Supplementary document to the paper "Optimal Offering of a Thermal Power Producer in Electricity Markets with Complex Block Orders"

Appendix A
Unit Commitment Model

This Appendix presents the detailed model of the unit commitment (UC) constraints and operating cost function of the thermal power producer examined in our paper, in which these constraints and cost function are presented in a compact form (through the set Π^{UC} and the function $C(p_{h,s}^{\text{SELL}})$), for space limitation reasons. This model is based on the approach of [1] and [2]. In order to comply with the stochastic framework of our paper, the feasibility of these constraints is independently respected per each price scenario s.

The power sold by the examined producer at hour h and scenario s is:

$$p_{h,s}^{\text{SELL}} = \sum_{q} p_{q,h,s}^{\text{BLK}}, \forall h, s, \tag{1}$$

where $p_{q,h,s}^{\text{BLK}}$ is the power output from block q of the producer at hour h and scenario s. The limits of this variable are:

$$0 \leq p_{q,h,s}^{\text{BLK}} \leq u_{h,s}^{\text{GEN}} \cdot P_{q}^{\text{MAX}}, \forall q, h, s, \tag{2}$$

where P_{q}^{MAX} is the maximum production limit of block q and $u_{h,s}^{\text{GEN}}$ is a binary variable expressing the UC status of the producer at hour h and scenario s (1 if it is on and 0 if it is off). The minimum stable generation (expressed by the parameter P^{MIN}) constraints of the examined producer are expressed by:

$$P^{\text{MIN}} \cdot u_{h,s}^{\text{GEN}} \leq \sum_{q} p_{q,h,s}^{\text{BLK}}, \forall h, s. \tag{3}$$

The ramp-down and ramp-up constraints of the examined producer are expressed by:

$$-R^{\text{DN}} \leq p_{h,s}^{\text{GEN}} - p_{h-1,s}^{\text{GEN}} \leq R^{\text{UP}}, \forall h, s, \tag{4}$$

where R^{DN} and R^{UP} represent the ramp-down and ramp-up rates of the examined producer. The start-up and shut-down cost constraints of the examined producer are expressed by:

$$C_{h,s}^{\text{UP}} \geq 0, \forall h, s, \tag{5}$$

$$C_{h,s}^{\text{DN}} \geq 0, \forall h, s, \tag{6}$$

$$C_{h,s}^{\text{UP}} \geq (u_{h,s}^{\text{GEN}} - u_{h-1,s}^{\text{GEN}}) \cdot K^{\text{UP}}, \forall h, s, \tag{7}$$

$$C_{h,s}^{\text{DN}} \geq (u_{h-1,s}^{\text{GEN}} - u_{h,s}^{\text{GEN}}) \cdot K^{\text{DN}}, \forall h, s, \tag{8}$$

where K^{UP} and K^{DN} represent the start-up and shut-down costs of the examined producer, while $C_{h,s}^{\text{UP}}$ and $C_{h,s}^{\text{DN}}$ are auxiliary variables expressing the start-up and shut-down costs incurred at hour h and scenario s. The minimum-up and minimum-down time constraints of the examined producer are expressed by:

$$\sum_{h=1}^{H^{\text{UP}}} (1 - u_{h,s}^{\text{GEN}}) = 0, \forall s, \tag{9}$$

$$\sum_{h=1}^{H^{\text{DN}}} u_{h,s}^{\text{GEN}} = 0, \forall s, \tag{10}$$

$$T^{\text{UP}} \cdot (u_{h,s}^{\text{GEN}} - u_{h-1,s}^{\text{GEN}}) \leq \sum_{j=h}^{h+T^{\text{UP}}-1} u_{j,s}^{\text{GEN}}, \quad \forall s, h \in [H^{\text{UP}}+1, ..., |H| - T^{\text{UP}} + 1], \tag{11}$$

$$-T^{\text{DN}} \cdot (u_{h,s}^{\text{GEN}} - u_{h-1,s}^{\text{GEN}}) \leq \sum_{j=h}^{h+T^{\text{DN}}-1} (1 - u_{j,s}^{\text{GEN}}), \quad \forall s, h \in [H^{\text{DN}}+1, ..., |H| - T^{\text{DN}} + 1], \tag{12}$$

$$\sum_{j=h}^{|H|} u_{j,s}^{\text{GEN}} - (u_{h,s}^{\text{GEN}} - u_{h-1,s}^{\text{GEN}}) \geq 0, \quad \forall s, h \in [|H| - T^{\text{UP}} + 2, ..., |H|], \tag{13}$$

$$\sum_{j=h}^{|H|} 1 - u_{j,s}^{\text{GEN}} - (u_{h-1,s}^{\text{GEN}} - u_{h,s}^{\text{GEN}}) \geq 0, \quad \forall s, h \in [|H| - T^{\text{DN}} + 2, ..., |H|], \tag{14}$$

where T^{UP} and T^{DN} represent the minimum-up and minimum-down time limits of the examined producer, while H^{UP} and H^{DN} express the number of hours the producer must initially remain on or off. The overall operating cost function of the examined producer is expressed by:

$$C(p_{h,s}^{\text{SELL}}) = \sum_{q} mc_{q}^{\text{G}} \cdot p_{q,h,s}^{\text{BLK}} + f^{\text{NL}} \cdot u_{h,s}^{\text{GEN}} + C_{h,s}^{\text{UP}} + C_{h,s}^{\text{DN}}, \forall h, s, \tag{15}$$

with the first term corresponding to the marginal cost (where mc_{q}^{G} is the marginal cost of block q), the second term corresponding to the no-load cost (including the no-load cost parameter f^{NL}), the third term corresponding to the start-up cost, and the fourth term corresponding to the shut-down cost.

APPENDIX B
CALCULATION OF NECESSARY NUMBER OF MODELED BLOCK ORDERS IN SECTION II-E

As discussed in our paper, when linked block orders (LBO) are included in the examined problem, we model more block orders than the market-regulated maximum number, i.e., $|B| > \chi^{\text{max}}$, in order to represent the augmented solution space. This becomes necessary in order to account for the fact that the optimal set of offered block orders may include only non-linked block orders (generally including regular and profile block orders) or any possible combination of parent-child bundles. This Appendix presents the calculation of this necessary number of modeled block orders $|B|$.

For a generic case where the maximum allowable number of child blocks per parent block is expressed by the parameter c, this calculation entails the following considerations:

- χ^{max} block orders should be modeled as potential parent blocks, linked to at least 0 child blocks, to account for the possibility that the optimal set of offered block orders includes only non-linked block orders,
- ψ_1 block orders should be modeled as potential parent blocks, linked to at least 1 child block, where $\psi_1 = \chi^{\text{max}}$ $\mathbf{div}$ 2,
- ...
- ψ_{c-1} block orders should be modeled as potential parent blocks, linked to at least c-1 child blocks, where $\psi_{c-1} = \chi^{\text{max}}$ $\mathbf{div}$ (c),
- ψ_c block orders should be modeled as potential parent blocks, linked to at least c child blocks, where $\psi_c = \chi^{\text{max}}$ $\mathbf{div}$ $(c+1)$.

In this context, ψ_c out of the χ^{max} potential parent blocks are linked to c child blocks, $(\psi_{c-1} - \psi_c)$ parent blocks are linked to $c-1$ child blocks, ... , $(\psi_1 - \psi_2)$ parent blocks are linked to 1 child block, and the residual $(\chi^{\text{max}} - \psi_1)$ parent blocks are not linked to any child blocks. Therefore, the total number of block orders we need to model is $|B|$ = $\left|B^{\text{PNT}}\right| + \sum_b \left|O_b^{\text{CH}}\right|$, where $\left|B^{\text{PNT}}\right| = \chi^{\text{max}}$ and $\sum_b \left|O_b^{\text{CH}}\right| = c \cdot \psi_c + (c-1) \cdot (\psi_{c-1} - \psi_c) + ... + 1 \cdot (\psi_1 - \psi_2)$. Considering for example the values of the market-regulated parameters assumed in the case studies of our paper ($\chi^{\text{max}} = 8$ and $c = 3$), the necessary number of modeled block orders is $|B| = 16$.

REFERENCES

[1] M. Carrión and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, 2006.

[2] Y. Ye, D. Papadaskalopoulos, J. Kazempour, and G. Strbac, "Incorporating Non-Convex Operating Characteristics into Bi-Level Optimization Electricity Market Models," *IEEE Transactions on Power Systems*, vol. 35, no. 1, pp. 163–176, 2020.