq \tau_1\tau_2\text{int}(\tau_1\tau_2\text{cl}(f_E)) \cup \tau_1\tau_2\text{cl}(\tau_1\tau_2\text{int}(f_E))$

Theorem 2.29 [25] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then

- (i) Every $(1, 2)^*$ -fuzzy soft pre open set is $(1, 2)^*$ -fuzzy soft b -open set.
- (ii) Every $(1, 2)^*$ -fuzzy soft b -open set is $(1, 2)^*$ -fuzzy soft β -open set.
- (iii) Every $(1, 2)^*$ -fuzzy soft semi open set is $(1, 2)^*$ -fuzzy soft b -open set.

Definition 2.30 [25] Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space and $f_E \in \widetilde{\mathcal{F}}(X, E)$.

- (i) $(1, 2)^*$ -fuzzy soft b -closure (briefly $(1, 2)^*$ -fbscl(f_E)) of a set f_E in (X, E, τ_1, τ_2) defined by $(1, 2)^*$ -fbscl(f_E) = $\bigcap \{g_E \mid f_E \subseteq g_E \text{ is a } (1, 2)^*\text{-fuzzy soft } b\text{-closed set in } (X, E, \tau_1, \tau_2)\}$
 - (ii) $(1, 2)^*$ -fuzzy soft b -interior (briefly $(1, 2)^*$ -fbsint(f_E)) of a set f_E in (X, E, τ_1, τ_2) defined by $(1, 2)^*$ -fbsint(f_E) = $\bigcup \{g_E \mid f_E \supseteq g_E \text{ is a } (1, 2)^*\text{-fuzzy soft } b\text{-open set in } (X, E, \tau_1, \tau_2)\}$.
- $(1, 2)^*$ -fbscl(f_E) is the smallest $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) which contains f_E and $(1, 2)^*$ -fbsint(f_E) is the largest $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) which is contained in f_E .

Definition 2.31 [3] Let $\widetilde{\mathcal{F}}(X, E)$ and $\widetilde{\mathcal{F}}(Y, K)$ be the families of all fuzzy soft sets over X and Y , respectively. Let $\phi : X \rightarrow Y$ and $\psi : E \rightarrow K$ be two functions. Then the pair (ϕ, ψ) is called a fuzzy soft mapping from X to Y , and denoted by $(\phi, \psi) : \widetilde{\mathcal{F}}(X, E) \rightarrow \widetilde{\mathcal{F}}(Y, K)$.

In the diagram, $f_A \in \widetilde{\mathcal{F}}(X, E)$, $g_B \in \widetilde{\mathcal{F}}(Y, K)$ and Let $\phi : I^X \rightarrow I^Y$ is the forward powerset operator (see e.g. [23]), that is $\phi^+(h) := \phi(h)$ for all $h \in I^X$. Since componentwise composition of two fuzzy soft functions (ϕ, ψ) from $\widetilde{\mathcal{F}}(X, E)$ to $\widetilde{\mathcal{F}}(Y, K)$ and (ϕ, ψ) from $\widetilde{\mathcal{F}}(Y, K)$ to $\widetilde{\mathcal{F}}(Z, T)$ is obviously a fuzzy soft function $(\phi \circ \phi, \psi \circ \psi)$ from $\widetilde{\mathcal{F}}(X, E)$ to $\widetilde{\mathcal{F}}(Z, T)$, where $\psi : E \rightarrow K$ and $\psi : K \rightarrow T$ and $\phi : I^X \rightarrow I^Y$ and $\phi : I^Y \rightarrow I^Z$ and the pair of identities (id_X, id_E) from $\widetilde{\mathcal{F}}(X, E)$ to $\widetilde{\mathcal{F}}(X, E)$ is the identical morphism.

(1) Let $f_A \in \widetilde{\mathcal{F}}(X, E)$. Then the image of f_A under the fuzzy soft mapping (ϕ, ψ) is the fuzzy soft set over Y defined by $(\phi, \psi)(f_A)$, where $\forall k \in \psi(E), \forall y \in Y$,

$$(\phi, \psi)(f_A)(k)(y) = \begin{cases} \phi(x) = y & \psi(e) f_A(e)(x), & \text{if } x \in \phi^{-1}(y); \\ \emptyset_E, & \text{otherwise.} \end{cases}$$

(2) Let $g_B \in \widetilde{\mathcal{F}}(Y, K)$. Then the inverse image of g_B under the fuzzy soft mapping (ϕ, ψ) is the fuzzy soft set over X defined by $(\phi, \psi)^{-1}(g_B)$, where $\forall e \in \psi^{-1}(K), \forall x \in X, \phi^{-1}(g_B)(e)(x) = g_B(\psi(e))(\phi(x))$. If both ϕ and ψ is injective then the fuzzy soft mapping (ϕ, ψ) is said to be injective. If both ϕ and ψ is surjective then the fuzzy soft mapping (ϕ, ψ) is said to be surjective. (ϕ, ψ) is said to be bijective if it is both injective and surjective.

3 $(1, 2)^*$ -fuzzy soft b -continuous maps

In this section we introduce the concept of $(1, 2)^*$ -fuzzy soft b -continuous functions and some of their properties are discussed.

We begin by the following concepts which will be used in the sequel.

Definition 3.1 A fuzzy soft mapping $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is said to be

- (i) $(1, 2)^*$ -fuzzy soft continuous (briefly $(1, 2)^*$ -fs-continuous) if the inverse image of every $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$ is a $\tau_1\tau_2$ -fuzzy soft open set in (X, E, τ_1, τ_2) .
- (ii) $(1, 2)^*$ -fuzzy soft semi-continuous (briefly $(1, 2)^*$ -fss-continuous) the inverse image of every $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft semi open set in (X, E, τ_1, τ_2) .
- (iii) $(1, 2)^*$ -fuzzy soft pre-continuous (briefly $(1, 2)^*$ -fsp-continuous) the inverse image of every $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$ is a $(1, 2)^*$ -fuzzy soft pre open set in (X, E, τ_1, τ_2) .
- (iv) $(1, 2)^*$ -fuzzy soft β -continuous (briefly $(1, 2)^*$ -fs β -continuous) the inverse image of every $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$ is a $(1, 2)^*$ -fuzzy soft β -open set in (X, E, τ_1, τ_2) .

Definition 3.2 A fuzzy soft mapping $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is said to be $(1, 2)^*$ -fuzzy soft b -continuous (briefly $(1, 2)^*$ -fs b -continuous) the inverse image of every $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$ is a $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) .

Example 3.3 Let $X = Y = \{x, y\}, E = K = \{e_1, e_2\}$. Let $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1E}\}, \tau_2 = \{\tilde{0}_E, \tilde{1}_E, f_{2E}\}$, where $f_{1E} = \{f_1(e_1) = \{x/0.5, y/0.0\}, f_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$f_{2E} = \{f_2(e_1) = \{x/0.0, y/0.7\}, f_2(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$.

It is clear that (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.

The $\tau_1\tau_2$ -fuzzy soft open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}\}$, where

$f_{3E} = f_{1E} \cup f_{2E} = \{f_3(e_1) = \{x/0.5, y/0.7\}, f_3(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$, and $(1, 2)^*$ -fuzzy soft b -open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}\}$, where f_{1E}, f_{2E}, f_{3E} are shown as above and

$f_{4E} = \{f_4(e_1) = \{x/0.5, y/0.0\}, f_4(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{5E} = \{f_5(e_1) = \{x/0.5, y/0.0\}, f_5(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{6E} = \{f_6(e_1) = \{x/0.5, y/0.0\}, f_6(e_2) = \{x/0.3, y/0.6\}\}$,

$f_{7E} = \{f_7(e_1) = \{x/0.0, y/0.7\}, f_7(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{8E} = \{f_8(e_1) = \{x/0.0, y/0.7\}, f_8(e_2) = \{x/0.3, y/0.6\}\}$,

$f_{9E} = \{f_9(e_1) = \{x/0.0, y/0.7\}, f_9(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{10E} = \{f_{10}(e_1) = \{x/0.5, y/0.7\}, f_{10}(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{11E} = \{f_{11}(e_1) = \{x/0.5, y/0.7\}, f_{11}(e_2) = \{x/0.0, y/0.6\}\}$.

Let $\sigma_1 = \{0_K, 1_K, g_{1K}, g_{2K}\}, \sigma_2 = \{\tilde{0}_K, \tilde{1}_K, g_{3K}\}$, where

$g_{1K} = \{g_1(e_1) = \{x/0.5, y/0.0\}, g_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$g_{2K} = \{g_2(e_1) = \{x/0.5, y/0.0\}, g_2(e_2) = \{x/0.3, y/0.0\}\}$,

$g_{3K} = \{g_3(e_1) = \{x/0.5, y/0.7\}, g_3(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$.

It is clear that (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.

The $\sigma_1\sigma_2$ -fuzzy soft open sets are $\{\tilde{0}_K, \tilde{1}_K, g_{1K}, g_{2K}, g_{3K}, g_{4K}\}$, where

$$g_{2K} = g_{1E} \tilde{\cup} g_{2E}, g_{3K} = g_{1E} \tilde{\cup} g_{3E},$$

$$g_{4K} = g_{2E} \tilde{\cup} g_{3E} = \{g_4(e_1) = \{x/0.5, y/0.7\}, g_4(e_2) = \{x/0.3, y/0.0\}\}.$$

It is clear that $(Y, K, \sigma_1, \sigma_2)$ is a fuzzy soft bitopological space.

Now define $\phi : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $\psi : E \rightarrow K$ such that

$$\phi(x, \mu_{f(e_i)}(x)) = (x, \mu_{g(e_i)}(x)), \psi(e_i) = e_i, i = 1, 2.$$

Then the inverse image of every $\sigma_1\sigma_2$ -fuzzy soft open sets are $\tau_1\tau_2$ -fuzzy b -soft open sets are $(1, 2)^*$ -fuzzy soft b -open sets in (X, E, τ_1, τ_2) . Thus $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous.

Theorem 3.4 Every $(1, 2)^*$ -fuzzy soft continuous map is $(1, 2)^*$ -fuzzy soft b -continuous.

Proof. Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -fuzzy soft b -continuous map. Let f_A be $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$. Since (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -continuous map, $(\phi, \psi)^{-1}(f_A)$ is $\tau_1\tau_2$ -fuzzy soft open set in (X, E, τ_1, τ_2) . And so $(\phi, \psi)^{-1}(f_A)$ is $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) . Therefore (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -continuous map. ■

The converse of the above theorem is not true as shown in the following example.

Example 3.5 Consider the Example 3.3, $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous but not $(1, 2)^*$ -fuzzy soft continuous.

Theorem 3.6 Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ be a fuzzy soft mapping. Then the following statements are equivalent:

- (i) (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -continuous.
- (ii) The inverse image of each $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$ is a $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) .

Proof. (i) $\Rightarrow$ (ii): Let g_K be a $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$. Then g_K^c is a $\sigma_1\sigma_2$ -fuzzy soft open set in $(Y, K, \sigma_1, \sigma_2)$. Thus $(\phi, \psi)^{-1}(g_K^c)$ is a $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) and $(\phi, \psi)^{-1}(g_K^c) = [(\phi, \psi)^{-1}(g_K)]^c$. Hence $(\phi, \psi)^{-1}(g_K)$ is a $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) .

(ii) $\Rightarrow$ (i): Let g_K be a $\sigma_1\sigma_2$ -fuzzy soft set in $(Y, K, \sigma_1, \sigma_2)$. Then g_K^c is a $\sigma_1\sigma_2$ -fuzzy soft closed set and by (ii), we have $(\phi, \psi)^{-1}(g_K^c)$ is $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) and $(\phi, \psi)^{-1}(g_K^c) = [(\phi, \psi)^{-1}(g_K)]^c$. Hence $(\phi, \psi)^{-1}(g_K)$ is a $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) . Therefore (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -continuous map. ■

Theorem 3.7 Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ be a fuzzy soft map. Then

- (i) Every $(1, 2)^*$ -fuzzy soft semi continuous map is $(1, 2)^*$ -fuzzy soft b -continuous.
- (ii) Every $(1, 2)^*$ -fuzzy soft pre continuous map is $(1, 2)^*$ -fuzzy soft b -continuous.
- (iii) Every $(1, 2)^*$ -fuzzy soft b -continuous map is $(1, 2)^*$ -fuzzy soft β -continuous.

Proof. (i) Assume that (ϕ, ψ) is $(1, 2)^*$ -fuzzy semi continuous and g_K be a $\sigma_1\sigma_2$ -fuzzy soft set in $(Y, K, \sigma_1, \sigma_2)$. Then $(\phi, \psi)^{-1}(g_K)$ is a $(1, 2)^*$ -fuzzy semi open set in (X, E, τ_1, τ_2) . Since every $(1, 2)^*$ -fuzzy semi open set is $(1, 2)^*$ -fuzzy b -open set and hence $(\phi, \psi)^{-1}(g_K)$ is a $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) . Therefore (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -continuous map.

The proof of (ii) and (iii) are similar to (i). ■

Remark 3.8 The converse of the above implications need not be true as shown in the following examples.

Example 3.9 Let $X = Y = \{x, y\}, E = K = \{e_1, e_2\}$. Let $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1E}\}, \tau_2 = \{\tilde{0}_E, \tilde{1}_E, f_{2E}\}$, where $f_{1E} = \{f_1(e_1) = \{x/0.0, y/0.7\}, f_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$f_{2E} = \{f_2(e_1) = \{x/0.0, y/0.7\}, f_2(e_2) = \{x/0.3, y/0.7\}\}$,

It is clear that (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.

The $\tau_1\tau_2$ -fuzzy soft open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}\}$ and $(1, 2)^*$ -fuzzy soft b -open sets are all fuzzy soft subsets in (X, E) i.e.

$\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}, f_{12E}, f_{13E}, f_{14E}\}$ where f_{1E}, f_{2E} are shown as above and

$f_{3E} = \{f_4(e_1) = \{x/0.5, y/0.0\}, f_4(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$f_{4E} = \{f_4(e_1) = \{x/0.5, y/0.7\}, f_4(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$f_{5E} = \{f_5(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, f_5(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{6E} = \{f_6(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, f_6(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{7E} = \{f_7(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, f_7(e_2) = \{x/0.3, y/0.6\}\}$,

$f_{8E} = \{f_8(e_1) = \{x/0.5, y/0.0\}, f_8(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{9E} = \{f_9(e_1) = \{x/0.5, y/0.0\}, f_9(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{10E} = \{f_{10}(e_1) = \{x/0.5, y/0.0\}, f_{10}(e_2) = \{x/0.3, y/0.6\}\}$,

$f_{11E} = \{f_{11}(e_1) = \{x/0.0, y/0.7\}, f_{11}(e_2) = \{x/0.3, y/0.0\}\}$.

$f_{12E} = \{f_9(e_1) = \{x/0.0, y/0.7\}, f_9(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{13E} = \{f_{10}(e_1) = \{x/0.5, y/0.7\}, f_{10}(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{14E} = \{f_{11}(e_1) = \{x/0.5, y/0.7\}, f_{11}(e_2) = \{x/0.0, y/0.6\}\}$.

and $(1, 2)^*$ -fuzzy soft semi open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}\}$

Let $\sigma_1 = \{0_K, \tilde{1}_K, g_{1K}\}, \sigma_2 = \{\tilde{0}_K, \tilde{1}_K, g_{2K}\}$, where

$g_{1K} = \{g_1(e_1) = \{x/0.0, y/0.7\}, g_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$g_{2K} = \{g_2(e_1) = \{x/0.0, y/0.7\}, g_2(e_2) = \{x/0.3, y/0.6\}\}$.

It is clear that (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.

The $\sigma_1\sigma_2$ -fuzzy soft open sets are $\{\tilde{0}_K, \tilde{1}_K, g_{1K}, g_{2K}, g_{3K}\}$, where

$g_{3K} = g_{1E} \cup g_{2E} = \{g_3(e_1) = \{x/0.5, y/0.7\}, g_3(e_2) = \{x/0.3, y/0.0\}\}$.

It is clear that $(Y, K, \sigma_1, \sigma_2)$ is a fuzzy soft bitopological space.

Now define $\phi : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $\psi : E \rightarrow K$ such that

$\phi(x, \mu_{f(e_i)}(x)) = (x, \mu_{g(e_i)}(x)), \psi(e_i) = e_i, i = 1, 2$. Then

$(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous. But the inverse image of $\sigma_1\sigma_2$ -fuzzy soft open sets is not a $(1, 2)^*$ -fuzzy soft semi open sets in (X, E, τ_1, τ_2) .

Since $(\phi, \psi)^{-1}(g_{1K}) = f_{1E}$ which is not $(1, 2)^*$ -fuzzy soft semi open sets in (X, E, τ_1, τ_2) .

Therefore $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is not $(1, 2)^*$ -fuzzy soft semi continuous.

Example 3.10 Consider the Example 3.2, $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous but not $(1, 2)^*$ -fuzzy soft pre continuous. Since the $(1, 2)^*$ -fuzzy soft pre open set in (X, E, τ_1, τ_2) are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}, f_{10E}, f_{11E}\}$ and $(\phi, \psi)^{-1}(g_2) = f_4$, which is not $(1, 2)^*$ -fuzzy soft pre open set in (X, E, τ_1, τ_2) .

Example 3.11 Let $X = Y = \{x, y\}$, $E = K = \{e_1, e_2\}$.

Let $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1E}\}$, $\tau_2 = \{\tilde{0}_E, \tilde{1}_E, f_{2E}\}$, where

$$f_{1E} = \{f_1(e_1) = \{x/0.5, y/0.0\}, f_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\},$$

$$f_{2E} = \{f_2(e_1) = \{x/0.0, y/0.7\}, f_2(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\},$$

It is clear that (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.

The $\tau_1\tau_2$ -fuzzy soft open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}\}$ where

$$f_{3E} = f_{1E} \cup f_{2E} = \{f_3(e_1) = \{x/0.5, y/0.7\}, f_3(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}, \text{ and } (1, 2)^*\text{-fuzzy}$$

soft b -open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}\}$, where

f_{1E}, f_{2E}, f_{3E} are shown as above and

$$f_{4E} = \{f_4(e_1) = \{x/0.5, y/0.0\}, f_4(e_2) = \{x/0.3, y/0.0\}\},$$

$$f_{5E} = \{f_5(e_1) = \{x/0.5, y/0.0\}, f_5(e_2) = \{x/0.0, y/0.6\}\},$$

$$f_{6E} = \{f_6(e_1) = \{x/0.5, y/0.0\}, f_6(e_2) = \{x/0.3, y/0.6\}\},$$

$$f_{7E} = \{f_7(e_1) = \{x/0.5, y/0.0\}, f_7(e_2) = \{x/0.3, y/0.6\}\},$$

$$f_{8E} = \{f_8(e_1) = \{x/0.0, y/0.7\}, f_8(e_2) = \{x/0.0, y/0.6\}\},$$

$$f_{9E} = \{f_9(e_1) = \{x/0.0, y/0.7\}, f_9(e_2) = \{x/0.3, y/0.6\}\},$$

$$f_{10E} = \{f_{10}(e_1) = \{x/0.5, y/0.7\}, f_{10}(e_2) = \{x/0.3, y/0.0\}\},$$

$$f_{11E} = \{f_{11}(e_1) = \{x/0.5, y/0.7\}, f_{11}(e_2) = \{x/0.0, y/0.6\}\}. \text{ and } (1, 2)^*\text{-fuzzy soft } \beta\text{-open}$$

sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}, f_{12E}, f_{13E}\}$,

where $f_{iE}; i = 1, 2, \dots, 11$ are shown as above and $f_{12E} = \{f_{12}(e_1) = \{x/0.5, y/0.0\}, f_{12}(e_2) =$

$$\{x/0.0, y/0.0\} = \tilde{0}\}, f_{13E} = \{f_{13}(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, f_{13}(e_2) = \{x/0.0, y/0.6\}\},$$

Let $\sigma_1 = \{\tilde{0}_K, \tilde{1}_K, g_{1K}, g_{2K}, g_{3K}\}$, $\sigma_2 = \{\tilde{0}_K, \tilde{1}_K, g_{4K}, g_{5K}\}$, where

$$g_{1K} = \{g_1(e_1) = \{x/0.5, y/0.0\}, g_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\},$$

$$g_{2K} = \{g_2(e_1) = \{x/0.5, y/0.0\}, g_2(e_2) = \{x/0.3, y/0.0\}\},$$

$$g_{3K} = \{g_3(e_1) = \{x/0.5, y/0.7\}, g_3(e_2) = \{x/0.3, y/0.0\}\},$$

$$g_{4K} = \{g_4(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_4(e_2) = \{x/0.3, y/0.0\}\},$$

$$g_{5K} = \{g_5(e_1) = \{x/0.0, y/0.7\}, g_5(e_2) = \{x/0.3, y/0.0\} = \tilde{0}\},$$

It is clear that $(Y, K, \sigma_1, \sigma_2)$ is a fuzzy soft bitopological space. The $\sigma_1\sigma_2$ -fuzzy soft open

sets are $\{\tilde{0}_K, \tilde{1}_K, g_{1K}, g_{2K}, g_{3K}, g_{4K}, g_{5K}\}$

Now define $\phi : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $\psi : E \rightarrow K$ such that $\phi(x, \mu_{f(e_i)}(x)) = (x, \mu_{g(e_i)}(x))$, $\psi(e_i) = e_i$, $i = 1, 2$. Then $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft β -continuous but not $(1, 2)^*$ -fuzzy soft b -continuous, since $(\phi, \psi)^{-1}(g_{5K})$ is not $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) .

Remark 3.12 The above discussions are summarized in the following diagram:

Theorem 3.13 Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous, then $(\phi, \psi)((1, 2)^*\text{-}f\text{scl}(f_E)) \subseteq \tau_1\tau_2\text{f}scl((\phi, \psi)(f_E))$ for every f_E in (X, E, τ_1, τ_2) .

Proof. Assume that (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -continuous and f_E be any fuzzy soft subset in (X, E, τ_1, τ_2) . Then $\tau_1\tau_2\text{f}scl((\phi, \psi)(f_E))$ is a $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$. Since (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -continuous, $(\phi, \psi)^{-1}(\tau_1\tau_2\text{f}scl((\phi, \psi)(f_E)))$ is $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) and it contains f_E . But $(1, 2)^*\text{-}f\text{scl}(f_E)$ is the intersection of all $(1, 2)^*$ -fuzzy soft b -closed sets containing f_E . Therefore $(1, 2)^*\text{-}f\text{scl}(f_E) \subseteq (\phi, \psi)^{-1}(\tau_1\tau_2\text{f}scl((\phi, \psi)(f_E)))$ and so $(\phi, \psi)((1, 2)^*\text{-}f\text{scl}(f_E)) \subseteq \tau_1\tau_2\text{f}scl((\phi, \psi)(f_E))$ for every fuzzy soft subset f_E in (X, E, τ_1, τ_2) . ■

Remark 3.14 The composition of two $(1, 2)^*$ -fuzzy soft b -continuous maps need not be $(1, 2)^*$ -fuzzy soft b -continuous as is shown in the following example.

Example 3.15 Let $X = Y = Z = \{x, y\}$, $E = K = T = \{e_1, e_2\}$.

Let $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1E}\}$, $\tau_2 = \{\tilde{0}_E, \tilde{1}_E, f_{2E}\}$, where

$f_{1E} = \{f_1(e_1) = \{x/0.5, y/0.0\}, f_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$f_{2E} = \{f_2(e_1) = \{x/0.0, y/0.7\}, f_2(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

It is clear that (X, E, τ_1, τ_2) is a fuzzy soft bitopological space.

The $\tau_1\tau_2$ -fuzzy soft open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}\}$ where

$f_{3E} = f_{1E} \cup f_{2E} = \{f_3(e_1) = \{x/0.5, y/0.7\}, f_3(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$, and $(1, 2)^*$ -fuzzy

soft b -open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}\}$, where

f_{1E}, f_{2E}, f_{3E} are shown as above and

$f_{4E} = \{f_4(e_1) = \{x/0.5, y/0.0\}, f_4(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{5E} = \{f_5(e_1) = \{x/0.5, y/0.0\}, f_5(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{6E} = \{f_6(e_1) = \{x/0.5, y/0.0\}, f_6(e_2) = \{x/0.3, y/0.6\}\}$,

$f_{7E} = \{f_7(e_1) = \{x/0.0, y/0.7\}, f_7(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{8E} = \{f_8(e_1) = \{x/0.0, y/0.7\}, f_8(e_2) = \{x/0.0, y/0.6\}\}$,

$f_{9E} = \{f_9(e_1) = \{x/0.0, y/0.7\}, f_9(e_2) = \{x/0.3, y/0.6\}\}$,

$f_{10E} = \{f_{10}(e_1) = \{x/0.5, y/0.7\}, f_{10}(e_2) = \{x/0.3, y/0.0\}\}$,

$f_{11E} = \{f_{11}(e_1) = \{x/0.5, y/0.7\}, f_{11}(e_2) = \{x/0.0, y/0.6\}\}$.

Let $\sigma_1 = \{\tilde{0}_K, \tilde{1}_K, g_{1K}\}$, $\sigma_2 = \{\tilde{0}_K, \tilde{1}_K, g_{2K}\}$, where

$g_{1K} = \{g_1(e_1) = \{x/0.5, y/0.0\}, g_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$g_{2K} = \{g_2(e_1) = \{x/0.0, y/0.7\}, g_2(e_2) = \{x/0.3, y/0.6\}\}$,

It is clear that $(Y, K, \sigma_1, \sigma_2)$ is a fuzzy soft bitopological space. The $\sigma_1\sigma_2$ -fuzzy soft open sets are $\{\tilde{0}_K, \tilde{1}_K, g_{1K}, g_{2K}\}$ and $(1, 2)^*$ -fuzzy soft b -open sets are

$\{\tilde{0}_K, \tilde{1}_K, g_{1K}, g_{2K}, g_{3K}, g_{4K}, g_{5K}, g_{6K}, g_{7K}, g_{8K}, g_{9K}, g_{10K}, g_{11K}, g_{12K}, g_{13K}, g_{14K}\}$ where g_{1K}, g_{2K} are shown as above,

$g_{3K} = \{g_3(e_1) = \{x/0.5, y/0.7\}, g_3(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,

$g_{4K} = \{g_4(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_4(e_2) = \{x/0.3, y/0.0\}\}$,

$g_{5K} = \{g_5(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_5(e_2) = \{x/0.0, y/0.6\}\}$,

$g_{6K} = \{g_6(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_6(e_2) = \{x/0.3, y/0.6\}\}$,

$g_{7K} = \{g_7(e_1) = \{x/0.5, y/0.0\}, g_7(e_2) = \{x/0.3, y/0.0\}\}$,

$g_{8K} = \{g_8(e_1) = \{x/0.5, y/0.0\}, g_8(e_2) = \{x/0.0, y/0.6\}\}$,

$g_{9K} = \{g_9(e_1) = \{x/0.5, y/0.0\}, g_9(e_2) = \{x/0.3, y/0.6\}\}$,

$g_{10K} = \{g_{10}(e_1) = \{x/0.0, y/0.7\}, g_{10}(e_5) = \{x/0.3, y/0.0\}\},$
 $g_{11K} = \{g_{11}(e_{11}) = \{x/0.0, y/0.7\}, g_{11}(e_2) = \{x/0.0, y/0.6\}\},$
 $g_{12K} = \{g_{12}(e_1) = \{x/0.0, y/0.7\}, g_{12}(e_2) = \{x/0.3, y/0.6\}\},$
 $g_{13K} = \{g_{13}(e_1) = \{x/0.5, y/0.7\}, g_{13}(e_2) = \{x/0.3, y/0.0\}\},$
 $g_{14K} = \{g_{14}(e_1) = \{x/0.5, y/0.7\}, g_{14}(e_2) = \{x/0.0, y/0.6\}\}.$
 Let $\eta_1 = \{0_T, 1_T, h_{1T}, h_{2T}, h_{3T}\}, \eta_2 = \{\tilde{0}_T, \tilde{1}_T, h_{4T}, h_{5T}\},$ where
 $h_{1T} = \{h_1(e_1) = \{x/0.5, y/0.0\}, h_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\},$
 $h_{2T} = \{h_2(e_1) = \{x/0.5, y/0.0\}, h_2(e_2) = \{x/0.3, y/0.0\}\},$
 $h_{3T} = \{h_3(e_1) = \{x/0.5, y/0.7\}, h_3(e_2) = \{x/0.3, y/0.0\}\},$
 $h_{4T} = \{h_4(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, h_4(e_2) = \{x/0.3, y/0.0\}\},$
 $h_{5T} = \{h_5(e_1) = \{x/0.5, y/0.0\}, h_5(e_2) = \{x/0.0, y/0.6\} = \tilde{0}\}.$

It is clear that (Z, T, η_1, η_2) is a fuzzy soft bitopological space. The $\eta_1\eta_2$ -fuzzy soft open sets are $\{\tilde{0}_T, \tilde{1}_T, h_{1T}, h_{2T}, h_{3T}, h_{4T}, h_{5T}\}$ and $(1, 2)^*$ -fuzzy soft b -open sets are $\{\tilde{0}_T, \tilde{1}_T, h_{1T}, h_{2T}, h_{3T}, h_{4T}, h_{5T}, h_{6T}, H_{7T}, h_{8T}, H_{9T}\}$ where $h_{1T}, h_{2T}, h_{3T}, h_{4T}, h_{5T}$ are shown as above,

$h_{6T} = \{h_1(e_1) = \{x/0.5, y/0.0\}, h_1(e_2) = \{x/0.3, y/0.6\}\},$
 $h_{7T} = \{h_2(e_1) = \{x/0.0, y/0.7\}, h_2(e_2) = \{x/0.3, y/0.0\}\},$
 $h_{8T} = \{h_3(e_1) = \{x/0.0, y/0.7\}, h_3(e_2) = \{x/0.0, y/0.6\}\},$
 $h_{9T} = \{h_4(e_1) = \{x/0.0, y/0.7\}, h_4(e_2) = \{x/0.3, y/0.6\}\},$

and $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ and $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ are defined as identity mappings. Hence (ϕ, ψ) and $(\check{\phi}, \check{\psi})$ are $(1, 2)^*$ -fuzzy soft b -continuous since $((\check{\phi}, \check{\psi}) \circ (\phi, \psi))^{-1}(h_{5T})$ which is not a $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) .

Theorem 3.16 Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ and $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ be two maps in fuzzy soft bitopological spaces. Then $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous, if (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -continuous and $(\check{\phi}, \check{\psi})$ is $(1, 2)^*$ -fuzzy soft b -continuous.

Proof. Let h_T be a $\eta_1\eta_2$ -fuzzy soft closed set of $(Z, T, \sigma_1, \sigma_2)$. Since $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous, then $(\check{\phi}, \check{\psi})^{-1}(h_T)$ is $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$. Now $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous and $(\check{\phi}, \check{\psi})^{-1}(h_T)$ is $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$, then $(\phi, \psi)^{-1}[(\check{\phi}, \check{\psi})^{-1}(h_T)] = ((\check{\phi}, \check{\psi}) \circ (\phi, \psi))^{-1}(h_T)$ is $(1, 2)^*$ -fuzzy soft b -closed in (X, E, τ_1, τ_2) . Hence $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous. ■

4 $(1, 2)^*$ -fuzzy soft b -irresolute maps

In this section we introduce the concept of $(1, 2)^*$ -fuzzy soft b -irresolute maps and discuss the relation with $(1, 2)^*$ -fuzzy soft b -continuous maps.

Definition 4.1 A fuzzy soft mapping $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is said to be $(1, 2)^*$ -fuzzy soft b -irresolute mapping (briefly, $(1, 2)^*$ -fuzzy soft b -irresolute) if $(\phi, \psi)^{-1}(g_K)$ is a $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) for every $(1, 2)^*$ -fuzzy soft b -closed set g_K in $(Y, K, \sigma_1, \sigma_2)$.

K, σ_1, σ_2).

Theorem 4.2 A fuzzy soft mapping $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -irresolute mapping if and only if the inverse image of every $(1, 2)^*$ -fuzzy soft b -open set in $(Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) .

Proof. $\Rightarrow$: Let g_K be a $(1, 2)^*$ -fuzzy soft b -open set in $(Y, K, \sigma_1, \sigma_2)$. Then g_K^c is a $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$. Thus $(\phi, \psi)^{-1}(g_K^c)$ is a $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) . But $(\phi, \psi)^{-1}(g_K^c) = [(\phi, \psi)^{-1}(g_K)]^c$, so that $(\phi, \psi)^{-1}(g_K)$ is $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) .

$\Leftarrow$: Let g_K be a $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$. Then g_K^c is a $(1, 2)^*$ -fuzzy soft b -open set in $(Y, K, \sigma_1, \sigma_2)$. Thus $(\phi, \psi)^{-1}(g_K^c)$ is $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) . Also $(\phi, \psi)^{-1}(g_K^c) = [(\phi, \psi)^{-1}(g_K)]^c$. Thus $(\phi, \psi)^{-1}(g_K)$ is $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) . Therefore $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -irresolute mapping. ■

Theorem 4.3 Every $(1, 2)^*$ -fuzzy soft b -irresolute mapping is $(1, 2)^*$ -fuzzy soft b -continuous.

Proof. Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -fuzzy soft b -irresolute mapping. Let g_K be $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$, then g_K is a $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$. Since (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -irresolute mapping, $(\phi, \psi)(g_K)$ is $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) . Therefore (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -continuous. ■

The converse of the above theorem is not true as is shown in the following example.

Example 4.4 Consider the Example 3.3, $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous but not $(1, 2)^*$ -fuzzy soft b -irresolute mapping.

Theorem 4.5 Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ and $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ be two maps in fuzzy soft bitopological spaces. Then $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is

(i) $(1, 2)^*$ -fuzzy soft b -irresolute, if (ϕ, ψ) and $(\check{\phi}, \check{\psi})$ are $(1, 2)^*$ -fuzzy soft b -irresolute maps.

(ii) $(1, 2)^*$ -fuzzy soft b -continuous, if (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -irresolute and $(\check{\phi}, \check{\psi})$ is $(1, 2)^*$ -fuzzy soft b -continuous.

Proof. (i) Let $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -irresolute and h_T be $(1, 2)^*$ -fuzzy soft b -closed set of (Z, T, η_1, η_2) . Since $(\check{\phi}, \check{\psi})$ is $(1, 2)^*$ -fuzzy soft b -irresolute; $(\check{\phi}, \check{\psi})^{-1}(h_T)$ is $(1, 2)^*$ -fuzzy soft b -closed set of $(Y, K, \sigma_1, \sigma_2)$. Also $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -irresolute. So $(\phi, \psi)^{-1}[(\check{\phi}, \check{\psi})^{-1}(h_T)] = ((\check{\phi}, \check{\psi}) \circ (\phi, \psi))^{-1}(h_T)$ is $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) . Thus $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is a $(1, 2)^*$ -fuzzy soft b -irresolute.

(ii) Let h_T be a $\eta_1\eta_2$ -fuzzy soft closed subset of (Z, T, η_1, η_2) . Since $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous, then $(\check{\phi}, \check{\psi})^{-1}(h_T)$ is $(1, 2)^*$ -fuzzy soft b -closed set of $(Y, K, \sigma_1, \sigma_2)$. Also $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft

b -irresolute. So every $(1, 2)^*$ -fuzzy soft b -closed set of $(Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -closed set of (X, E, τ_1, τ_2) . Hence $(\phi, \psi)^{-1}[(\check{\phi}, \check{\psi})^{-1}(h_T)] = ((\check{\phi}, \check{\psi}) \circ (\phi, \psi))^{-1}(h_T)$ is $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) . Hence $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous. ■

5 $(1, 2)^*$ -fuzzy soft b -open(closed) maps

In this section, we introduce and discuss the characterization of $(1, 2)^*$ -fuzzy soft b -open(closed) maps and studied some of their properties.

Definition 5.1 A fuzzy soft map $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is said to be
(i) a $(1, 2)^*$ -fuzzy soft b -open (briefly, $(1, 2)^*$ -fsb-open) map if the image of every $(1, 2)^*$ -fuzzy soft b -open set in (X, E, τ_1, τ_2) is $(1, 2)^*$ -fuzzy soft b -open set in $(Y, K, \sigma_1, \sigma_2)$.
(ii) a $(1, 2)^*$ -fuzzy soft b -closed (briefly, $(1, 2)^*$ -fsb-closed) map if the image of every $(1, 2)^*$ -fuzzy soft b -closed set in (X, E, τ_1, τ_2) is $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$.

Theorem 5.2 If $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ is $(1, 2)^*$ -fuzzy soft b -closed map and $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -closed map. Then $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -closed map.

Proof. Let f_E be $(1, 2)^*$ -fuzzy soft closed set in (X, E, τ_1, τ_2) , $(\phi, \psi)(f_E)$ is $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$. Since $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -closed map, $(\check{\phi}, \check{\psi})((\phi, \psi)(f_E))$ is $(1, 2)^*$ -fuzzy soft closed set in (Z, T, η_1, η_2) and $(\check{\phi}, \check{\psi})((\phi, \psi)(f_E)) = (\check{\phi}, \check{\psi}) \circ (\phi, \psi)(f_E)$. Therefore $(\check{\phi}, \check{\psi}) \circ (\phi, \psi)$ is $(1, 2)^*$ -fuzzy soft b -closed map. ■

Theorem 5.3 Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ and $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ be two fuzzy soft maps such that $(\check{\phi}, \check{\psi}) \circ (\phi, \psi)$ is $(1, 2)^*$ -fuzzy soft b -closed map.

(i) If (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft continuous and surjective, then $(\check{\phi}, \check{\psi})$ is $(1, 2)^*$ -fuzzy b -closed map.

(ii) If $(\check{\phi}, \check{\psi})$ is $(1, 2)^*$ -fuzzy soft b -irresolute and injective, then (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -closed map.

Proof. (i) Let g_K be $\sigma_1\sigma_2$ -fuzzy soft closed set in $(Y, K, \sigma_1, \sigma_2)$. If (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft continuous and surjective, then $(\phi, \psi)^{-1}(g_K)$ is a $\tau_1\tau_2$ -fuzzy soft closed set in (X, E, τ_1, τ_2) . Since $(\check{\phi}, \check{\psi}) \circ (\phi, \psi)$ is $(1, 2)^*$ -fuzzy soft b -closed map, $(\check{\phi}, \check{\psi}) \circ (\phi, \psi)((\phi, \psi)^{-1}(g_K)) = (\check{\phi}, \check{\psi})(g_K)$ is $(1, 2)^*$ -fuzzy soft b -closed map in (Z, T, η_1, η_2) . Hence $(\check{\phi}, \check{\psi}) : (Y, K, \sigma_1, \sigma_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -closed map.
(ii) Let f_E be $\tau_1\tau_2$ -fuzzy soft closed subset of (X, E, τ_1, τ_2) . If $(\check{\phi}, \check{\psi}) \circ (\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Z, T, \eta_1, \eta_2)$ is $(1, 2)^*$ -fuzzy soft b -closed map; then $(\check{\phi}, \check{\psi}) \circ (\phi, \psi)(f_E)$ is a $(1, 2)^*$ -fuzzy soft b -closed set in (Z, T, η_1, η_2) , and so $(\check{\phi}, \check{\psi})^{-1}((\check{\phi}, \check{\psi}) \circ (\phi, \psi)(f_E)) = (\phi, \psi)(f_E)$ is $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$. Since $(\check{\phi}, \check{\psi})$ is $(1, 2)^*$ -fuzzy soft b -irresolute and injective, then (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -closed map. ■

Proposition 5.4 For any bijection fuzzy soft map $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ the following statements are equivalent.

(i) The inverse fuzzy soft map $(\phi, \psi)^{-1} : (Y, K, \sigma_1, \sigma_2) \rightarrow (X, E, \tau_1, \tau_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous.

(ii) (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -open map.

(iii) (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -closed map.

Proof. (i) $\Rightarrow$ (ii): Given that $(\phi, \psi)^{-1} : (Y, K, \sigma_1, \sigma_2) \rightarrow (X, E, \tau_1, \tau_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous. Let f_E be a $\tau_1\tau_2$ -fuzzy soft open set in (X, E, τ_1, τ_2) . Then $((\phi, \psi)^{-1})^{-1}(f_E) = (\phi, \psi)(f_E)$ is $(1, 2)^*$ -fuzzy soft b -open set in $(Y, K, \sigma_1, \sigma_2)$.

(ii) $\Rightarrow$ (iii): Let $(\phi, \psi) : (X, E, \tau_1, \tau_2) \rightarrow (Y, K, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -fuzzy soft b -open map. Let f_E be a $\tau_1\tau_2$ -fuzzy soft open set in (X, E, τ_1, τ_2) . Then f_E^c be a $\tau_1\tau_2$ -fuzzy soft closed set in (X, E, τ_1, τ_2) . Thus $(\phi, \psi)(f_E^c)$ is $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$. Therefore (ϕ, ψ) is $(1, 2)^*$ -fuzzy soft b -closed map.

(iii) $\Rightarrow$ (i): Let (ϕ, ψ) is a $(1, 2)^*$ -fuzzy soft b -closed map. Let f_E be a $\tau_1\tau_2$ -fuzzy soft closed set in (X, E, τ_1, τ_2) , then $((\phi, \psi)^{-1})^{-1}(f_E) = (\phi, \psi)(f_E)$ is $(1, 2)^*$ -fuzzy soft b -closed set in $(Y, K, \sigma_1, \sigma_2)$. Therefore $(\phi, \psi)^{-1} : (Y, K, \sigma_1, \sigma_2) \rightarrow (X, E, \tau_1, \tau_2)$ is $(1, 2)^*$ -fuzzy soft b -continuous. ■

6 Conclusion

In this paper, we introduced the concepts of $(1, 2)^*$ -fuzzy soft b -continuous maps, $(1, 2)^*$ -fuzzy soft b -irresolute maps and the relations with other weak forms of fuzzy soft continuous maps in fuzzy soft bitopological spaces. Also we introduced and investigated the basic properties of $(1, 2)^*$ -fuzzy soft b -irresolute map and $(1, 2)^*$ -fuzzy soft b -open (closed) maps. We hope that this paper will be important for researchers to studying many other concepts and also the generalization for some important results in topology.

Compliance with ethical standards

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