Yut

When making a choice or playing a game, we employ methods such as tossing a coin, rolling dice, or, throwing a Yut stick. Yut sticks are roughly half-cylindrical in shape and as such it is much easier to bias the outcome of up vs. down than it is for a coin or dice. If the floor is soft, then the stick is likely to have its flat side up, as the round side has a larger surface. If the floor is hard and the stick is ‘rolled’, the stick is likely to have its round side up.

How does the probability of the outcomes depend on the relevant parameters?

Chances of flat side up

- Binomial distribution
  - n is number of throws

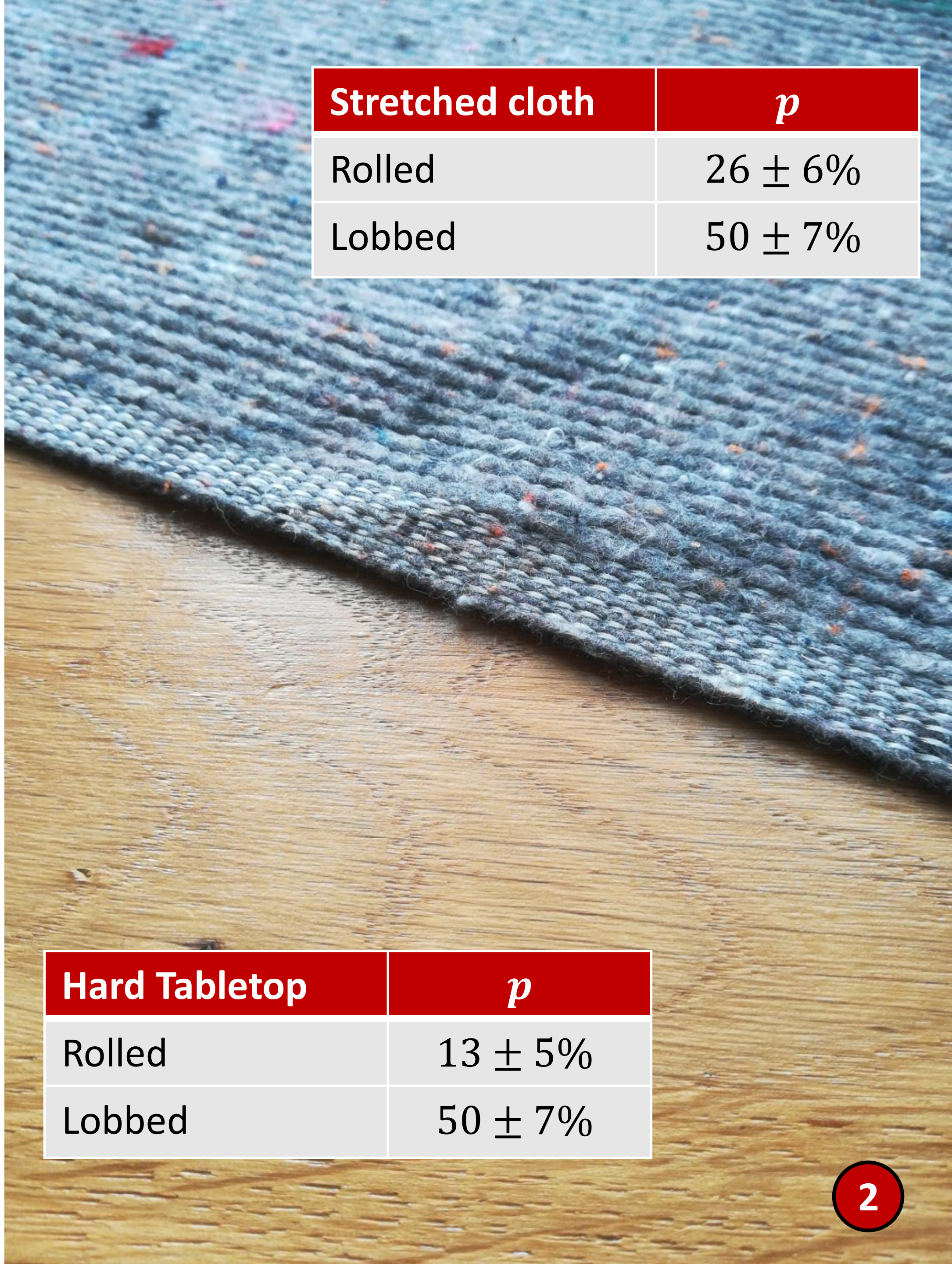
$$n = 200$$

  - v is number of successes
  - p is probability of success

$$p = \frac{v}{n}$$

  - σ is uncertainty

$$\sigma = \sqrt{np(1-p))}$$

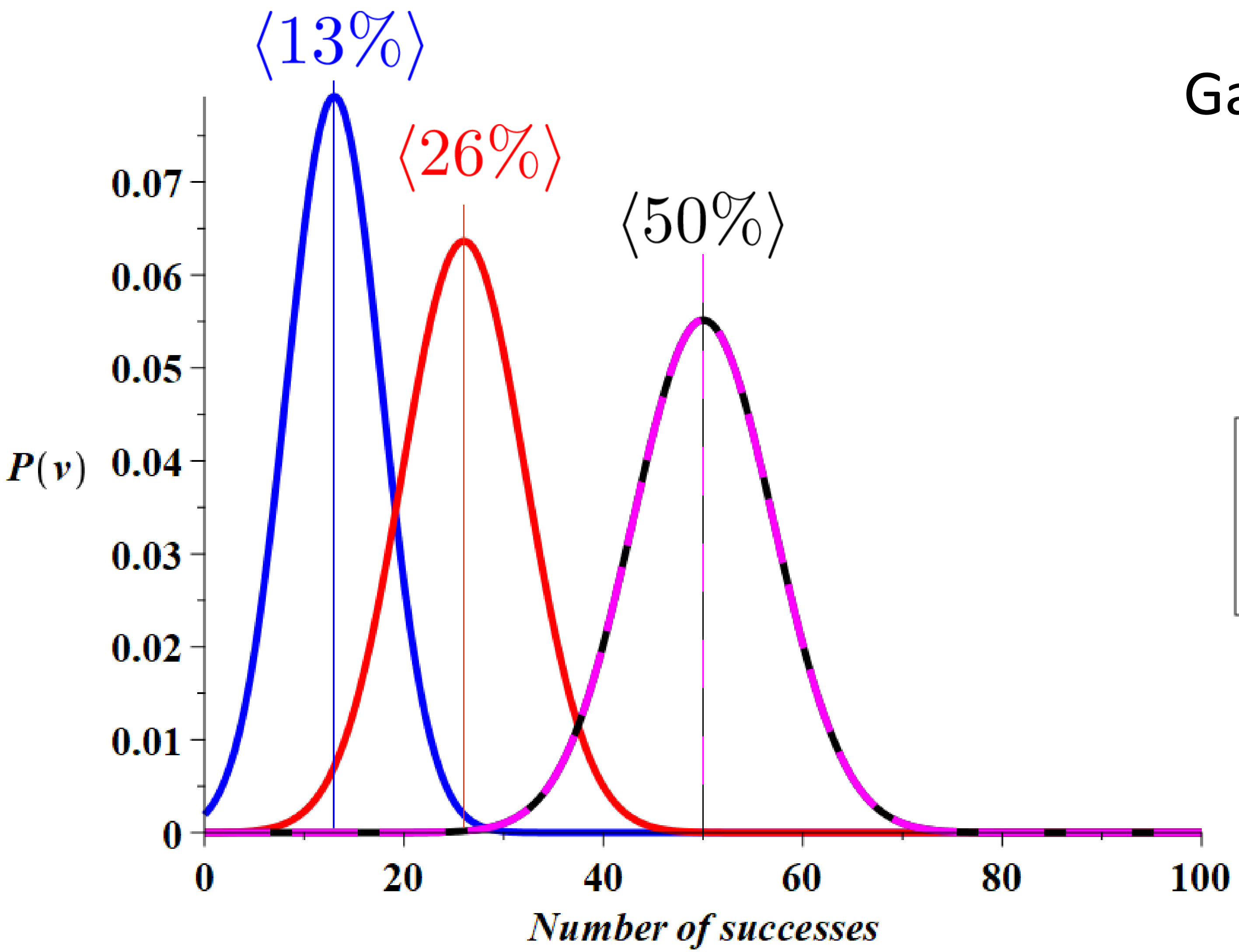


Stretched cloth	p
Rolled	$26 \pm 6\%$
Lobbed	$50 \pm 7\%$

Hard Tabletop	p
Rolled	$13 \pm 5\%$
Lobbed	$50 \pm 7\%$

Source:
An Introduction to Error Analyses, John R. Taylor

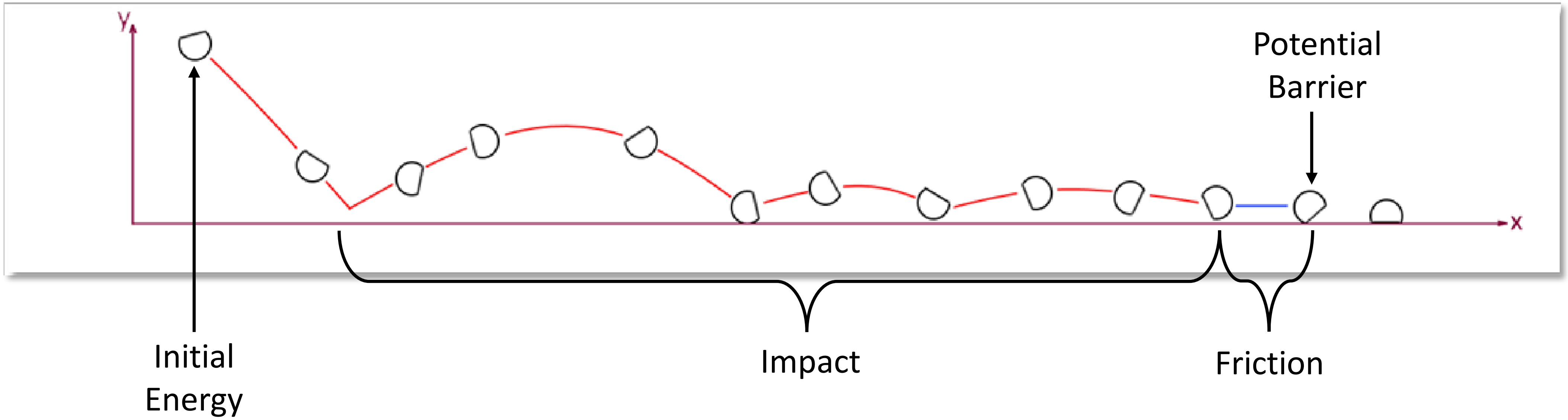
Normal Distribution



Gaussian Approximation

$$G_{v,\sigma_v} \approx \frac{1}{\sigma_v\sqrt{2\pi}} e^{-\frac{(v-\bar{v})^2}{2\sigma_v^2}}$$

Source:
An Introduction to Error Analyses, John R. Taylor



- Assume there is no drag.
- Simplified to a motion in two directions.

- Seperation of problem
  - Initial energy
  - Impact sequence
  - Friction
  - Potential Barrier

Geometry

- The displacement of the centre of mass

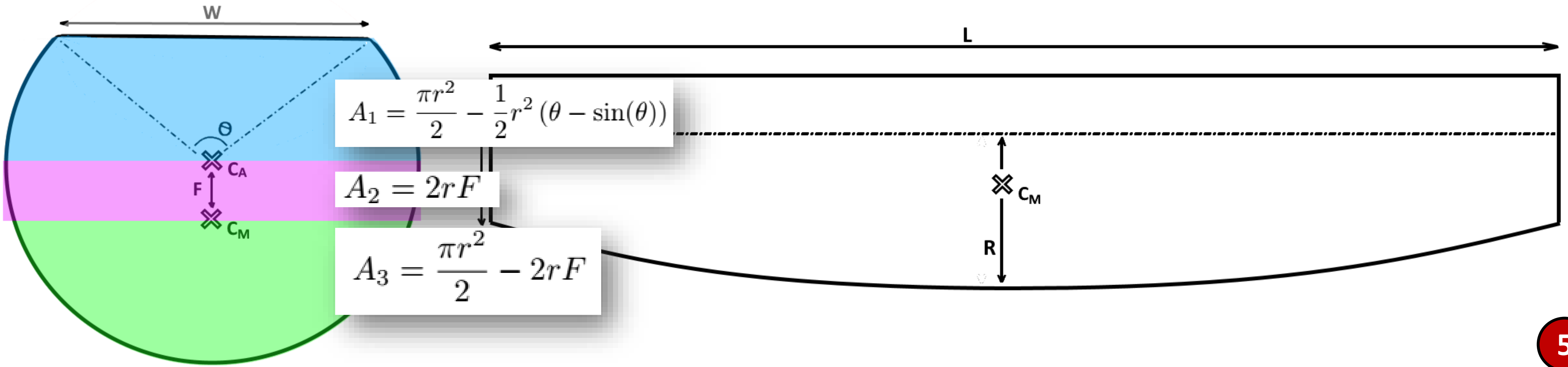
$$A_1 + A_2 = A_3 \longrightarrow F = \frac{r}{8}\left(\theta - \sin(\theta)\right)$$

$$F \approx 1.3\,\mathrm{mm}$$

- Moment of inertia

$$I_{Axis} \approx m\left(\frac{1}{2}R^2 + F^2\right) \approx 2 \times 10^{-6}\,\mathrm{m}^2\,\mathrm{kg}$$

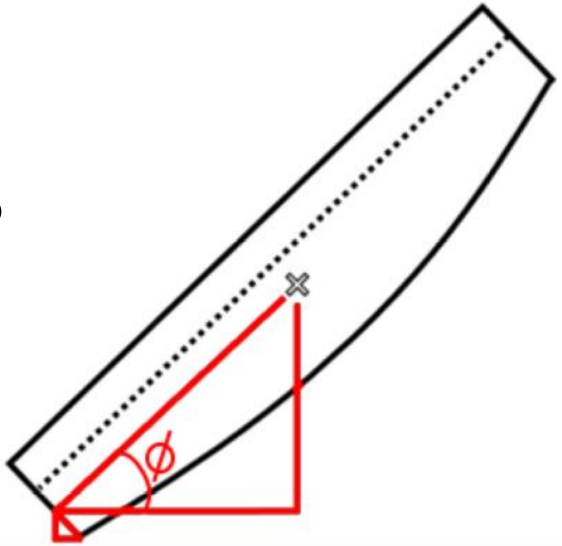
$$I_{Length} \approx \frac{1}{12}mL^2 \approx 8.8 \times 10^{-5}\,\mathrm{m}^2\,\mathrm{kg}$$



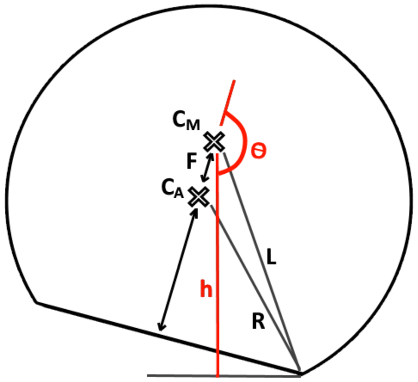
Source:
Physics for Scientist and Engineers, Jewett and Serway

Height of the center of mass

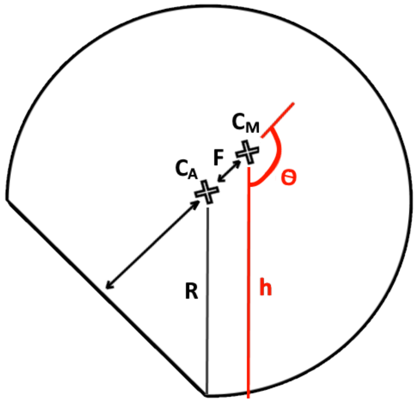
- Height depends on 2 degrees of freedom:
  - Around its own axis (θ)
  - To the floor (Ø)



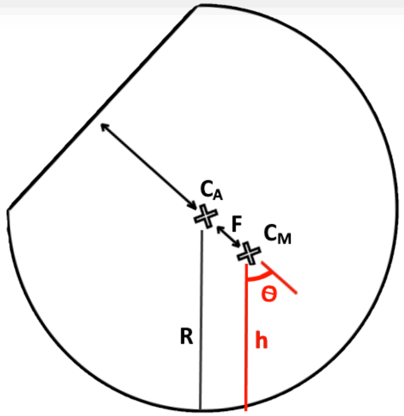
$$h_{tot} = \frac{L}{2} \cdot \sin(\phi) + h \cdot \cos(\phi)$$



$$h = L \cdot \cos\left(\theta - \left(\pi - \cos^{-1}\left(\frac{d+F}{L}\right)\right)\right)$$



$$h = R + F \cdot \sin(\theta - \frac{\pi}{2})$$

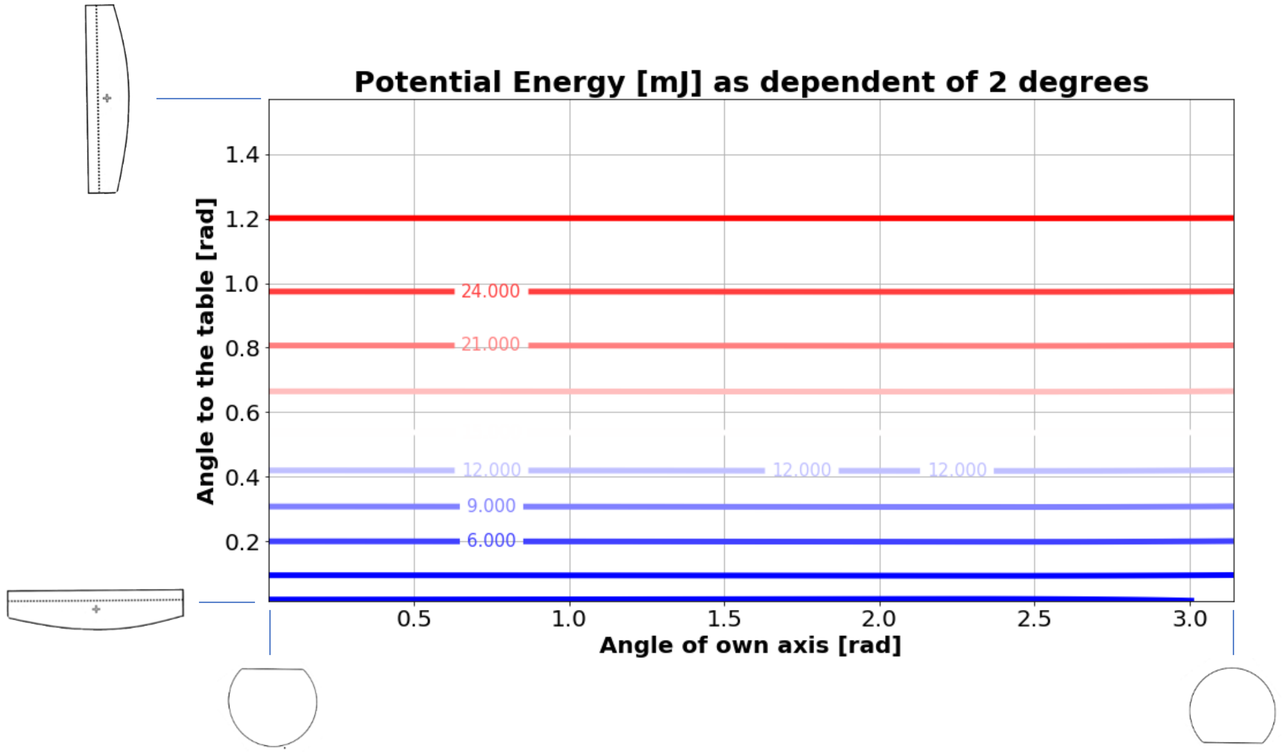


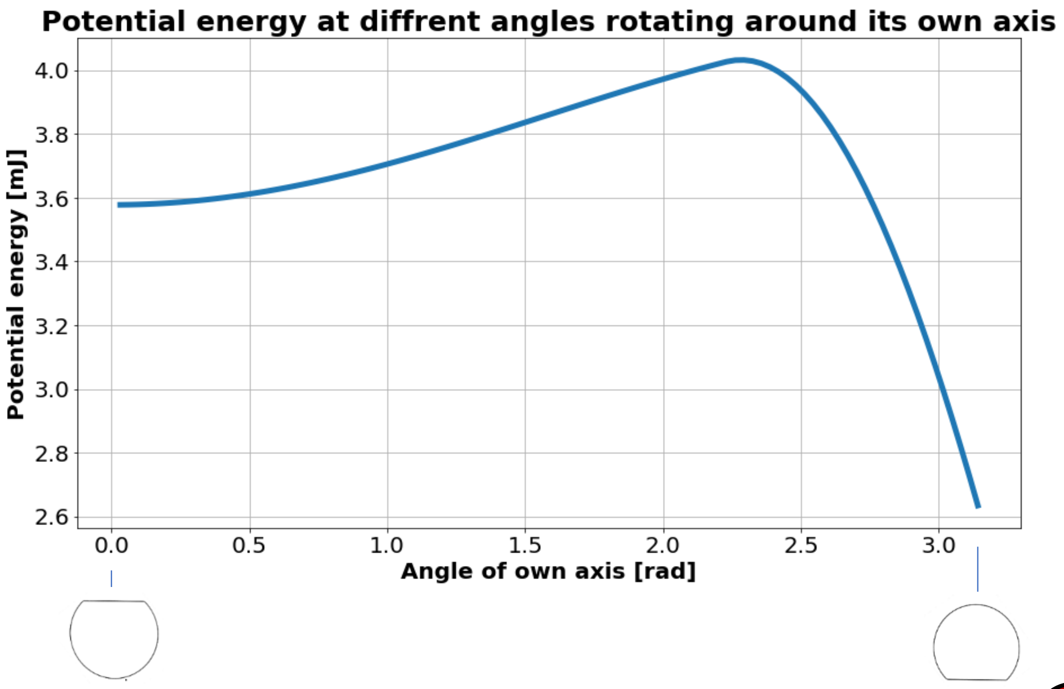
$$h = R - F \cdot \cos(\theta)$$

Potential Energy

- Map of potential energy.

$$E_{pot} = m \cdot g \cdot h$$

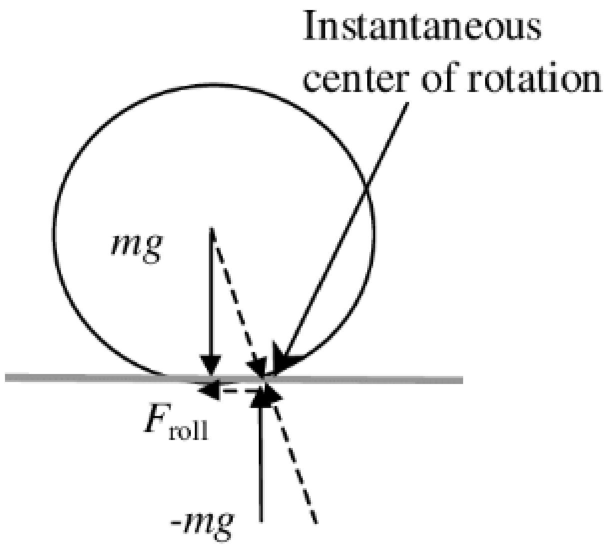




Rolling friction

- Friction Coefficient depends:
  - Deformation of underlayer
  - Deformation of the Yut stick

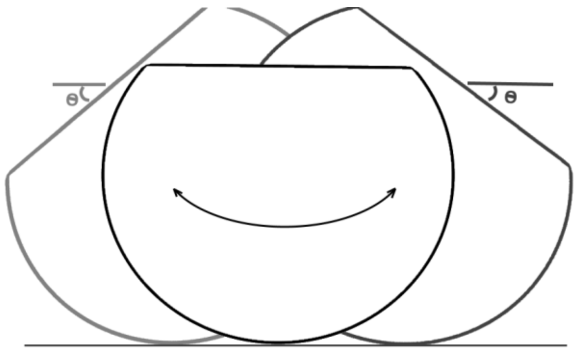
$$F = C_r \cdot N$$



Experiment:

Determination Coefficient

- High uncertainty
- Measured the angle of which it travels



Surface	$E_f/_{rad}$	$E_f/_{rot}$
Hard	0.02 mJ	0.10 mJ
Soft	0.14 mJ	0.66 mJ

Impact

- On impact:
  - Some translational energy leaves the system
  - Horizontal translational energy is converted into rotational energy.
- Newtons experimental law

$$C_I = \frac{v_f}{v_i}$$

  - Coefficient of restitution
  - $C_I = 1$ correspond to an elastic collision
  - $C_I = 0$ correspond to an inelastic collision
- Dependent of landing angle
  - Rotational energy typical not affected

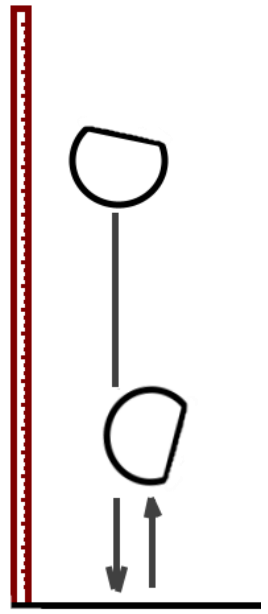
Experiment:

Determination Coefficient

- Drop from 1 meter
- Measured with Slow-motion camera

$$E_f = E_{pot} + E_{rot}$$

$$C_I = \sqrt{\frac{E_f}{E_i}}$$



Surface	c_I	σ_I
Hard	0.57	0.088
Soft	0.47	0.029

Initial energy

- Potential energy and the vertical translational energy determines the number of jumps.

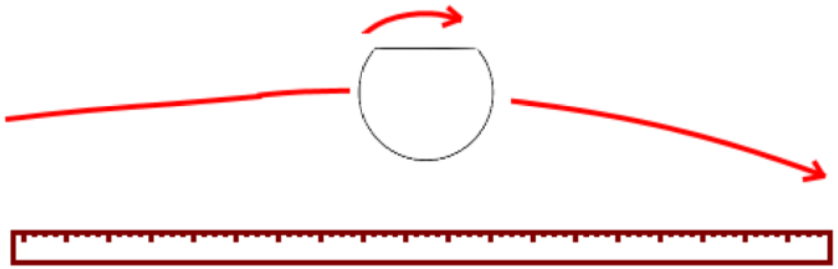
- Horizontal translational energy and rotation energy is the primary source of increasing the chance of round side up

Experiment:

Magnitude of initial energy

- Using Slow-motion camera to measure:
  - v_x
  - T

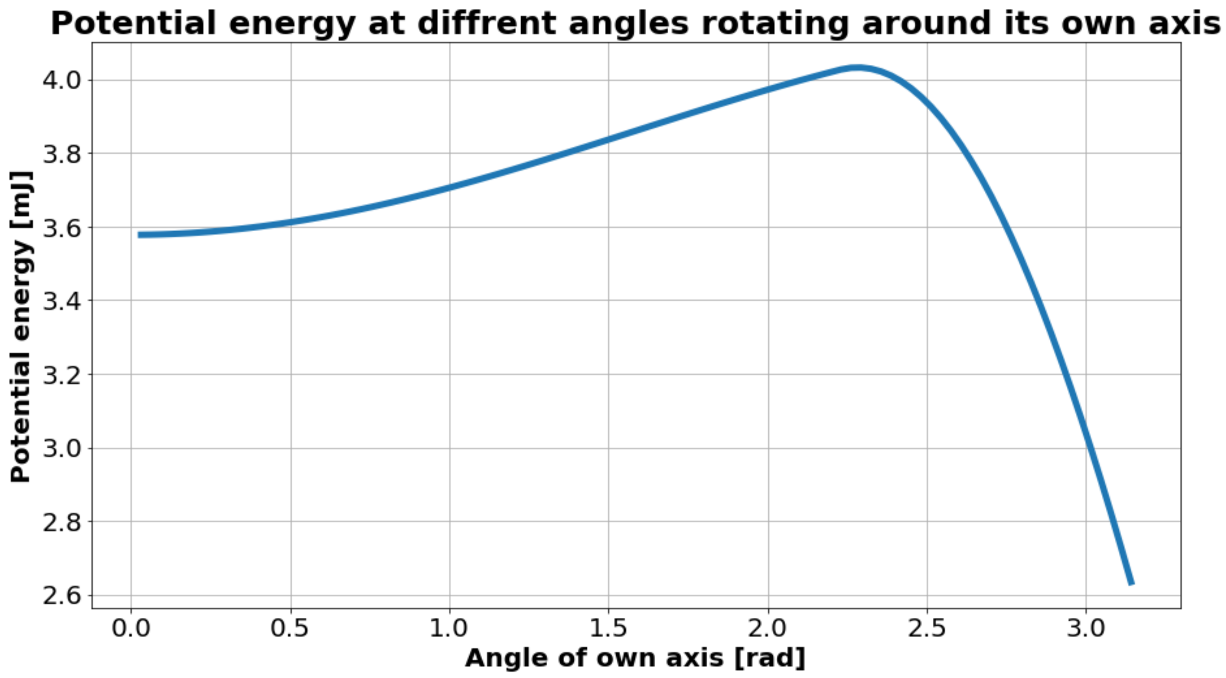
$$E_{Tra} = \frac{1}{2} m v_x^2 \qquad E_{Rot} = \frac{1}{2} I \omega^2$$



	E_i	σ_i
Initial energy	0.83 mJ	$3.2 \cdot 10^{-4}$

Conclusion

- For the hard surface, friction is insignificant
- Soft surface, friction can affect the roll

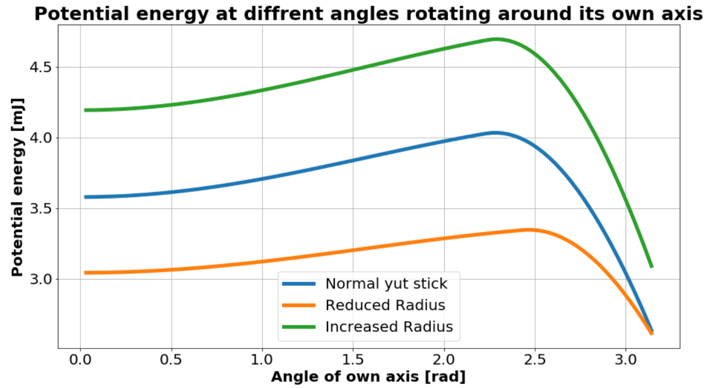


	E_i	σ_i
Initial energy	0.83 mJ	$3.2 \cdot 10^{-4}$

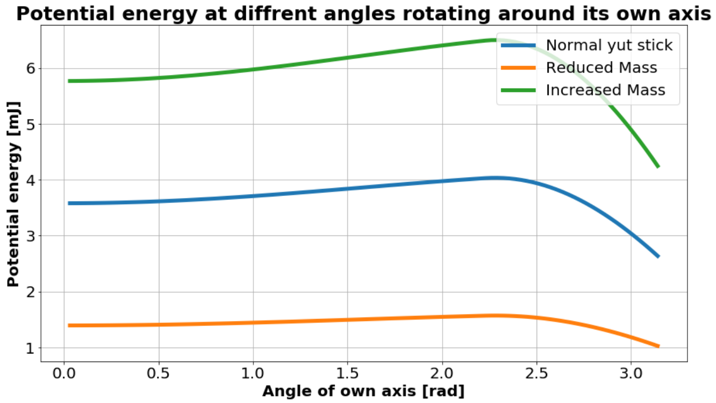
Surface	$E_f/_{rad}$	$E_f/_{rot}$
Hard	0.02 mJ	0.10 mJ
Soft	0.14 mJ	0.66 mJ

Appendix:

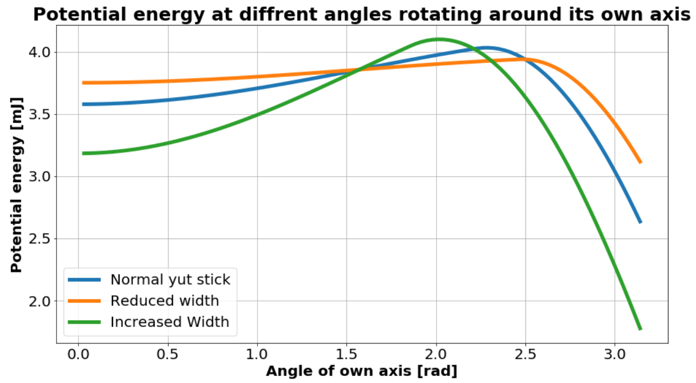
Parameters alteration



Result	W [mm]	p	σ_v
Wide	16.5	17%	3.7
Normal	12.5	7%	2.5
Narrow	10.0	14%	3.5



Result	L [mm]	p	σ_v
Long	209	7%	2.6
Normal	185	7%	2.5
Short	151	9%	2.9

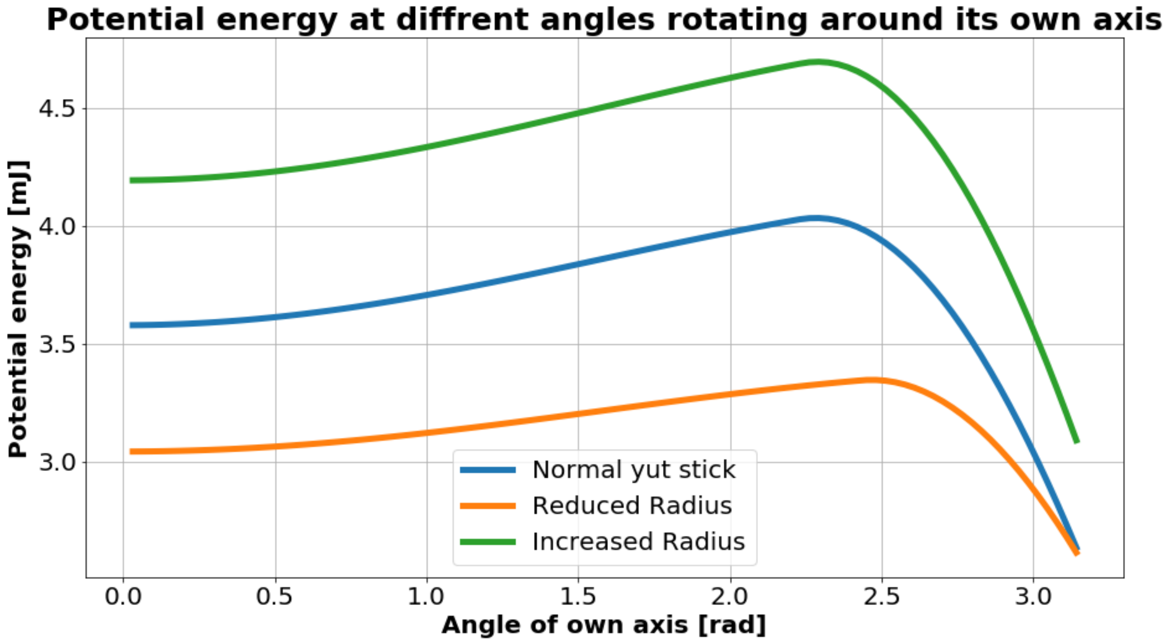


Result	D [mm]	p	σ_v
Thick	26.0	9%	2.9
Normal	23.0	7%	2.5
Thin	19.6	33%	4.7

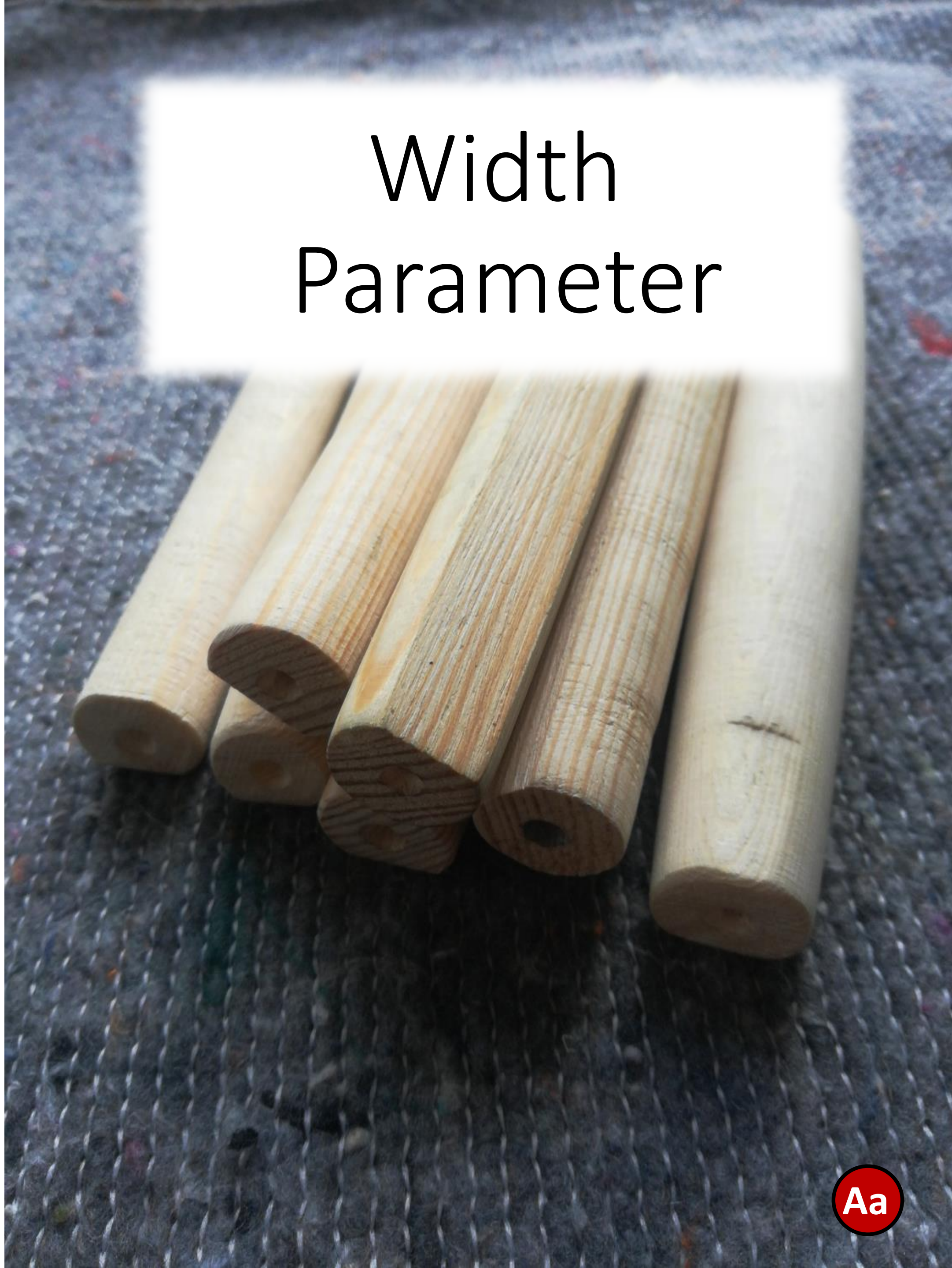
A

Result	W [mm]	p	σ_v
Wide	16.5	17%	3.7
Normal	12.5	7%	2.5
Narrow	10.0	14%	3.5

Data for 100 throws



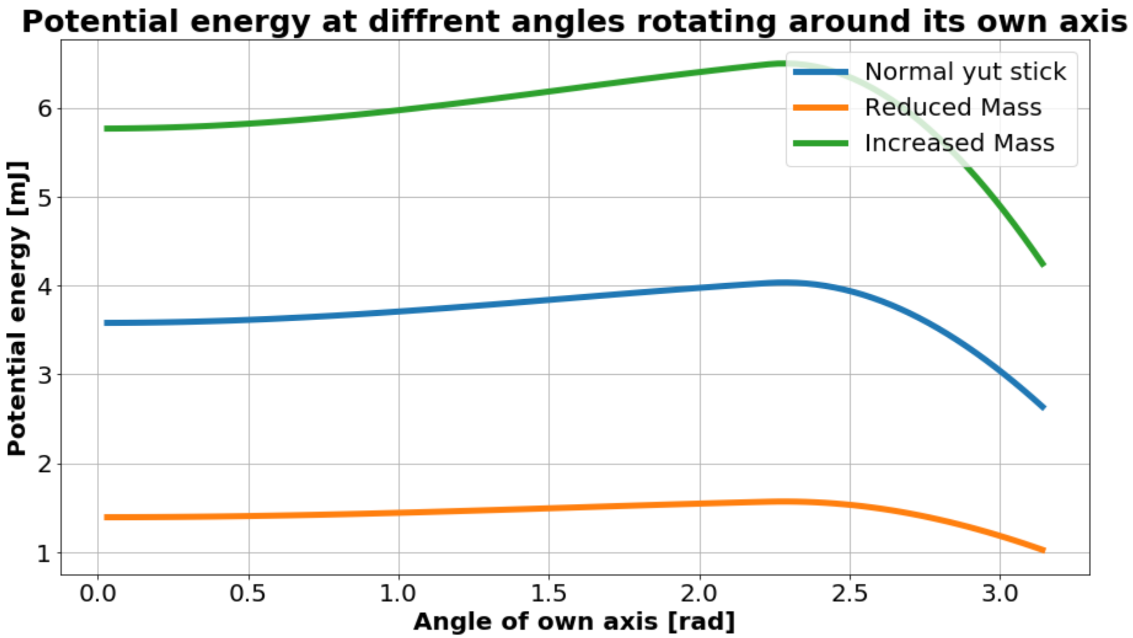
Width Parameter



Length Parameter

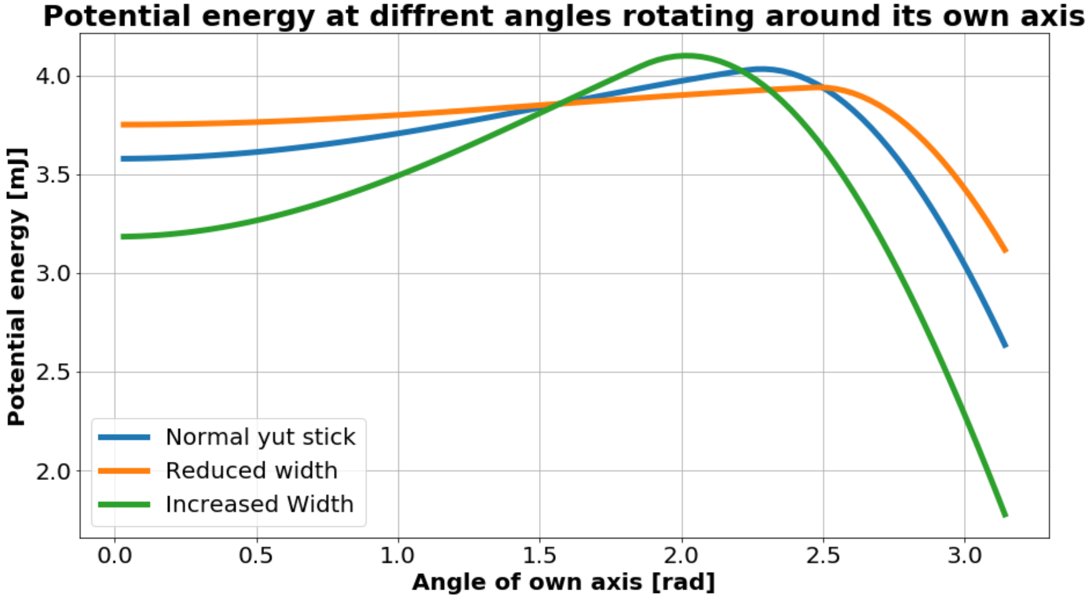
Result	L [mm]	p	σ_v
Long	209	7%	2.6
Normal	185	7%	2.5
Short	151	9%	2.9

Data for 100 throws



Result	D [mm]	p	σ_v
Thick	26.0	9%	2.9
Normal	23.0	7%	2.5
Thin	19.6	33%	4.7

Data for 100 throws



Large radius Parameter

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Appendix:

Drag

Appendix:

Geometrical Approximations