Model-Aware Collision Resolution for High-Order Orthogonal Modulations

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Abstract

A novel method is proposed to resolve collisions in slotted-ALOHA random access schemes that rely on orthogonal modulations at the physical layer, a common choice in wireless sensor networks. The algorithm includes solutions from machine learning, regularized optimization and transportation theory to exploit the symmetry structure that characterizes the signal model at a reasonable computational cost. Simulation results corroborate the effectiveness of the new method as compared to previous solutions.

Index Terms

Collision resolution, orthogonal modulations, clustering, optimal transport, norm-1 regularization, ADMM.

I. INTRODUCTION

The Internet of Things has brought a renewed interest in Wireless Sensor Networks (WSNs): A plethora of new applications call for a comprehensive and unrelenting deployment of sensors that measure all aspects of our lives without the burden of cables. A large amount of research effort is thus being devoted to providing sensors with a reliable wireless link to the central unit while minimizing energy consumption and, in turn, maximizing battery duration [1].

In this context, orthogonal modulations represent a valuable solution due to their robustness against noise and the possibility to design simple, noncoherent detection schemes that do not require the presence of pilots for channel estimation, thus reducing overhead and transmitted energy. Note that, in WSNs, the inherently low spectral efficiency of orthogonal modulations is usually not a major drawback, due to the low information rate required by the sensors. As a matter of fact, a number of current WSN systems employ orthogonal modulations at the physical layer. For instance, Semtech widely used LoRa long-range wireless communications protocol is based on a chirp spread spectrum modulation [2].

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A critical weakness of orthogonal modulations is that, despite the extra degrees of freedom used during transmission, no characteristic signature is assigned to the sources for their identification. As a result, collisions in multiple access channels are a knotty problem that leads to packet loss and requires the implementation of an acknowledgment–retransmission scheme, which impinges on the battery life. To mitigate this issue, we propose an algorithm that leverages tools from multiple disciplines (machine learning, regularization and transportation theory) to reduce the complexity of an exhaustive search for the transmitted messages in a multiple access scenario.

Notation: Bold upper- and lower-case letters denote matrix and vector quantities, respectively. Superscripts T and H stand for regular and Hermitian transpose of a matrix, respectively. $\mathbf{I}_X$ is the identity matrix of size X , while $\mathbf{1}_X$ is a vector of length X with all entries equal to one. $\|\cdot\|_F$ denotes the Frobenius norm. For any complex-valued vector $\mathbf{x}$, its norm-1 is defined as the sum of its elements’ magnitudes, namely $\|\mathbf{x}\|_1 = \sum_i |x_i|$.

II. SIGNAL MODEL

A. Orthogonal Signaling

We consider a slotted-ALOHA random access scheme supported by an orthogonal modulation at the physical layer. In M -ary orthogonal signaling, each transmitted symbol occupies only one dimension in the M -dimensional signal space, according to the symbol value. For instance, the symbol value determines the frequency profile in chirp spread spectrum modulation, the transmission tone in frequency-shift keying modulation, or the pseudo-noise code in direct-sequence spread spectrum modulation. In order to stress this fact, transmitted symbols are often represented as one-hot vectors of size M . More specifically, if $s^k[n] \in \{1, 2, \dots, M\}$ denotes the symbol transmitted by device k at time n , then the corresponding symbol vector is $\mathbf{x}^k[n] = \mathbf{e}_{s^k[n]}$, with $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_M\}$ the Euclidean canonical basis. In what follows we will also assume $n \in \{1, 2, \dots, N\}$ and $k \in \{1, 2, \dots, K\}$, with N the total number of symbols in an access slot and K the number of sources transmitting during the considered slot.

At the receiver side, a bank of M correlators extracts the components corresponding to each signal dimension, resulting in the M -dimensional complex signal vector

$$\mathbf{y}[n] = \sum_{k=1}^K h^k \mathbf{x}^k[n] + \mathbf{w}[n] \quad (1)$$

where $h^k \in \mathbb{C}$ denotes the channel coefficient of source k and $\mathbf{w}[n]$ are i.i.d. additive white Gaussian noise (AWGN) vectors with $\mathbf{w}[n] \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$. Note that we are assuming that the channel effect is the same over all signal dimensions and over all symbols in an access slot. This frequency flat, block fading channel model is well justified in, e.g., sensor networks, where terminals are static and transmissions are narrowband (that is, with low-throughput traffic characteristics).

B. Detection: General Considerations

When a single device is transmitting, i.e. $K = 1$, one readily sees that the signal model in (1) is suitable for a noncoherent detection scheme. Indeed, the entries of signal vector $\mathbf{y}[n]$ are given by

$$y_i[n] = \begin{cases} h^1 + w_i[n] & \text{if } i = s^1[n] \\ w_i[n] & \text{otherwise} \end{cases}$$

Fig. 1. Symbol vectors $\mathbf{y}[n]$ are permutations of one another (top). Message detection is equivalent to aligning the signal vectors while keeping track of the required moves among elements (bottom). Symbol collisions may cause ambiguities.

and the receiver can detect the transmitted symbol by picking the entry with the highest magnitude:

$$\hat{s}^1[n] = \arg \max_i |y_i[n]|.$$

It can be proven [3, Section 3.8] that this scheme corresponds to the optimal maximum likelihood (ML) detector regardless of the channel distribution.

Conversely, when the received signal is the combination of multiple device contributions, noncoherent detection does not work anymore since the different sources cannot be separated by simply analyzing the magnitudes of the correlator outputs. The next section introduces a novel blind approach that exploits the assumption that the channels are constant over the entire slot duration and along all signal dimensions [cf. (1)].

III. PROPOSED ALGORITHM

A. Intuition and Main Ideas

In order to grasp the intuition behind the detection algorithm proposed next, let us consider first the noiseless case. From (1), it is straightforward to realize that signal vectors $\mathbf{y}[n]$ are permutations of one another when $\mathbf{w}[n] = 0$, $n = 1, 2, \dots, N$. More specifically, each vector $\mathbf{y}[n]$ has up to K nonzero entries, whose values correspond to the channel gains of the K transmitting sources. The position of the active entries is given by the symbol transmitted by the corresponding source. (Exceptions may occur when two or more devices transmit the same symbol during time n , that is $s^k[n] = s^{k'}[n]$ for some n and $k \neq k'$, as commented in more detail later on.) Fig. 1 provides a graphic representation of this property. The problem of detecting all transmitting sources is equivalent to the one of aligning the signal vectors, that is to matching the vector entries that correspond to the same source. According to the representation of Fig. 1, aligned signals show each source on a single row. By keeping track of the required permutations, one can readily recover the transmitted symbol sequences.

Note that, even if $M!$ possible permutations exist, the alignment of two M -long vectors can be cast as an optimal transport problem for which efficient algorithms have been designed [4]. Specifically, we will employ the

Hungarian method [5], which solves the alignment problem in polynomial time $O(M^3)$. In the next subsection we will see how the dimension of the search space can be reduced from M to a much lower number exploiting the fact that most of the signal vector entries are filled with noise only.

The alignment process can be considerably impaired by the presence of additive noise, especially in sensor networks where the signal-to-noise ratio is typically very low. Robustness can be gained by properly choosing the order in which signal vectors $\mathbf{y}[n]$ are aligned. Intuitively, one should choose the vector that most clearly aligns to the already processed ones.

Inspired by hierarchical clustering, a classic machine-learning algorithm [6, Chapter 14], we propose here an algorithm that incrementally builds a cluster of aligned vectors by including, at each iteration, the closest vector to the current cluster according to a newly defined distance measure. We will also show that, as the cluster dimension increases, the centroid of the cluster gives a more refined estimation of the channel coefficients.

Concerning the signal vectors where a symbol collision happens (see Fig. 1), they obviously cannot be aligned to match the other vectors. The probability of such a collision increases with the number of transmitting sources or as we reduce the modulation order. Nonetheless, we can assume that collisions are rare in the targeted WSN protocols, where sensor data packets are sporadic and where the modulation size is rather high (in LoRa, e.g., $M = 2^{SF}$, with $SF \in \{6, 7, \dots, 12\}$). For instance, when $M = 128$, the probability of symbol collision is 0.008 with two sources and 0.023 with three sources.

Finally, it is worth mentioning that our approach shows some analogies with the one presented in [7], where classic maximum-likelihood coherent detection is preceded by a clustering-based channel-estimation phase. However, in [7], clusters are built from the separate entries of the signal vectors, as opposed to aligning the entire vectors. This choice, which does not exploit the inner structure of the signal, is less robust to noise.

B. Algorithm Details

The first step towards the formalization of the proposed algorithm is the definition of the distance¹ measure used for clustering.

Definition 1: Let $\mathbf{Y}_1 = [\mathbf{y}[n_{1,1}] \ \mathbf{y}[n_{1,2}] \ \dots \ \mathbf{y}[n_{1,N_1}]]$ and $\mathbf{Y}_2 = [\mathbf{y}[n_{2,1}] \ \mathbf{y}[n_{2,2}] \ \dots \ \mathbf{y}[n_{2,N_2}]]$ be two clusters of signal vectors of size N_1 and N_2 , respectively, with $n_{i,j} \in \{1, 2, \dots, N\}$. Also, let $\mathbf{Y}_1$ and $\mathbf{Y}_2$ be disjoint, i.e. $\{n_{1,i}\}_{i=1}^{N_1} \cap \{n_{2,i}\}_{i=1}^{N_2} = \emptyset$. Hereafter, the distance between $\mathbf{Y}_1$ and $\mathbf{Y}_2$ is defined as

$$d_c(\mathbf{Y}_1, \mathbf{Y}_2) = \min_{\mathbf{h} \in \mathbb{C}^M, \mathbf{P} \in \mathcal{P}_M} \left\| \begin{bmatrix} \mathbf{Y}_1 & \mathbf{P}\mathbf{Y}_2 \end{bmatrix} - \mathbf{h}\mathbf{1}_{N_1+N_2}^T \right\|_F^2 \quad (2)$$

s.to $\|\mathbf{h}\|_1 \leq c$

for some real $c > 0$ and where $\mathcal{P}_M$ is the set of $M \times M$ permutation matrices (i.e., all matrices whose rows and columns have a single element equal to one and all others are equal to zero).

In plain words, the distance between two clusters is measured in terms of the best mean squared error that can be achieved when a rank-1 matrix approximates the combination of the two clusters after alignment (recall the signal structure depicted in Fig. 1). The regularization constraint $\|\mathbf{h}\|_1 \leq c$ has been introduced to promote a sparse solution: Indeed, by tuning the value of $c > 0$, we can control the number of nonzero elements of

¹Note that the term “distance” is used in a loose informal sense and not in the rigorous mathematical one.

the estimate $\hat{\mathbf{h}}$ [8]. This trick allows us to make the solution aware that only few sources are colliding, thus diminishing the harmful effect of the noise.

The optimization problem in (2) can be solved very efficiently as shown in Section III-C. Before that, the following remarks will motivate our choice and give more insight into the corresponding detection algorithm.

First of all, in the noiseless case, there certainly exists $\mathbf{P}_{n_1, n_2} \in \mathcal{P}_M$ such that $\begin{bmatrix} \mathbf{y}[n_1] & \mathbf{P}_{n_1, n_2} \mathbf{y}[n_2] \end{bmatrix} = \mathbf{y}[n_1] \mathbf{1}_2^T$, for all pairs (n_1, n_2) . This fact remains true when we replace $\mathbf{y}[n_1]$ and $\mathbf{y}[n_2]$ by two clusters of aligned signal vectors. Hence, the (unconstrained) minimization problem follows straightforwardly.

The second remark is about the regularization constraint $\|\mathbf{h}\|_1 \leq c$, for some real $c > 0$. Besides its role of controlling the number of detected sources, as commented before, this assumption can also be exploited to speed up the computation. Indeed, once sparsity is in place, one can reduce the permutation space from $\mathcal{P}_M$ to $\mathcal{P}_{2\hat{K}}$, where $\hat{K} \ll M$ is the number of nonzero entries allowed by the regularizer (see further Section III-C for all the details).

Finally, assume that $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are clusters of aligned signal vectors. Also, let $\hat{\mathbf{h}}$ and $\hat{\mathbf{P}}$ denote the optimal point of the minimization problem in (2). Then, $\begin{bmatrix} \mathbf{Y}_1 & \hat{\mathbf{P}} \mathbf{Y}_2 \end{bmatrix}$ is also a cluster of aligned signal vectors. Moreover, $\hat{\mathbf{h}}$ is the (sparse) minimum mean square error estimate of the channel coefficients.

With our definition of distance between clusters of signal vectors, which also leads to the permutation required to align two clusters (each vector can be considered as a singleton cluster), Algorithm 1 is a natural solution to the collision resolution problem. This algorithm formalizes the approach loosely introduced in Section III-A: At each iteration, the cluster of aligned vectors is enlarged by including the closest signal in terms of the distance in (2). The first cluster is obtained by pairing the two signals that are closest to one another. The algorithm halts when all signals have been processed and returns the detected messages as well as an estimate of the channel coefficients.

Indeed, if no error is introduced in the computation of the distances, cluster $\hat{\mathbf{Y}}$ at step 10 is aligned, meaning that each source corresponds to a single row of matrix $\hat{\mathbf{Y}}$. This justifies the proposed estimation of the channel coefficients as the row-wise average of the matrix entries, which corresponds to the minimum mean square error (MMSE) estimator (assuming there are no alignment errors). For all indices $n \in \{1, 2, \dots, N\}$, the transmitted symbols can be recovered by tracking back the source rows to their original positions according to permutation matrices $\mathbf{P}_{n_1, n_j}$ (note that all permutations are relative to the first selected symbol vector $\mathbf{y}[n_1]$).

C. Distance Computation and Vector Alignment (Steps 1 and 7 of Algorithm 1)

According to Definition 1, the distance between two clusters can be computed by solving minimization problem (2). Note that, when $c = +\infty$ (that is, no constraint is imposed), the solution can be written as

$$\hat{\mathbf{P}} = \arg \min_{\mathbf{P}} \text{tr} \left(\mathbf{P} \mathbb{R}e \left\{ -\bar{\mathbf{y}}_2 \bar{\mathbf{y}}_1^H \right\} \right) \quad \hat{\mathbf{h}} = \alpha_1 \bar{\mathbf{y}}_1 + \alpha_2 \hat{\mathbf{P}} \bar{\mathbf{y}}_2$$

where $\alpha_i = N_i / (N_1 + N_2)$ and where $\bar{\mathbf{y}}_i = N_i^{-1} \mathbf{Y}_i \mathbf{1}_{N_i}$ is the centroid of cluster i . One readily sees that the minimization in $\mathbf{P}$ is an assignment problem that matches pairs of elements in $\bar{\mathbf{y}}_1$ and $\bar{\mathbf{y}}_2$ to maximize the real part of the resulting inner product $\mathbb{R}e\{\bar{\mathbf{y}}_1^H \mathbf{P} \bar{\mathbf{y}}_2\}$. Even though this is a combinatorial problem, it is well known that it can be solved in polynomial time $O(M^3)$ by means of the Hungarian method [4], [5]. However, a complexity of order $O(M^3)$ can still be too high in our case, given the modulation order M in practical

Algorithm 1 Proposed detection algorithm.

- 1: compute the distances $d_c(\mathbf{y}[n], \mathbf{y}[n'])$ according to (2) for all possible pairs $n \neq n'$ and record the corresponding optimal permutation $\mathbf{P}_{n,n'}$
 - 2: form the cluster $\hat{\mathbf{Y}} = [\mathbf{y}[n_1] \quad \mathbf{P}_{n_1,n_2}\mathbf{y}[n_2]]$, where (n_1, n_2) is the pair of indices corresponding to the signal vectors with minimum distance to one another
 - 3: set $\mathbf{y}[n_1]$ as the alignment reference vector (and $\mathbf{P}_{n_1,n_1} = \mathbf{I}_M$)
 - 4: store $\mathbf{P}_{n_1,n_2}$ $\triangleright$ alignment of $\mathbf{y}[n_2]$ w.r.t. $\mathbf{y}[n_1]$
 - 5: $\Omega \leftarrow \{1, 2, \dots, N\} \setminus \{n_1, n_2\}$
 - 6: **for all** $i \in \{3, 4, \dots, N\}$ **do**
 - 7: compute the distances $d_c(\hat{\mathbf{Y}}, \mathbf{y}[n])$ between the current cluster $\hat{\mathbf{Y}}$ and all the unused signal vectors $\mathbf{y}[n]$, $n \in \Omega$, according to (2), and record the corresponding $\mathbf{P}_{n_1,n}$
 - 8: let n_i be the index of the closest vector to cluster $\hat{\mathbf{Y}}$
 - 9: store $\mathbf{P}_{n_1,n_i}$ $\triangleright$ alignment of $\mathbf{y}[n_i]$ w.r.t. $\mathbf{y}[n_1]$
 - 10: $\hat{\mathbf{Y}} \leftarrow [\hat{\mathbf{Y}} \quad \mathbf{P}_{n_1,n_i}\mathbf{y}[n_i]]$
 - 11: $\Omega \leftarrow \Omega \setminus \{n_i\}$
 - 12: **end for**
 - 13: $\hat{\mathbf{h}} \leftarrow \frac{1}{N} \hat{\mathbf{Y}} \mathbf{1}_N$ $\triangleright$ channel MMSE estimation
 - 14: estimate the number of active sources from the magnitudes of the entries of $\hat{\mathbf{h}}$
 - 15: the transmitted codewords can be reconstructed from the rows corresponding to the active sources of all permutation matrices $\mathbf{P}_{n_1,n_i}$ used to build the final cluster $\hat{\mathbf{Y}}$
-

modulations (e.g., up to 4096 for LoRa) and the number of distances that are computed to align the entire message. The iterative approach outlined in this section builds on the signal structure to reduce complexity significantly.

Recall that the square of the Frobenius norm is separable along the columns of the argument matrix and invariant with respect to permutations of its rows. Then, it is straightforward to see that problem (2) is equivalent to the following consensus problem:

$$\begin{aligned}
 & \underset{\mathbf{g}, \mathbf{h} \in \mathbb{C}^M; \mathbf{P} \in \mathcal{P}_M}{\text{minimize}} \quad \|\mathbf{Y}_1 - \mathbf{h} \mathbf{1}_{N_1}^T\|_F^2 + \|\mathbf{Y}_2 - \mathbf{g} \mathbf{1}_{N_2}^T\|_F^2 \\
 & \text{s.to} \quad \|\mathbf{h}\|_1 \leq c, \quad \|\mathbf{g}\|_1 \leq c, \quad \mathbf{h} = \mathbf{P} \mathbf{g}.
 \end{aligned}$$

Note that the objective function consists of two convex terms, each of them depending on either one of the two variables $\mathbf{h}$ and $\mathbf{g}$, which are then coupled by the affine (for fixed $\mathbf{P}$) constraint $\mathbf{h} = \mathbf{P} \mathbf{g}$. The regularization constraints $\|\mathbf{h}\|_1 \leq c$ and $\|\mathbf{g}\|_1 \leq c$ are also convex. Then, the minimization problem in $(\mathbf{h}, \mathbf{g})$ is in the standard form for applying the alternating direction method of multipliers (ADMM) [9], which iterates between a convex minimization problem in $\mathbf{h}$, a convex minimization problem in $\mathbf{g}$ and, finally, an update of the dual variable (denoted by $\mathbf{u}$ hereafter) that drives both $\mathbf{h}$ and $\mathbf{g}$ towards a primal feasible point where $\mathbf{h} = \mathbf{P} \mathbf{g}$.

On the other hand, given two vectors $\mathbf{h}$ and $\mathbf{g}$, finding the permutation matrix $\mathbf{P}$ that achieves the best possible consensus $\mathbf{h} \approx \mathbf{P} \mathbf{g}$ is again an assignment problem that can be solved by the Hungarian method.

These observations suggest that we can build a set of ADMM-inspired iterations to search alternatively between variables $(\mathbf{h}, \mathbf{g})$ and $\mathbf{P}$. Specifically, we first need to relax the consensus constraint and build the resulting augmented Lagrangian. For any penalty parameter $\rho > 0$, the augmented Lagrangian of our minimization problem is defined as

$$\mathcal{L}_\rho(\mathbf{h}, \mathbf{g}, \mathbf{P}, \mathbf{u}) = \|\mathbf{Y}_1 - \mathbf{h}\mathbf{1}_{N_1}^T\|_F^2 + \|\mathbf{Y}_2 - \mathbf{g}\mathbf{1}_{N_2}^T\|_F^2 + \rho\|\mathbf{h} - \mathbf{P}\mathbf{g}\|_2^2 + 2\rho\Re\{\mathbf{u}^H(\mathbf{h} - \mathbf{P}\mathbf{g})\}.$$

Next, the Lagrangian is minimized with respect to each variable while keeping the other ones fixed. The corresponding iterations, indexed by t , thus read

$$\begin{aligned}\mathbf{h}^{t+1} &= \arg \min_{\mathbf{h} \in \mathbb{C}^M} \mathcal{L}_\rho(\mathbf{h}, \mathbf{g}^t, \mathbf{P}^t, \mathbf{u}^t) \quad \text{s. to } \|\mathbf{h}\|_1 \leq c \\ \mathbf{g}^{t+1} &= \arg \min_{\mathbf{g} \in \mathbb{C}^M} \mathcal{L}_\rho(\mathbf{h}^{t+1}, \mathbf{g}, \mathbf{P}^t, \mathbf{u}^t) \quad \text{s. to } \|\mathbf{g}\|_1 \leq c \\ \mathbf{P}^{t+1} &= \arg \min_{\mathbf{P} \in \mathcal{P}_M} \mathcal{L}_\rho(\mathbf{h}^{t+1}, \mathbf{g}^{t+1}, \mathbf{P}, \mathbf{u}^t) \\ \mathbf{u}^{t+1} &= \mathbf{u}^t + \mathbf{h}^{t+1} - \mathbf{P}^{t+1}\mathbf{g}^{t+1}.\end{aligned}$$

After some algebra, they can be shown to be equivalent to

$$\begin{aligned}\mathbf{h}^{t+1} &= \arg \min_{\substack{\mathbf{h} \in \mathbb{C}^M \\ \|\mathbf{h}\|_1 \leq c}} \left\| \mathbf{h} - \frac{N_1 \bar{\mathbf{y}}_1 - \rho(\mathbf{P}^t \mathbf{g}^t - \mathbf{u}^t)}{N_1 + \rho} \right\|_2^2 \\ \mathbf{g}^{t+1} &= \arg \min_{\substack{\mathbf{g} \in \mathbb{C}^M \\ \|\mathbf{g}\|_1 \leq c}} \left\| \mathbf{g} - \frac{N_2 \bar{\mathbf{y}}_2 - \rho(\mathbf{P}^t)^T(\mathbf{h}^{t+1} + \mathbf{u}^t)}{N_2 + \rho} \right\|_2^2 \\ \mathbf{P}^{t+1} &= \arg \min_{\mathbf{P} \in \mathcal{P}_M} \text{tr} \left(\mathbf{P} \Re \left\{ -\mathbf{g}^{t+1}(\mathbf{h}^{t+1} + \mathbf{u}^t)^H \right\} \right) \\ \mathbf{u}^{t+1} &= \mathbf{u}^t + \mathbf{h}^{t+1} - \mathbf{P}^{t+1}\mathbf{g}^{t+1}.\end{aligned}$$

Note that, because of the update on $\mathbf{P}$, it is possible that the method converges to a local minimum (as opposed to the global one). However, for a sufficiently small ρ , simulation results show an excellent performance. Also, for the purpose of our problem, there is no need for a perfect consensus between $\mathbf{h}$ and $\mathbf{g}$, since we are mainly interested in the two vectors having a common support, which is typically achieved after a reasonable number of iterations even for small ρ .

The solution to the minimization problems that return the updates of $\mathbf{h}$ and $\mathbf{g}$ is given by the (complex) soft-thresholding operator applied element-wise, which typically returns a sparse vector. Namely, in the case of $\mathbf{h}$ and for any $\mathbf{z} \in \mathbb{C}^M$,

$$\arg \min_{\mathbf{h} \in \mathbb{C}^M} \|\mathbf{h} - \mathbf{z}\|_2^2 \quad (\text{s.to } \|\mathbf{h}\|_1 \leq c) = \mathcal{S}_\lambda(\mathbf{z})$$

where

$$\mathcal{S}_\lambda(z_i) = \begin{cases} 0 & \text{if } |z_i| \leq \lambda \\ \frac{z_i}{|z_i|}(|z_i| - \lambda) & \text{if } |z_i| > \lambda \end{cases}$$

with $\lambda > 0$ the largest value such that $\|\mathcal{S}_\lambda(\mathbf{z})\|_1 = c$.

Concerning the update of $\mathbf{P}$, we readily see that the current formulation still corresponds to an assignment problem whose complexity is of order $O(M^3)$. However, it is worth remarking that $\mathbf{h}^{t+1}$, $\mathbf{g}^{t+1}$ and $\mathbf{u}^t$ are sparse vectors and, thus, it is enough to run the algorithm on the union of all their supports, whose dimension will be much smaller than M .

Fig. 2. Average word error rate as a function of the instantaneous SNR for different coding rates. (Left) $\text{RS}(56, 46)_{128}$. (Right) $\text{RS}(56, 28)_{128}$.

IV. NUMERICAL RESULTS

With the purpose of testing the proposed approach, we have simulated a system with $K = 3$ colliding sources. The modulation order is fixed to $M = 128$ and each slot has a length of $N = 56$ symbols. We also assume that each slot corresponds to the transmission of a codeword from a Reed-Solomon code² $\text{RS}(56, L)_{128}$, with $L \in \{28, 46\}$. The two coding rates have been chosen to be close to the minimum and maximum coding rates used by LoRa (i.e., $1/2$ and $4/5$, see [2]) and are capable of correcting up to 14 and 5 symbol errors, respectively. For all three sources and for all the transmitted codewords, the channel coefficients are drawn from a circularly symmetric complex Gaussian distribution with zero mean and unitary variance.

In step 14 of Algorithm 1 we adopt a greedy approach: sources are decoded in the descending order of their estimated channel magnitudes, stopping at the first decoding error (we assume that the transmitted message includes a CRC). Note that this approach may not even try to decode some sources with very weak channels: these are counted as errors in the performance evaluation.

The word error rate for any one of the three sources (the system is symmetric) is depicted in Fig. 2 as a function of the current received signal-to-noise ratio $\text{SNR} = |h^k|^2 / \sigma^2$, $k = 1, 2, 3$, for unitary transmit power. This metric allows us to isolate the performance degradation due to other sources' interference from the effect of channel fading. It is worth remarking that the SNR is measured after the correlators and, thus, it includes the spreading gain (around 21 dB in our case). The curves are obtained by running batches of one thousand simulated messages at various values of the noise variance σ^2 and computing each message received SNR. As a term of comparison, we also plot the results for a simple benchmark detector that works as follows.

- 1) For each received signal vector, we only keep the K entries with the highest magnitude and discard the other $M - K$ ones (in this case, we assume that the number of active sources is known at the receiver).

²Such a code can be easily obtained by shortening a proper $\text{RS}(127, 117)_{128}$ code. Note that Reed-Solomon codes are a pretty natural choice for orthogonal modulations, since it is straightforward to map modulation symbols to code symbols, that is to finite-field elements in $\mathbb{GF}(M)$, when M is an integer power of a prime number—in our case $M = 128 = 2^7$ [10].

- 2) The resulting purged vectors are aligned by means of the Hungarian method, as it is done in the proposed algorithm. In this case, however, the vectors are processed in the order they are received, as opposed to cherry-picking them according to the minimum-distance order.

Finally, the “Ho-Tan” curve represents the performance corresponding to the algorithm in [7]. As a reference, the plot also reports the WER curve for the classic single-source noncoherent scheme, typical of orthogonal signaling [3, Section 3.8].

One readily sees that the proposed scheme gains significantly with respect to the benchmark one, for both coding rates. As explained in more detail above, the reason resides in the fact that the clustering-based alignment algorithm processes the symbol vectors in an order that promotes a lower MSE in the estimation of the channel coefficients. Also, the norm-1 regularization in (2) selects the active entries of the symbol vectors in a “soft” fashion, seeking the sparse representation that is the best joint approximation of the two symbol vectors (or clusters thereof) that are being aligned. The first step of the benchmark algorithm, on the contrary, carries out a “hard” selection on each separate symbol vector, without looking for hidden similarities between the vectors.

Of course, the gain comes at the expense of an increased complexity, both in terms of memory (the sink must buffer the entire slot before detecting the symbols) and computation time. However, the algorithm only concerns the receiver, whose processing power is typically high, while is totally transparent for the sources, whose resources are much more reduced.

The poor performance of the Ho-Tan algorithm at low SNR was expected, since their channel estimation algorithm does not exploit the signal structure outlined in Section III-A; instead, for each symbol, the outputs of the M correlators are classified independently of one another. Conversely, at higher SNR where the channel estimation errors become negligible, the Ho-Tan solution shows a better trend. This behavior is due to the fact that the algorithm includes a maximum-likelihood multi-source detector as a final step, which we chose not to implement but that can be added to improve performance.

V. CONCLUSIONS

For a moderate complexity penalty at the receive side, the algorithm presented in this paper offers a valuable solution to the problem of colliding packets in sensor networks that rely on orthogonal modulations. Indeed, simulation results show that all messages can be recovered with a reasonable reliability as long as the channel can be considered frequency flat and constant over the entire message.

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