

Novel quantitative push gravity theory poised for verification

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Abstract

[Note: The edits of the current version (v7) are typed in teal color font]. New work provides compelling evidence for a genuine re-appraisal of an old way to explain gravity, which has been sidelined in the periphery of science for a long time. A novel quantitative push gravity theory has been advanced on the basis of a set of primary principles (postulates), from which the derivation of classical acceleration and force by stationary massive bodies in the steady state is possible. In contrast to prior conceptions, it is shown that the absorption of gravity particles by matter need not be extremely weak and linear, in order to derive and explain the observed classical laws of gravity. Any value of the absorption coefficient by a uniform spherical mass produces a gravitational field obeying the inverse square of distance law. The gravitational constant (big G), is itself a function of the ratio of the absorption coefficient over the density of matter. The latter ratio (mass attenuation coefficient) now becomes the new universal gravitational constant of the cosmos, whilst G can vary in different locations of the universe. The measured mass of planets and stars is only an effective or apparent mass actually smaller than the real mass due to a self-shadowing or shielding effect of the absorption of gravitational particles. Any given mass appears quantitatively different depending on its spatial distribution. We now find that Newton's gravitational law uses only the apparent (or effective) masses with a potentially variable G, but the inverse square distance relationship is locally preserved in the cosmos. The radiant flux of energetic particles being uniform over a region of space creates a maximum acceleration of gravity for all material bodies in that region, so that any further mass accretion over a certain upper limit does not create additional acceleration; this limit is reached when practically all gravitational particles are absorbed (saturation state) by the massive body above a saturation mass. The latter limit should be measurable, for which some tentative situations and experiments are proposed for prospective experiments and tests. The internal field of a spherical mass and the external field of a two layered sphere have been derived. The superposition principle of gravity fields has been reformulated and the Allais effect explained and **calculated**. The equivalence principle can now be properly understood and explained in a way that the principle per se becomes redundant under the theory being self-consistent. Matter, inertia and mass can be properly defined and understood. For moving bodies, the established relationships from special and general relativity may continue to operate within the gravitational fields created by push particles, but may need to be adapted and re-aligned within the greater framework of push gravity principles operating at any distance. These advances constitute the main part of this report purported to become a valid mathematical formulation for a basic physical interpretation or embodiment of gravity poised for verification. An attempt is made to overcome the main remaining objection of presumed catastrophic thermal accretion of absorbed particles. A further attempt is made also for the push-gravity principles to explain the vastly higher intensity gravitational fields of white dwarfs, neutron stars and black holes. It is proposed that the field of white dwarf stars is created also by push particles but of a different kind, namely, by those responsible for mediating the electric field. In the same way, the field of neutron stars is created by yet a third kind of push particles, namely, those responsible for mediating the nuclear field. The effective mass attenuation coefficient is variable around those massive bodies. In general, push particles may exist with different energy (or mass) having different mean free paths as they traverse different concentrations of masses like black holes, neutron stars, dwarfs, stars, planets, ordinary masses, atoms, nuclei, protons and all the known or unknown sub-nuclear particles. The invariable principle of momentum transfer (push) by particles directly relating to their absorption rate by the various concentrations (density) of masses could be the basis and the starting principle for a prospective unification theory of everything. **Further work seems to explain a common underlying mechanism manifesting itself as effective mass and force, both of which are caused by the rate of push particles absorption. Intrinsic effective mass of lone bodies and variable effective mass of interacting bodies are liaised with a force always obeying the inverse square distance law. The general constitution equations of push gravity are now provided.**

1 Introduction

Scientific theories are built from concepts and discoveries that have come before, and are constantly evolving and being refined. When it comes to the theory of gravity, there should be no difference. While maintaining established laws, e.g. in relativity, re-assessing sidelined theories of push gravity may help fill-in gaps of our understanding. This paper humbly seeks to re-assess what this author views as compelling evidence for push gravity and its only goal is to have related theories re-evaluated and ultimately incorporated into more extensive scientific writings.

The push (or shadow) gravity theory (PG) is not generally widely known, by and large, despite of it dating back since Newton's time. It remains outside the mainstream of established theories of gravity and is not generally accepted or introduced, even in a negative way, to primary, secondary or tertiary schooling systems. Whilst fully acknowledging its shortcomings and reasons for which it continues to receive little attention, the present paper reports new advances of this theory, which should provoke a renewed consideration beyond prior attempts to break out of the hitherto impasses in science. Push gravity is now developed on a new basis thought to overcome at least most, if not all, of the prevailing objections.

It would be helpful and productive, if we initially avoid the existing stalemate imposed by the existing criticisms and objections against the push gravity theory by patiently examining the mathematical or quantitative relationships newly derived in this work. The results and conclusions produced are important in their own right. We have opted to work out (or rework) a number of significant derivations of fundamental importance first and then follow with appropriate discussion here and elsewhere. Afterwards, we can re-visit all known arguments and objections to PG, most of which may be shown to be invalid, redundant, or not necessarily valid. Triggering a protracted series of arguments and counter-arguments from the outset could be counter-productive for the ordinary reader to proceed and appreciate the main findings and purpose of this report.

In the above sense, we initially assume the validity of a set of PG principles or postulates, which allow a novel derivation of the laws of gravity in the steady state of initially stationary (or slow moving) material bodies. In particular and based on these principles, Newton's law of gravity can be easily derived but with a new understanding of the universal constant of gravity. This is followed by the general case, where the same principles are used to derive some novel relationships beyond Newtonian mechanics, from whereby is shown that Newton is the limiting case for very weak absorption of the push particles. A universal law for the acceleration of gravity is produced, which reveals the most fundamental physical quantities involved. Both the gravitational field around a material sphere and the force between two material spheres is established. An attempt is then made to use and propose tests, observations and experiments to verify the new physics. By this methodology, we will provide completely independent means of describing some fundamental physics and phenomena providing better explanations than hitherto existing theories.

This approach then, overall, will further assist towards invalidating most of the objections and at least neutralizing the others, or placing them on a rational speculation for an interim period. This will allow experts in the areas of particle physics, theoretical physics, astrophysics and mathematics to find new and fruitful ground for further progress to both use and advance the presented theory to its logical conclusions. Ultimately, work can include general relativity, a generalized theory of fields and a unified theory of everything.

In setting out such an ambitious goal, it should be clarified from the outset that the presented PG theory is thought to by no means be in conflict with the theories of relativity, by and large. The mathematical tools of the general relativity may still be applicable and useful to PG in the case of moving bodies close to the speed of light, but this is left for later works, i.e. when the time dependence phenomena are considered under PG. For the present work, we start only with the steady state condition of push particle flows around stationary bodies, which is sufficient to reveal some important misses of existing theories in general and, perhaps, of the general relativity in particular.

Unavoidably, we include a certain amount of speculation and heuristic findings, which should be separated out from the fundamental derivations of PG. For this purpose, we divide this report in two parts, the first of which concentrates on the definite new mathematical derivations of PG, whilst the second part expands the first to include possible applications under certain conditions and speculations.

For the above purposes, there is no extensive literature relating to the novel developments of PG in the first part. However, by attempting in the second part to apply the new findings to existing data and theories, the task overwhelms the expertise of this author who takes a great risk in possible misapplication of what otherwise can be a valid PG theory. Therefore, this report does not contain an exhaustive search of literature on all aspects touched upon, but only a limited reference to prior works as needed or known to the present author, who also resorts and refers to Wikipedia to indirectly provide a more extensive list of references. These misses may be excused, whilst they could also be rectified by others in the spirit of further progress along the path ushered by the following work.

Part One (1)

2 Early push gravity theory

Nicolas Fatio de Duillier is considered to be the first who proposed an explanation of the phenomenon of gravity. That subject was one among many and various interests that he worked on around the same time that Newton developed his own laws of gravity. Fatio's works are not readily available in present journals for direct accessibility and reference, but can be found in a Wikipedia article (Wikipedia contributors, 2018b) containing numerous references (de Duillier, 1929; Gagnebin, 1949) and further reading on push gravity. His theory is "*based on minute particles streaming through space and pushing upon gross bodies...*" via collisions between ordinary matter and ethereal corpuscles, which was thought to be his greatest work. This theory was later reworked and presented also by Le Sage (Wikipedia contributors, 2018a). However, Fatio's mechanical theory of gravity soon fell into oblivion, chiefly because no drag by the aether on the motion of the planets could be detected in celestial motions; it was finally abandoned on account of a number of serious objections by renowned scientists around the beginning of the 20th century. As we know, Special Relativity (SR) and General Relativity (GR) have become the prevailing or established theories for over a century to date.

A few works have appeared from time to time attempting to revive PG. However, the latter still remains outside the mainstream physics.

3 Push Gravity (PG) principles

The fundamental principle or assumption of push gravity (PG) as understood or proposed in this report is that the forces we experience by an assumed gravitational field attached to material bodies is actually generated by the flow and absorption of a radiation energy in the form of elementary particles or waves, or both, traveling randomly but homogeneously in all directions in the interstellar/interplanetary space, or at least in regions of the order of magnitude of solar systems. This is a form of radiant flux, the nature of which need not be specified at the outset, but which, for convenience, we can initially assume that it consists of elementary particles to be called **gravions** (gravity + ion (from "ίόν" meaning "going")) and are characterized by the following postulated properties:

1. They rarely interact (or collide) between themselves resulting in relatively very long mean free paths as compared with planetary size orbits.
2. They interact with material objects at any point at a rate in direct proportion to the density of the matter they traverse.
3. During their interaction with matter, they become partially or totally absorbed but re-emitted in a different form of particles (energy) with much shorter mean free path so as not to pertain (mediate) further to gravitational force, but likely to pertain to other types of forces or reactions.
4. Conservation of momentum: During their interaction with matter, they transfer momentum to the material body, a process that appears as a force acted upon the material body.

We further use two complementary provisional assumptions to connect the theory with existing theories, namely:

- 5 The gravions are relativistic.
- 6 The speed of gravions is the speed of light (photons).

The third principle (#3) is analyzed and discussed in considerable detail in Part 2.

The sum total of all gravion absorption by a material body results in a depletion zone around the said body, a process that appears as a gravitational field acting on any other material body inside the said field.

The nature of gravions and the nature of their interaction with matter remains to be found, so that "particle" and "matter" are as yet undefined entities, as they may pertain to energy or mass in particle or waveform according to established ideas and principles in physics.

Dibrov (2011) called the particles fations, or other names may be found, but we opted to use a fresh term for good reasons, such as to dissociate, not critically, the presented theory here from previous ones on this subject. The gravions may be identical to the known gravitons from elsewhere, but a new term [aims](#) to avoid possible conflicts or transferring properties from existing theories not necessarily needed or assumed by the PG as presented here. After all, gravions and gravitons might be the same thing, except that we attempt to

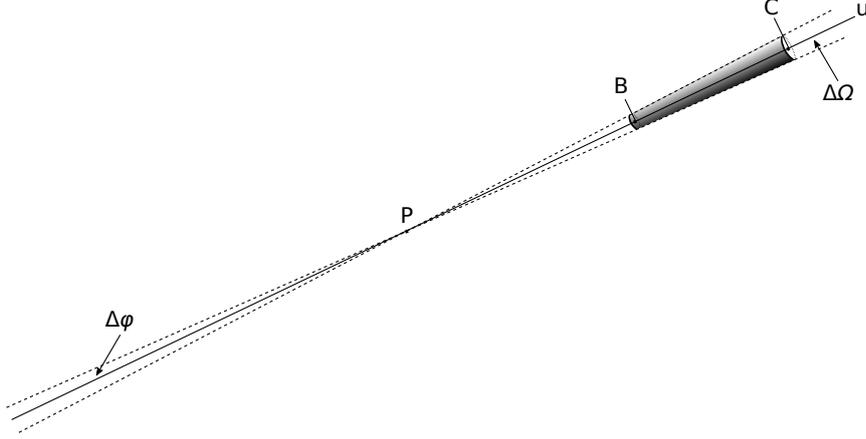


Figure 1: *Push Gravity principle.*

start afresh (i.e. be independent) in this work. The term gravion has been coined by incorporating the root word "ion" meaning "going" or "traveling" from ancient Greek (hence, by denoting particles or energy that is flowing).

3.1 Formulation of principle

The preceding principles can be formulated as follows:

Let us denote the radiant flux of gravions (energy) by Φ_0 , which is the radiant energy emitted, transmitted or received anywhere per unit time (in Watts), i.e. the rate of flow of particles/energy by gravions. The radiant flux received by a surface, per unit solid angle Ω , per unit area S in a particular direction defines the radiance L_0 by:

$$L_0 = \frac{\partial^2 \Phi}{\partial \Omega \partial S} \quad (1)$$

At any point in space, we will need to find and use the flux density J_0 (also called intensity), namely, the flux per unit area received within a solid angle $\Delta\Omega$

$$J_{0\Delta\Omega} = L_0 \Delta\Omega \quad (2)$$

If within this solid angle there is a finite material body, the received flux will be diminished due to absorption. Referring to Fig. 1, the radiance and the flux density at any point in free space is initially the same from all directions resulting in zero force, except when at a point P the flux density is affected by the presence of matter in the direction u within a cone with small semi-angle $\Delta\varphi$ subtending a small solid angle $\Delta\Omega$. Due to the absorption of gravions by matter contained in the distance BC, there is a deficiency from that direction and hence an excess flux from the opposite direction within the same angle.

We can treat the problem as we use the general case of any radiation absorption by matter and write the well known equations of absorption. In the general case, the flux density (intensity) $J(u)$ at any point u along the line u diminishes in proportion to product $J(u)du$

$$dJ_{\Delta\Omega} = -kJ_{\Delta\Omega}(u)du \quad (3)$$

where the constant of proportionality k is the coefficient of absorption for gravions (or attenuation coefficient in the Beer-Lambert law). Upon integration, we obtain the classical exponential transmission equation

$$J_{\Delta\Omega} = J_{0\Delta\Omega} \exp(-ku) \quad (4)$$

where $J_{0\Delta\Omega}$ is the incident (initial) intensity per above. The absorbed intensity $J_{a\Delta\Omega}$ is simply the difference

$$J_{a\Delta\Omega} = J_{0\Delta\Omega} - J_{\Delta\Omega} \quad (5)$$

and the corresponding absorption fraction $f_{a\Delta\Omega}$ in the small solid angle $\Delta\Omega$ is:

$$f_{a\Delta\Omega} = \frac{J_{0\Delta\Omega} - J_{\Delta\Omega}}{J_{0\Delta\Omega}} = 1 - \exp(-ku) \quad (6)$$

For the case in Fig. 1, by setting $AB = \ell$ we simply write

$$f_{a\Delta\Omega} = 1 - \exp(-k\ell) \quad (7)$$

where k is constant if the density is uniform.

We note that for very small values of $k \ll 1$, Eq. 4 reduces to

$$J_{\Delta\Omega} = J_{0\Delta\Omega}(1 - k\ell) \quad (8)$$

and

$$J_{a\Delta\Omega} = J_{0\Delta\Omega} - J_{\Delta\Omega} = J_{0\Delta\Omega}k\ell \quad (9)$$

and

$$f_{a\Delta\Omega} = k\ell \quad (10)$$

The above equation is the basic assumption of Fatio's theory and all subsequent theories of push gravity, i.e. the absorption of gravions by a planet is very weak and linear, because only then could they reproduce Newton's equation of gravity.

In the above and subsequent notation, we use the subscript "a" to denote the presence of absorption so that f_a is a shorthand notation for the absorption fraction of gravions per unit area inside a finite solid angle:

$$f_a = \int f_{ad\Omega} d\Omega \quad (11)$$

This fraction will be used later for finding the total energy absorbed by a sphere.

4 Newton's gravity law

Based on the given PG principle, we can derive Newton's equation of gravity in a simple way as follows:

Referring to Fig. 2, let us consider a point O at distance r from the center of a sphere at point P with radius R . We draw a straight line u from point O traversing the sphere along the chord AB, the length $\ell(\varphi)$ of which is given by:

$$AB = 2(AM) = 2\sqrt{R^2 - r^2 \sin^2 \varphi} = 2r\sqrt{a^2 - \sin^2 \varphi} \equiv \ell(\varphi) \quad (12)$$

since

$$OM = r \sin \varphi \quad (13)$$

and

$$a = \frac{R}{r} = \sin \varphi_0 \quad (14)$$

while we want these quantities expressed as a function of the angle φ in the range

$$0 \leq \varphi \leq \varphi_0 \quad (15)$$

We also need the initial $u_1 = OA$ and final $u_2 = OB$ lengths on the line OAB along u corresponding to points A and B

$$u_1(\varphi) = r \cos \varphi - \sqrt{R^2 - r^2 \sin^2 \varphi} = r \left(\cos \varphi - \sqrt{a^2 - \sin^2 \varphi} \right) \quad (16)$$

and

$$u_2(\varphi) = r \cos \varphi + \sqrt{R^2 - r^2 \sin^2 \varphi} = r \left(\cos \varphi + \sqrt{a^2 - \sin^2 \varphi} \right) \quad (17)$$

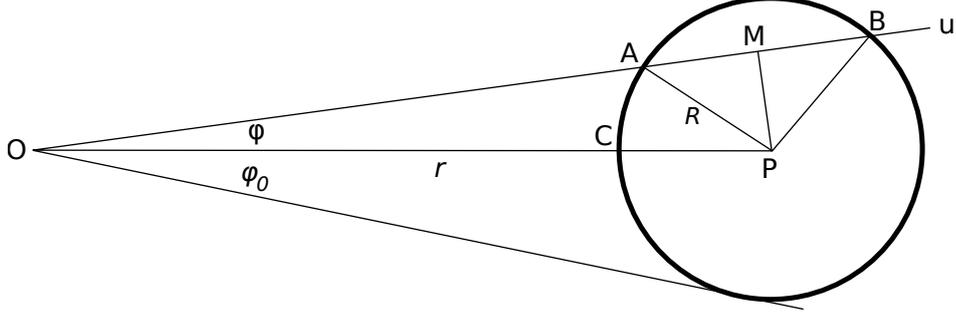


Figure 2: *Derivation of push gravity around a sphere.*

We note that the above equations describe the given circle in polar coordinates, when the origin O lies away from the circle, which then it is simplified to just the chord length, when the origin lies on the surface ($r = R$) by the well known cosine equation:

$$\ell(\varphi) = 2r \cos \varphi \quad (18)$$

The elementary annular solid angle $d\Omega$ at angle φ around the axis OP is

$$d\Omega = 2\pi \sin \varphi d\varphi \quad (19)$$

Gravions arrive at point P from all directions uniformly in the absence of any mass around. However, if the sphere contains a uniform mass we can initially assume that some gravions are absorbed by the mass in direct proportion to the elementary solid angle and the length of the chord AB at angle φ . This creates a depletion of gravions from that direction, from which the total depletion (fractional absorption) of gravions is found by the double integral

$$f_a = \int_0^{\varphi_0} \int_{u_1}^{u_2} 2\pi \sin \varphi d\varphi k du \quad (20)$$

where we use the previously defined absorption coefficient being $k \ll 1$ along the length $\ell(\varphi)$. Integrating with u along the $\ell(\varphi)$, we get

$$f_a = \int_0^{\varphi_0} 2\pi \sin \varphi d\varphi \cdot k(u_2 - u_1) = 2\pi \int_0^{\varphi_0} \sin \varphi k \ell(\varphi) d\varphi \quad (21)$$

or

$$f_a = 4\pi k r \int_0^{\varphi_0} \sin \varphi \sqrt{a^2 - \sin^2 \varphi} d\varphi \quad (22)$$

Now, since the flux of gravions is a directional quantity (vector) at the test point O, the components normal to direction OP cancel out and only the components along OP add to a total directional flux for the generation of an acceleration of gravity g . The latter components are integrated by multiplying the above integrand by $\cos \varphi$:

$$f_g = 4\pi k r \int_0^{\varphi_0} \sin \varphi \cos \varphi \sqrt{a^2 - \sin^2 \varphi} d\varphi \quad (23)$$

to find the total component of accelerating fraction f_g below:

$$f_g = \left[-\frac{4\pi k r}{3} (a^2 - \sin^2 \varphi)^{3/2} \right]_0^{\varphi_0} \quad (24)$$

By substituting the integration limits on account of the above relationships, we finally get:

$$f_g = \frac{4\pi k r}{3} a^3 = \frac{4\pi k R^3}{3r^2} \quad (25)$$

By introducing an average density ρ of the spherical mass, the last result becomes :

$$f_g = \frac{k}{\rho} \frac{4\pi\rho R^3}{3r^2} = \frac{k}{\rho} \frac{M}{r^2} \quad (26)$$

where M is the total mass of the sphere. This is essentially Newton's law of gravity subject to a proportionality constant to yield the force of the gravions on a test unitary mass, which is the acceleration at point O.

In the above and subsequent notation, we use the subscript "g" to mean the component of absorption responsible for the generation of acceleration g .

It should be noted that the ratio $\Lambda = \frac{k}{\rho}$ is the mass attenuation coefficient of the Beer-Lambert law in any absorption situation written in alternative form as a function of the area density (or mass thickness) $\lambda = \rho\ell$, that applies also in flux density attenuation in PG, i.e.

$$J = J_0 \exp\left(-\frac{k}{\rho}\rho\ell\right) \equiv J_0 \exp(-\Lambda\lambda) \quad (27)$$

The fraction f_g as initially derived above is a pure (dimensionless) parameter involving only geometrical parameters (Euclidean geometry) that appears to be a fundamental property of nature. The inverse square of distance law appears from the outset together with the sphere diameter and the absorption co-efficient k , which implies an absorbing entity like the mass, or density of the mass to appear in the next step.

This fraction was obtained by integrating over all absorption possible around the axis of symmetry defined by points O and P and yielding the simplest solution for a sphere. However, for any other shape, we should integrate around three normal independent axes (x,y,z) and add the corresponding acceleration fractions vectorially, as is shown in Appendix B.

5 Beyond Newton

Next, we obtain the expected acceleration from the previous derivation, as a consequence of the push gravity principle.

5.1 Universal gravitational constant in weak absorption regime

The simple derivations above can already lead to a better understanding of the universal constant G (or bigG).

From the absorption and acceleration fractions f_a and f_g introduced in the previous section, we convert to the corresponding fractions of absorption and acceleration for the flux density J_a and J_g below:

$$J_a = J_0 f_a \quad (28)$$

$$J_g = J_0 f_g \quad (29)$$

where J_a is the flux density absorbed by the presence of a mass (here spherical uniform mass) and J_g is the component of J_a in the direction of the axis of symmetry responsible for the generation of acceleration.

We now proceed to find the constant of proportionality to reproduce Newton's gravitational law from Eq. 26 by

$$J_g = J_0 \frac{k}{\rho} \frac{M}{r^2} = J_0 \Lambda \frac{M}{r^2} \quad (30)$$

using the newly introduced constant Λ .

The physical meaning of this constant is the number of absorption events per unit density of matter in units of inverse mass-thickness (m^2/kg). In other words, it is the number of absorption events per kilogram per square meter. The inverse ($1/\Lambda$) is the mass-thickness (or area density) per absorption event. This is a new cosmic constant the magnitude of which remains to be found.

It is generally known in flow problems that the product of pressure times the velocity of the flow yields the flux intensity. Thus, if we divide J_g by the velocity c of the radiant flux (gravions), we obtain the pressure p_g exerted by the gravions at O:

$$p_g = \frac{J_g}{c} \quad (31)$$

An elementary test mass dm is located at point O with a surface area dS and thickness x having a density ρ' with corresponding absorption coefficient k' . The force dF on this test mass is then given by:

$$dF = p_g dS \cdot k' x = \frac{J_g}{c} dS \cdot k' x \quad (32)$$

where we multiply by $k'x$ to allow only for the fraction of gravions absorbed by the test mass, considering that k' , in general, is the number of absorption events per unit length. The force per unit mass, i.e. the acceleration g is then

$$g = \frac{dF}{dm} = \frac{\frac{J_g}{c} dS \cdot k' x}{\rho' dS \cdot x} = \frac{J_0 k M k'}{c \rho r^2 \rho'} = \frac{J_0}{c} \Lambda^2 \frac{M}{r^2} \quad (33)$$

The above equation is exactly Newton's law, where the factors of proportionality between g and M/r^2 must correspond to the universal constant G :

$$G = \frac{J_0}{c} \Lambda^2 \quad (34)$$

The above is already an important derivation for the universal gravitational constant in terms of other constants, namely, the gravion speed and intensity of the neighboring universe, and the mass attenuation coefficient (new universal constant). Eqs. 33 and 34 are thought to be new fundamental derivations beyond Newton even within the realm of Newtonian mechanics for weak absorption.

5.2 General gravitation law in any absorption regime

Having considered the case of weak absorption, we now proceed to investigate what happens if absorption is strong, or to any arbitrary degree, i.e. the absorption coefficient can take any value. This actually means that we allow gravitational shielding inside a material body and between bodies. We may also refer to this condition as self-shadowing within the bulk of a massive body. In other words, we allow "gravitational shielding" as a core condition of a general push gravity theory, as opposed to considering it a case for rejecting PG, as has been done by the hitherto critics. This ushers a novel approach to push gravity.

In the general case, where self-shadowing (shielding) is caused by a significant k , we follow the same initial procedure as previously with reference to Fig. 2: The force is proportional again to the elementary annular solid angle $2\pi \sin \varphi$, but now multiplied by the absorption fraction of the flux intensity along the length AB (Fig. 2) provided by Eq. 6; we also multiply by $\cos \varphi$ to allow, as previously, only for the component of force along the direction OP, so that we only need to integrate with respect to angle as follows:

$$f_g = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi d\varphi \cdot [1 - \exp(-k\ell(\varphi))] \quad (35)$$

and

$$f_g = 2\pi \int_0^{\varphi_0} \sin \varphi \cos \varphi \left[1 - \exp\left(-2kr\sqrt{a^2 - \sin^2 \varphi}\right) \right] d\varphi \quad (36)$$

The final integration of the above expression in the given subtended angle φ_0 by the sphere is fortunately an analytical expression of the form:

$$f_g = 2\pi \left[\frac{\sin^2 \varphi}{2} - \frac{\exp\left(-2kr\sqrt{a^2 - \sin^2 \varphi}\right) \left(2kr\sqrt{a^2 - \sin^2 \varphi} + 1\right)}{4k^2 r^2} \right]_0^{\varphi_0} \quad (37)$$

and with the given values of integration from 0 to φ_0 , we finally obtain

$$f_g = \pi \left[R^2 - \frac{1}{2k^2} + \frac{\exp(-2kR)(2kR + 1)}{2k^2} \right] \frac{1}{r^2} \equiv \frac{\pi A}{r^2} \quad (38)$$

where we have now a new parameter A , which is a function of k and R only (i.e. independent of r):

$$A = \left[R^2 - \frac{1}{2k^2} + \frac{\exp(-2kR)(2kR + 1)}{2k^2} \right] \quad (39)$$

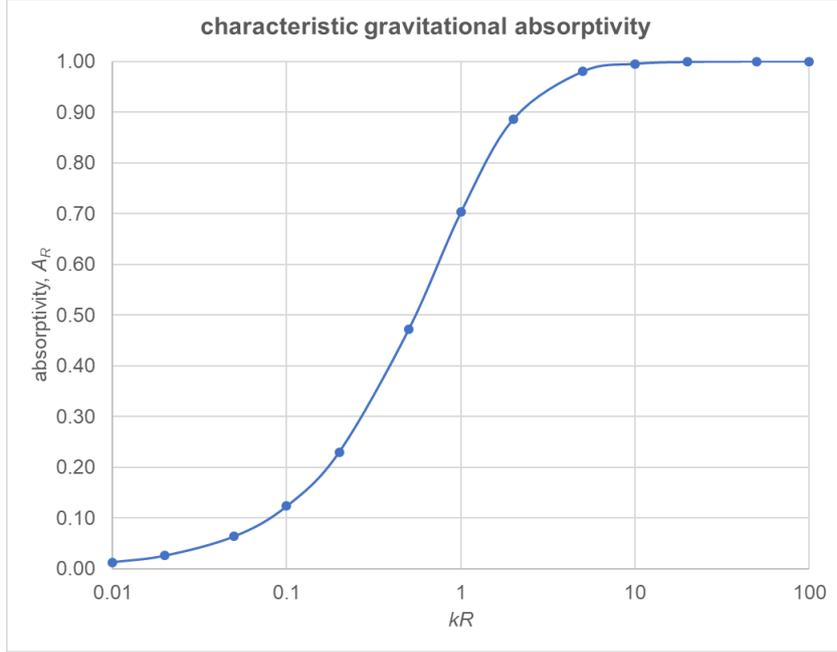


Figure 3: *Dependence of characteristic gravitational absorptivity A_R on kR .*

Like in Eq. 25, we find that the fraction f_g derived in the general case of strong absorption is again a dimensionless parameter (quantity) that appears to be a fundamental property of nature and that the inverse square of distance law is preserved. This law is a consequence of the geometry alone (Euclidean) by any uniform flux propagated and absorbed in space. It is the law in the steady state around any absorbing medium (mass), whilst the time dependence remains to be introduced at a later stage of PG development.

Now, we follow the same procedure, as previously, to obtain the acceleration: For the test mass acted upon by a pressure p_g , Eq. 29 now becomes

$$J_g = J_0 \frac{\pi A}{r^2} \quad (40)$$

In view of above, Eq. 33 is modified to become:

$$g = \frac{dF}{dm} = \frac{J_g dS \cdot k'x}{\rho' dS \cdot x} = \frac{J_0 k' \pi A}{c \rho' r^2} = \frac{J_0 \Lambda \pi A}{c r^2} \quad (41)$$

The above provides the equation of acceleration in PG, which again preserves the inverse square of distance law. However, the factor(s) of proportionality between g and M/r^2 is different from the corresponding PG derivation in Newton's equation, the significance of which will be described later. To understand the difference, we need to first investigate the properties of the newly derived parameter A .

6 Investigation, consequences and new physics with parameter A

6.1 PG versus Newton

From from Eqs. 25 and 38, we see that the corresponding f_g (or g , or force) is always proportional to $1/r^2$ regardless of the values of k and R . The general assumption by previous proponents of PG that the gravion absorption should be very weak (in order to produce Newton's Law), is now found to be redundant together with the allegation that the "gravitational shielding" is a reason to reject PG. On the contrary, this is now found to be a fundamental underlying mechanism of PG. This is already an important finding.

It is helpful and instructive to normalize the distance r over the radius of the sphere R :

$$n_R = \frac{r}{R} \quad (42)$$

whereby we re-write the newly found expression as

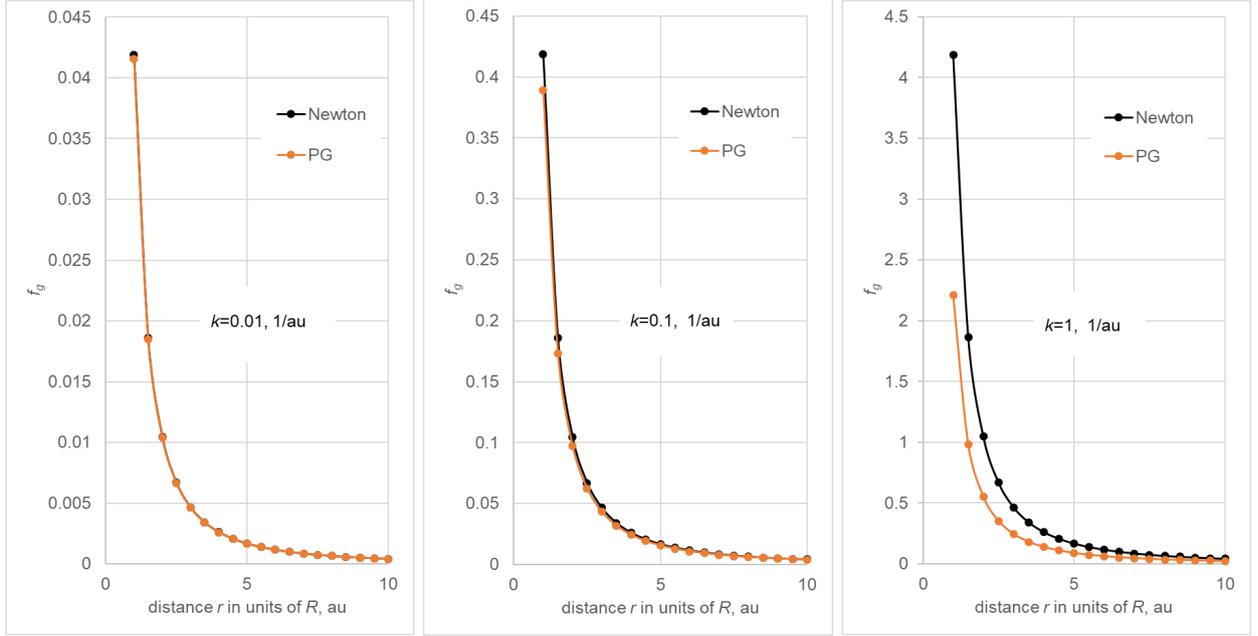


Figure 4: Gravity acceleration factor f_g vs. distance r in units of radius R for three different absorption coefficients k using linear (Newton) and exponential (PG) absorption.

$$\frac{f_g}{\pi} = \left[1^2 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR)(2kR + 1)}{2k^2 R^2} \right] \frac{1}{n_R^2} = \frac{A_R}{n_R^2} \quad (43)$$

by introducing the characteristic parameter A_R :

$$A_R = 1 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR) \cdot (2kR + 1)}{2k^2 R^2} \quad (44)$$

The latter parameter depends only on the product kR and it is plotted in Fig. 3. This shows that A_R is monotonically (absolutely) increasing, as it should, but it reaches a saturation limit at very high values of kR . For simplicity, we may also set $R = 1$ and plot against k , or set $k = 1$ and plot against R , in either case reproducing the same curve. The important observation is that most of the change in value of A_R takes place over the span of about four orders of magnitude of k , or R , or kR . Overall, the magnitude varies from near zero to unity. This means that increasing the radius of a sphere, the shadowing parameter f_g (and acceleration) at the surface will reach a saturation value as opposed to infinity predicted by Newton. Likewise, by increasing the absorption capacity (density, or k) of a constant radius sphere, the shadowing parameter f_g (and acceleration) at the surface will again reach a saturation value, not the infinity provided by Newton. This new parameter A_R characterizes the absorbing ability of a spherical mass or planet and may be referred to as *characteristic gravitational absorptivity*, or absorptivity, for short.

For a direct comparison, we plot simultaneously f_g against normalized distance r_R for $r_R > 1$, i.e. by setting $R = 1$ au (arbitrary unit) in Eqs. 25 and 38 as shown in Fig. 4 for three fixed values of k in a range spanning three orders of magnitude. Initially, we may avoid the involvement of mass M and density ρ by investigating only the quantity f_g . For very low values of k , the pair of curves are indistinguishable. We note that as we increase k , the shadowing derived from PG is increased **absolutely** (see actual values), as it should, because more absorption by the gravitating mass means more **net** push by gravions. However, the curve lies below the corresponding expected Newtonian force, as it should. This is to be expected from the general absorption Eqs. 6 and 10, whereby the second equation is a straight line tangent to the first near (or at) the origin (at very short distance, or very low k), always yielding a higher value above the downward concave line of PG absorption. The latter is a consequence of the self-shadowing (gravitational shielding) effectively creating a hidden mass, which, if it could exert an “attractive” force (per Newton), it would be greater than the corresponding PG force found.

The above analysis is also consistent with a comparison between Newton and PG as provided in Fig. 5 by plotting the ratio of $f_{gPG}/f_{gNewton}$ from (Eq. 38)/(Eq. 25) vs. k for a constant sphere radius $R = 1$ au. The absorption ratios by PG/Newton approaches unity for very small values of k ($k < 0.01$), as it should, but vanishes for very large values of k , which means that f_g becomes infinity in Newton, whilst it reaches a saturation value in PG. This is reasonable and helpful in understanding the mechanism of shielding. Noted

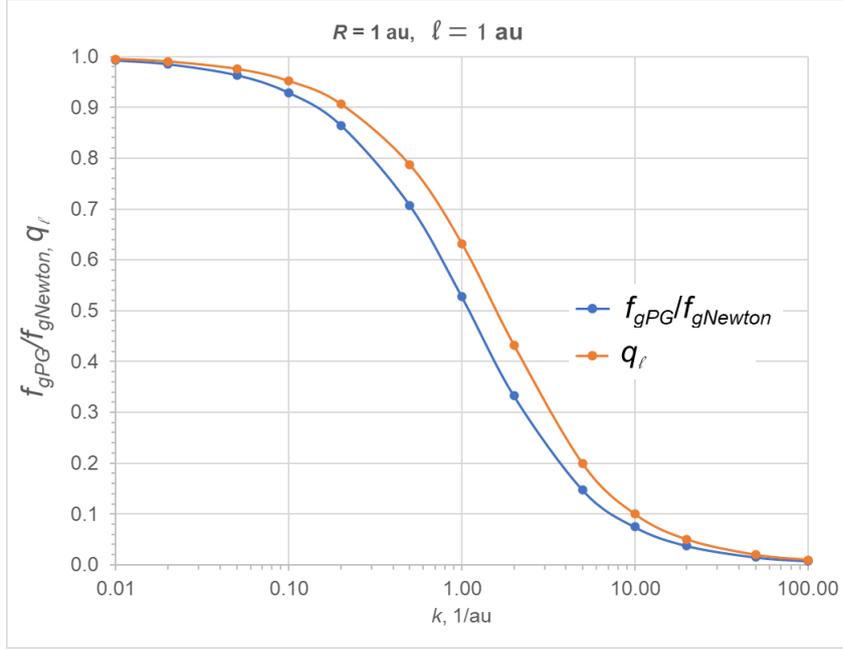


Figure 5: Dependence of ratio $f_{gPG}/f_{gNewton}$ and q_l on k

that the horizontal axis is logarithmic tending to uplift (concave up) the initial straight line (Newtonian) but eventually tending to reach a saturation value asymptotically (concave down). PG is the overriding physics in all cases, whilst Newtonian physics is an approximation in the limiting case of very low values of k . The above ratios are given by:

$$q = \frac{f_{gPG}}{f_{gNewton}} = \frac{g_{PG}}{g_{Newton}} = \frac{3A_R}{4kR} \quad (45)$$

which will be referred to as contraction factor, or factor q . The significance and use of this factor will become apparent in further development of PG theory.

If we also take the ratio from the integrands of Eqs. 23 and 36, i.e. the ratio of the differential accelerations (or factors df_g) inside an elementary solid angle $d\Omega$, we obtain another factor q_l , to be referred to as length contraction factor:

$$q_l = \frac{df_{gPG}}{df_{gNewton}} = \frac{1 - \exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right)}{2kr\sqrt{a^2 - \sin^2\varphi}} = \frac{1 - \exp(-k\ell)}{k\ell} \quad (46)$$

and is plotted on the same Fig. 5.

This is the ratio of an effective length $\ell_e(\varphi)$ in PG divided by the real length in Eq. 12 of the chord traversing the sphere at angle φ from the origin O in Fig. 2:

$$\ell_e(\varphi) = \frac{1}{k} \left[1 - \exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right) \right] \quad (47)$$

It is a contracted or compressed length, with which we may construct a virtual volume (body) by replacing the points defined by Eq. 17 with new ones defined by

$$u_{e2} = u_1 + \ell_e \quad (48)$$

or correspondingly replacing the points by Eq. 16 with new ones defined by

$$u_{e1} = u_2 - \ell_e \quad (49)$$

We may refer to these shapes as *gravitoids*, which are helpful for our theoretical understanding of the underlying workings of PG. Further details and analysis are provided in Appendix A.

Note 1: The current practice to find the mass of a planet is to place an artificial satellite around it and measure the period and radius of orbit. However, we now find that the actual mass still remains unknown by such measurements. This is not a trivial finding.

Note 2: For small values of k or R , we revert to Newtonian mechanics, which can also be seen by expanding the exponential to a Taylor series $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$.

$$A_{RkR \rightarrow 0} = \frac{4}{3}kR(1 - kR)_{kR \rightarrow 0} \approx \frac{4}{3}kR \quad (50)$$

$$\text{i.e. for very small } kR: \pi A_R = \frac{4}{3}\pi kR = f_{g\text{Newton}} \quad (51)$$

the latter reproducing Eq. 25 for f_g in Newton derivation at $r = R$.

6.2 Universal gravitational “constant” in any absorption regime vs. a new cosmic constant

We note that in Eq. 25 the multiplier preceding the factor $1/r^2$ divided by k provides the volume V of the gravitating sphere. Likewise, in Eq. 38, the multiplier preceding $1/r^2$ divided by k also provides an *effective* sphere volume V_e with the same center:

$$V_e \equiv \frac{\pi A}{k} \quad (52)$$

The real volume, real density and real mass are designated by V , ρ , and M . The measured (effective, measured, or apparent) density ρ_e is the effective mass M_e divided by the real volume

$$\rho_e = \frac{M_e}{V} = \frac{\rho V_e}{V} \quad (53)$$

and

$$\rho_e V = \rho V_e = M_e \quad (54)$$

also

$$\frac{\rho}{\rho_e} = \frac{V}{V_e} = \frac{M}{M_e} \quad (55)$$

We can now continue from Eq. 38 by multiplying with $V\rho_e$ both numerator and denominator as follows:

$$f_g = \frac{\pi A}{V\rho_e} \frac{V\rho_e}{r^2} = \frac{kV_e}{V\rho_e} \frac{M_e}{r^2} = \frac{kV_e}{V_e\rho} \frac{M_e}{r^2} = \frac{k}{\rho} \frac{M_e}{r^2} \quad (56)$$

which is identical to Eq. 26, except that we use the real density and not the effective (fictitious) one used (or implied) in Newton’s equation. Based on this, we can repeat the same steps to establish the force on a testing mass and derive an identical form of equation as in 33

$$g = \frac{dF}{dm} = \frac{\frac{J_g}{c} dS \cdot k'x}{\rho' dS \cdot x} = \frac{J_0}{c} \frac{k}{\rho} \frac{M_e}{r^2} \frac{k'}{\rho'} = \frac{J_0}{c} \Lambda^2 \frac{M_e}{r^2} \quad (57)$$

but where again we use the real density of the gravitating body in Λ . We repeat the same equations in order to stress that they are different in the meaning of ρ and Λ , whereby we derive the same expression for the universal gravitational constant:

$$G = \frac{J_0}{c} \Lambda^2 \quad (58)$$

This is the same equation arrived at for weak graviton absorption, so that Λ is the new universal constant for the cosmos. From this and the known density of a given mass, we derive the absorption coefficient k . The universal constant G is proportional to the ratio of k/ρ squared, where ρ is the real density. For very low values of k the real density becomes very close to or is indistinguishable from the measured (effective) density. From this, we learn that G is constant only to the extent that J_0/c is constant in the neighboring universe. As pointed out earlier, Λ expresses the number of scattering events per unit length per unit mass density anywhere and provides a more tangible constant parameter to have. Thus, G may be found to be relatively more variable than previously suspected, according to further investigations by PG.

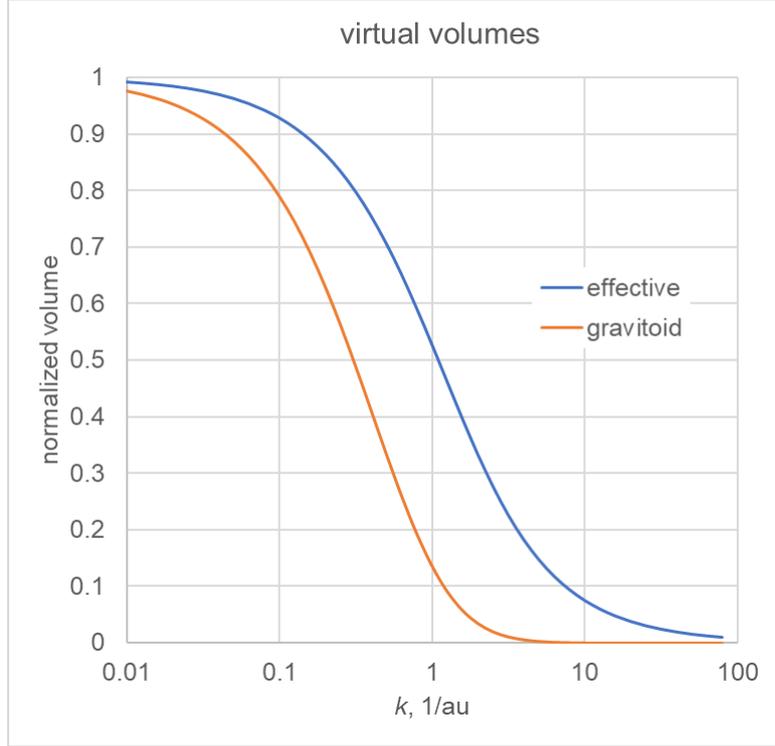


Figure 6: *Virtual volumes, gravitoid and effective, normalized over the real spherical volume.*

The effective volume introduced above is plotted in Fig. 6 against k after it is normalized over the real volume. As expected, it coincides with the real volume (at very low k), but then monotonically decreases to a vanishing value at very large k .

6.3 Maximum universal acceleration

We can try to use known values of planet parameters to derive the Λ , k , ρ and J_0 . Basically, we need to know the flux intensity J_0 , or the absorption coefficient k , on which all other parameters depend. Conversely, from the known physical parameters of a planet, we may assume values for J_0 in any given range and derive the other new parameters of Λ , k and ρ as a function of J_0 . In practice, we may proceed as follows:

The acceleration of gravity g_R at the surface of a sphere, i.e. at $r = R$, is given by Eq. 41 as:

$$g_R = \frac{J_0}{c} \Lambda \frac{\pi A}{R^2} = \frac{\pi J_0}{c} \Lambda A_R = \frac{\pi G}{\Lambda} A_R \quad (59)$$

From Eqs. 41, 44 and Fig. 3, there is a maximum possible acceleration $g_{maximum} \equiv g_0$ in the surrounding universe to be manifested on the surface of a star (sphere) with sufficiently large product kR , i.e. with $A_R = 1$, given by any of the following equations:

$$g_0 = \pi \frac{J_0}{c} \Lambda = \frac{\pi G}{\Lambda} = \frac{\pi \rho G}{k} \quad (60)$$

In subsequent work, we will be using values of g_0 in a tentative range to obtain an idea of the expected magnitude of various parameters and anticipated measurements. That is, until we establish the actual value of g_0 , we may obtain the new constant Λ and hence k from the known density of a mass, for any given value g_0 .

It is useful to write Eqs. 57 and 59 correspondingly as:

$$g = G \frac{M_e}{r^2} = \frac{G}{\Lambda} \frac{\pi A}{r^2} = g_0 \frac{A}{r^2} \quad (61)$$

$$g_R = g_0 A_R \quad (62)$$

Now, given the measured acceleration g_R on the surface of a spherical body, we can find k by solving the equation below:

$$g_0 A_R - g_R = g_0 \left[1 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR)(2kR + 1)}{2k^2 R^2} \right] - g_R = 0 \quad (63)$$

as a function of g_0 . Then, for any given g_0 , we can find in turn ρ , Λ and J_0 from Eq. 60. We will return to the question of g_0 in a subsequent section

6.4 Commonality and departure between Newton and PG

It is not fortuitous that both Newtonian and PG mechanics share a common limiting case but depart thereafter. Let us start from the derivation of the volume of the sphere as seen from point O in Fig. 2. For the elementary volume we have

$$dV = d\Omega u^2 du \quad (64)$$

which multiplied by the density ρ gives the elementary mass and, divided by the inverse square distance, yields the Newtonian acceleration:

$$dg_{Newton} = \frac{d\Omega u^2 du}{u^2} \rho \quad (65)$$

In PG, we use the factor f_g from which we obtain the same elementary acceleration by :

$$dg_{PG} = \frac{g_0}{\pi} d\Omega k du \quad (66)$$

which is identical to Newton above except for the proportionality constants. This initial similarity is not trivial, because it explains the fundamental difference at the root of the two theories (approaches), as we increase the absorption coefficient: In Newton, it is given that the acceleration is inversely proportional to the square of distance, whilst in PG this is a consequence of the solid angle (geometry) incorporating the inverse square distance relationship. In Newton, this is the result of an assumed radiance (of gravity) emanating from the elementary mass, whilst in PG the same field emanates from the radiance of the surrounding universe. Whilst the analogy might seem trivial simply shifting the problem of origin of the elusive gravity from the inside to the outside of a given mass, the consequences diverge from the two approaches as we increase the absorption coefficient to any level. That is, when considering very large masses or densities, Newton and PG provide very different solutions and outcomes: Newton provides a linear cumulative radiance of gravity by simple summation of all the constituent masses/volumes, whilst PG allows for shadowing (shielding) of the universal radiance traversing the mass, which, in turn, results in an asymptotic limit to the total shielding and hence to the total acceleration or force. We may think of this limit as effectively integrating the Newtonian law linearly but within contracting upper limits of a volume per Eq. 66, which defines the said gravitoid. This shape would produce the same Newtonian force with a mass having the actual (real) density. The above integration has been performed numerically and plotted against k in Fig. 6 after it is normalized by dividing by the sphere volume, as was done for the effective spherical volume defined by Eq. 52. For comparison, this is also plotted against k in Fig. 6. We note that it is generally lying above the gravitoid, as it should, because it is further away from the gravitoid. They both have the same real density and both yield the correct value of acceleration for the real gravitating sphere.

Hence, these are important findings for cosmological considerations in relation to what happens as we keep adding mass (accretion) to a star (dwarfs, black holes, etc.). We will discuss this again later.

An important conclusion here is that there is more mass in the universe than Newton's Law measures. This is a form of dark matter but not exactly in the sense considered by existing theories to date, in accounting for the observed celestial motions. We now find weaker forces, not greater. However, the greater forces, if needed, may be accounted for by forces originating from the outside now predicted from PG theory, not from the inside anticipated by Newton. At very large distances, forces are exerted by the gravions in the universe, so there is no need to attribute them to an attraction by dark matter. However, dark matter should assume a different meaning now by the shadowing effect (gravitational shielding) in PG. Thus, breaking up a planet to dust would appear to create new matter (out of shadow - see redistribution of density in later Section 10), which gives a kind of credence to the creationist theory of matter, except that no new matter actually is created other than new matter coming out of the shadows (literally). All this and more creates new understanding and new physics that will become clearer as we develop and prove the novel PG presented in this work. As we investigate next, the bigG is a function of the gravion density in the universe, which should vary between regions inside a galaxy and in intergalactic regions. So, if we need extra forces, these may arise from the variation of bigG alone. The sum total of the effects caused by hidden masses and the variation of bigG might explain or replace the hypothesized dark matter and dark energy of current theories. PG may offer the new physics needed.

6.5 Summary of new parameters and relationships

We have already expressed various relationships in alternative forms, which we may further re-arrange for easy reference in later derivations here or elsewhere as follows: By combining Eqs. 41 and 57 we derive:

$$\pi A = \Lambda M_e \quad (67)$$

and

$$\Lambda = \frac{\pi A}{M_e} = \frac{\pi}{M_e} \left[R^2 - \frac{1}{2k^2} + \frac{\exp(-2kR)(2kR + 1)}{2k^2} \right] = \frac{\pi R^2 A_R}{M_e} \quad (68)$$

We obtain a further insight of the above parameters by re-writing the above as

$$A_R = \Lambda \frac{M_e}{\pi R^2} = \Lambda \lambda_e \quad (69)$$

by defining an effective mass-thickness λ_e (or area-density) with

$$\lambda_e = \frac{M_e}{\pi R^2} \quad (70)$$

The neighborhood prevailing graviton pressure p_g should be handy to have (per Eq. 58) as

$$p_g = \frac{J_0}{c} = \frac{G}{\Lambda^2} \quad (71)$$

If two spheres (planets) 1 and 2 have equal surface acceleration g_R , it follows from Eq. 59 that the product kR for both spheres is the same

$$k_1 R_1 = k_2 R_2 \quad (72)$$

Also we have the universal (cosmic) constancy for Λ giving:

$$\Lambda = \frac{k_1}{\rho_1} = \frac{k_2}{\rho_2} = \frac{\pi G}{g_0} = \frac{c g_0}{\pi J_0} = \text{constant} \quad (73)$$

so that we obtain

$$\rho_1 R_1 = \rho_2 R_2 \quad (74)$$

The above equations apply in PG theory with real densities ρ_1 and ρ_2 . In Newtonian mechanics, we similarly obtain for the effective (apparent) densities ρ_{e1} and ρ_{e2} , i.e. if the g_R is equal for both spheres (at their surface):

$$g_R = \frac{4}{3} G R_1 \rho_{e1} = \frac{4}{3} G R_2 \rho_{e2} \quad (75)$$

$$\rho_{e1} R_1 = \rho_{e2} R_2 \quad (76)$$

We obtain the ratios of real to effective densities as

$$\frac{\rho_1}{\rho_{e1}} = \frac{\rho_2}{\rho_{e2}} \quad (77)$$

From a given value for g_0 , we find the corresponding k from Eq. 63 and then ρ from Eqs. 60:

$$\rho = g_0 \frac{k}{\pi G} \quad (78)$$

and then the ratio ρ/ρ_e from the known effective (measured) density. This ratio is also provided directly from:

$$\frac{\rho}{\rho_e} = \frac{4}{3} k R \frac{g_0}{g_R} \quad (79)$$

We also derive relationships including the frequently encountered factor A_R :

$$M_e = \frac{\pi \rho R^2}{k} A_R \quad (80)$$

$$V_e = \frac{\pi R^2}{k} A_R \quad (81)$$

Finally, it is important to note that the parameter g_0 or the factor f_g yield the acceleration g , via Eqs. 43, 60 and 61 in a simple form by a summary of equations:

$$g = f_g \frac{g_0}{\pi} = f_g \frac{J_0}{c} \Lambda = f_g \frac{G}{\Lambda} = g_0 \frac{A}{r^2} = g_0 \frac{A_R}{n_R^2} \quad (82)$$

The maximum (or limiting) universal constant g_0 now takes on a tangible significance in establishing the quantitative relationships in PG, and it may substitute the constant G accordingly. We should stress that new universal constant of the cosmos Λ is given by Eq. 68, which in words states that the hitherto universal constant G is proportional to the new universal constant (maximum acceleration) g_0 both with reference to a region of the universe, so that together they yield the cosmic (overall universal) constant Λ . It should be noted that we attempt to distinguish the term “universal” from the term “cosmic” with reference to the neighboring universe or to the “entire” universe (=cosmos).

With the new parameters now introduced, it is useful to re-write Eq. 45 of the ratio of accelerations as a function of tentative values for g_0 :

$$q = \frac{g_{PG}}{g_{Newton}} = \frac{3A_R}{4kR} = \frac{3A_R}{4\pi G \rho R} g_0 \quad (83)$$

This factor characterizes a multitude of parameters, not only the acceleration, with reference to PG and Newton. For example we can write

$$q = \frac{\rho_e}{\rho} = \frac{m_e}{m} = \frac{V_e}{V} = \frac{m_e}{m} \quad (84)$$

7 Force between two spherical masses

For the force between two spherical masses, we can formulate the problem entirely from gravion absorption considerations, carry out four integrations and produce the force law, as we did for the acceleration at a point for a single sphere. This would be an independent way, from first principles, to derive the required relationship. However, we can still arrive at the same desired result in a much simpler way as follows:

Since we already have established the relationships between all the parameters needed for the PG force equations, we can apply a “reverse engineering” approach. Now, in the knowledge that Newton is correct except for the masses used, we can start with the Newtonian law of force by using the effective masses provided by PG theory together with preceding equations between various parameters:

$$F = G \frac{M_{e1} M_{e2}}{r^2} = G \frac{\pi A_1}{\Lambda} \frac{\pi A_2}{\Lambda} \frac{1}{r^2} = \frac{J_0}{c} \frac{\pi A_1 \pi A_2}{r^2} = \frac{g_0}{\Lambda} \frac{\pi A_1 A_2}{r^2} \quad (85)$$

This is consistent with our hitherto understanding of the meaning of the parameters involved. The importance is that Newton’s law now involves the effective masses M_{e1} and M_{e2} , not the real masses assumed, but not used, in prior mechanics. The above equations is a far reaching conclusion. Now we can write, or start with the PG force law as

$$F = \pi \frac{g_0}{\Lambda} \frac{A_1 A_2}{r^2} \quad (86)$$

where we do not need the masses, but equivalently we need the more intrinsic parameter of absorption coefficients (relating to mass), the radii (geometry), the new cosmic constant Λ and the prevailing maximum acceleration g_0 in the neighboring universe, or equivalently the pressure J_0/c exerted by the radiant energy in our neighborhood. We may further rearrange the above to provide a more tangible idea of how the force is derived by

$$F = p_g A_{R1} A_{R2} \frac{\pi R_1^2 \pi R_2^2}{r^2} \quad (87)$$

which states that the force is proportional to the pressure exerted by the gravions times the absorptivities times the cross-sections of the spheres while still being inversely proportional to the square of the distance. It seems like we can separate one group of factors pertaining to geometry alone and another group of factors

pertaining to matter (energy) and its interactions involving the local system of two masses interacting with the universal pressure of gravions. The two masses do nothing by themselves except for the mediating flow of gravions.

Therefore, the above equations provide variant expressions of the law of gravitational force in the new physics of PG. They are particularly appealing by their consistency and symmetry of parameters beyond Newtonian physics.

The mathematical derivation from first principles of the radiant intensity absorption involving multiple integrations involves a number of simplifications and cancellation of terms by appropriate choice of a reference system of co-ordinates. The multiple integrations may also be done by numerical means via relatively simple Python codes run in parallel to shorten the computation times. Integrals involving absorption along lines crossing a single sphere cancel out leaving only integrals crossing both spheres simultaneously. This work has been omitted from [this Section](#) to avoid needless congestion that could potentially distract from the important finding above and beyond. This section may be expanded to form a self contained chapter with sufficient detail to qualify for publication, which, however, can await at least for an initial response by the established scientific community on the current report as is. [In the meantime, we have added more details by presenting the constitutional equations of the interaction of two material spheres in Appendix C, etc.](#) Numerical integration of those equations has lead to important results and conclusions.

Some concern might arise by a later proposal in Section 14.5, namely, that the effective mass being involved in the generation of force may be variable with its distance from another mass due to the perturbation of the surrounding gravion flux. However, these concerns have been eased by finding that the above presented fundamental equation is still valid. The envisaged variation of mass is coordinated by the mathematics of absorption in a way that the inverse square of distance law is always preserved at any absorption regime. We have tested ordinary solar system bodies (near Newtonian) as well as “artificial” ones with extremely dense and massive bodies. These outcomes are better presented together with a new analysis of effective mass generation and variation with distance in Sections 15.7 and 16.

IMPORTANT: The new gravitational law in PG expressed by Eq. 87 has far more repercussions than being a simple substitution of old parameters with new. It states that the force between two masses is not simply proportional to the effective masses but proportional to the associated absorptivities of the masses. This means that non-spherical masses exert a different force for different relative orientation of the same masses (bodies). If, as they move towards or away from each other they change orientation, they also change gravitational absorptivity and hence the trajectories would not be as expected from Newtonian mechanics. The dependence of acceleration on the density distribution will become more clear in Section 10. Further analysis is presented under the Equivalence Principle in Section 14.3.

8 Internal spherical field

So far, we have examined the field generated externally to a spherical body, but we now proceed to find the field also inside the sphere. With reference to Fig. 7, the acceleration at any point X inside a sphere with radius $AP = R$ is provided by integrating the absorption along the lengths of mass inside the differential solid angles indicated on either side of the point X along any direction of the line u . We note that the absorption length $XA = BC$ leaving only the length XB to yield a net absorption, which is the same as that of the sphere with radius R_X crossing the point X. Therefore, we have the same situation as that experienced by Newtonian mechanics, in that a hollow sphere would exert zero force inside its cavity. Now, the acceleration at this internal point is given by (see Eq. 82)

$$g_X = g_{0X} \left(1 - \frac{1}{2k^2 R_X} + \frac{\exp(-2kR_X) \cdot (2kR_X + 1)}{2k^2 R_X} \right) \equiv g_{0X} A_{R_X} \quad (88)$$

where A_{R_X} is the familiar A_R factor at the surface of the internal sphere with radius $PB = R_X$ and $g_{0X} < g_0$ due to the shielding of the outer layer from X.

We can find g_{0X} by resorting to the usual absorption factor f_{g_X} at point X by the following steps:

The exponential absorption factor in the direction XBC is

$$1 - \exp(-k \cdot XB(\varphi) - k \cdot BC(\varphi))$$

and in the direction XA is

$$1 - \exp(-k \cdot XA(\varphi))$$

so that we take their difference in the integral:

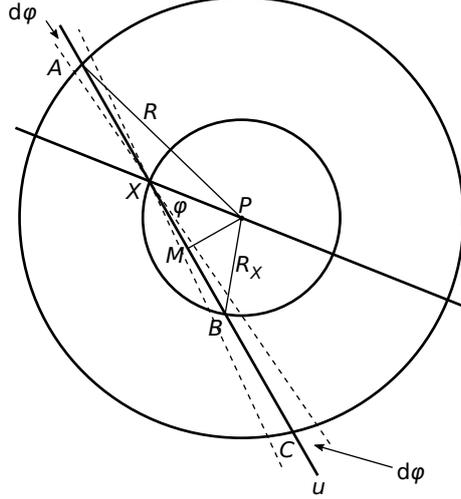


Figure 7: Derivation of the internal field of a uniform density sphere.

$$f_{gX} = \int_0^{\pi/2} 2\pi \sin \varphi \cos \varphi d\varphi \cdot [\exp(-k \cdot XA(\varphi)) - \exp(-k \cdot XB(\varphi) - k \cdot BC(\varphi))] \quad (89)$$

and integrate with respect to angle φ from 0 to $\pi/2$ as can be seen in the referenced diagram.

From the geometry shown and M being the mid-point of the chord AC, we find and replace the lengths accordingly with:

$$XB(\varphi) = 2R_X \cos \varphi$$

$$XA(\varphi) = BC(\varphi) = \sqrt{R^2 - (R_X \sin \varphi)^2} - R_X \cos \varphi$$

to obtain the integral formula:

$$f_{gX} = \int_0^{\pi/2} 2\pi \sin \varphi \cos \varphi d\varphi \cdot \left[\exp\left(-k\sqrt{R^2 - (R_X \sin \varphi)^2} + kR_X \cos \varphi\right) - \exp\left(-k\sqrt{R^2 - (R_X \sin \varphi)^2} - kR_X \cos \varphi\right) \right] \quad (90)$$

From this found, we can derive the acceleration at X by the factor g_0/π and equate it to its value given above by Eq. 88:

$$g_X = \frac{g_0}{\pi} f_{gX} = g_{0X} A_{R_X} \quad (91)$$

from which we can find the relationship between the internal g_{0X} and external g_0 .

$$g_{0X} = \frac{g_0 f_{gX}}{\pi A_{R_X}} \quad (92)$$

The expected Newtonian acceleration at X is given by:

$$g_{XN} = \frac{4}{3}\pi G \rho_e R_X \quad (93)$$

No analytical relationship was found for f_{gX} , so that we may resort to numerical means for this parameter. For practical application, we also need to see the difference of PG from Newton acceleration against various depths from the surface of the Earth by replacing the internal radius as a function of depth.

$$R_X = R - \text{depth} \quad (94)$$

$depth=$	2500	5000	7500	10000	12500
$g_N=$	9.816404	9.812551	9.808697	9.804844	9.800990
g_0	Δg_X				
300	7.29E-05	1.45E-04	2.18E-04	2.90E-04	3.61E-04
500	4.32E-05	8.62E-05	1.29E-04	1.72E-04	2.14E-04
1000	2.14E-05	4.27E-05	6.39E-05	8.50E-05	1.06E-04
2000	1.07E-05	2.13E-05	3.18E-05	4.23E-05	5.28E-05
5000	4.25E-06	8.48E-06	1.27E-05	1.69E-05	2.11E-05
10000	2.12E-06	4.24E-06	6.34E-06	8.43E-06	1.05E-05
20000	1.06E-06	2.12E-06	3.17E-06	4.21E-06	5.26E-06
30000	7.08E-07	1.41E-06	2.11E-06	2.81E-06	3.50E-06
50000	4.24E-07	8.47E-07	1.27E-06	1.69E-06	2.10E-06

Table 1: *Difference of acceleration Δg between Newton and PG at various depths in Earth.*

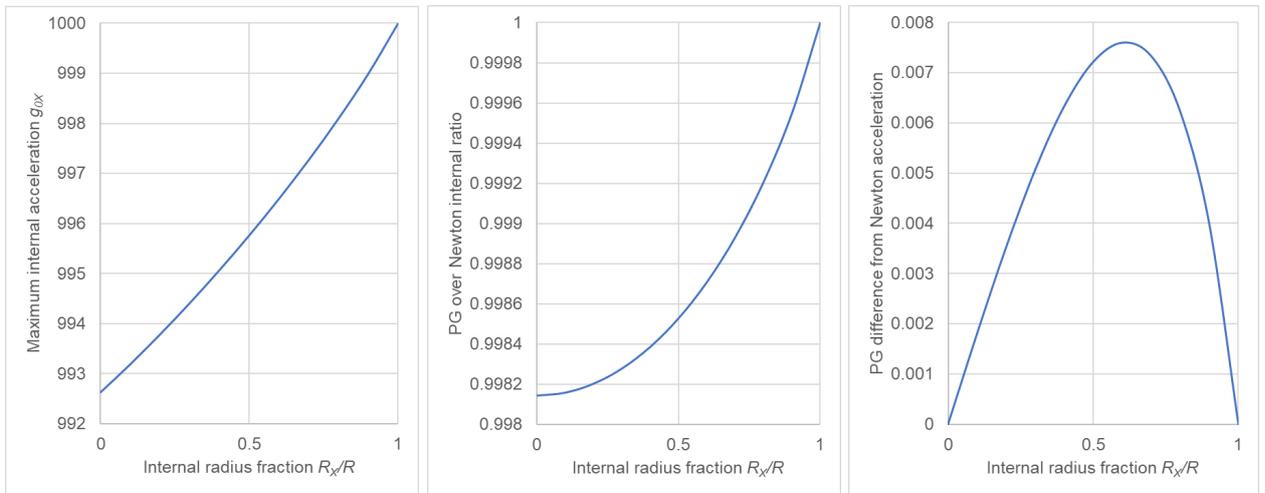


Figure 8: *Internal maximum acceleration g_{0X} , internal ratio of PG over Newton accelerations and internal difference of PG from Newton acceleration in Earth.*

We present some values as in Table 1 for Earth by using average values for density and absorption coefficient taken from the Table 3 as used for various planets in the following section. Tentatively, we initially use the value $g_0 = 1000 \text{ m/s}^2$. The results provide the expected deviation of measurements from Newtonian physics at various depths, if the Earth's crust had uniform density and a spherical shape. We can do measurements in a very deep mine or in a deep ocean, however, we would need to re-calculate the local acceleration in both Newton and PG cases. In practice, measurements of this kind would be complicated by influences of the local variations of density and time dependent fluctuations of the local acceleration, but the given table provides a first idea of the order of magnitude of expected deviation from Newton for a prospective careful experiment. It seems that these deviations should be measurable by a sensitive gravimeter with sufficient confidence if g_0 has a sufficiently low value. In turn, by establishing true measurements of the acceleration at various depths, we can deduce the unknown parameter g_0 .

For theoretical considerations, we can also see the variation of maximum internal acceleration g_{0X} , the ratio of g_{0X}/g_{NX} (PG/Newton) and the difference of PG from Newton acceleration in Fig. 8 at any depth (fractional radius) again for a tentative external $g_0 = 1000 \text{ m/s}^2$ in the case of planet Earth using the same parameters.

Establishing the variation of the maximum acceleration factor g_0 inside a planet, it also suggests that this parameter may not be so constant even in our relatively "small" area of the universe even inside the heliosphere, since there is a significant mass within the heliosphere itself, whilst our planets are just internal points within this sphere. This might explain the Pioneer anomaly for the deviation of gravity measurements from expected values from Newtonian mechanics. This then points also to the alternative possibility of purposefully sending a spacecraft to more accurately measure the same effect while eliminating (preventing) other already proposed explanations.

8.1 The Greenland experiment anomaly

Shortly before publication of the manuscript of this work, it has been accidentally found a report on "*The Greenland Gravitational Constant Experiment*" Zumberge *et al.* (1990) dealing exactly with the measurement of gravity in a bore hole in the ice-sheet. A deviation (shortfall) from Newton has been found in the range of between 1-4 mGal. This report appears particularly comprehensive in dealing with all possible sources of error and still found to establish a gravitational anomaly that cannot be explained by known theory stating in the abstract that: "*An anomalous variation in gravity totaling 3.87 mGal ($3.87 \times 10^{-5} \text{ m/s}^2$) in a depth interval of 1460 m was observed. This may be attributed either to a breakdown of Newtonian gravity or to unexpected density variations in the rock below the ice*". Although these measurements cannot be used "as is" to do any quantitative connection to the PG predictions in this Section, we do note that the order of magnitudes match well with those of Table 8. This is particularly encouraging to organize a similar experiment, perhaps, best suited in an ocean, where the local variations of gravity may be less or more easily predictable and the depth measurements about one order of magnitude greater.

However, variant reports by Zumberge and coworkers have failed to reproduce this anomaly at various oceanic depths, which might be attributed to either (a) the experimental error involved overwhelms the anticipated effect, which is of the order of magnitude tentatively deduced from the Allais effect per following Section 12.4, or (b) the mathematical treatment used for the Newtonian derivation may need reworking, else the direct method used in PG computations needs to be applied for the specific mass distribution at the location tested and integrated with the whole planet.

9 Application to the solar system

We can tentatively apply the equations of PG so far, like in Sections 6.3 and 6.5, to the solar system by assuming values of the maximum prevailing acceleration g_0 in our area of the universe.

We first plot the density ratios for many bodies of the solar system in Fig. 9 in the given hypothetical range of values for g_0 between 300-50000 m/s^2 . These graphs show that we get practically identical curves for Mercury and Mars having close to same surface accelerations, whilst all else are proportionally separated in accordance to their surface gravity. The graphs indicate the degree of departure from real densities depending on the chosen value of g_0 .

For better appreciation of the magnitudes involved, numerical results are also presented, as example, for three bodies (Sun, Earth and Moon) in Table 2 in the same range of g_0 . Some typical values of Λ for the same range of g_0 are also given being universal for all bodies per Eq. 73.

For any given set of mass and radius of each planet, we have derived the corresponding surface acceleration and effective density rather than using random (published) values from different sources. This is necessitated by the need to be consistent and accurate in these calculations and avoid discrepancies. These parameters are

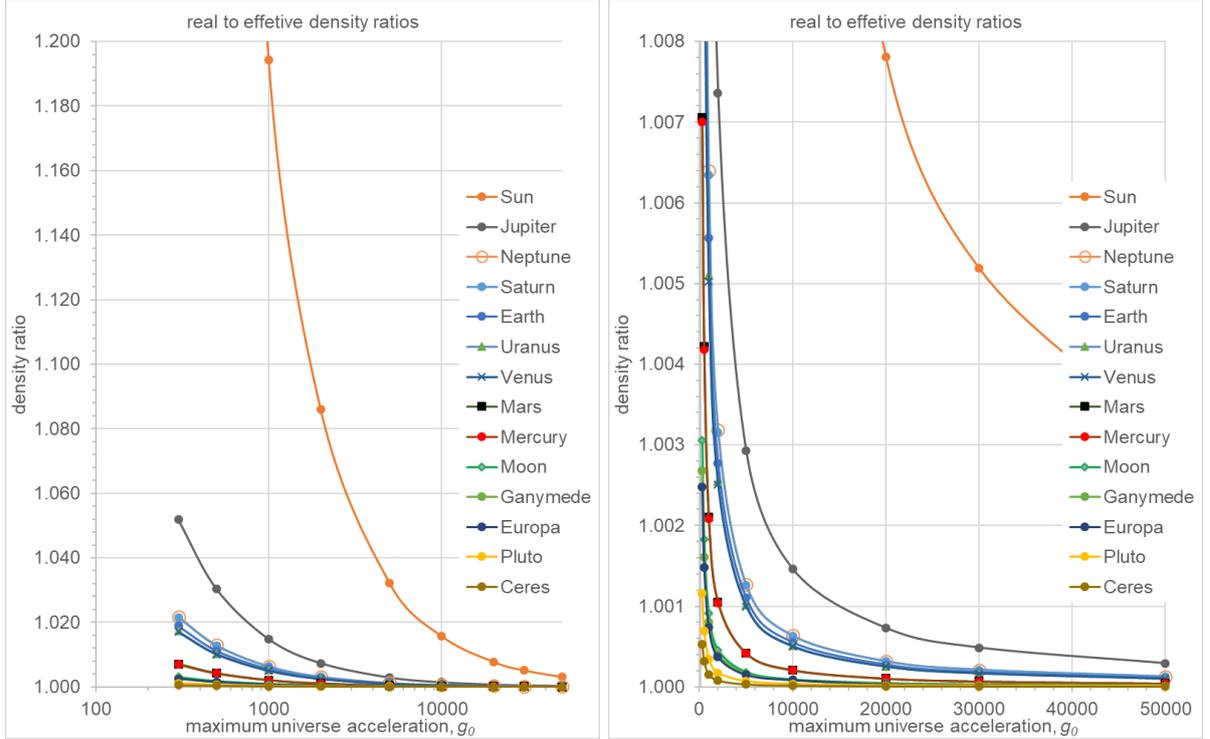


Figure 9: *Ratio of real to effective (measured) densities for planets, moons and the Sun.*

sensitive to very small changes of the input data. Small bodies are even more sensitive and round-off errors in the calculations are significant. Excel sheets were used for formatting the plotted figures, which initially necessitated the use of an “Add-in” (xlPrecision) to increase accuracy beyond 15 decimal places. Likewise, in a later use of Python code to reproduce the same output, we had to use increased computational precision to avoid serious round-off errors. In Table 3, we provide the initial data used for various bodies here and in all calculations elsewhere in this report. We also quote in parenthesis some variant values of the surface acceleration that were found from different sources for comparison, but not applied.

In all above, the derived and used parameters are based on the average density of the chosen bodies, which strictly speaking cannot produce the correct (actual) PG result, unless we knew in advance the radial density distribution for any given body. However, we obtain some first order of magnitude idea of the new important parameters introduced in this work. It should be noted that the density ratios approach unity as we increase g_0 .

9.1 Further analysis

To better understand the meaning of the real density expected for a planet, we can plot what the acceleration on the surface would be if the measured (effective) density were used as the real density. Let’s use the data for the Sun given by Table 3 and plot g_R against g_0 in Fig. 10 using Eqs. 59 and 60. We note that the Sun’s real acceleration is approached asymptotically at very high values of g_0 . The latter is as expected, because increasing g_0 decreases k , which makes the PG value to become Newtonian, i.e. to reduce to $g_R = 274.825$ m/s² as given in Table 3. The same can be deduced by taking the limit of Eq. 59 as $k \rightarrow 0$.

We have already found that by increasing the radius of a planet by adding mass at constant density, the surface acceleration reaches a saturation limiting value, namely, g_0 , i.e. when A_R becomes unity. This is at variance with Newtonian prediction of infinity by Eq. 75.

Likewise, with increasing the density by keeping the radius constant, the Newtonian prediction is infinity. However, in PG the factor A_R being a function of the product kR becomes a product also of ρR meaning that $A_R \rightarrow 1$ by increasing ρ with constant R and A . Similarly, by shrinking a star (sphere) with constant mass, we obtain unity for A_R as the density becomes fast very large (the density being inversely proportional to the third power of radius). In other words, the eventual surface acceleration reaches the saturation value of g_0 in clear distinction from Newtonian mechanics.

Last in this connection, we should also consider what happens at a fixed point in space away from a sphere (star), when the sphere shrinks with constant mass. By Newton, the acceleration remains constant

	Sun		Earth		Moon		
g_0 , m/s ²	k, 1/m,	ρ/ρ_e	k, 1/m	ρ/ρ_e	k, 1/m	ρ/ρ_e	Λ , m ² /kg
300	3.4245E-09	3.4640	3.9262E-09	1.01886	2.3464E-09	1.003060	6.99E-13
500	9.1141E-10	1.5366	2.3380E-09	1.01121	1.4061E-09	1.001833	4.19E-13
1000	3.5419E-10	1.1943	1.1625E-09	1.00556	7.0242E-10	1.000915	2.10E-13
2000	1.6103E-10	1.0860	5.7963E-10	1.00277	3.5105E-10	1.000457	1.05E-13
5000	6.1225E-11	1.0322	2.3146E-10	1.00111	1.4038E-10	1.000183	4.19E-14
10000	3.0125E-11	1.0158	1.1567E-10	1.00055	7.0185E-11	1.000091	2.10E-14
20000	1.4944E-11	1.0078	5.7818E-11	1.00028	3.5091E-11	1.000046	1.05E-14
30000	9.9371E-12	1.0052	3.8542E-11	1.00018	2.3393E-11	1.000030	6.99E-15
50000	5.9499E-12	1.0031	2.3123E-11	1.00011	1.4036E-11	1.000018	4.19E-15

Table 2: Calculated absorption coefficient k and ratio of real ρ over effective ρ_e density for the Sun, Earth and Moon in an assumed range of g_0 values.

planet	radius R	mass M_e	density ρ_e	g_R (other)
Sun	6.95E+08	1.989E+30	1.41446E03	274.825 (273.7)
Jupiter	6.9911E7	1.8982E27	1.326E3	25.9204 (24.79)
Neptune	2.4622E7	1.02413E26	1.6379344E3	11.27456624 (11.15)
Saturn	5.8232E7	5.6834E26	6.87123E2	11.1860(10.44)
Earth	6.371E6	5.97237E24	5.5136E03	9.82026 (9.807)
Uranus	2.5362E7	8.6810E25	1.27037E3	9.00729 (8.69)
Venus	6.0518E6	4.8675E24	5.243E3	8.87009 (8.87)
Mars	3.3895E6	6.4171E23	3.93408E03	3.727854(3.720)
Mercury	2.4397E6	3.3011E23	5.42701E3	3.70150 (3.7)
Moon	1.73700E06	7.34767E22	3.34705E03	1.62533 (1.625)
Ganymede	2.634E6	1.4819E23	1.93590E3	1.42554 (1.428)
Europa	1.560E6	4.799844E22	3.01832E03	1.316343805 (1.315)
Pluto	1.1883E6	1.303E22	1.85386E03	0.615862 (0.62)
Ceres	4.730E05	9.393E20	2.161E3	0.280203 (0.28)
Callisto	2.4103E6	1.075938E23	1.8344	1.235

Table 3: Numerical constants of planets, moons, and the Sun used in calculations of preceding tables and graphs.

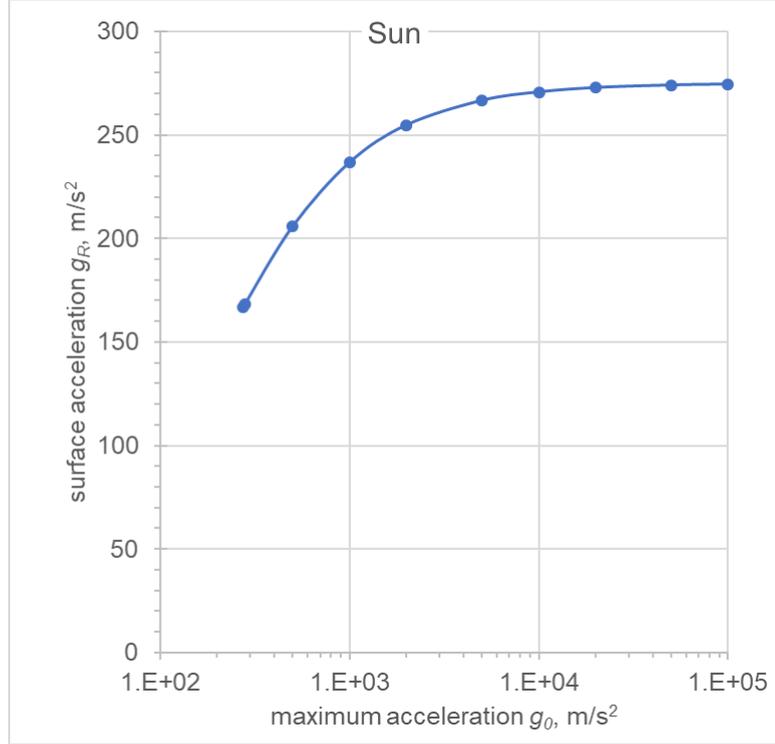


Figure 10: *Expected surface acceleration on the surface of the Sun against maximum g_0 using the measured density as real density.*

at that point, but by PG this is not the case: The acceleration monotonically becomes smaller, due to self shadowing (k increases much faster than the radius) by

$$g_{fixed_r} = g_0 \frac{\pi A}{r^2} \quad (95)$$

noting that A varies as:

$$A = R^2 \left[1 - \frac{R^4}{2C^2} + \frac{R^4}{2C^2} \exp(-2C/R^2) \cdot (2C/R^2 + 1) \right] \quad (96)$$

where the constant C is defined during the k substitution below:

$$k = \frac{3GM}{g_0 R^3} \equiv \frac{C}{R^3} \quad (97)$$

Noted also that the PG equation of acceleration reduces to Newton's equation, as expected, for very small values of k :

$$g_A = g_0 A_{A_{k \rightarrow 0}} = g_0 \frac{4}{3} k R = \frac{4}{3} \pi G \rho R = G \frac{M}{R^2} \quad (98)$$

Furthermore, we can substitute k accordingly and find g_A for a white dwarf and a neutron star. The extreme accelerations reported for these bodies pose for now a serious question on whether PG could ever be directly measurable or detectable if g_0 needs to be too high. This would constitute a new serious challenge for PG by not being able to detect it experimentally, unless those extremely high values of acceleration are generated by yet another type of push particle. We will discuss this issue again in Part 2 of this report.

10 Concentric spheres with different densities

We now consider the case of two concentric spheres of different density as depicted in Fig. 11. The inner sphere has a radius R_1 with density ρ_1 , mass M_1 and absorption coefficient k_1 , and the outer sphere has a

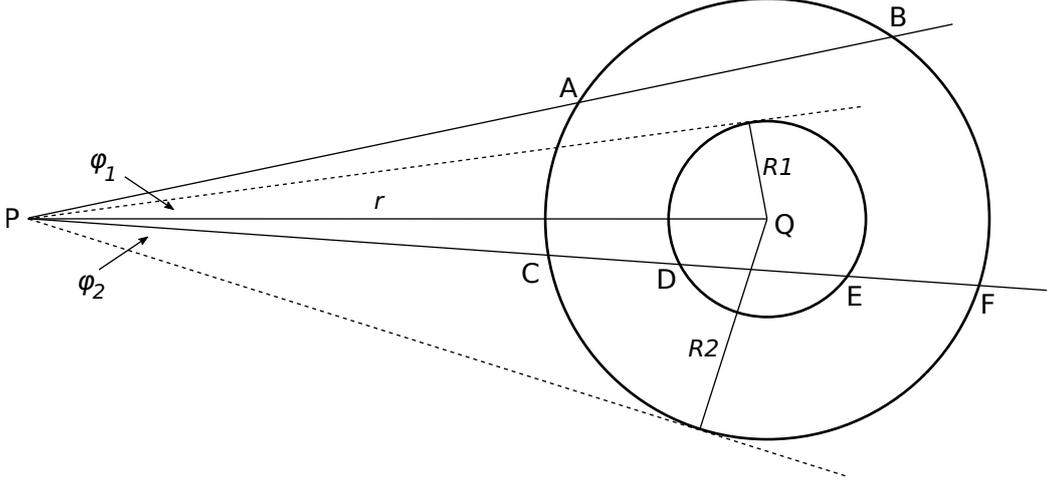


Figure 11: *Derivation of acceleration from concentric spheres with different densities.*

radius R_2 with density ρ_2 , mass M_2 , and absorption coefficient k_2 . There are two cases of PG absorption, namely, one along a typical chord AB traversing only the outer sphere, and another traversing segment CD of the outer sphere then a chord DE of the inner sphere and then segment EF of the outer sphere again.

To find the acceleration at point P being at a distance $r = PQ$, we follow the integration steps as in the first place for PG (Eq. 37), but for the two parts described above:

Part one involves integration in the angle between φ_1 and φ_2 for the outer spherical layer.

$$f_{g2} = 2\pi \left[\frac{\sin^2 \varphi}{2} - \frac{\exp\left(-2kr\sqrt{a^2 - \sin^2 \varphi}\right) \left(2kr\sqrt{a^2 - \sin^2 \varphi} + 1\right)}{4k_2^2 r^2} \right]_{\varphi_1}^{\varphi_2} \quad (99)$$

where

$$a = \frac{R_2}{r} = \sin \varphi_2 \quad (100)$$

After substituting the integration limits, we get a familiar relationship as follows:

$$f_{g2} = \pi \left[R_2^2 - R_1^2 - \frac{1}{2k_2^2} + \frac{\exp\left(-2k_2\sqrt{R_2^2 - R_1^2}\right) \left(2k_2\sqrt{R_2^2 - R_1^2} + 1\right)}{2k_2^2} \right] \frac{1}{r^2} \quad (101)$$

Part two then involves the following steps starting with the general PG Eq. 37, where we have for the inner sphere

$$a = \frac{R_1}{r} = \sin \varphi_1 \quad (102)$$

and need to replace the exponential having length ℓ in the exponent with three exponential factors corresponding to the three consecutive absorption layers (lengths) in EF, DE and CD:

$$f_{g1} = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi d\varphi \cdot [1 - \exp(-k_2 \cdot EF(\varphi)) \cdot \exp(-k_1 \cdot DE(\varphi)) \cdot \exp(-k_2 \cdot CD(\varphi))] \quad (103)$$

That is

$$f_{g1} = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi d\varphi \cdot [1 - \exp(-2k_2 \cdot EF(\varphi) - k_1 \cdot DE(\varphi))] \quad (104)$$

Because

$$2EF = CF - DE \quad (105)$$

Jupiter	radius R	mass M_e	density ρ_e
whole planet	6.9911E7	1.8982E27	1326
core	1.6E7	4.289E26	25000
outer	6.9911E7	1.4693E27	1039
g_0	g_{RPG}	$g_R - g_{RPG}$	ρ/ρ_e
300	18.97555828	6.944841717	1.051873063
500	18.87716456	7.043235439	1.030304147
1000	18.80444874	7.115951257	1.014860051
2000	18.76839103	7.152008972	1.007359332
5000	18.74684295	7.17355705	1.002927051
10000	18.73967389	7.180726112	1.001460768
20000	18.73609184	7.184308161	1.000729697
30000	18.73489819	7.185501814	1.000486312
50000	18.73394339	7.186456606	1.000291714

Table 4: A two-layered sphere model of Jupiter with same real mass redistributed to the corresponding radii provided; Surface accelerations with PG and difference from Newton in a range of g_0 values.

and using Eq. 12 for each of the spheres, we can easily replace with:

$$f_{g1} = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi d\varphi \cdot \left[1 - \exp \left(-2k_2 r \sqrt{a_2^2 - \sin^2 \varphi} - 2(k_1 - k_2) r \sqrt{a_1^2 - \sin^2 \varphi} \right) \right] \quad (106)$$

for which unfortunately the anti-derivative could not be found analytically. The total acceleration is given by the usual factor as:

$$g = g_0(f_{g1} + f_{g2})/\pi \quad (107)$$

As usual, we equate $r = R_2$, when we need to find the acceleration g_R at the surface of a sphere.

We may appreciate the relative magnitudes involved, if we were to take, for example, Jupiter as consisting of two concentric spheres with the tentative (arbitrary) parameters provided in Table 4. Jupiter's core constitution is uncertain, so that the values are only indicative for the present purposes and chosen among various values in the literature (<https://sciencing.com/jupiters-core-vs-earths-core-21848.html>). The Jupiter mass is actually layered with variable densities, but the best we can demonstrate at this stage is to start with a uniform mass equal to the total one actually measured (M_e) from its corresponding acceleration $g_R = 25.92 \text{ m/s}^2$ (already used here). We then find the real mass M , as we did for various planets before, by first solving the equation of the parameter A_R for k with any given g_0 , from which we establish the real density ρ and density ratio ρ/ρ_e . Next, we redistribute this mass in the two spheres in the same proportion as initially provided in this table, namely 0.2259599008534 fraction of the total is compressed inside the inner sphere (core) and the remainder fraction is contained by the outer spherical layer. The real densities ρ_1 and ρ_2 in the two layers are readily found, from which the corresponding parameters k_1 and k_2 are calculated and used in Eq. 11. The results for the acceleration on the surface of the planet are given in numerical form in Table 4 again as a function in the typical range of g_0 .

In Newtonian mechanics, the redistribution would have no effect on the surface acceleration g_R , but in PG the surface acceleration g_{RPG} is very different, as we can see it is significant. The lower values obtained from PG indicate that the final actual densities should be increased in order to yield the real measured surface acceleration. In other words, there is a significant amount of hidden mass by the mere fact of having a dense core over and above (in addition to) the hidden mass also present in a uniform distribution. This is important, which means that any attempt to redistribute the mass of Jupiter along the radius should take into account the new physics revealed by PG. This also means that all previous calculations assuming an average constant density for the planets produces only approximate results. The difference becomes more

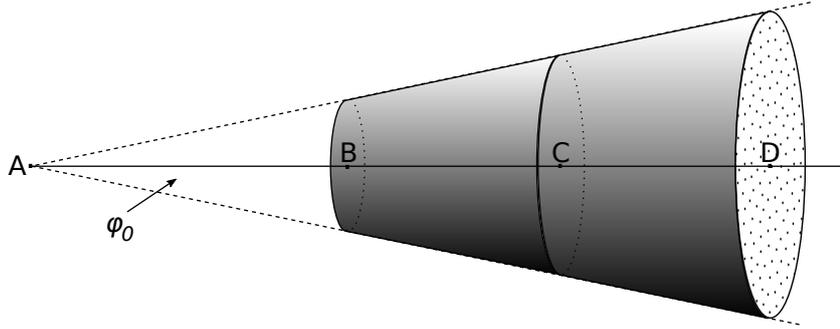


Figure 12: *Coaxial truncated spherical cones (sections) with fixed and equal height.*

important with the increase of the planet or star size. Noted also that the main (dominant) component of the PG acceleration comes from the diluted outer layer mass for the chosen mass redistribution - if the two components are considered separately. Hence in general, all prior attempts dealing with assumed mass and mass distributions should be re-appraised accordingly. In fact, artificial satellites orbiting Jupiter have reported anomalous orbits with a noticeable wobble, which may be attributed to Moons of Jupiter being shadowed by a different core density from the outer planet. We now have a new basis to re-evaluate and explain many phenomena already on record.

We may generalize and conclude that the radial distribution of density in a spherical body is critical in the generation of acceleration at the surface of the sphere and beyond according to PG, whereas this distribution makes no difference in the Newtonian acceleration lumping the mass at the center of gravity (i.e. center of the sphere). In an arbitrary shape with an arbitrary density distribution then, the only correct way is to derive the acceleration and force by integration of the graviton absorption around three coordinate axes yielding the three components of the vector of acceleration.

11 The superposition principle revisited and revised

The superposition principle, also known as superposition property, states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually. This applies to Newtonian gravity. However, this is not valid in general PG, unless the absorption coefficient k is relatively small.

Let us now consider Fig. 12, where we draw two co-axial truncated cones subtending the same solid angle at point A with semi-angle φ_0 and with equal height, namely, $BC = CD = \ell$. Each material cone creates the same amount of graviton shadowing, if considered separately, i.e. without the presence of the other. In other words, PG provides an insight first with an immediate result that all truncated cones of constant angle and equal height will produce the same acceleration of gravity regardless of their distance from a common convergence point; this result can be derived at without any computation or integration of the elementary masses constituting these shapes.

However, when they act in series as depicted, the inner (nearest to A) cone is shadowed by the outer one and absorbs a lesser amount from the decreased output of graviton intensity by the outer cone. In the special case where the absorption is linear, which is the case when k is sufficiently small, then we can superpose their separate absorption like in Newtonian superposition of gravity.

Now, we consider the general case of PG again in Fig. 13(a), which is essentially the same as the previous figure but the truncated cones subtend a very small angle $\Delta\varphi$, which allows the shifting of the inner cone as in 13(b) by the same small angle without practically changing the direction of the vector of shadowing (acceleration), i.e. both are considered to retain the same direction at point A. By this, we get a simplified derivation in the case of exponential absorption of gravitons (i.e. general PG) below.

Each truncated cone constitutes a material layer with thickness ℓ and absorption coefficient k , so that the transmitted intensity is given by Eq. 6. When the layers are in series as shown in (a), the total absorption through the double thickness is

$$\Delta J_{series} = \Delta J_0 (1 - \exp(-k2\ell)) \quad (108)$$

However, if the inner cone is shifted as in (b) with the vectors of acceleration practically lined up, we can add them numerically for the total absorption according to Eq. 6 as

$$\Delta J_{parallel} = \Delta J_0 2 (1 - \exp(-k\ell)) \quad (109)$$

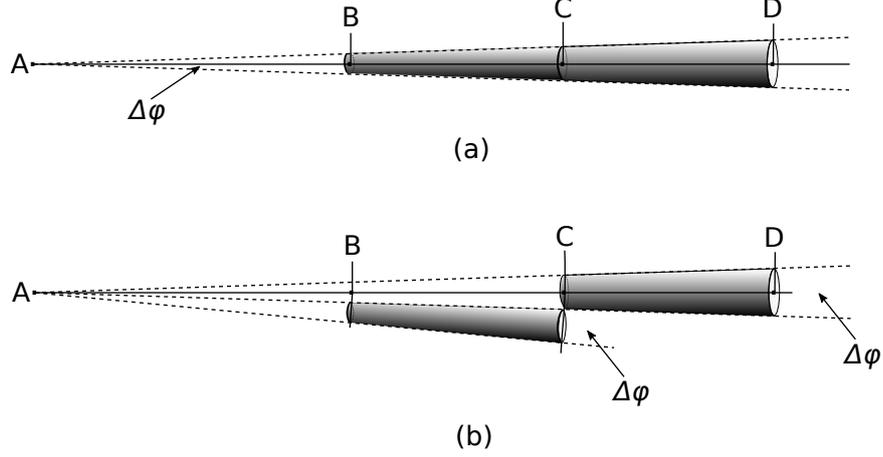


Figure 13: *Elementary truncated cones of equal height in series (a) and in parallel (b).*

The difference between these cases then becomes

$$\Delta J_{parallel} - \Delta J_{series} = \Delta J_0 (1 + \exp(-k2\ell) - 2\exp(-k\ell)) \quad (110)$$

which is a positive number and indicates that the total shadowing (acceleration) by the two layers is stronger when they are in parallel than when in series by one shielding the other.

The same can be expressed also in terms of absorption fractions:

$$f_{parallel} - f_{series} = 1 + \exp(-k2\ell) - 2\exp(-k\ell) \quad (111)$$

and in terms of acceleration:

$$\Delta g = g_{parallel} - g_{series} = g_0 (1 + \exp(-k2\ell) - 2\exp(-k\ell)) \quad (112)$$

12 On direct measurement of PG

12.1 Sphere

We can inquire about the difference of acceleration derived by Newton and PG on the surface of a sphere of known density to determine if it is practically possible to detect and measure the new PG parameters directly. If the Newton acceleration on the surface is g_{RN} and the PG acceleration g_R , their difference is given by

$$\Delta g = g_{RN} - g_R = \frac{4}{3}\pi G\rho R - g_0 A_R \quad (113)$$

for which we need k in A_R given from Eq. 60 as

$$k = \frac{\pi G\rho}{g_0} \quad (114)$$

from assumed values of g_0 and the real density of the sphere. We can plot the difference like we plotted the ratio of accelerations in Fig. 5, but we prefer to see directly some numerical outputs in Table 5 by choosing, say steel with $\rho = 7500 \text{ kg/m}^3$.

We may also further work on the equation above to produce:

$$\Delta g = g_0 \left(\frac{4}{3}kR - A_R \right) \quad (115)$$

which is a function of the product kR .

For very high kR , the difference is very high, but for very small kR the difference is very small but finite. By expanding the exponential to a Taylor series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ and taking the limit for small kR , we obtain for the difference of accelerations

$R=$	10	100	1000	10000	100000
$g_N=$	2.097E-05	2.097E-04	2.097E-03	2.097E-02	2.097E-01
g_0	Δg				
300	8.243E-13	8.243E-11	8.243E-09	8.243E-07	8.243E-05
500	4.946E-13	4.946E-11	4.946E-09	4.946E-07	4.946E-05
1000	2.473E-13	2.473E-11	2.473E-09	2.473E-07	2.473E-05
2000	1.236E-13	1.236E-11	1.236E-09	1.236E-07	1.236E-05
5000	4.946E-14	4.946E-12	4.946E-10	4.946E-08	4.946E-06
10000	2.473E-14	2.473E-12	2.473E-10	2.473E-08	2.473E-06
20000	1.236E-14	1.236E-12	1.236E-10	1.236E-08	1.236E-06
30000	8.243E-15	8.243E-13	8.243E-11	8.243E-09	8.243E-07
50000	4.946E-15	4.946E-13	4.946E-11	4.946E-09	4.946E-07

Table 5: *Difference of acceleration between Newton and PG on the surface of an iron sphere with density 7500 kg/m³.*

$$A_{RkR \rightarrow 0} = \frac{1}{6}kR(8 - 6kR + 4k^2R^2) \quad (116)$$

$$\Delta g_{kR \rightarrow 0} = \frac{1}{3}g_0k^2R^2(3 - 2kR)_{kR \rightarrow 0} = g_0k^2R^2 \quad (117)$$

For the numerical example of the table, we see that we could have used Eq. 117 for small kR , which provides that the difference is proportional to g_0 and to the square of the radius of the sphere. The practical outcome is that, for the smallest sphere, we would need an extremely sensitive gravimeter with an accuracy up to 10 orders of magnitude smaller than the expected Newtonian value. The situation improves fast as we increase the sphere diameter, except that such spheres are out of any practical use. The situation improves with decrease of g_0 .

12.2 Cone

We have further investigated whether the same reference spherical masses used above, if reshaped properly, they could yield any improved (i.e. greater) difference between Newton and PG for a possible measurement from a known density mass. This has been investigated for truncated and spherical cones with negative results (i.e. no improvement). However, interestingly enough it was found that there is an optimum cone angle yielding maximum acceleration difference at their apex, but still very close to, (but less than) the spherical shape. There is no need to present these results at present in order to give priority to more mundane issues below.

12.3 Cube

Perhaps, a large steel (or other heavy material) cube shape might be more feasible to construct by bricks, which would reduce cost by later disassembling and re-use of the steel material. The Newtonian gravitational fields has already been provided analytically by Chappel *et al.* (2012). Measurements of some gravity contour (or point) around the cube may be done with the most sensitive gravimeter to investigate possible "anomaly". With a positive outcome, we can then calculate the corresponding PG gravity contour (or point) by integrating the shading of gravions per established theory. From the known density, we will then be able to directly derive all other PG parameters.

12.4 The Allais effect

The previous finding on gravity superposition in PG can be used for explaining the known "Allais effect" recorded during total eclipses of the Sun. According to this, the gravity on Earth is increased during the

eclipse, namely, the Moon+Sun have less attraction on Earth during an eclipse than just before or after the eclipse.

Qualitatively, we can say that when the Moon stands between the Sun and the Earth, it shields the shadowing of the Sun reducing the sum total of the shadows of the Moon and Sun separately prior to them being aligned. The principle of this effect, now for spherical bodies, is derived quantitatively below by PG theory.

The derivation is facilitated, since by coincidence the Sun and the Moon subtend practically about the same average solid angle of 0.53 and 0.52 degrees, respectively. Based on Fig. 14, we use R_s for the Sun radius and R_m for the Moon radius, located at distances $PQ = r_s$ and $PQ' = r_m$ from point Q.

When the two spheres are lined up, then the integral of their PG gravity acceleration is given by

$$f_g = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi d\varphi \cdot [1 - \exp(-k_s \ell_s(\varphi) - k_m \ell_m(\varphi))] \quad (118)$$

where we have simply added the lined-up chord lengths $AB = \ell_s$ and $A'B' = \ell_m$ of the Sun and the Moon. Using the established lengths for these chords, we substitute as follows:

$$f_g = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi d\varphi \cdot \left[1 - \exp\left(-2k_s r_s \sqrt{a_s^2 - \sin^2 \varphi} - 2k_m r_m \sqrt{a_m^2 - \sin^2 \varphi}\right) \right] \quad (119)$$

However, because both spheres are taken to subtend equal angles, we have

$$a_1 = \frac{R_m}{r_m} = \frac{R_s}{r_s} = a_2 = \sin \varphi_0 = a \quad (120)$$

and the integrand is simplified below:

$$f_g = 2\pi \int_0^{\varphi_0} \sin \varphi \cos \varphi d\varphi \cdot \left[1 - \exp\left(-2(k_s r_s + k_m r_m) \sqrt{a^2 - \sin^2 \varphi}\right) \right] \quad (121)$$

which is of the same form as Eq. 36 by setting $kr = k_s r_s + k_m r_m$. Thus, we obtain from

$$f_g = 2\pi \left[\frac{\sin^2 \varphi}{2} - \frac{\exp\left(-2kr \sqrt{a^2 - \sin^2 \varphi}\right) \left(2kr \sqrt{a^2 - \sin^2 \varphi} + 1\right)}{4k^2 r^2} \right]_0^{\varphi_0} \quad (122)$$

the accelerating absorption fraction

$$f_g = 2\pi \left[\frac{\sin^2 \varphi}{2} - \frac{\exp\left(-2(k_s r_s + k_m r_m) \sqrt{a^2 - \sin^2 \varphi}\right) \left(2(k_s r_s + k_m r_m) \sqrt{a^2 - \sin^2 \varphi} + 1\right)}{4(k_s r_s + k_m r_m)^2} \right]_0^{\varphi_0} \quad (123)$$

By substituting the limits of integration and in view of Eq. 120, we get

$$f_g = \pi \left[\frac{R_s^2}{r_s^2} - \frac{1}{2(k_s r_s + k_m r_m)^2} + \frac{\exp(-2(k_s R_s + k_m R_m)) \cdot (2(k_s R_s + k_m R_m) + 1)}{2(k_s r_s + k_m r_m)^2} \right] \quad (124)$$

The above result applies during the eclipse, i.e. when the spheres are in “series”.

When the spheres are in “parallel” to each other, i.e. just before or just after the eclipse, we use the form of Eq. 38 to sum the separate contributions of each as

$$f_g = \pi \left[R_s^2 - \frac{1}{2k_s^2} + \frac{\exp(-2k_s R_s) \cdot (2k_s R_s + 1)}{2k_s^2} \right] \frac{1}{r_s^2} + \pi \left[R_m^2 - \frac{1}{2k_m^2} + \frac{\exp(-2k_m R_m) \cdot (2k_m R_m + 1)}{2k_m^2} \right] \frac{1}{r_m^2} \quad (125)$$

The Allais effect then should appear as the difference of acceleration in the above positions (equations)

$$g_{Allais} = g_{parallel} - g_{series} \quad (126)$$

where we have multiplied Eqs. 124 and 125 by G/Λ to obtain the factor g_0 and hence the corresponding accelerations.

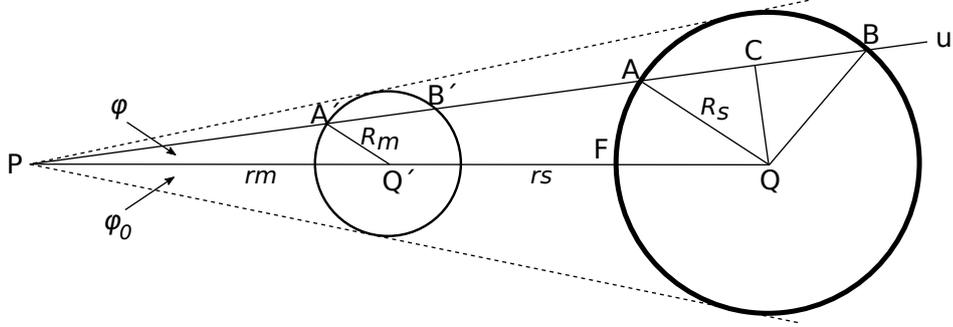


Figure 14: *PG diagram during Sun-Moon eclipse.*

g_0 , m/s ²	k , 1/m, Sun	ρ , kg/m ³	ρ/ρ_e
300	5.865327998802280E-09	8392.131445	5.459718
500	1.027835151863200E-09	2451.050109	1.594594
1000	3.894839466528660E-10	1857.583228	1.208499
2000	1.758800091947190E-10	1677.664808	1.091448
5000	6.665703761053060E-11	1589.551972	1.034124
10000	3.276671652759630E-11	1562.757685	1.016692
20000	1.624742838322570E-11	1549.791755	1.008257
30000	1.080184118890250E-11	1545.531147	1.005485
50000	6.466902304965150E-12	1542.144338	1.003282

Table 6: *Variation of Sun parameters against increasing values of g_0 with adjusted radius $R = 6.76002E08$ m corresponding to effective density $\rho_e = 1537.0998312$ kg/m³ and $g_R = 290.489290112956$, but the same mass $M_e = 1.989E30$*

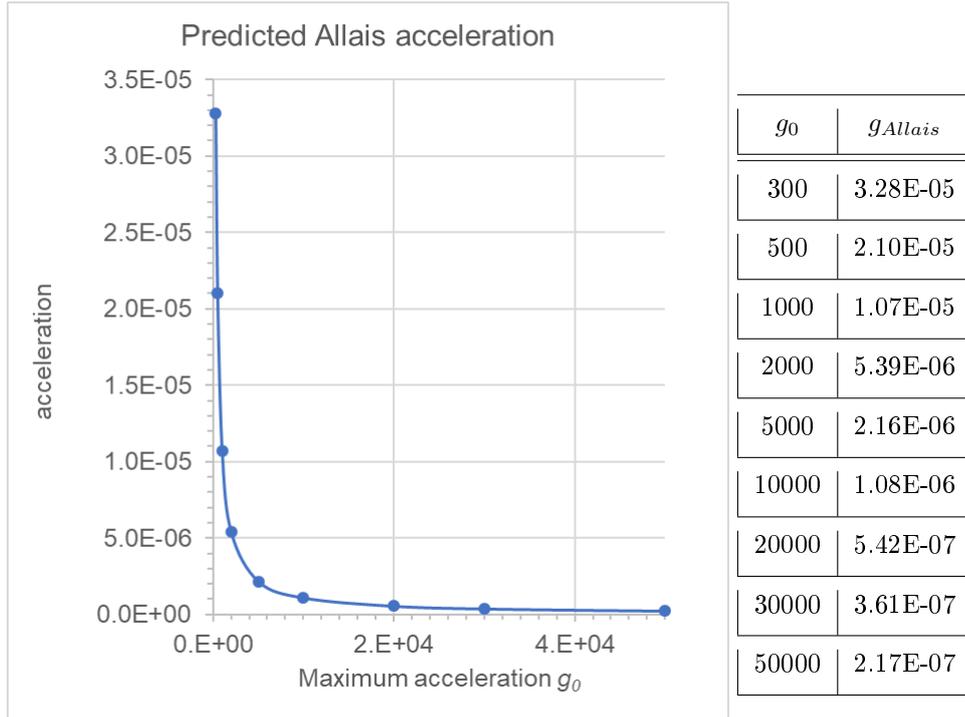


Figure 15: Predicted variation of the Allais effect acceleration vs. assumed values of g_0 .

We can plot calculated values of g_{Allais} against g_0 to establish for which values of g_0 we find the measured g_{Allais} . However, because we are dealing with very small numbers in these calculations, it is necessary not only to use increased accuracy in the mathematical tools employed and also to eliminate the small but significant difference arising from the fact that the Moon and Sun do not subtend exactly equal angles. In other words, it was found that the small difference of the actual subtended angles overwhelms the Allais effect altogether by resulting in significant inconsistencies in the calculations. In order to test the validity of the principle, at least, we need to take the real values for one sphere and project (adjust) slightly the other so as to satisfy the condition of Eqs. 120. This is done only to be able to use the derived Allais equations above, in order to demonstrate the effect; otherwise we would be faced with considerable complexity to use PG with the actual angles, which is not needed for the present purposes. A high accuracy theoretical derivation would also require the availability of experimental measurement of the Allais effect with commensurate accuracy. Unfortunately, such measurements do not exist, because the Allais effect has been reported with significant inconsistencies. We believe that these inconsistencies are caused by the interference of various other effects on Earth, such as tidal effects and others (may be atmospheric, etc.) as well as variability of the time and position during this event. In view of these practical difficulties, it would be in vain to apply derived formulas for a direct practical outcome, except that we can use them to establish the principle and an order of magnitude. Refinement of the theory and practice of the Allais phenomenon is left for future work. Therefore, we opted to adjust the radius of the Sun to match the subtended angle of the Moon, while keeping its mass constant. We have done this and present the numerical values in Table 6, as opposed to the values presented in Table 2. We can then use these values of k for the Sun and the corresponding values from Table 3 for the Moon in Eq. 126, which we plot in Fig. 15 with tabulated numerical values of the same graph in the inserted table.

From a paper by Lorenzen (2017), we obtain an average value $g_{Allais} = 3.5E-7 \text{ m/s}^2$, which corresponds to $g_0 = 30800 \text{ m/s}$. If that were to be correct, then we would have derived the fundamental constant g_0 for our neighborhood universe. However, this is only a tentative value, most likely to be revised later. It may be that similar measurements taken from the Moon during a solar eclipse by the Earth could provide more reliable values; clearly, in that case, we should derive another equation taking into account the actual solid angles subtended by Earth and the Sun at the Moon.

With the proposed tests, PG could be verified but not disproved in the event of a very high level of g_0 reducing the effect beyond the measuring ability of our instruments at present. It is hoped that "anomalies" of Newtonian mechanics will be firmly measured and established to provide an affirmation of PG once and

for all in the near future.

12.5 Other verification means for PG

We have already proposed measurements of the internal field of Earth's crust, variation of gravity in the heliosphere and other methods above. Another test may be by using very sensitive gravimeters to measure the variation of gravity on Earth during a 24 hour rotation preferably during a new Moon (or better before, during and after a solar (better total) eclipse), whereby the Earth is shadowing the Sun+Moon system overnight. The deviation from predicted values using Newtonian mechanics should provide an indication and perhaps an evaluation of the PG parameters.

Alternatively, a similar to the previous observation could be made by the variation of the orbit of an Earth satellite during a new Moon (or better before, during and after a solar (better total) eclipse, whereby the Earth is shadowing the Sun and Moon systems during the night passage of the satellite, presuming that the effect on the orbit could be measurable. Could then such a variation explain the variations (wobble) observed by Juno during orbiting Jupiter?

13 Discussion on gravitational law

The finding that the gravitational force is inversely proportional to distance constitutes a universal relationship now derived from the principles of static PG theory. It is unlikely that this is a fortuitous derivation, although we must wait to find the same consistency with dynamic PG. It is likely that PG can provide a genuine platform to re-work many other relationships with new physics.

We have derived some fundamental but novel relationships yielding the classical acceleration and force but revealing a different relationship with the actual mass. The classical (Newtonian) mass is now understood to be only an apparent or effective manifestation of the real mass. The inverse square of distance law is preserved, whilst the classical gravitational constant G is itself a function of another constant like Λ , J_0 or g_0 , all of which are characteristic of any given region of space. It is important that these relationships are not merely empirical, but are based on a simple principle or premise of particles uniformly traveling in all directions, while they are absorbed by matter at a rate in proportion to the density of the matter. This provides a more “tangible” explanation of the gravity, which, however, shifts the problem to the understanding of the nature of these particles, not less mysterious than the elusive gravity to date. Nevertheless, it looks like we can narrow down the fundamental problem of gravity to a “lesser” entity bringing us closer to the goal of a unification theory. After all, forces are already attributed to the exchange of different kind of particles: Gluons for the strong nuclear force, photons for the electromagnetic force, the bosons for the weak nuclear force and speculated gravitons for gravity. Quantum chromodynamics aims to find the smallest building block of nature and the forces that hold them together. PG may not be seen in conflict but rather it may offer a general platform to remold and hopefully unify current quantum gravity and graviton, superstring theory, loop quantum gravity and blending quantum gravity with quantum mechanics for a theory of everything.

The validity of the gravitational law derived is further subject to ensuring that the involved gravitating bodies exist in “free space”, otherwise the space itself is filled with matter albeit of extremely low density. For in the latter case, we deal effectively with an internal field as found in Section 8. Then, g_0 is variable and a function of distance r from some center of mass, hence the overall gravitational law ceases to be strictly inversely proportional to the square of distance. That means that the gravitational constant $G(r)$ can be a weak function of distance from the said center of mass. In the latter case, the force will be slightly weaker than Newton's law resulting in precession for an orbiting planet. No attempt is made here to evaluate the magnitude (significant or negligible for an elliptical orbit) of possible variation of $G(r)$, before we can establish the theory PG itself. Eventually, relativistic effects may also have to be included in addition to the classical derivation of precession under PG, [but this is subject to further PG analysis and development.](#)

In one aspect, the [form](#) of Newton's gravitational law could be considered correct with regard to $1/r^2$ (being universal), but [if the mass becomes effective mass as revealed by PG](#), then such a law is incorrect. This is further analyzed under Section 16. Furthermore, it is an irony that the “universal gravitational constant G ” is neither universal nor constant. In fact, we may not need to travel very far to realize that its value varies significantly within the heliosphere and more so as we approach the neighborhood of the Sun. Close enough to the Sun, there may be a lot of mass emitted to a significant level, which makes the closest planets effectively experiencing an “internal” gravitational field per Section 8. Mercury may be significantly affected by the variation of $G(r)$ during its orbit. It remains to derive its precession based on this theory and see if a satisfactory explanation can be found.

This discussion applies to gravity around stars and planets, but can we still call gravity the field around dwarfs, neutron stars and black holes, if it is caused by the different proposed types of push particles,

according to a subsequent proposal? Each of these fields would have its own mass attenuation coefficient Λ with a different value from that corresponding to gravions. Beyond white dwarfs, a neutron star would have a variable Λ by superposition of two types of push particles, whilst black holes by superposition of more types of push particles. We may then have to introduce other terms (semantics) to differentiate the fields around these massive bodies from our familiar gravity field. In fact, we should expect to have a mixed or variable effective Λ parameter, which we might wish to denote or index with a different subscript. In those fields then the inverse square of distance law may break down again, but the math remains to be worked out. Correct terminology is important, because, when we say that the gravitational law is preserved in the cosmos, we mean that it applies to gravity due to gravions under the postulates at the outset, which is correct only regionally in the cosmos.

Last but not least is to discuss the mean free path (m.f.p.) of gravions postulated at the outset of PG. Whereas the m.f.p. is assumed to be much greater than the size of the gravitating bodies, no further qualifications were made. How much greater is it in reality? We have no knowledge of this yet, so we can only discuss the various main possibilities: (a) the m.f.p. is infinite, i.e. gravions never interact between themselves, (b) the m.f.p. is of intergalactic order and (c) the m.f.p. is of intra-galactic order. These orders of magnitude are not the only possibilities, but are sufficient for a general discussion in Part 2 of this report. These ranges of m.f.p. define corresponding regions of space, where the gravitational law varies. As soon as the postulated m.f.p. ceases to apply, push gravity behaves differently and is governed by different relationships and laws. It is of great importance to know also the forces (fields) at the transition from one region to the next and beyond. Part 1 has not dealt with such regions yet. Pending such work, we can only speculate at this stage what happens. The inadequacy of PG theory of Part 1 for those regions may correspond to the inadequacy of GR (general relativity) also at long distances, except that PG can be readily expanded and advanced in ways discussed further in the next part of this report.

Part Two (2)

The following presentation is an integral part of the whole report, but it is separated out because it contains a significant amount of speculative theory, which may have to be revised or rejected without affecting the preceding Part One (1). Part 1 should remain valid at least as a mathematical development of PG based on a set of postulates, barring inadvertent but rectifiable errors. In Part Two (2) specifically, it is proposed that the theory can be extended to a general push gravity (GPG) by borrowing some mathematical tools of general relativity (if and to the extent needed), astrophysics and other cosmological theories, or that all theories may complement one another. However, no analytical steps have been undertaken yet, whilst it is hoped that this would be achieved better by experts in the corresponding science fields. It is only initially attempted to introduce PG in astrophysics in the case of white dwarfs, neutron stars and black holes, but again it is hoped that this task would be best undertaken by others. The application of PG in particle physics and cosmology is barely but humbly mentioned, or discussed, in the hope that it might spark further discussion and research for a unified field theory and a theory of everything: It makes sense to assume that all fields are created by particles, now by push particles, i.e. all with a common denominator as the only way to achieve unity.

As more material is added, re-organization of this report and its parts may become due. Until this is done, it is hoped that no misunderstanding arises from the way it is currently presented.

14 Towards a dynamic push gravity

The introduction of time in PG for moving bodies should constitute another chapter of PG dynamics still to be developed. However, an attempt to introduce some elements and ideas of it here is thought to help prepare towards a proper dynamics theory, but also address possible questions arising from the static PG of Part 1.

Since we already established that Newton's gravitational law can actually be derived from first principles of PG, we may wonder, if we should accept the other laws of Newton by way of principle (granted), or they may also be derived wholly or in part. For clarity, we understand that Newton's first law states that "*an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force*", in other words, material bodies have "*inertia*". The second law states that "*the force F on an object is equal to the mass ' m ' of that object multiplied by the acceleration ' a ' of the object, i.e. $F=ma$* ". The third law states that "*action = reaction, i.e. when a body exerts a force on a second body, the second body simultaneously exerts a force equal and opposite in direction on the first body*".

Regarding a fourth law stating that "*forces exerted by different bodies add up (superimpose) like vectors, i.e. the forces obey the principle of superposition.*", we already found that it does not apply in PG per Section 11.

Now, it is not clear, if we can mix and match the above first three laws with PG, or we should strive to derive them also from the first principles already adopted at the outset in Part 1. For example, is it legitimate to inquire as to the intrinsic meaning of inertia? Is it an apriori physical attribute for all material bodies or just a convenient empirical entity to express an experience mathematically per second law of gravity? Connected to this question is also, why "action-reaction" takes place.

Furthermore, it would be unwise to rely entirely on Newton's laws alone without regard to subsequent revolutionary developments in many fields of physics, and in particular relativity. We already assume, at least provisionally, the existence of moving particles, the gravions, at the speed of light as an added principle of PG, hence we have to take into account at least the special theory of relativity (SR). By no means do we imply exclusion of the general theory of relativity (GR), but we can try to determine how far we can reach initially without it. We may also mix and match various attempts with all theories of gravity, by trial and error, iteratively, in order to arrive at some understanding as to how things pan out under the framework of PG. In this course, we may need to use much of the existing tools (math) and insights of other theories without prejudice or fear. The subsequent part of this section serves only as a beginning along these lines.

14.1 Relative and absolute reference frames

Throughout Part 1, we considered only stationary material bodies relative to each other but implicitly also relative to the gravions as a whole. We already discussed the possibility of gravions with different mean free paths defining different regions of the universe. Gravions then may be treated like a gas or an aether. Such an aether, though, is not a passive medium for the propagation of light like the classical (conventional) aether in prior physics. It is an absolutely energetic medium interacting with material bodies. This medium

“fills”, or better forms the space, which acts on material bodies, while at the same time this space is acted upon by the material bodies. Based on this primary interaction between space and matter, in turn, there arises interaction between matter and matter giving rise to displacement of matter (bodies) relative to it (i.e. relative to each other body) and relative to space (aether). In this way, the gravions, or space (aether) is now endowed with a privileged rest frame of reference, against which all other movements can be measured. While each body is stationary in its own reference frame and moves relative to the reference frame of other moving bodies, they all interconnect via the primary or privileged reference frame of the aether. Distance and time are now interconnected via the reference frame of the aether. If aether inflates or streams in the universe, so is its frame of reference. The analogy is the same as with the motion of an expanding (better, inflating) universe carrying with it its clocks and length-measuring-sticks, and operating under the presumably tested laws of relativity. If we can continue using some tools and concepts of relativity, we can now also flesh it out with a material (energy) content, namely, that of the gravions incessantly moving in all directions while defining an absolute frame of reference for time and distance. The aether of gravions is a source of energy, from which all other forms of energy emanate and to which they return in a perpetual cycle of cosmos.

It is important to note that the “aether of gravions” can co-exist with a host of other types of particles as proposed in Section 18.

“Statics” in physics is the branch of mechanics concerned with bodies at rest and forces in equilibrium. We may provisionally use this term in PG too, but with the qualification that there is continuous action by the relativistic moving gravions underlying the emerging forces. Until we may coin another term (if needed), we use “static PG” to describe the theory as in Part 1. Furthermore, to avoid possible confusion between the terms of “kinematics” and “kinetics”, let’s use the existing term of “dynamics” to describe the motion of bodies including its causes or not. So “dynamic PG” refers to the theory including both kinetics and kinematics. If there are disagreements with such a terminology, we may defer a possible resolution for later. After all, there may not be any need to distinguish “statics” from “dynamics” in PG.

14.2 The Equivalence Principle

We can easily reproduce Newton’s attraction force by push gravity and hopefully all other observed relationships (a task by later work). Furthermore, under the understanding of PG, we can now say that the well known equivalence principle (EP) is not violated. In fact, it is better explained as an identical process in the two systems being referred to, namely, one in a gravitational field and another accelerated by an equal force in space outside the gravitational field. That is, whether a mass is pushed “by hand” (or pulled via a rope in an elevator in free space), or the same mass is pushed by gravions by an equal force, then the outcome should be the same, namely, the mass will travel distances proportional to the square of time (t^2). Push gravity creates a force by streaming gravions through the entire mass dragging every mass element concurrently, the sum total being a force no different from a push (pull) force acting by a spring with measurable deformation on a solid mass (or an imaginary accelerating force experienced inside an elevator in space). The gravitational push force is distributed throughout the mass, whilst the spring force acts on the external surface of a **rigid** (for argument’s sake) mass and indirectly transmitted and distributed to all body elements producing an identical outcome. Then the same mass being acted by an equal force would accelerate by the same amount, i.e. we would measure distances proportional to the square of time, from the moment the mass is set free to travel (in free space or in the neighborhood of the gravitating body). If the mass is held stationary by some stationary “wall”, then the mass experiences the force (by the gravions or the spring) without moving (like pushing on or pushed by a stationary wall). The gravion force appears as a mysterious attraction force by Newton, which necessitated the adoption an “equivalence principle” to explain the observable equal outcomes by the same mass acted upon by the Earth’s gravity, or by “ the rope on an enclosed elevator encompassing the same total mass”. With the insight readily provided by PG (streaming gravions), the "equivalence principle" need not be a "principle" at all any more; it is just an identity as seen by PG, **it is** the same thing.

The Equivalence Principle (EP) is a mere and easily understood consequence of the hidden reality of the PG gravity principle, hence there is no need to postulate the EP any longer. The self-shadowing (shielding) causing an underestimation of the real mass does not refute the above understanding: To the extent that part of the mass is shielded from the action of gravions, if we push it “by hand” by the same force, as Newton would have us to use, then we would correspondingly measure the same distances. The actual mass (bigger than the apparent one) would be acted upon with an equal force, in both cases, of a falling body due to gravity or moving in space outside gravity. In both systems (cases) the same force acts on the same mass being real or effective, producing the same outcome.

Summary: PG does not require an equivalence principle, since everything exists in a real “elevator” being pushed by streaming gravions, not requiring a fictitious (gedanken) second

elevator as theorized to date.

The above arguments are upgraded with an updated Section 16.

14.3 Falling bodies and Flyby anomaly

From the above description and understanding of the EP and if there is no distinction between effective and real mass (as per PG), then it follows that the gravitational and inertial mass are equivalent, actually equal. The latter equality then forms an alternative form of the Principle. In other words, the ratio of gravitational to inertial mass of any object is equal to some constant C, if and only if all objects fall at the same rate in a given gravitational field, so that C=1. The latter form of the Principle must be distinguished from its original “gedanken” description stating that “*the gravitational force we experience on Earth is identical to the force we would experience were we sitting in a spaceship accelerating at 1g*”.

However such an equality of masses is clearly at variance with PG: As understood and described above, the effective mass corresponds to the gravitational mass, the force from which is transmitted to the real (entire) mass of the body, i.e. to the inertial mass of the body.

Thus, applying the PG parameters as developed so far, let us designate by M_e the mass of a large gravitating body (sphere), so that it is considered stationary, when other much smaller bodies with effective mass m_e fall to it. We consider only the case, where the falling trajectory is radial, so that the assumed steady state of PG is thought to be practically retained. A uniform (parallel) gravitational field allows the use of the static force during fall without time effects. We can use the effective mass as in Newtonian mechanics for the potential energy GM_em_e/r around the gravitating mass. We obtain the potential energy by integrating the corresponding acting force times the elementary path lengths of the falling body. Likewise, we integrate for the work done by the **same** force on the total (real/inertial) mass m to obtain the additional kinetic energy as the body falls from point (radius) r_1 to r_2 and apply the conservation of energy equation:

$$\frac{1}{2}mu_{PG}^2 = \frac{GM_e}{r_1}m_e - \frac{GM_e}{r_2}m_e \quad (127)$$

from which the final velocity u_{PG} is given by

$$u_{PG} = \sqrt{2GM_e \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \frac{m_e}{m}} \quad (128)$$

It there is no distinction between the two masses above, then, by Newtonian mechanics, the corresponding final velocity u_N would be

$$u_N = \sqrt{2GM_e \left(\frac{1}{r_1} - \frac{1}{r_2} \right)} \quad (129)$$

The ratio between the above two velocities is then immediately obtained as

$$\frac{u_{PG}}{u_N} = \sqrt{\frac{m_e}{m}} \quad (130)$$

From Section 6.5, we substitute the ratio of masses to obtain:

$$\frac{u_{PG}}{u_N} = \sqrt{\frac{3A_R g_0}{4\pi G \rho R}} = \sqrt{q} \quad (131)$$

as a function of the unknown parameter g_0 . The ratio of masses in Eq. 130 is independent of the gravitating center (body) and it is equal to the accelerations ratio given by Eq. 83 on the moving (gravitated) sphere (the contraction factor q). We already listed the difference of the two velocities in Table 5 for steel spheres. We repeat the same, but for the above ratios of a steel sphere with radius $R = 10$ m and density $\rho = 7500$ kg/m³ in the typical range of g_0 in Table 7.

We can apply the above figures for reported flyby velocities (at perigee) and find that the difference between velocities is of the order of mm/s. This is consistent with observed flyby anomalies and it might help further explain them, i.e. in addition to or in lieu of various other proposed explanations. The Oumuamua anomaly (Bialy & Loeb, 2018) might be another candidate to re-examine as a flyby effect.

Theoretically, a spacecraft on an elliptical orbit could experience a greater force on its inbound direction than on its outbound one by changing direction of its disk-like (for example) shape thus exhibiting a greater effective mass in one part of the orbit than in the other. This would result in incremental accretion of energy

g_0	PG over Newton velocity ratios
300	0.999999980343
500	0.999999988206
1000	0.999999994103
2000	0.999999997051
5000	0.999999998821
10000	0.999999999410
20000	0.999999999705
30000	0.999999999803
50000	0.999999999882

Table 7: *Ratio of falling velocities by PG over Newton.*

until it can reach escape velocity and then repeat the same process around a bigger planet (e.g. Jupiter), or the Sun. Similarly, mass distribution in a fan-like configuration might optimize the flyby effect by opening and closing the fan accordingly. In Section 12.2, we report that there is an optimum angle for spherical and truncated cone shape, whilst other shapes may be further investigated later. This might have little practical application, but it remains to be seen, if there is some benefit in furthering such an investigation. For an elliptical orbit, time effects on the shadow (push force) of relativistic gravions become important in PG, a problem not yet formulated.

In general, this effect says that a steel ball and a feather do not fall at the same speed inside a vacuum chamber: Let us consider a flat feather falling with its plane parallel or vertical to the direction of the gravitational field. In both cases it has the same real mass but different effective mass. When it falls with its plane vector parallel to the field, the effective mass is greater than when it falls with its plane vector normal to the field. In both cases, we have the same object (mass) and the same inertial mass. However, it will fall faster in the first case than in the second. The maximum speed (and acceleration) will be when the effective mass is practically equal to the real mass, i.e. when the feather can be spread out as much as possible (e.g. by further thinning it down). Let us then consider a steel sphere and a very thin steel disk of the same mass; we can achieve this by first using the sphere and then the same object is flattened out to a very thin disk. Like with the feather, the steel sphere will achieve a slower final velocity than the same mass in the shape of disk. Now, the fine steel disk and the fine feather will fall at the same speed if they are both thin enough to expose their real mass to the field, and will fall in accordance with Newtonian mechanics, because they both use the real (total) mass. However, the steel sphere will be slower than the feather, because the sphere displays an effective mass further away from its real mass than the feather does. The effect of orientation of a falling body is thus a new finding by PG, an extremely small effect to measure in the laboratory, but it may become cumulative and observable during a fall towards a planet or star from a significant distance.

Corollary: All bodies fall at equal rates inside a uniform gravitational field, if and only if they all expose their real mass to gravions, or if they expose the same ratio of effective-to-real-mass, i.e. if and only if they have the same contraction factor q .

The flyby difference (referred to as an anomaly to date) might be used purposefully for the measurement of the unknown value g_0 in our solar system. Furthermore, the presented perceptions on EP itself from the perspective of PG theory might help us better understand the Principle itself and its implications in past and future physics.

It must be stressed that the above derivations of velocities were used for “falling bodies” acted upon by forces generated in the steady state of gravion flow, so that the time effect is presumably small and the validity of equations is tacitly assumed.

As a further approximation in this section, we have been tempted to include the flyby anomaly, but for which the time effects must be ultimately included, as it is also discussed in the next Section.

14.4 Advance theoretical solution

If we use the above reasoning in a similar fashion for an orbiting body in circular motion (for simplicity), we equate the inertial and gravitational force (initially) in PG:

$$m \frac{u_{PG}^2}{r} = G \frac{m_e M_e}{r^2} \quad (132)$$

where again m_e and m are the effective and real masses moving around a large (hence stationary) effective mass M_e , yielding

g_0	velocity ratios of PG over Newton	velocity difference, m/s
300	0.990700461	-276.9402851
500	0.994442855	-165.4917729
1000	0.997229786	-82.49698282
2000	0.998616965	-41.18677725
5000	0.999447281	-16.45996439
10000	0.999723723	-8.227530833
20000	0.999861882	-4.113155718
30000	0.999907926	-2.741968791
50000	0.999944759	-1.645082695

Table 8: *Earth velocity ratios and differences by PG and Newton.*

$$u_{PG} = \sqrt{G \frac{M_e m_e}{r m}} \quad (133)$$

In Newton we have:

$$u_N = \sqrt{G \frac{M_e}{r}} \quad (134)$$

so that we again get for the ratio of velocities:

$$\frac{u_{PG}}{u_N} = \sqrt{\frac{3A_R g_0}{4\pi G \rho R}} = \sqrt{q} \quad (135)$$

Now, if we apply this to the Sun-Earth system for simplicity assuming circular orbit, there is a significant slower than experience velocity component. We find this from the density ratio values in previous Table 2 and list them again together with the ratios and differences between PG and Newton in Table 8.

We have used the Earth's average speed of 29.78 km/s. The tabulated outcome is clearly incompatible with experience: With $g_0 = 50000 \text{ m/s}^2$, the orbit length would be by 51.876 km shorter in one year. We have the option of increasing g_0 until we bring the difference to an acceptable level, but first we have to modify the above equations to include time effects. The equations used above assume instantaneous transmission of the push force, which is incorrect. After we derive the correct equations for orbital motion, we can find the required value of g_0 to bring the velocity u_{PG} to an acceptable level and consistent with experience. This would constitute an advance theoretical solution to the problem of finding the prevailing maximum acceleration g_0 in our solar system, over and above the proposed experiments throughout this report. This work has not been done yet, whilst it is not clear how it will pan out. At present, this objective falls outside the scope of the present report and beyond the resources available to this author.

We have reached a critical point in the development of a general PG theory for moving bodies. In the following Section 15.3, we discuss the possibility of using the tools of general relativity (GR) to develop a *dynamic* PG, or further develop GR in the framework of PG. Now, this might appear to be inconsistent with GR from the outset, because PG breaches the equivalence principle, if stated as equality between inertial and gravitational mass, which is a cornerstone of GR. To reconcile this contradiction, we may inquire that the postulated equation:

$$m_{gravitational} = m_{inertial} \quad (136)$$

be replaced with the equation:

$$m_{gravitational} = qm_{inertial} \quad (137)$$

which is prompted from the corresponding relationship between effective (gravitational) and real mass (inertial) in PG:

$$m_e = qm \quad (138)$$

In other words, can we introduce the contraction factor q to redefine (or replace) the EP and carry on with a modified GR? This is where the subtle differences in the understanding of EP become important. This

leaves the inquiry open on how to integrate relativity with PG. Gravions are assumed relativistic and we need to develop a relativistic theory of PG. Then, we could also address the objection listed in the following Section 15.6. Time effects must also include the almost helical Earth trajectory, as the Sun moves around the center of galaxy, which makes the overall formulation more complex.

The breakdown of the EP expressed in terms of differing inertial and gravitational mass seems to be necessitated also in new quantum theory (Kajari *et al.*, 2010), so that our finding here is not alone or alarming. In fact, coming to the same conclusion from an entirely different perspective, namely, from quantum mechanics, provides a strong reason to correlate the corresponding theories in an effort to unify quantum mechanics and gravitational field.

Should any further difficulties appear or remain in the development of a general PG theory, then we may have to look for some other counteracting (compensating) mechanism for the shortfall in orbital motion, before we can confidently abandon PG. For example, in Sections 15.7 and 15.8, an attempt is made to account for the postulated exiting forms of the absorbed gravions, not yet knowing if they have some second order perturbation on the gravitational field. Other compensating mechanisms may also be present.

At any rate, we can always resort to high enough g_0 in order to establish compatibility between theory and measurements, i.e. by bringing the fraction q much closer to unity. This alternative solution always remains on the table for consideration, except that it would make the prospect of measurements more difficult. In this case then, all the referenced gravitational anomalies (Allais effect, Greenland gravity anomaly, Pioneer anomaly, flyby anomaly, etc.) must be re-visited and conclusively discounted as been anomalies of gravity, namely, deviations from Newton and/or GR. This strengthens our proposal of the need to undertake some decisive experimental tests in the event that static PG theory (for stationary bodies) can be confirmed and measured.

The case of very high g_0 values, if needed, must also be considered in the spirit of discussion in Section 18 dealing with white dwarfs, neutron stars and black holes. Increasing g_0 only resolves the problem for our nearby solar system, but it shifts the importance of distinction between effective and real mass for much larger, or denser bodies and systems, like binary systems, black holes, etc, whereby compatibility of PG with such systems must be established. The difference between real and effective masses must be very high for such bodies, which also means that the EP would be grossly violated in terms of great inequality between gravitational and inertial mass. The proposal of establishing momentum or push gravity as the universal and unifying cause of all types of acceleration in Section 17 provides a reasonable platform to relate to the new quantum theory mentioned above (Kajari *et al.*, 2010).

14.5 Matter, inertia and mass

In continuation to the previous analysis, we can bring it to its logical conclusion below.

We intuitively identified the real mass $m \equiv m_{real}$ with the inertial mass $m_{inertial}$ and the gravitational mass $m_{gravitational}$ with the effective mass $m_{effective}$. However, this need not be necessarily so. It may be that, after all, the inertial mass is equal with the gravitational mass making the equivalence principle absolutely inviolable in all its expressions. Such a finding could lead to either of two outcomes:

(a) The PG becomes unsustainable, unless:

(b) Both PG as advanced in this report and the EP are true, even if EP includes equality of masses.

Then we have to accept some inexorable conclusions, even if they are counter-intuitive at first.

To avoid possible confusion, we write the subscripts of various masses explicitly by a full word. In case (b), it is not the entire m_{real} responsible for the phenomenon of inertia, i.e. a resistance to change in kinetic state (to move faster or slower). In reality then, it should be only the $m_{effective}$ that manifests inertia. At the same time, we can continue identifying $m_{effective} \equiv m_{gravitational}$. This implies that there is a fraction of the real mass, namely, the difference

$$m_{passive} = m_{real} - m_{effective} \tag{139}$$

being passive, oblivious and not resisting to the application of the gravitational force on the effective fraction of the total mass. If this were to be true, it would revolutionize our understanding and perceptions about the hitherto meanings of matter and mass. Newton defined (or identified) mass as the amount of matter:

$$mass_{Newton} \equiv matter = m_{inertial} \tag{140}$$

which would need to be re-appraised, if case (b) is true.

In fact, upon further considering this idea, we may also bring some intuition one way or another. In one way, we could think of the gravions constituting a sort of a "lattice" that activates the effective part of the mass. In doing so, it is this lattice that resists in changing the kinetic state, or inertia of the body. The passive part of the mass is ineffective and does not care (does not resist) moving along with the active part

($m_{effective}$) of the mass without actually offering any resistance. We could then safely identify the total mass of a body with its matter:

$$matter \equiv m_{real} \neq m_{inertial} \quad (141)$$

Yet, by further iterative thinking, we can make the inventive step that, instead of the gravion-lattice **activating** the effective mass to resist, it is the lattice itself that resists the movement of the body (matter) by engaging via the effective (active) part of the mass. The effective mass is passive by itself, except that it is somehow tied to the activating gravions. In consequence then, we can safely state that the entire mass is actually passive and hence it has no inertia; what appears as inertia of the mass (or part thereof), it is actually the resistance of the gravions opposing the mass to change its kinetic state.

The concept of gravion-lattice may take on various embodiments and conceptualizations: Gravions continuously penetrating and being absorbed through a mass could be likened to rolling ropes (albeit very inefficient way) constraining the mass from changing momentum. By whatever means and ways to describe the gravion-mass interaction, we can generally state that it is the gravions that are responsible for what appears as inertia of matter. When we try hard to move a sledge on slippery ice, it is the gravions, which resist invisibly to us, but we only experience the force on the tangible sledge. By such thinking, we may have ultimately deciphered the mystery of controversial inertia. We may know why bodies resist, now saying that bodies actually do not resist, but it is the energetic gravions that want to “push back” on us via the mass ($m_{effective}$). There remains to better conceptualize how they do this and why they only do it when we accelerate or decelerate a body. For the time being, we can summarize our possibly new understanding as follows:

$$m_{inertial} = m_{gravitational} = m_{effective} = qm_{real} \equiv q \cdot matter \quad (142)$$

We note that the above equation is similar to Eq. 137 except for the semantics on masses, i.e. which mass is which and what they do. If the above is true, the consequences would be immense. For example, the inertial mass of a very dense body, like a neutron particle, or a white dwarf, or a neutron star, or a black hole is much-much smaller that it could be if the same body is expanded (dilated) to produce an effective mass close to its real mass. The *dynamics* of an exploding star would be far different from what we would derive by allowing for a constant mass. Mass, inertia and matter now (in PG) mean different and variable entities. As another example, the flyby gravitational anomaly still applies, so that if we are able to vary the effective mass by a large factor minimizing it during the outbound trajectory, we could hurdle a body into space at huge velocities.

In all above, we made no mention of relativity implying that we considered only low speeds. When we go to relativistic speeds, then we have to expand on additional notions of masses, namely, that of rest mass m_{rest} and that of relativistic mass $m_{relativistic}$. In doing so, we may not be in conflict with GR and we may just carry on with established relativistic theory. Actually, it seems that we may even have a better understanding of the meaning of relativistic mass, which has been often misconstrued by many GR proponents for over a century. Relativistic mass has been so confusing even among notables in relativity, that it has been called “the pedagogical virus” (Okun, 2006). For consistency with our introduced terminology and semantics, we should set for the rest mass:

$$m_{rest} \equiv m_{effective} \quad (143)$$

so that the relativistic mass can be given by:

$$m_{relativistic} = \gamma m_{effective} \quad (144)$$

with the usual relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (145)$$

There remains now only to conceptualize and formulate how exactly the relativistic mass comes into being. Nevertheless, the important conclusion must be that the gravions remain responsible for this mass too, which is not a “mass” per se, i.e. it is not matter (stuff), but only an inertial mass. Such a conceptualization is then closely consistent with the “orthodox” and rigorous teachings of the theory of relativity, namely, that the relativistic mass is not a “mass”. However, this is a close agreement with GR but not a total agreement, because GR teaches that the “*only true mass is the rest mass*”. We may now have found that even the rest

mass is not true mass, because it is only an effective mass, which can vary with density and orientation for any given body. The only true mass is actually the real mass as has been established by the present PG theory, which is non other than the matter of the body. In any case, non of all these masses has an inertia (a will to resist), but the energetic gravions are responsible for the quantity (parameter) of mass that enters our equations in physics. Therefore, gravions create both gravitational fields and apparent masses. This should be in happy agreement with GR originating from an “opposite” direction.

From the preceding analysis, it seems that PG reaches a critical point as soon as we apply the concept of effective and real mass to moving bodies. This could be either the end of PG, or a long awaited breakthrough in physics. The latter might occur in one sense, if we are prepared to review and re-appraise the notion of “inertia”. It is interesting to note that the dictionary synonyms of “inert” are “dormant, immobile, impotent, inactive, listless, motionless, paralyzed, passive, powerless”. However, in physics, we associate inertia with a resistance or refusal of a body to change its kinetic status, which is not passivity or inactivity. A resistance to the change of movement implies a power, or will, or action to resist, i.e. a reaction. Where does this power for objection to an action comes from? It might have been a misnomer to use the word “inertia” to describe our experience, when we try to change the kinetic state of a body. A more appropriate word might have been “reactivity”. In chemistry, it is more appropriate to call an inert element so, because the element does nothing by way of (chemical) reaction; it is action-neutral, However, in physics, all bodies are not action-neutral, when prompted to move or stop or change velocity. They all present reactivity and not “inertia” per the outside-of-physics use of the word. “Inertia” means inaction, the same as in other languages, e.g. in Greek inertia $\iff \alpha\delta\rho\acute{\alpha}\nu\epsilon\iota\alpha \iff$ inaction. Of course, word-play does not make physics, except that we may have fortuitously come to use the word “inertia” for what it actually means for the real mass. As a result, the word “effective” mass may now assume the role of the formerly “inertial” mass.

In the preceding analysis, we reached the dilemma of either abandoning PG, or abandoning the classical inertia meaning. We also used the word “passive” for a passive mass $m_{passive}$ of a material body.

We can better appreciate why, the EP stated as per “gedanken elevator”, is a different thing than stated as equality of masses. We may provisionally use the equality of masses in PG to learn that the “effective” mass plays the role of both the “inertial” and the “gravitational” mass of prior physics. If we can established such a finding by other means, then there is no distinction between those two prior masses, and the EP becomes redundant again. A good way to this end is to start by experimentally verifying the static PG, namely, the existence of effective mass as already proposed, or by some other experimental means. Theoretical means are also welcome, but practice is the ultimate criterion of truth.

14.6 Mass, energy and black holes

As a result of the previous potential discovery of possible properties of the effective and real mass, we further investigate the consequences on mass and energy of bodies with increasing density all the way to the creation of black holes. We have already considered the effect of increasing density, but we also need to account for the distribution of effective mass inside a given material sphere at very high density.

14.6.1 Material sphere

We need first to clarify and summarize some previous findings to help us make an important step without laborious cross referencing: If the real density ρ is known in advance, we can find the absorption factor k directly from $k = \pi\rho G/g_0$, which substituted in Eq. 44 yields the contraction factor

$$q = \frac{3A_R}{4kR} = \frac{3}{4kR} \left(1 - \frac{1}{2k^2R^2} + \frac{\exp(-2kR) \cdot (2kR + 1)}{2k^2R^2} \right) = \frac{\rho_e}{\rho} = \frac{m_e}{m} \quad (146)$$

From the above, we obtain the effective mass and effective density:

$$m_e = \frac{3A_R}{4kR} m = \frac{\pi R^2 A_R}{k} \rho = \frac{\pi R^2}{\Lambda} A_R = \frac{g_0}{G} R^2 A_R \quad (147)$$

$$\rho_e = \frac{3A_R}{4kR} \rho = \frac{3A_R}{4\Lambda R} = \frac{3g_0 A_R}{4\pi G R} \quad (148)$$

The above equations state that, we as increase the real mass arbitrarily inside any fixed radius sphere, the corresponding effective mass increases monotonically to an asymptotic value as the absorptivity A_R approaches unity. At the same time, the contraction factor vanishes to a zero value but never reached.

We can also arrive at the same equations by starting with a given (known from Newtonian mechanics) effective mass, or effective density. Then, we find the coefficient k directly by solving Eq. 63 written as:

$$g_0 A_R - g_R = g_0 \left[1 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR)(2kR + 1)}{2k^2 R^2} \right] - G \frac{m_e}{R^2} = 0 \quad (149)$$

from which we immediately obtain again (as from Eq. 80):

$$m_e = \frac{g_0}{G} R^2 A_R = \frac{\pi R^2}{\Lambda} A_R \quad (150)$$

The above findings state that we cannot pack any arbitrary Newtonian mass (i.e. effective mass) inside a given radius sphere for a given universal constant Λ , or a combination of constants G with g_0 ; there is a limit approached asymptotically as A_R approaches unity. That limit is given by

$$m_{emax} = \frac{g_0}{G} R^2 = \frac{\pi R^2}{\Lambda} \quad (151)$$

and

$$\rho_{emax} = \frac{3g_0}{4\pi GR} = \frac{3}{4\Lambda R} \quad (152)$$

By way of example and comparison, we use a sphere with the radius of Earth ($R = 6.37 \times 10^6$ m) and $g_0 = 1000$ m/s², and find the maximum possible effective M_{emax}

$$M_{emax} = \frac{g_0}{G} R^2 = \frac{1000}{G} \cdot (6.37E + 06)^2 = 6.08 \times 10^{26} \text{ kg} \quad (153)$$

That limit would be achieved, if a near infinite amount of real mass could be accreted. For the particular example here, the ratio of that limiting value over the Earth's effective (Newtonian) mass is $M_{emax}/M_e = 101.8303172467580$.

Even risking of becoming pedantic, we need some further clarifications, because there is a bigger risk from misusing the two densities in the new situation of PG. The contraction factor q was defined for a condition outside a sphere. Thus, initially we assumed that the density is known and real, so that, if we use it in both PG and Newton, we arrive at different outcomes correspondingly, the ratio of which is provided by $q(\rho)$ being a function of real density. Subsequently, we introduced the effective density to match Newton with PG. If we use the two densities at the same time in the contraction factor, we obtain unity:

$$q(\rho, \rho_e) = \frac{g_{PG}(\rho)}{g_N(\rho_e)} = \frac{g_0 A_R}{\frac{4}{3}\pi GR \rho_e} = 1 \quad (154)$$

after substituting ρ_e from Eq. 148. This is useful background to correctly understand the internal accelerations ratio used in Section 8 for a low (typical) density-of-the-Earth example. There, we found the internal parameters of g_{0X} , $q_X = g_X/g_{XN}$ and the difference $\Delta g_X = g_{XN} - g_X$ at any point X inside a sphere per Fig. 7 defining the internal radius R_X and the fractional radius R_X/R of the sphere with radius R . Explicitly, we have:

$$q_X(\rho, \rho_e) = \frac{g_X}{g_{XN}} = \frac{3g_{0X} A_{R_X}}{4\pi G \rho_e R_X} \quad (155)$$

where we use both the real and effective densities already established for the material sphere from the outside, whilst they do not change once found.

With q_X known per process in Section 8, we can find the corresponding density ρ_{eX} at point X required to balance the PG acceleration at that internal point for the internal sphere, for which we have:

$$q_X = \frac{\rho_{eX}}{\rho_e} \quad (156)$$

This finds the effective density for any internal sphere with radius R_X , which, in effect, provides also the desired overall distribution of the total effective mass for the entire sphere as a function of R_X . Thus, we can verify that for $R_X = R$, we get $q_X = q = \rho_e/\rho_e = 1$ as expected. Therefore, q_X provides solution to our inquiry.

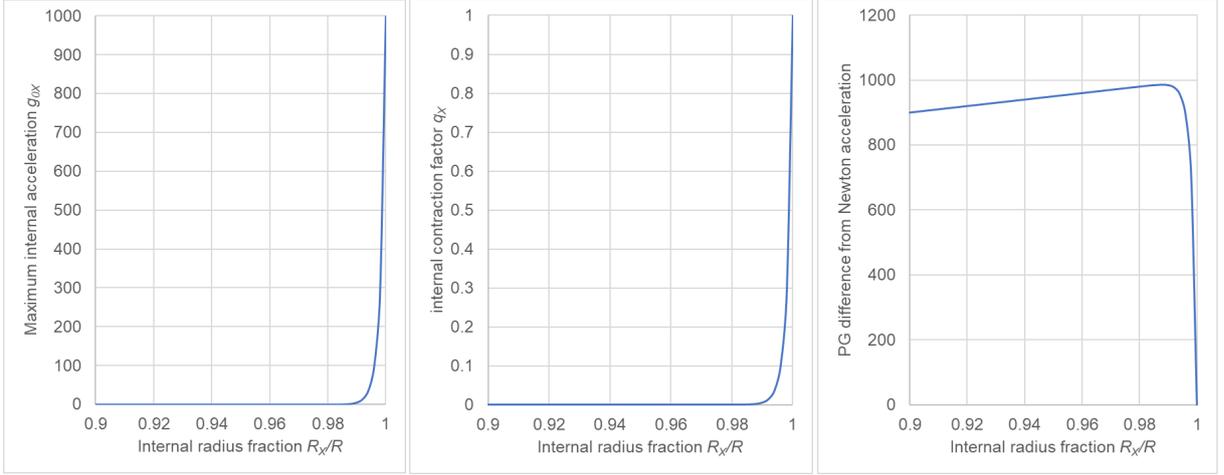


Figure 16: *Internal maximum acceleration g_{0X} , contraction factor q_X and difference $g_{NX} - g_{0X}$ near maximum effective mass limit $M_{e-test} = 101.83M_e$ using Earth's diameter and $g_0 = 1000 \text{ m/s}^2$.*

Now, we are ready to plot the same internal parameters as per Fig. 8, but close to (i.e. a little under) the limiting value of M_{emax} found above. This is done in Fig. 16 using $M_{e-test} = 101.83M_e$, for which the corresponding densities are $\rho_{e-test} = 561449.92 \text{ kg/m}^3$ and $\rho_{test} = 299899725.44 \text{ kg/m}^3$ with a contraction factor $q_{test} = 0.01829587874417$. The latter factor indicates that the total effective mass is only $\approx 1.83\%$ of the total mass, i.e. relatively low but over $100\times$ the Earth's mass. The graph of the same factor internally decreases extremely fast from the surface to the center of the sphere. The effective mass is practically concentrated in the top 1% of the radius forming a very thin active layer very close to the surface.

If the hypothesis that the real mass is inactive and passive (i.e. without classical inertia), whilst the only bearer of active (reactive) mass is the effective mass, it might provide us with what looks like a black hole. There are generally different ideas about what happens inside a black hole. Especially from our reference frame, it is generally unknown what happens, other than a singularity to in-falling material. Some say nothing happens at all, not even a vacuum, it has no properties and it is not even a hole; whilst matter approaches the black hole, it slows down and finally stops at the event horizon. They also say that all mass becomes concentrated at the two-dimensional surface of the event horizon. The latter is very close to what we also find in Fig. 16. They also say that it seems that a black hole destroys energy, which is again similar to what we say, i.e. that the real mass has no energy, but also no inertia (new finding). Actually, with our approach, there is no paradox, except that we might have been misguided about the meaning of our experienced inertia. The Schwarzschild envelope or boundary and the Schwarzschild radius may be exactly, what we find for the limiting case by Eq. 151.

Actually, a close examination of Eqs. 150 and 151 may be full of meaning: The effective mass is proportional to the surface area of the sphere (πR^2) over the universal constant Λ and how close the absorptivity A_R is to unity. It is a simple equation and, hopefully, it is also true.

We further understand that: By increasing g_0 , say by a factor of $10\times$ or $100\times$ with all other parameters constant, the distribution of the effective mass is pushed inward toward the center of the sphere. However, if we also increase the effective mass by the same factor, then we recover the same distribution, i.e. resembling the event horizon. So far, it is arbitrary to keep the sphere radius constant while accreting mass, unless we can find or propose a mechanism to achieve exactly that. For this, we need to consider what happens in the formation of high surface acceleration on white dwarfs, neutron stars and black holes together with an interim presentation of some other issues below. We continue with more on mass, energy and black holes in Section 18.1

14.6.2 Material line segment

Based on what appears to be an important development for a high density material sphere, we need to repeat a similar formulation for a material line segment; all this will help us derive a possible relationship between the contraction factors in PG with the Lorentz contraction factor in GR. We could take a similar approach for a material thin rod, but we greatly simplify the mathematical formulation required by using the shape of Fig. 1, within a very small (differential) solid angle $d\Omega$, length $\ell = BC$, absorption coefficient k

and real density ρ . All corresponding referenced parameters for this case will have the subscript ℓ font. The contraction factor q_ℓ for a material line segment was given by Eq. 46 actually written for chord lengths of a sphere. However, it is better to repeat and review all needed PG parameters with a fast-track derivation below.

We should point out that the material body in Fig. 1 is traversed by gravions in all possible directions interacting with it, but all such interactions are not transmitted to an observer at point O. Point O is affected by all gravions arriving from all possible directions in a full 4π solid angle at that point; they all have a null effect except for those inside the bi-directional elementary solid angle subtended by the material object in the drawing.

Axial external points of line segment: We aim to find the (external) contraction factor along the axis at any distance up to and including the end points of the line length together with other parameters needed for further work and analysis.

The elementary absorption factor df_a (no need for f_g here) is given by Eq. 7 in PG:

$$df_{aPG} = [1 - \exp(-k\ell)] d\Omega \equiv f_{\ell PG} \quad (157)$$

where, for convenience, we abbreviate the differential absorption factor within the differential solid angle $d\Omega$ by $f_{\ell PG}$; the above equation also defines the corresponding absorptivity A_ℓ of the line body with

$$A_\ell = 1 - \exp(-k\ell) \quad (158)$$

It is worth noticing that the ratio A_ℓ/k now is an effective length ℓ_e as opposed to the spherical parameter ratio A/k producing an effective volume V_e per Eq. 52.

The corresponding absorption factor for Newton (i.e. with extremely small k) is:

$$df_{\ell N} = k\ell d\Omega \equiv f_{\ell N} \quad (159)$$

The length contraction factor is the ratio of the two accelerations:

$$q_\ell(\rho) = \frac{f_{\ell PG}}{f_{\ell N}} = \frac{1 - \exp(-k\ell)}{k\ell} = \frac{\ell_e}{\ell} \quad (160)$$

which is the same as for the length contraction found for the chords of the sphere in Eq. 46.

Next, we need to introduce the relationship of effective to real density ratio. For this, we follow the same steps as for a sphere by introducing a small test mass to find the accelerations, and easily obtain the corresponding equations:

$$g_{\ell PG}(\rho) = \frac{J_0}{c} \Lambda A_\ell = \frac{G}{\Lambda} A_\ell = G\rho \frac{A_\ell}{k} = G\rho\ell_e \quad (161)$$

$$g_{\ell N}(\rho) = G\rho\ell \quad (162)$$

Thus

$$q_\ell = \frac{g_{\ell PG}(\rho)}{g_{\ell N}(\rho)} = \frac{\ell_e}{\ell} \quad (163)$$

By introducing an effective density for Newton equation to produce the same acceleration as with PG:

$$g_{\ell N}(\rho_e) = \frac{J_0}{c} \Lambda^2 \rho_e \ell = G\rho_e \ell \quad (164)$$

we obtain the ratio of accelerations being unity:

$$\frac{g_{\ell PG}}{g_{\ell N}} = \frac{\rho\ell_e}{\rho_e\ell} = \frac{\ell_e}{\ell} \frac{\rho_e}{\rho} = 1 \quad (165)$$

and the absorptivity and ratio of densities given by

$$A_\ell = \Lambda\rho_e\ell \quad (166)$$

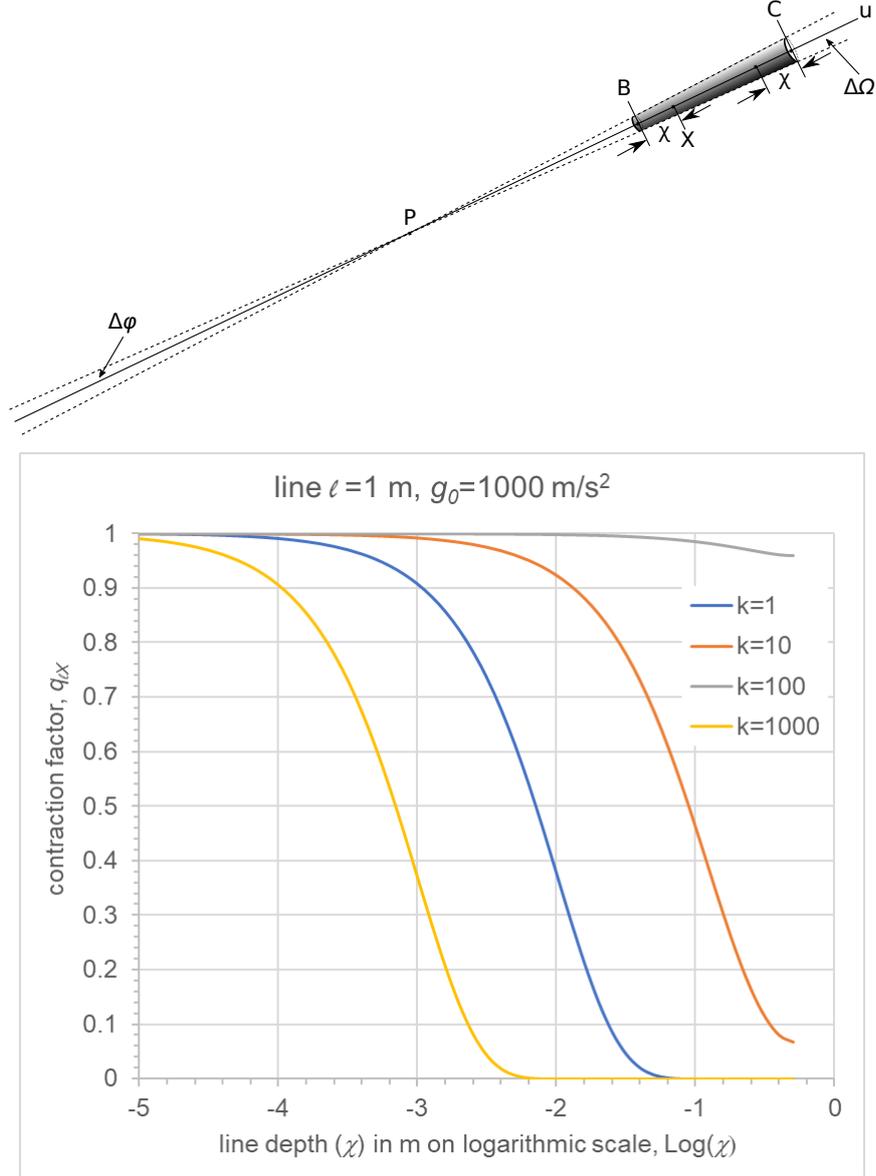


Figure 17: Line segment geometry (upper) and internal contraction factor q_{lX} against depth χ (lower), for line length ℓ , maximum acceleration g_0 and fixed absorption factors k indicated

$$\frac{\rho_e}{\rho} = \frac{\ell_e}{\ell} = \frac{A_\ell}{k\ell} = q_\ell \quad (167)$$

This tells us that the ratio of densities for an elongated shape, like the thin truncated cone of Fig. 1, is different from a sphere. A material line body has different PG effects, whilst all other shapes should have PG effects between the extreme cases of a line segment and a sphere.

With sufficiently large $k\ell$, we get $A_\ell = 1$, so that there is a maximum acceleration $g_{\ell_{max}}$ at the end of the length, or at any distance away from it on the axis, $g_{\ell_{max}} \equiv g_{\ell 0} = g_0$

$$g_0 = \frac{G}{\Lambda} = \frac{G\rho}{k} \quad (168)$$

which corresponds to Eq. 60 without the factor π . From any given k , we obtain the real density for a material line segment:

$$\rho = \frac{g_0}{G} k \quad (169)$$

and for the effective density:

$$\rho_e = q_\ell \rho = \frac{A_\ell}{k\ell} \rho = \frac{1 - \exp(-k\ell)}{k\ell} \rho = \frac{g_0 [1 - \exp(-k\ell)]}{G\ell} \quad (170)$$

Finally, the distribution of the effective mass along the line segment is the derivative of $f_{\ell PG}$ in Eq. 157 with respect to fractional distance $h = \chi/\ell$ from the end of the line segment in the range $0 \leq h \leq 1$

$$\frac{df_{\ell PG}}{dh} = k\ell \exp(-k\ell h) \quad (171)$$

from which the normalized over $k\ell$ distribution is

$$\frac{1}{k\ell} \frac{df_{\ell PG}}{dh} = \exp(-k\ell h) \quad (172)$$

Axial internal points of line segment: We aim also to find the internal contraction factor at any point inside the line length together with other important parameters needed for further work and analysis.

The upper drawing in Fig. 17 is the same material line object: At any point X inside at a distance (depth) χ from either end point, there is a net length $\ell_X = \ell - 2\chi$ responsible for the net absorption at that point, because the absorption by the two outer layers χ cancel out. Thus, at point X, we have:

$$g_{\ell X PG} = g_{0X} [1 - \exp(-k\ell_X)] \equiv g_{0X} A_{\ell_X} \quad (173)$$

where A_{ℓ_X} is the familiar A_ℓ factor but at the end point of the internal line length ℓ_X , and $g_{0X} < g_0$ due to the shielding of the outer layer length from X.

$$A_{\ell_X} = 1 - \exp(-k\ell_X) = 1 - \exp(-k(\ell - 2\chi)) \quad (174)$$

We can find g_{0X} by resorting to the usual absorption factor $f_{\ell X PG}$ at point X simply by

$$f_{\ell X PG} = [\exp(-k\chi) - \exp(-k\ell + k\chi)] d\Omega \quad (175)$$

without the need to integrate over the sphere as previously.

From this found, we can derive the acceleration $g_{\ell X}$ at X by the product with the factor g_0 and equate it to its value given above by Eq. 173:

$$g_{\ell X PG} = g_0 f_{\ell X PG} = g_{0X} A_{\ell_X} \quad (176)$$

from which we can find the relationship between the internal g_{0X} and external g_0 .

$$g_{0X} = \frac{g_0 f_{\ell X PG}}{A_{\ell_X}} \quad (177)$$

The expected Newtonian acceleration at X per Eq. 164 is given by:

$$g_{\ell X N} = G\rho_e \ell_X = G\rho_e(\ell - 2\chi) \quad (178)$$

The ratio of PG over Newton accelerations at point X per above provides the corresponding internal contraction factor for the line length:

$$q_{\ell X} = \frac{g_{\ell X PG}}{g_{\ell X N}} = \frac{g_0 [\exp(-k\chi) - \exp(-k\ell + k\chi)]}{G\rho_e(\ell - 2\chi)} \quad (179)$$

With $q_{\ell X}$ known per above, we can find the corresponding effective density ρ_{eX} at point X required to balance the PG acceleration at that internal point for the internal line length ℓ_X , namely:

$$g_{\ell X PG} = g_{\ell X N}(\rho_{eX}) = G\rho_{eX}(\ell - 2\chi) \quad (180)$$

from which we have:

$$q_{\ell X} = \frac{\rho_{eX}}{\rho_e} \quad (181)$$

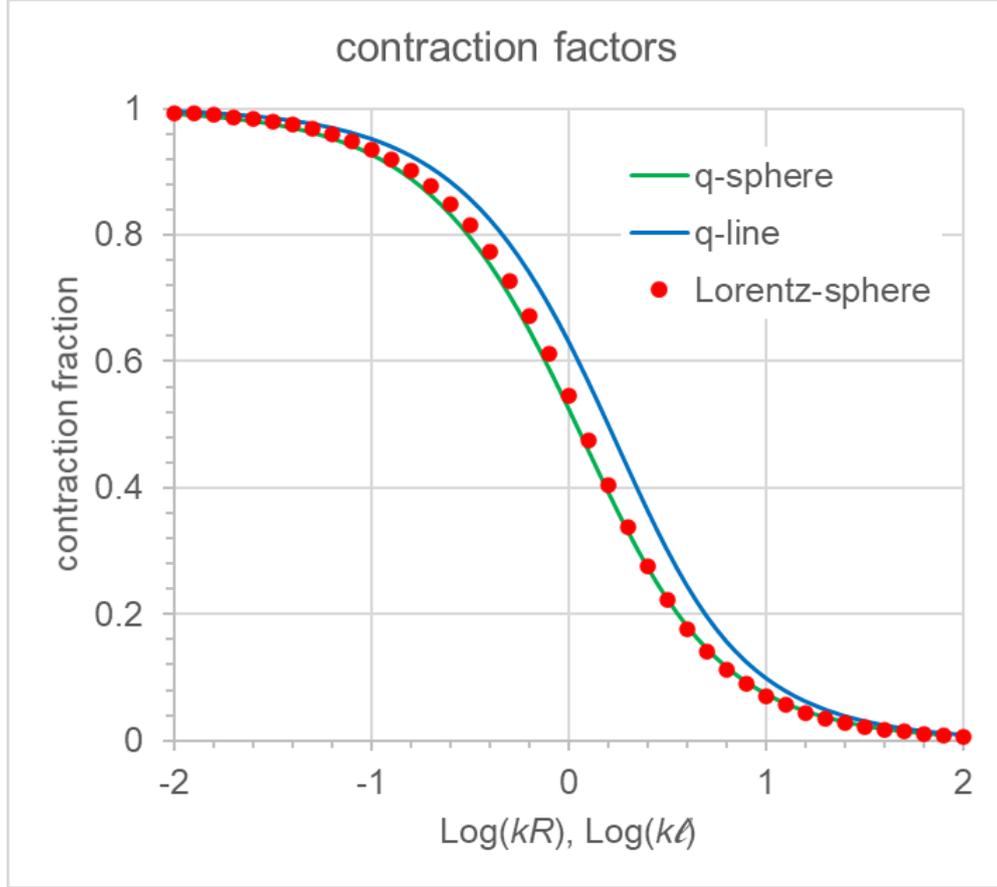


Figure 18: Comparison of PG contraction factors with Lorentz factor fitted with A_R .

This finds the effective density for any internal line length ℓ_X , which, in effect, provides also the desired overall distribution of the total effective mass for the entire length as a function of ℓ_X , or the depth χ with $0 \leq \chi \leq \ell/2$; we can verify that for $\ell_X = \ell$, we get $q_{\ell X} = q_\ell = \rho_e/\rho_e = 1$, as expected. We plot the internal contraction factor for a line length $\ell = 1$ m and $g_0 = 1000$ m/s² as a demonstration in Fig. 17 with some fixed values of the absorption coefficient k . This is a monotonically decreasing function of χ , which decreases extremely fast at very high values of the absorption coefficient k (or the real density ρ). This means that there is a look-like “event horizon” at the two ends of the material line segment, like with the sphere found before. Of course, this is only a theoretical outcome, because any “rod”-like structure would collapse to a spherical geometry at high accretion of mass. Nevertheless, it is useful to consider this contraction too in the following presentation.

14.7 PG contraction factors versus Lorentz factor

The previous suggestion that we may have already discovered an alternative explanation for the Schwarzschild event horizon in black holes prompts us to have a closer investigation of the meaning of the contraction factors given by Eqs. 45 and 46. We further attempt to establish a possible relationship with the Lorentz contraction factor. These attempts are made in a kind of round-about-way, not strictly building the dynamics of PG theory from ground up yet. Trialing such attempts involves a mix-and-match of prior principles and understandings. We already acted like that in arriving at the proposal that the effective mass could be the same as the prior “inertial” mass. This potential conclusion was based on the use of the EP despite its possible redundancy in PG. We are fully aware about this on/off relationship with the principle. Redundant does not mean invalid. Valid or invalid will be determined as we develop the theory and practice of PG. With this proviso, we should be entitled to continue trialing various novel possibilities now open with PG, which are not yet well understood or finally accepted. We aim at eventually using as fewer postulates or “principles” as possible, which entails or presupposes better knowledge of the physical processes behind the principles.

Let us rehash some of the things already learned from Part 1, as a prelude to make an important next step towards a dynamic PG theory. The presumably event horizon in PG was deduced by observing the sphere's internal mass distribution at an ever increasing density. However, for an observer (or small test mass) on the surface or away from the sphere, the experience would be described differently. As we increase the mass of the sphere, the observer would feel an increased attraction from the direction of the sphere in proportion to the factor f_g . If the observer was trained only in Newtonian mechanics, he/she would report that the sphere was increasing its mass to an amount equal to what we call effective mass. However, if the observer could view and count all the gravions arriving at the test particle, then he/she would report no change in all directions but from those in the solid angle subtended by the sphere. In more detail, it would be reported that a maximum variation (depletion) comes from the center of the sphere. In the case of observation from a point at 100 radii from the gravitating sphere, the directional depletion of gravions would correspond to the gravitoid shapes calculated by PG theory in Fig. 28. A PG observer would report that a net push is experienced from the opposite direction of depletion. The Newtonian observer would report only an attraction by the visible spherical geometry (real one) with an apparent mass coming from the entire sphere, although it could be pointed out (with PG hindsight) that an equal effect could result from the real mass contained in the said gravitoid (unbeknown to the Newtonian observer).

Since we introduced the possibility of the effective mass playing the role of "inertial" mass in Section 14.5, which was not anticipated when the concept was first introduced in Part 1, it is helpful to review and clarify the following (even with some repetition): The effective mass and density are initially distributed uniformly over the real volume of a sphere and produce the measured (Newtonian) acceleration (or force F) at any external point. However, there exist a greater real mass and density also distributed uniformly but with a part of it being shielded from gravion action and without inertia; this part is probably the "stuff" of black holes, but possibly also ever present along with "ordinary" effective mass. We have also devised an effective spherical volume (smaller than the real volume), which, filled with real mass, produces the same Newtonian force F . Furthermore, we have devised gravitoids, which, filled also with real mass, produce the same force F . Since we attached a special interest to the effective mass, we have also become interested in finding its actual distribution inside a sphere (or line segment). Effective mass is created, where a gravion is absorbed. The outer layers are the most active with diminishing effect towards the center of the sphere. We have found the internal distribution for a stationary sphere relative to an observer (or small test mass) inside the sphere. However, for an external observer, the distribution of effective mass starts with highest concentration at the opposite end of the chord relative to the observer. Eq. 172 was derived for this purpose. Plotting the latter distribution (no need to be shown here) yields corresponding curves as in Fig. 17 but for the full line (chord) length. It is this distribution, which directly describes an important physical process, whilst other parameters are only mathematical tools and notions helping in the development of PG. We also note that we introduce a kind of relativity with respect to the observer's location: At a point outside the sphere, only the interactions of gravions inside the subtended solid angle by the gravitating body enter in making the force F , whereas at a point inside the sphere all gravions from all directions are involved in finding the internal force F . With all these clarifications, we realize that increasing the density of a material sphere produces a Moon-like meniscus of effective mass towards the outer surface away from the observer, or correspondingly, a gravitoid (imaginary) meniscus of real mass towards the near side of the sphere to the observer. For an observer inside the sphere, we report a maximum concentration of effective mass towards an envelop close to the surface of the sphere (probably the "event horizon"). In all cases, the geometrical integrity of the sphere remains invariant, whilst it is only the amount and distribution of effective mass that varies by Eq. 147 towards some extreme state (and effective shape), which might correspond to certain mathematical outcomes by GR.

The above is according to PG theory about the effects of increasing the mass (and acceleration) to very high levels, which is an analog to the paradigm of relativity, but without establishing any relationship between the two theories yet. We only establish a concept of contraction in PG, more precisely a concentration of effective mass, possibly corresponding to a contraction of length in relativity. Clearly, these two kinds of contraction are two different things, but they may share a common underlying process, which GR is not telling us about, but which PG is being built on. It is said(?) that in GR time dilation and length contraction near a massive body are not the same as time dilation and length contraction at relativistic speeds. Correspondingly in PG, we may not say from the outset that the contraction factors derived for a stationary body are the same for a moving body at relativistic speeds. However, it is reasonable to envisage that as we increase the speed of a moving sphere, the amount of gravion intensity traversing and interacting with the sphere increases with concomitant increase of the effective mass and variation of its distribution (see Eq. 147). That means that there is a correlation and possibly a relationship between speed v and kR . It is the task of PG to establish such a relationship, if it exists. Pending such rigorous development though, it can be helpful to attempt and try some intuitive steps as a kind of advance scouting exercise.

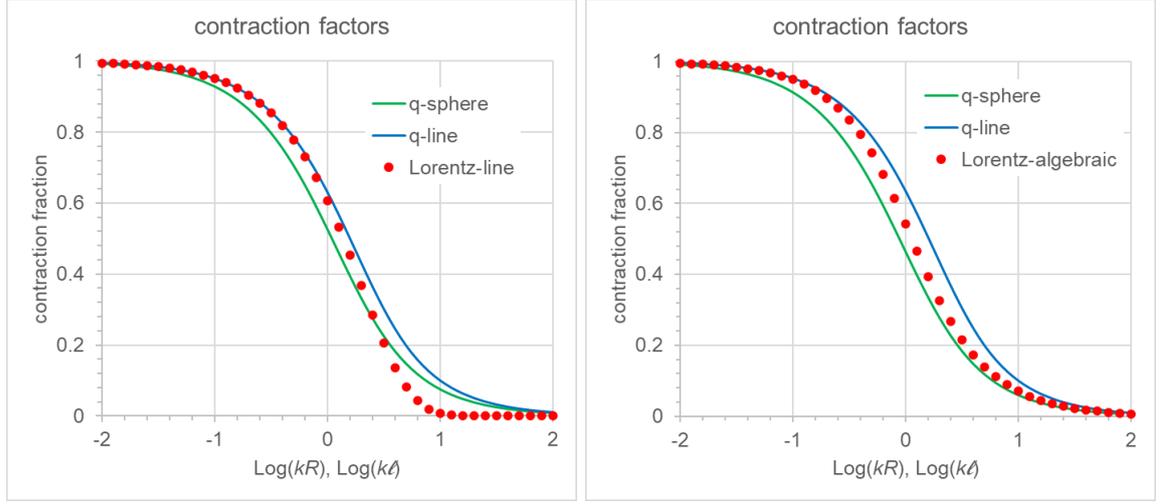


Figure 19: Comparison of PG contraction factors q -sphere (q) and q -line (q_ℓ) with Lorentz factor fitted with A_ℓ from line segment (left) and algebraic sigmoid (right)

For a possible connection between PG and relativity, we can initially try to express the PG contraction factor as a function of velocity, or the velocity as a function of kR , since we already have the function of $q(kR)$ (per Eq. 45). We try the latter by proposing some impromptu functions for the ratio of velocities v^2/c^2 in the Lorentz contraction factor $L(v^2/c^2)$

$$L = \gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}} \quad (182)$$

We note that both types of contraction (in PG and relativity) are functions of the same form (sigmoids) varying between 1 and 0. We inquire, if v^2/c^2 can be expressed as a function of kR preferably without a “fudge” coefficient to bring both $q(kR)$ and $L(kR)$ to agreement. Fudge coefficients are often detested and preferably avoided. We further note that the absorptivity factor A_R has a sigmoid form without any arbitrary constant to “fudge” with, which conveniently prompts us to try first by simply setting:

$$\frac{v^2}{c^2} = A_R = 1 - \frac{1}{2(kR)^2} + \frac{\exp(-2kR) \cdot (2kR + 1)}{2(kR)^2} \quad (183)$$

The result is plotted in Fig. 18 with curve points (in red) labeled “*Lorentz-sphere*” along with the two PG contraction factors for sphere $q(kR)$ and line segment $q_\ell(k\ell)$. There is an immediate very good-to-excellent agreement between “*Lorentz-sphere*” and “*q-sphere*”. This is very encouraging and may be used as guidance to proceed further with PG.

We can also try to replace the Lorentz velocities ratio with the absorptivity factor of line length per Eq. 158:

$$\frac{v^2}{c^2} = A_\ell = 1 - \exp(-k\ell) \quad (184)$$

and plot the outcome (*Lorentz-line*) together with q -line and q -sphere as before in Fig. 19 (left). We now see a significant deviation from both q -sphere and q -line. Actually, there is good agreement with about the first half of the q -line curve. The deviation is surprising at first, but considering that distribution of mass and shape in a body are important in PG, the outcome may be justified. It could be that both q -line and q -sphere are correct, whilst all other body shapes may be characterized by curves lying in-between those two. In that case, if PG can express the correct contraction for any shape, then the Lorentz contraction may be a good approximation of reality, either for sphere or line segment, but not an exact one. The shape of the accelerating body does not appear in relativity(?).

For good measure, we have also tried several other sigmoid fitting forms, like the so called “generalized logistic”, “hyperbolic” and “algebraic” sigmoid functions. All failed to produce any reasonable or better fit, except for the algebraic function:

$$\frac{v^2}{c^2} = f(z) = \frac{z}{\sqrt{1+z^2}} \quad (185)$$

where z is the product of some characteristic *length* times the absorption coefficient: $z = k \cdot \text{length}$. The outcome is provided in Fig. 19 (right). Interestingly, the Lorentz factor fits well with $q - \text{line}$ at low values of z and well with $q - \text{sphere}$ at high values of z , with transition values in-between.

If the “*Lorentz-sphere*” fitting curve in Fig. 18 is the correct one, then we could write that:

$$q = \frac{3A_R}{4kR} \approx \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \quad (186)$$

which could herald an initial (tentative) expression of the PG contraction factor as a function of velocity. If in any way it can be shown that the PG contraction is equivalent or near equivalent to the relativistic contraction, then it could have enormous consequences in physics. It is impractical to exhaustively mention and discuss all those consequences here. A lot of work should follow. In the meantime, we may provide some tentative thoughts about the significance of the above findings.

First, some serious questions arise, which can have critical repercussions not only for PG but probably also for GR. The discussion of these questions will determine if PG and GR can coexist and complement each other, or one of them has to give way to the other.

In one aspect and by way of Eq. 186, it follows that an increased speed is accompanied by an increase of the absorption coefficient k for a fixed body radius, or length. This means an increase in mass or density. This means that the relativistic mass is not simply a mathematical intervention to enforce the light speed limit. The relativistic mass can be an effective mass increasing with the speed of the body.

It may be argued that no such new mass is consistent with experiment. For example, in the Large Hadron Collider at CERN, both Xe and Pb ions are accelerated to energies of about 2–4 TeV and nothing seems to happen to the ions. However, we may wonder, if all is taken correctly into account in arriving at such a conclusion. By no means do we challenge that conclusion here, except to draw attention to the possibility of other parameters playing a role too. For example, it is said that two “up/down quarks” are found in “ordinary” matter, but another four “other types of quarks” are found only in accelerator collisions. It is not clear why this happens (at least to the present author). Could then accretion of mass take place in nuclear and sub-nuclear structures that do not alter the macroscopic appearance of matter? If the PG “real mass” contains one active fraction (effective mass) and one inactive fraction without classical inertia, could we then allow and account for mass accretion in an accelerated ion beam in the LHC experiments?

Therefore, it seems that accelerating a body to relativistic speeds is equivalent to increasing its mass to an asymptotic upper limit, but not to infinity. We may initially surmise that the effective mass is gradually created and redistributed preferentially in the direction of motion. The increase of effective mass could only come about by a concomitant increase of the total real mass at fixed g_0 . The length itself in the direction of motion does not actually contract, but the amount of effective mass is compressed close to the head of the moving body and away from its tail. This is a point of fundamental departure from GR preaching that the actual mass does not increase, since there is only one mass, namely, the rest mass, whilst the physical length contracts. Theorists insist that relativistic mass per se has been thoroughly deprecated from the outset of GR, whilst it was introduced mathematically only to make the limit of speed of light look natural. However, PG can accommodate, literally, an actual increase of mass, namely, real mass, part of which constitutes an increasing amount of effective in lieu of relativistic mass. There is a balance between the rate of accreted and the rate of re-emitted mass according to a forthcoming Eq. 198. This novel finding of PG could resolve a persisting debate (or misunderstanding) and revolutionize the understanding of relativity.

There is no doubt that the problem of mass is one of the key problems of modern physics, whilst one wonders why the “debate” or corrective steps on the meaning of relativistic mass has continued since the inception of GR. Even notables like Penrose and Hawking did not come clear on this issue for whatever reasons (Okun, 2006). The relativistic mass may not be a “pedagogical virus”, after all, it might be a common sense reality. [More about the concepts of mass and force have been worked out in Section 16 with a possible explanation on how new matter could be generated in particle accelerators. That is, the particle mass can acquire an effective mass over and above the maximum permitted outside the accelerator. The latter creates an artificially increased value of \$g_0\$, so that when the particle decelerates at the end of its journey, it has to shed the extra mass in the form of new particles inside the accelerator. This re-adjustment of particle mass continues as long as they violate the PG law of maximum effective mass.](#)

In another aspect and by way of Eq. 186, we may have another more serious conflict between PG and GR: The Lorentz factor necessitates an arbitrary increase of the relativistic mass, as we approach the speed of light. That would require PG to be able to also increase the effective mass to infinity, correspondingly. However, PG anticipates an upper acceleration limit g_0 for a stationary body. If accelerating a body by

motion is equivalent to accelerating it by a nearby massive body, then either PG, or GR, or both should be adapted to produce equivalent outcomes. It is unclear if this is possible at this stage of development, especially if GR rules out(?) relativistic-speed and nearby-massive-body equivalence. May be this equivalence breaks down at relativistic speed extremely close to the speed of light. May be GR cannot be verified too close to the speed of light. May be GR needs to modify its prediction from an infinite relativistic mass to some maximum (limiting) value relativistic mass corresponding to a PG upper limit of acceleration and effective mass. The alternative for PG is to think of a way (formulation or whatever) that increases the amount of absorbed gravions to infinity as we get too close to the speed of light, something waiting to be worked out. As a last resort for PG would be to apply superluminal speeds of the body to sweep and absorb an arbitrarily increasing amount of gravions. After all these combinations of possibilities, it is also quite possible that all three curves in Fig. 18 are valid considering that contraction factors may not be the same for a moving body at relativistic speeds and for a nearby massive body! This means that the Lorentz contraction factor for a moving body remains to be found in PG.

In summary of above ideas, PG may anticipate that all gravions swept at the head of the moving body are (near) totally absorbed, whilst (nearly) no gravions are absorbed at the tail, when motion is very close to the speed of light. Total absorption occurs at the limit of maximum acceleration g_0 , at some maximum effective mass $m_{emax-moving}$, or density $\rho_{emax-moving}$ and provided that the real geometric integrity of the body is preserved, i.e. the radius for a sphere remains constant; otherwise, the situation becomes more complex. The details for such outcomes remain to be worked out.

Whilst GR has been verified on many occasions, it is not known (at least to the present author), if it has been verified at speeds somewhere sufficiently close to the speed of light. Could it be that the Lorentz factor is an approximate manifestation of another “contraction” process as now described by PG? It may be that one of the two theories is the true one, whilst the other is an approximation. They both appear to converge (agree) at low enough speeds (whilst disagreeing on the meaning of relativistic mass), but they are in conflict too close to the speed of light. For a proper answer, we have to wait until PG is put to the test for verification or not, while we also continue to develop it theoretically.

We are aware that we did not derive the Lorentz contraction factor above from PG principles, except to demonstrate the possibility that the PG contraction factors may already describe what the Lorentz factor exists for and much more. We have derived, hopefully, equivalent contraction factors and the Schwarzschild envelope without even resorting to relativity yet. At the outset, we have added the postulates (or principles) #5 and #6 provisionally on the assumption that PG may be built as an expansion of relativity taken for granted. Furthermore, a significant discussion on postulate #3 is presented and proposed in the following sections of Part 2, which could make this principle redundant too. Part 2 of this report is an open-ended discussion towards elaboration of a fully fledged, self contained PG theory and practice.

We are pioneering a totally new ground with PG necessitating a re-examination of a large number of problems in physics. For another example, the new concept of PG contraction factors could provide another understanding of the Michelson-Morley experiment. There is a need to re-trace the founding steps of relativity in order to juxtapose them with those of PG and explain why the two theories result in similar but also mutually exclusive outcomes in an increasing number of cases. There is both commonality and departure between the two, like between PG and Newton, and like GR and Newton. Which one is the bigger one? There is a lot of work (rework) to be done by re-visiting a lot of outstanding or seemingly established topics in physics. For one thing, though, so far PG demonstrates a lot of promise with fresh ideas and outcomes.

In the preceding analysis, we advanced some bold assumptions and assertions not necessarily exhausting the gamut of possibilities under PG. That means that we may continue to try and reconcile the aspects of PG with prior prevailing theories on the nature of matter, mass, energy and inertia. For example, see some additional aspects possible in Section ?? .All options remain on the table. For this reason, it should be appreciated that the current single-author advancement of PG ideas has its limits, which can be overcome by the participation of the broader scientific community.

15 Response to criticisms

As mentioned at the outset of this report, there [have been](#) numerous objections to the idea of push gravity since the original proposal by Fatio. This has applied to all hitherto variants of PG, but it is hoped that all these objections may be overcome in part by the preceding findings and in part by some new arguments and models presented in this Part 2. Most of the objections may be overcome without further ado, but the main problem of energy absorption and mass accretion can only be tackled speculatively at this point, if we have to face the dilemma of abandoning the preceding findings, or advancing forward on those findings. The best known objections, [as outlined in this referenced version of Wikipedia contributors \(2018a\)](#), are discussed next.

15.1 Weak absorption, range and gravitational shielding

Whilst early conceptions of push gravity maintained that it was mandatory to assume very minimal absorption of gravions in order to avoid the objectionable gravitational shielding, it is exactly the opposite consideration that frees push gravity and explains some of its intrinsic workings. Gravitational shielding or self-shadowing by mass is now at the core of the workings and understanding of PG. This is not something to object to, because via and by its presence we can actually derive the gravitational law, in fact, in a new form that can account for a lot of missing information in Newtonian mechanics including singularities.

By the same token, gravitational shielding leads to a distinction between real and effective mass, which, in turn, may lead to a re-appraisal or re-think of the notion of inertia and mass per Section 14. When we can faithfully describe the observed motion of bodies with a theory of the dynamics of PG, we will finally put this objection to rest, unless we better achieve verification of PG by experimental means first.

15.2 Equivalence Principle

The allegation that PG would violate the Equivalence Principle must have been a misconception in view of our previous explanation. PG actually frees us from having to resort to the Equivalence Principle, which arose out of the need to understand the nature of force initially perceived as arising either from a gravitational field or from a moving mass under acceleration by an applied external force. PG finds no distinction between these two kinds of force, as the flow of gravions produces the same force in both situations (systems). For the first time, we have a tangible explanation of the **phenomenon** of equivalence of force. The gravitational force experienced by a body as attraction (pull) is actually a push force, namely, the sum total of all elementary push forces distributed in the bulk of the mass and arising from absorbed gravions. The latter force is of the same nature as the push force applied to a hypothetical elevator in free space, inside which we would experience an equivalent force. This equivalent force need not be such (equivalent) axiomatically, because it is “prima-facie” push in nature. Hence the Equivalence Principle per se vanishes without ever being in conflict with PG.

Nevertheless, if the Principle emanates from (or is based upon) the equality of the gravitating mass to the inertial mass, then PG is clearly at variance. However, PG quantifies the variance as being extremely small to easily detect in the human laboratory, but, hopefully, big enough to measure in flyby experiments and planetary orbits. If the EP is described as equality of masses (gravitational and inertial) then PG clearly violates it. Conversely, if PG provides the true relationship between the two masses via $m_e = qm$, then EP violates nature and becomes redundant under PG. Expressed differently, if “equivalence” means true proportionality but not equality, then PG provides exactly this proportionality in a tangible, physical and explanatory form.

The above ideas summarize the analysis of Section 14.2, but we should also stress that the EP seems to be only an arbitrary approximation for scales of our immediate experience. This approximation breaks down at very large masses, or densities. Large densities occur in white dwarfs, neutron stars and black holes (large scales), but they also occur at nuclear and sub-nuclear particles (small scales). Thus, the finding that EP clearly breaks down at quantum levels (Kajari *et al.*, 2010) may not be a mere coincidence, but a great consistency with our independent findings on EP. The inertial and gravitational masses are not equal, whilst the original “gedanken” conception of the EP is now redundant needlessly locking down further development of physics for over a century.

Finally, in view of the possibility to decipher the meanings of matter, inertia and mass per Sections 14.5 and 16, PG appears to be on strong ground but not necessarily consistent with the EP, or with the theory of relativity in every respect.

15.3 Theory of relativity

It has been argued that PG is incompatible with the established theories of relativity. It is often argued that since the general theory of relativity (GR) is continually verified by contemporary measurements with great accuracy, PG not emanating from within GR must be wrong. However, the counter-argument may be that PG is a re-appraisal of classical Newtonian mechanics, upon which to build and extend the current relational developments of relativity. PG explains the generation of a gravitational field around a mass that presumably can be observed and measured identically with existing data. We would suggest that it is prejudicial to think that PG has to arise out of (or fit in) GR, whilst the opposite might be true. Therefore, the two theories may not have to be in conflict upon closer examination.

If gravions travel with the speed of light, then in the steady state, they establish a pushing field that fictitiously appears as an attractive field around the shadowing (gravitating) mass. This field is being established at the speed of light without emanating from the mass, but rather emanating from the surrounding

universe. If the mass starts moving at speeds comparable to the speed of gravions (and light), then there will be a disturbance of the surrounding shadowing or warped space (field) due to a time lag that propagates at the gravion speed. This disturbance would be consistent with the gravitational waves scientists are trying to detect.

An analogy may be found in solid state physics near a PN-junction, where “holes” are formed from the absence of electrons on one side of the junction with an equal amount of excess electrons on the other side. These holes are treated, or behave, like a positive current of charge moving in the opposite direction to electrons with negative current. GR then is like it is treating gravity as gravion-holes apparently emanating from (or associated with) the mass, whilst in reality it is the real gravions (particles) moving in the opposite direction towards the gravitating mass that should be considered, or equivalently considered. The end result (force and acceleration) appears to be the same by both approaches. Both ways of creating a field around a mass presumably create identical apparent outcomes.

We propose then that the gravitational field described by PG and the field described by GR are quantitatively identical at every point around a stationary material body. The difference is that PG tells us how/why this is formed (i.e. its origins), whilst GR remains mute about the origin of the same field, but it yields verifiable measurements, anyway. The latter is sometimes described like “GR generates correct results for the wrong reasons”. However, knowing the origins of gravity is a fundamental difference between GR and PG that could get us over the existing barriers in physics.

When we start applying PG to moving bodies with significant speed relative to the speed of gravions, then we may be able to borrow the mathematical tools already developed for relativity, special and general, to describe the same resulting effects and measurements. There is probably no restriction to the importation of Special Relativity as is. The mathematical derivations and achievements of GR may also be transferred and used in PG, in particular as they relate space and time. This transfer might be particularly useful where GR actually succeeds and discarded where GR fails (e.g. at very long distances, etc). The present work has only dealt with PG in the steady state without ever involving time effects yet. Therefore, it might be premature to argue that the two theories are in conflict.

Arguments of the type, for example, that because the Mercurial precession can be explained by GR is proof and manifestation of the success of GR should by no means be used to oppose PG. The same fields being established also by PG should arrive at the same outcomes. In fact, PG provides a new framework to re-appraise the contributions of other planets on Mercury’s precession by expanding classical mechanics with PG, which may produce a further refinement of the same calculations taking into account the real density and mass distributions of all the planets contributing to this precession. The other argument that the Sun bends the star light is not the privilege of GR only, because PG can do the same thing on photons by the pushing gravions presumably at the same (correct) deflection angle.

If at first sight the above assertions might seem simplistic, it is because there is a large volume of phenomena to be understood under PG, before we make further assertions. For example, could the temporal part of the metric in relativity, which determines the rate at which clocks tick and is responsible for Newtonian gravity, relate to the rate of gravion flux intensity? Could the increase of mass (relativistic mass) of a moving body as it approaches the speed of light be tied and explained in the new terms of real and effective mass? Should we, perhaps, re-appraise the meaning of inertial mass in conjunction with the meaning of matter and “stuff”? In general, could the theoretical concepts of relativity achieve an embodiment in PG theory?

The above important questions together with issues raised in Section 14 may now be better understood in the hope to further an inquiry into the novel PG theory. We may be faced with any of the following outcomes: (a) PG and GR may complement each other, (b) GR may be expanded to incorporate PG ideas, (c) PG may replace GR as an all inclusive description of experience, or (d) PG becomes unsustainable. The examination of these possibilities is the next challenge that we face for building a dynamic PG. The preceding trials may serve to provide some indication of what may come next. Clearly, such a task is huge and falls outside the capacity and resources of a single author. Hopefully, the learned adherents of GR can make a critical contribution.

In summary to the related objections, relativity may not be presented as reason for rejection of PG. Even if it appears that PG is not consistent with certain established ideas of GR, the “jury remains out” until sufficient experimental evidence is gathered in support or not of PG. *It may ultimately be that there is a substantial overlap or correspondence of ideas and conclusions between PG and GR, but also with a fundamental departure between the two theories from some point onward (see Section 16).*

15.4 Drag

It has been argued that push particles (original ultramundane corpuscles) would introduce a drag force on the orbiting Earth, eventually slowing down the planet to ever closer orbits around the Sun. This would indeed be a consequence, if the particles were acting like classical mechanical balls. However, the gravions

are relativistic with no difference in speed relative to the planet motion. Gravions are not expected to make a difference over any effects already experienced with photons over the broadest spectrum of wavelengths originating from outer space.

In view of the attempted explanations on the nature of matter, mass, energy and inertia in Section 14, it is implied that the gravions have no drag on the effective mass being energized (generated) by the very same gravions. This is an assumption pending further investigation on the nature of gravions and their interactions with real mass and with themselves.

15.5 Superluminal speed

During the early stages of push gravity theories, the hypothetical corpuscles were required to have some superluminal speed to reduce the expected drag to a practically ineffective minimum. However, this is not required after the advent of relativity and in the light of the present arguments.

15.6 Orbital aberration

It has been further argued that PG would introduce orbital aberration due to the finite speed of gravity created by gravions. This aberration would tend to accelerate an orbiting body away from the other, unless gravity propagates much faster than the speed of light, or must not be a purely central force. It has been further argued that the same finite speed of gravity problem is almost exactly canceled by the mathematics in GR. Now, it is not clear why PG cannot overcome this problem in the same way, if GR can. It is proposed that we may continue to use and adapt aspects and derivations of GR, or postulate an equivalence between GR and PG (at least in part), until it can be finally clarified if this is at all appropriate, or under what conditions.

Nevertheless, recent measurements report that planetary orbits are widening faster than if this were solely through the Sun losing mass by radiating energy. This results in an anomalous increase of the astronomical unit, which might then be explained by the above PG criticism pending further analysis of the situation.

As discussed in Section 14.4, and until we can quantify time effects, PG theory remains incomplete. Any verdict can be postponed, until at least some tests are done to possibly verify the principle of PG.

15.7 Energy and mass considerations

15.7.1 Planetary absorption

Basically, the most serious criticism arises from the need that the gravions must be absorbed in order to produce a force, but the amount of energy absorption would then be so high as to be unsustainable by the gravitating body. This is the main reason, for which notables like Kelvin, Maxwell and Poincaré (Wikipedia contributors, 2018a; Poincaré, 1908), after initial consideration, moved away from PG. There is no obvious or immediate solution to this major problem haunting any PG theory. For this reason, we based the entire development of PG on the assumption that the absorbed energy is somehow re-emitted. However, until some experiments provide encouragement at least, we are entitled to speculate with some improved models in continuation to previous attempts to overcome this hurdle. Let's first formulate the energy absorption problem based on derivations in Part 1.

We find the total energy passing per unit surface area of a sphere and absorbed by the bulk in the sphere: We start with the absorbed gravions (energy) inside the solid angle subtended by the sphere at point O (see Fig. 2), which is given by the previously defined J_a (not J_g):

$$J_a = J_0 f_a = J_0 \int_0^{\varphi_0} 2\pi \sin \varphi d\varphi \cdot [1 - \exp(-k\ell(\varphi))] \quad (187)$$

$$J_a = 2\pi J_0 \int_0^{\varphi_0} \sin \varphi \left[1 - \exp\left(-2kr\sqrt{a^2 - \sin^2 \varphi}\right) \right] d\varphi \quad (188)$$

The above provides the per unit area absorption at each gravion trace direction (not the per unit area of the surface of the sphere). However, at the surface of the sphere (with $r = R$ and $a = 1$), for the absorbed flux density per unit area of the sphere, we must apply the cosine law for oblique incidence and multiply by $\cos \varphi$ yielding the parameter J_{aR} :

$$J_{aR} = 2\pi J_0 \int_0^{\varphi_0} [1 - \exp(-2kR\cos\varphi)] \sin\varphi \cos\varphi d\varphi \quad (189)$$

which can be integrated analytically as per Eq. 36 including the established absorptivity A_R :

$$J_{aR} = \pi J_0 A_R \quad (190)$$

The above provides the absorbed density flux per unit area of the sphere from all directions inside a hemispherical solid angle, i.e. by integrating from 0 to $\pi/2$. Thus, we multiply by the surface area of the sphere to obtain the total absorbed density flux, i.e. the total energy per unit time, or power W as:

$$W = 4\pi^2 J_0 A_R R^2 \quad (191)$$

By replacing J_0 from Eqs. 68 and 73:

$$J_0 = \frac{cg_0}{\pi\Lambda} = \frac{cg_0 M_e}{\pi^2 R^2 A_R} \quad (192)$$

we finally obtain

$$W = 4cg_0 M_e \quad (193)$$

from which we have an energy absorption rate per unit effective mass W_{M_e}

$$W_{M_e} = \frac{W}{M_e} = 4cg_0 \quad (194)$$

If want to use $E = mc^2$, the above energy is equivalent to a mass accretion rate of:

$$mass_accretion_rate = \frac{4g_0}{c} \quad (195)$$

from which, depending on the prevailing g_0 , we find the absorbed energy. With a moderate level of $g_0 = 10^4$ m/s², we would get about 1.3×10^{-4} kg for every kg of the sphere (say, Earth) every second. This is clearly an enormous amount of energy (mass) that cannot be accounted for by our experience on the planet. An early criticism leveled against PG claimed that the absorbing mass would be doubling every second. This criticism is generally valid even with our much lower accretion rate found [here](#).

The above derivations are the most telling reason for the rejection of PG, which has been the case to date. Therefore, this constitutes a critical point whether to continue with this theory or come to an end of this investigation once more. The present author is of the opinion to persist in finding some way(s) to push through this barrier, literally. That is because the preceding findings have produced a system of consistent outcomes with Newtonian mechanics as the limiting case, and because it promises to resolve many other cosmological problems on a new basis. We may recall an analog situation early in the 20th century, when the orbiting electrons should be emitting electromagnetic radiation, the lack of which did not deter the then visionary scientists to introduce and accept the orbital model of the atom. Thus, instead of rejecting the PG theory, we may have to accept that the dissipated energy by gravions manages somehow to escape out of the absorbing mass in a different form of radiating particles. A new motto could then be “*what goes in must come out*”, but catchphrases don’t make science on their own, unless they are confirmed without leading to another impasse: The above demand allegedly leads to another violation, namely, of the second law of thermodynamics, an objection discussed separately next.

Equivalent alternative formulations to derive the gravion absorption rate by a single (lone) material sphere are provided in Appendix D.2.1, etc.

15.8 Second law of thermodynamics

It has been argued that the gravions, if re-emitted as different particles to carry away the dissipated energy, would violate the second law of thermodynamics, which was the reason for rejecting the re-emission of particles/energy as initially (tentatively) proposed but abandoned by notables such as Kelvin, Poincaré, Lorentz and Thomson (Wikipedia contributors, 2018a). However, if we look closer at the intrinsic meaning of this law, it may not necessarily be violated overall. This arises from the fact that the law relates to the most probable state of a closed system having the maximum entropy. The entropy S relates with the number of accessible states Ω via

$$\Omega = \exp(S/k) \tag{196}$$

(k here is the Boltzmann constant) and the probability P of finding the system in that state is

$$P \sim \Omega = \exp(S/k) \tag{197}$$

Now, when the system has a relatively small number of accessible states, the fluctuations can be very frequent, wide and repeatable, i.e. recurrence of unstable states may be quite feasible within the time scale of graviton frequency absorption. The consequence of this is that the system can often be found “momentarily” in a state of decreased entropy favoring the emission of some augmented (with accreted mass) particle out of the system. This happens when by random redistribution of mass and energy within the subsystem generates a sub-particle capable of overcoming the constraints that keep the subsystem together. When enough quantitative material and energy accumulation has occurred (accreted), the subsystem bounces emitting a new particle, all of this on an extremely short time-scale (appearing to us). The particles of the subsystem co-operate to get rid of and push out one of their own members every-now-and-then, often or not in the time scale of the subsystem. In other words, the second law of thermodynamics does not prevent us from accepting that matter/energy can be re-emitted after a number of trial fluctuations following a certain number of graviton absorption inside a proton, electron, neutron or any other nuclear, sub-nuclear, or elementary particle (subsystem). Thus, what was initially conceived by the critics as thermal dissipation inside matter in general, it will not appear as known chemical (molecular) heat that would melt and evaporate the planet. It would only appear as internal energy of a particle that is not thermally coupled with an atom or molecule via some sort of recoil action during the said re-emission. The re-coil produced by the proposed re-emission is taken up and averaged out by the subsystem behaving under the established quantum mechanical laws. In fact, it might be that the underlying mechanism of quantum mechanical randomness may be caused exactly by such re-coil of the subsystems of particles. Electrons and nuclear particles move about randomly per quantum mechanics. This model further assumes that the re-emitted particles are also penetrating the surrounding matter out of the planet with a long enough mean free path as not to heat the planet catastrophically but not long enough as to act like gravitons in generating gravity (i.e. canceling out gravity). It is only the very long mean free paths of gravitons that generate gravity among planets and stars, while the second generation emitted particles, as proposed here, behave like a diffusing gas out of the planet, perhaps, with some but not catastrophic heat dissipation. It may be that part of, if not all, the heat in the core of planets is generated by this mechanism in an analogous way, in addition to, or in lieu of, the heat being produced by radioactivity per prevailing theories.

We need not at this point specify the exact nature of the particles being re-emitted, other than for them to be able to carry away the absorbed graviton energy, or a critical part thereof. It is left for further investigation by particle and nuclear physics to establish if any of the known particles qualifies to play this role, as for example, neutrinos might (or might not) serve this purpose. Alternatively, we may build on a new model to describe the properties and consequences of this second generation of particles emanating from the primary graviton flow.

In support of the above general proposal, we may cite a similar situation that explains radioactivity. Particles can rearrange in the nucleus, or change from one type to another statistically over time. Random quantum fluctuations can promote relaxation to a lower energy state and decay via quantum tunneling. Radioactive decay half-life varies over many orders of magnitude on a timescale down to 10^{-23} seconds (Wikipedia contributors, 2019d). In our proposed analog, we may envisage all sub-nuclear particles including protons, electrons, positrons, etc. to undergo such statistical fluctuations inside themselves at even extremely smaller time scales beyond the range of our measuring instruments, in effect, appearing like providing a continuous absorption of gravitons and re-emission of secondary type-II particles diffusing in the surrounding material space without causing further gravitation or catastrophic heat. This continuous absorption then is tantamount to a continuous push without the feared catastrophic melting down.

The above proposed model should not be less plausible than the latest quantum fluctuation theories (Wikipedia contributors, 2019c). It is in accord with the fluctuation theorem and the ongoing discussion, research and experiments relating to Maxwell’s demon.

Thus, the present framework in understanding gravity should not be inconsistent with modern theories. Quantum field theory is about very small stuff, small particles (the standard model). Gluons bind quarks together. Quantum gravity considers loops of gravitational force, then we get knots, loop quantum gravity and time disappears (problem of frozen time). These quantum states of space fluctuate, fluctuations in the quantum states of space create the appearance of time. These loops exist on the scale of Planck length. A proton contains 10^{65} quantum volumes, whilst gravitons is said to carry the force of gravity by exchanging them, (the photon carries the electromagnetic force, so the graviton carries the gravitational force), but gravitons are thought to be pseudo-force particles according to loop theory. The quantum nature of space

does not allow singularities, whilst the universe did not come about with a bang but with a big bounce [Jim Baggott: <https://www.youtube.com/watch?v=dW7J49UTns8>]. All these latest conceptions might be further adjusted and advanced by the new understanding of PG, so that our approach should not be less plausible than all these other modern models and proposals. In fact, PG seems to be consistent with the above theories so that PG may act as a resolution by binding together of the best of elements in those theories.

Finally, if the main concern of PG theory has been the huge amount of energy or mass required to be absorbed in order to result in the measured acceleration on a planet, the likely revelation in Section 14.5 may help. In view of the idea of the creation of effective mass (Newtonian mass) by gravion absorption, the absorption becomes a blessing rather than a curse: The large absorption is consistent with a large presence of planet mass, however, subject to a balancing rate of outflow of the same energy. We proposed a mechanism for such an outflow via the fluctuation theorem above. In the steady state, the outflow must be proportional to the steady state effective mass m_e as in the general formulation we have found in Eqs. 193 and 194:

$$\frac{dE}{dt}(\text{inflow}) = \frac{dE}{dt}(\text{outflow}) = \text{constant} \cdot M_e \quad (198)$$

16 Two-sphere variation of mass and force

In continuation to and based on preceding analysis, we can derive some important relationships between mass or energy and force for two material spheres. The energy absorption rate given for a single sphere by Eq. 191 is derived by the universal gravion intensity J_0 times a factor S_a . For a single (lone) sphere and according to Eq. 191 the latter factor is:

$$S_a = 4\pi^2 R^2 A_R \quad (199)$$

and is involved in the known relationships:

$$S_a J_0 = 4cg_0 M_e \quad (200)$$

$$S_a \frac{cg_0^2}{\pi^2 G} = 4cg_0 M_e \quad (201)$$

$$S_a = 4\pi^2 G \frac{M_e}{g_0} \quad (202)$$

The S_a is a characteristic surface area pertaining to the omnidirectional absorption of gravions from all directions around the sphere with radius R and can be also computed directly from the constitutional equations of gravion absorption provided in the Appendix. We will later see that this relates via a factor 4π to an effective cross-section for a classical unidirectional absorption of a beam of particles.

With S_a derived (or computed), we can then find the corresponding effective mass as:

$$M_e = \frac{g_0}{4\pi^2 G} S_a = \frac{S_a}{4\pi \Lambda} \quad (203)$$

We can readily ascertain that, S_a being an effective absorption area and the universal constant Λ being an area per unit mass, correctly yields a mass M_e . It is an important finding and understanding that the effective mass is the ratio of these two parameters.

Up this point, we have considered a single (or lone) material sphere, but this begs the question what happens in the presence of another material sphere at a distance r in the neighborhood. The answer is found by considering the constitutional equations of absorption simultaneously for the two spheres. This is done in considerable detail in the Appendix. We apply those methods here to three representative cases with a brief description of the approach. To discuss and understand what happens, we have computed in one case, by way of example, the Earth-Moon interaction as we vary the distance. In a second case, we repeat the same for two very dense spheres, one of which is very small. In the third case, we use two very dense sphere with both being also extensive (large).

We have developed and used two equivalent methods in the Appendix, namely, one integrating through the bulk of each sphere, the other around the surface of the sphere. This is possible by noting that for every internal point (in the bulk), all possible graviton traces must cross the surface. Conversely, all graviton traces through every point on the surface account for the entire bulk points twice (note that in Eq. 191, we used only one direction of flow at each point, so that we summed over the entire surface without dividing by 2).

With reference to the Appendix and related figures, we compute various fractions of graviton absorption pertaining to two kinds or classes of graviton traces or paths through each sphere. There is one class of traces crossing only one sphere, to which we refer with the term “single”, or “lone” trace. The other class of traces belongs to (i.e. they cross) both spheres, to which we refer with the term “joint” trace. Gravitons along single paths produce no net force as they travel in opposite directions in equal numbers. Nevertheless, they are absorbed by a certain amount in each direction along the chord length of a sphere, the total amount of which relates to certain effective mass. Gravitons along joint paths produce a net force along the trace due to the unequal amount of gravitons entering from the two ends of the chord. In addition, they are also absorbed by a certain amount in each direction, the total of which again relates to a certain effective mass. The task is to find both the net force exerted on one of the spheres by the other and at the same time the total effective mass of the sphere “created” by (or related to) the absorbed gravitons.

At the outset, we need to choose one of the two spheres for investigation of force and total absorption. Let this be sphere_2 and let’s apply the “surface” method of investigation to find the total gravitons absorbed (we investigate the force afterwards). We consider what happens at every point O on the surface of this sphere and integrate over the entire surface for the total result of absorption relating to force or effective mass. A chord OC traversed by “single” traces yields an absorption factor $2(1 - \exp(-k_2 OC))$. However, if chord OC is traversed by “joint” graviton traces, the absorption factor becomes $(1 - \exp(-k_2 OC))(1 + \exp(-k_1 AB))$, where A_1B_1 is the jointly traced chord of the other sphere_1, with corresponding absorption coefficients k_2 and k_1 . We integrate at point O separately all single and all joint traces with respect to the azimuth and zenith angles around the axis normal to the surface of the sphere at point O:

$$M_{ar-single} = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} (2(1 - \exp(-k_2 OC))) \sin \varphi_z \cos \varphi_z d\theta_z d\varphi_z \quad (204)$$

The above outcome is the same for all points on an elementary surface annulus by rotation around the axis joining the two spheres at angle ω . This elementary surface is $2\pi \sin \omega R_2^2 d\omega$, so that by a final integration over this angle, we obtain the end result:

$$S_{ar-single} = \int_0^\pi M_{ar-single} \cdot 2\pi \sin \omega R_2^2 d\omega \quad (205)$$

We follow the same steps for the “joint” traces of sphere_2:

$$M_{ar-joint} = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} (1 - \exp(-k_2 OC))(1 + \exp(-k_1 AB)) \sin \varphi_z \cos \varphi_z d\theta_z d\varphi_z \quad (206)$$

$$S_{ar-joint} = \int_0^\pi M_{ar-joint} \cdot 2\pi \sin \omega R_2^2 d\omega \quad (207)$$

The total characteristic absorption surface is:

$$S_{ar-total} = S_{ar-single} + S_{ar-joint} \quad (208)$$

The notation “ ar ” in the subscripts above stands for “absorption” at distance “ r ”. Now, there is an important new quantity to consider. That is the difference between the total absorption of sphere_2 being at distance r from sphere_1 and the absorption that sphere_2 has when it is at infinite distance from the other, in other words, when it is away from the influence of any other bodies. We have already found the latter by Eq. 199. It is instructive to write the corresponding equations for this quantity starting with the corresponding factor for a chord OC. The “single” contributions cancel out leaving only the difference of the

$r, \text{ m}$	$M_{er-single}, \text{ kg}$	$M_{er-joint}, \text{ kg}$	$M_{er-loss}, \text{ kg}$	$M_{e-sum}, \text{ kg}$	<i>Asymptotic loss</i>
12742000	5.9134E+24	5.8939E+22	$4.7876E+19$	5.9724E+24	$4.5098E+19$
25484000	5.9583E+24	1.4058E+22	$1.1431E+19$	5.9724E+24	$1.1274E+19$
50968000	5.9689E+24	3.4775E+21	$2.8281E+18$	5.9724E+24	$2.8186E+18$
101936000	5.9715E+24	8.6712E+20	$7.0524E+17$	5.9724E+24	$7.0465E+17$
203872000	5.9722E+24	2.1664E+20	$1.7620E+17$	5.9724E+24	$1.7616E+17$
407744000	5.9723E+24	5.4152E+19	$4.4043E+16$	5.9724E+24	$4.4041E+16$
815488000	5.9724E+24	1.3537E+19	$1.1010E+16$	5.9724E+24	$1.1010E+16$

Table 9: Variation of various fractions of the Earth effective mass versus distance using $R = 6371000 \text{ m}$, $k = 1.16248157479707E - 09 \text{ m}^{-1}$ and $g_0 = 1000 \text{ ms}^{-2}$ (CASE_1)

$r, \text{ m}$	$M_{er-single}, \text{ kg}$	$M_{er-joint}, \text{ kg}$	$M_{er-loss}, \text{ kg}$	$M_{e-sum}, \text{ kg}$	<i>Asymptotic loss</i>
12742000	6.3579E+22	9.8495E+21	$4.7876E+19$	7.3477E+22	$4.5098E+19$
25484000	7.1141E+22	2.3241E+21	$1.1431E+19$	7.3477E+22	$1.1274E+19$
50968000	7.2900E+22	5.7361E+20	$2.8281E+18$	7.3477E+22	$2.8186E+18$
101936000	7.3333E+22	1.4295E+20	$7.0524E+17$	7.3477E+22	$7.0465E+17$
203872000	7.3441E+22	3.5710E+19	$1.7620E+17$	7.3477E+22	$1.7616E+17$
407744000	7.3468E+22	8.9259E+18	$4.4043E+16$	7.3477E+22	$4.4041E+16$
815488000	7.3474E+22	2.2314E+18	$1.1010E+16$	7.3477E+22	$1.1010E+16$

Table 10: Variation of various fractions of the Moon effective mass versus distance using $R = 1737000 \text{ m}$, $k = 7.02425385602087E - 10 \text{ m}^{-1}$ and $g_0 = 1000 \text{ ms}^{-2}$ (CASE_1)

“joint” terms between the two situations. This is easily found to be $(1 - \exp(-k_2OC))(1 - \exp(-k_1AB))$, with which we can write the corresponding integrations:

$$M_{ar-netloss} = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} (1 - \exp(-k_2OC))(1 - \exp(-k_1AB)) \sin \varphi_z \cos \varphi_z d\theta_z d\varphi_z \quad (209)$$

$$S_{ar-netloss} = \int_0^\pi M_{ar-netloss} \cdot 2\pi \sin \omega R_2^2 d\omega \quad (210)$$

We use the term “net loss” to refer to this absorption in an effort to distinguish it from present (current) absorption taking place at distance r , because it does not exist, i.e. it is not current, but was present when the sphere was “lone” but now this amount of graviton absorption has gone missing. This is caused by the perturbation of the universal graviton flux by the other sphere. We can then write the more general equation with these parameters as:

$$S_a \equiv S_{a-sum} = S_{ar-total} + S_{ar-netloss} = S_{ar-single} + S_{ar-joint} + S_{ar-netloss} \quad (211)$$

All this will be better understood and discussed looking at the results of the following three cases.

CASE_1 (Earth-Moon). The masses and radii used are taken from Table 3 together with their absorption coefficients from Table 2. We performed the above integrations separately for the Earth and Moon, each one being the sphere of investigation with regards to its own graviton absorption. The numerical results of the above integrals are better shown first in table form for all three factors for Earth and Moon in Tables 9 and 10. The distance is varied in multiples of Earth radius from 2 to 128 radii doubling each distance. Actually, we have converted the S_- parameters to an effective mass via the conversion Eq. 203, in order to make it more tangible with our familiar (perhaps) “mass” concept. These masses $M_{er-single}$, $M_{er-joint}$, $M_{er-loss}$ and $M_e = M_{e-sum}$ correspond to the terms of above Eq. 211.

We note that the variation of mass is small but significant within a few radii distance, but it approaches fast and asymptotically the “lone” mass, as we increase the distance; the sum of the three fractions of mass is constant with distance and equal to the lone mass, as it should. The “single” component increases, whilst the “joint” component decreases as we increase the distance, so that their sum is always much closer to the

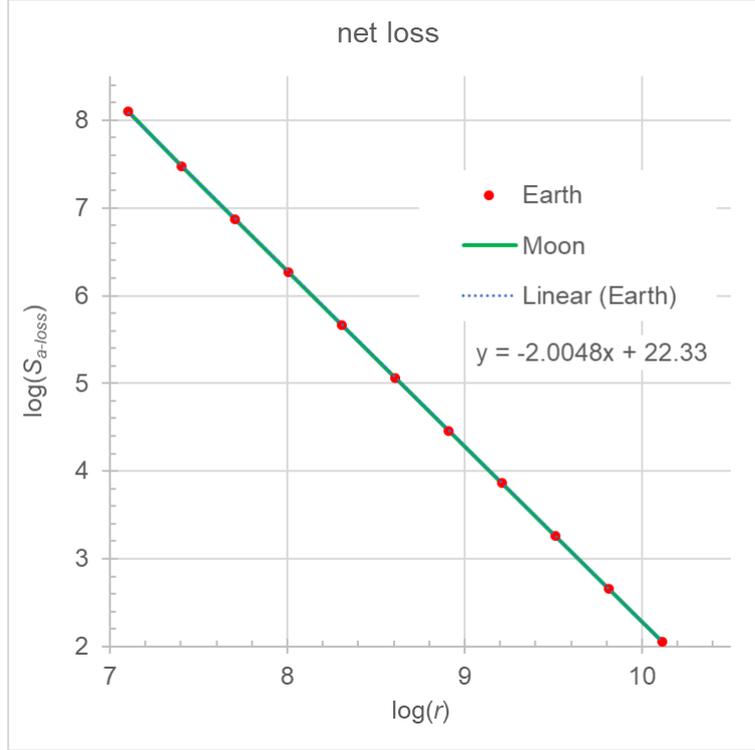


Figure 20: Absorption loss of Earth and Moon versus distance (CASE_1)

“lone” mass. We can see this by the variation of mass loss being 5 orders of magnitude for the Earth and 3 orders for the Moon at the shortest distance below the “lone” mass, whilst it becomes 8 and 6 orders of magnitude at 128 Earth radii. We further note that the net loss is the same for the Moon and Earth, as should be expected on account of the symmetry in the chords of the net loss factor . We investigate the mass loss also in graph form in Fig. 20 but by plotting the net loss of absorption factor $S_{a-netloss}$ on logarithmic scales for both axes. We have also added an extra four point up to 2048 radii distance to assist the look of a trendline. The fitted straight line ($y = -2.0048x + 22.33$) is indistinguishable from the computed curve indicating a strong inverse square distance relationship. We suspected that this overlap may be only due to a Newtonian approximation for this case; the absorptivity for Earth being $A_{R-earth} = 0.00982$ and for Moon $A_{R-moon} = 0.00162$ with $g_0 = 1000 \text{ ms}^{-2}$ clearly place the case very early on the A_R graph in Fig. 3. This prompted us to repeat the same investigation for other denser and/or more extensive spheres below.

CASE_2 (Small-and-large-dense-spheres). An extreme case at the other end of Newtonian regime is to use $A_R = 0.99$ (near saturation absorption). We can do this using the Earth radius R_2 and corresponding $k_2 = 1.10987744324188E - 06 \text{ m}^{-1}$. We can pack a large amount of effective mass inside the given radius depending on the g_0 that we can choose according to Eq. 203. For a near white dwarf situation, we chose $g_0 = 10^5 \text{ ms}^{-2}$. For the second interacting sphere, we chose a very small one with radius $R_1 = 1 \text{ m}$ but with the same $A_R = 0.99$ corresponding to $k_1 = 7.07102919089469E + 00 \text{ m}^{-1}$. The results for each sphere are given in Tables 11 and 12. Then likewise, we plot the net loss factor $S_{a-netloss}$ in Fig. 21.

Now, we can make the following observations: The mass losses for the large gravitating sphere are practically (relatively) negligible being 14 orders of magnitude below that of the lone mass, as is also the “joint” fraction of mass, whilst the “single” fraction is practically equal to the lone (not enough decimal places are provided in the limited width of the table). However, the joint and net loss masses of the small sphere are of the same order of magnitude as the lone, whilst the single mass is a few orders of magnitude less. This was to be expected, since the small sphere, when brought only 2 m above the surface of the large sphere, is practically shielded from about half the gravions from the direction of the large sphere. The unforeseen behavior is that of the net loss, which is better shown in graph form in the provided figure. The curve looks for the most part close to a straight line especially at long distance, but it significantly deviates from the straight line at close range. Further and most importantly, we have now found that the curve becomes asymptotic with a special straight line (on log-log scales) given by:

$r, \text{ m}$	$M_{er-single}, \text{ kg}$	$M_{er-joint}, \text{ kg}$	$M_{er-loss}, \text{ kg}$	$M_{e-sum}, \text{ kg}$	<i>Asymptotic loss</i>
6371002.0	6.0209E+28	7.0314E+14	<i>6.8921E+14</i>	6.0209E+28	<i>3.6713E+14</i>
6.3711E+06	6.0209E+28	7.0065E+14	<i>6.8887E+14</i>	6.0209E+28	<i>3.6712E+14</i>
6.3716E+06	6.0209E+28	6.8829E+14	<i>6.8678E+14</i>	6.0209E+28	<i>3.6706E+14</i>
6377371.0	6.0209E+28	6.3098E+14	<i>6.7466E+14</i>	6.0209E+28	<i>3.6640E+14</i>
7008100.0	6.0209E+28	4.3435E+14	<i>4.2575E+14</i>	6.0209E+28	<i>3.0341E+14</i>
7645200.0	6.0209E+28	3.3379E+14	<i>3.2718E+14</i>	6.0209E+28	<i>2.5495E+14</i>
8919400.0	6.0209E+28	2.2439E+14	<i>2.1995E+14</i>	6.0209E+28	<i>1.8731E+14</i>
12742000.0	6.0209E+28	1.0029E+14	<i>9.8300E+13</i>	6.0209E+28	<i>9.1782E+13</i>
25484000.0	6.0209E+28	2.3783E+13	<i>2.3312E+13</i>	6.0209E+28	<i>2.2946E+13</i>
50968000.0	6.0209E+28	5.8751E+12	<i>5.7588E+12</i>	6.0209E+28	<i>5.7364E+12</i>
101936000.0	6.0209E+28	1.4645E+12	<i>1.4355E+12</i>	6.0209E+28	<i>1.4341E+12</i>
203872000.0	6.0209E+28	3.6586E+11	<i>3.5861E+11</i>	6.0209E+28	<i>3.5852E+11</i>
407744000.0	6.0209E+28	9.1447E+10	<i>8.9637E+10</i>	6.0209E+28	<i>8.9631E+10</i>
815488000.0	6.0209E+28	2.2861E+10	<i>2.2408E+10</i>	6.0209E+28	<i>2.2408E+10</i>

Table 11: Variation of various fractions of the effective mass of the large dense sphere versus distance, i.e with $R_2 = 6371000 \text{ m}$ and $k_2 = 1.10987744324188E - 06 \text{ m}^{-1}$ (CASE_2)

$r, \text{ m}$	$M_{er-single}, \text{ kg}$	$M_{er-joint}, \text{ kg}$	$M_{er-loss}, \text{ kg}$	$M_{e-sum}, \text{ kg}$	<i>Asymptotic loss</i>
6371002.0	1.1659E+12	7.9297E+14	<i>6.8921E+14</i>	1.4834E+15	<i>3.6713E+14</i>
6371063.7	6.6336E+12	7.8784E+14	<i>6.8887E+14</i>	1.4834E+15	<i>3.6712E+14</i>
6371637.1	2.0976E+13	7.7560E+14	<i>6.8678E+14</i>	1.4834E+15	<i>3.6706E+14</i>
6377371.0	6.6288E+13	7.4240E+14	<i>6.7466E+14</i>	1.4834E+15	<i>3.6640E+14</i>
6434710.0	2.0822E+14	6.5665E+14	<i>6.1848E+14</i>	1.4834E+15	<i>3.5990E+14</i>
7008100.0	6.1796E+14	4.3964E+14	<i>4.2575E+14</i>	1.4834E+15	<i>3.0341E+14</i>
7645200.0	8.1995E+14	3.3622E+14	<i>3.2718E+14</i>	1.4834E+15	<i>2.5495E+14</i>
8919400.0	1.0381E+15	2.2527E+14	<i>2.1995E+14</i>	1.4834E+15	<i>1.8731E+14</i>
9556500.0	1.1056E+15	1.9105E+14	<i>1.8668E+14</i>	1.4834E+15	<i>1.6317E+14</i>
12742000.0	1.2846E+15	1.0043E+14	<i>9.8300E+13</i>	1.4834E+15	<i>9.1782E+13</i>
25484000.0	1.4362E+15	2.3790E+13	<i>2.3312E+13</i>	1.4834E+15	<i>2.2946E+13</i>
50968000.0	1.4717E+15	5.8755E+12	<i>5.7588E+12</i>	1.4834E+15	<i>5.7364E+12</i>
101936000.0	1.4805E+15	1.4645E+12	<i>1.4355E+12</i>	1.4834E+15	<i>1.4341E+12</i>
203872000.0	1.4826E+15	3.6586E+11	<i>3.5861E+11</i>	1.4834E+15	<i>3.5852E+11</i>
407744000.0	1.4832E+15	9.1448E+10	<i>8.9637E+10</i>	1.4834E+15	<i>8.9631E+10</i>
815488000.0	1.4833E+15	2.2861E+10	<i>2.2408E+10</i>	1.4834E+15	<i>2.2408E+10</i>

Table 12: Variation of various fractions of the effective mass of the small dense sphere versus distance, i.e with $R_1 = 1 \text{ m}$ and $k_1 = 7.07102919089469E+00 \text{ m}^{-1}$ (CASE_2)

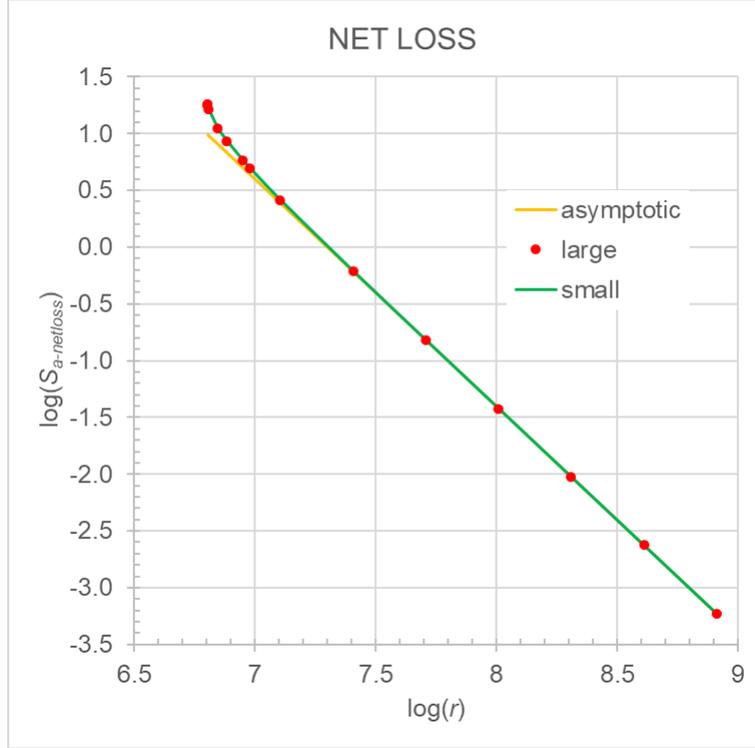


Figure 21: Net loss of small and large dense spheres with asymptotic loss versus distance (CASE_2)

$r, \text{ m}$	$M_{er-single}, \text{ kg}$	$M_{er-joint}, \text{ kg}$	$M_{er-loss}, \text{ kg}$	$M_{e-sum}, \text{ kg}$	<i>Asymptotic loss</i>
12742000	6.0010E+28	1.0051E+26	$9.8516E+25$	6.0209E+28	$9.1782E+25$
25484000	6.0162E+28	2.3793E+25	$2.3322E+25$	6.0209E+28	$2.2946E+25$
50968000	6.0197E+28	5.8757E+24	$5.7593E+24$	6.0209E+28	$5.7364E+24$
101936000	6.0206E+28	1.4645E+24	$1.4355E+24$	6.0209E+28	$1.4341E+24$
203872000	6.0208E+28	3.6586E+23	$3.5861E+23$	6.0209E+28	$3.5852E+23$
407744000	6.0208E+28	9.1448E+22	$8.9637E+22$	6.0209E+28	$8.9631E+22$
815488000	6.0209E+28	2.2861E+22	$2.2408E+22$	6.0209E+28	$2.2408E+22$

Table 13: Variation of various fractions of the effective mass of the bigger dense sphere versus distance with $R_2 = 6371000 \text{ m}$ and $k_2 = 1.10987744324188E - 06 \text{ m}^{-1}$ (CASE_3)

$$S_{F0} \equiv \frac{\pi A_1 \pi A_2}{r^2} = \frac{\pi^2 A_{R_1} A_{R_2} R_1^2 R_2^2}{r^2} \quad (212)$$

which is identical with the factor in deriving the force between the two spheres in Eq. 85. This is an extraordinary novel finding in PG, which we discuss and analyze after we tabulate the corresponding results of an additional extreme case below.

CASE_3 (Two-large-and-dense-spheres). We again choose a near saturation regime with $A_R = 0.99$, but with both spheres being extensive, i.e. with $R_1 = 1000000 \text{ m}$, $R_2 = 6371000 \text{ m}$ and corresponding $k_1 = 7.07102919089467E-06 \text{ m}^{-1}$, $k_2 = 1.10987744324188E - 06 \text{ m}^{-1}$. The results are given in Tables 13 and 14. Also, the net loss is plotted in Fig. 22.

Due to the chosen radii, we can't have the situation of a very close range equivalent to that of case_2, so that the asymptotic in case_3 is hardly distinguishable on the plot. However, we can compare at the same short distance of $r = 2R_2$. We note that the deviation of the mass loss from the asymptotic line at $r = 2R_2$ distance is about the same in all three cases, namely, the ratio of $M_{e-loss}/M_{e-asymptotic}$ is 1.062, 1.070 and 1.073 as can be found from the corresponding tables. That is, the variation of mass loss is nearly inversely proportional to the square of distance almost regardless of density, whilst it is the distance, or better the geometry that is the main controlling parameter.

$r, \text{ m}$	$M_{er-single}, \text{ kg}$	$M_{er-joint}, \text{ kg}$	$M_{er-loss}, \text{ kg}$	$M_{e-sum}, \text{ kg}$	<i>Asymptotic loss</i>
1.2742E+07	1.2842E+27	1.0065E+26	<i>9.8516E+25</i>	1.4834E+27	<i>9.1782E+25</i>
2.5484E+07	1.4362E+27	2.3800E+25	<i>2.3322E+25</i>	1.4834E+27	<i>2.2946E+25</i>
5.0968E+07	1.4717E+27	5.8761E+24	<i>5.7593E+24</i>	1.4834E+27	<i>5.7364E+24</i>
1.0194E+08	1.4805E+27	1.4646E+24	<i>1.4355E+24</i>	1.4834E+27	<i>1.4341E+24</i>
2.0387E+08	1.4826E+27	3.6586E+23	<i>3.5861E+23</i>	1.4834E+27	<i>3.5852E+23</i>
4.0774E+08	1.4832E+27	9.1448E+22	<i>8.9637E+22</i>	1.4834E+27	<i>8.9631E+22</i>
8.1549E+08	1.4833E+27	2.2861E+22	<i>2.2408E+22</i>	1.4834E+27	<i>2.2408E+22</i>

Table 14: Variation of various fractions of the effective mass of the smaller dense sphere versus distance with $R_1 = 1000000 \text{ m}$ and $k_1 = 7.07102919089467\text{E-}06 \text{ m}^{-1}$ (CASE_3)

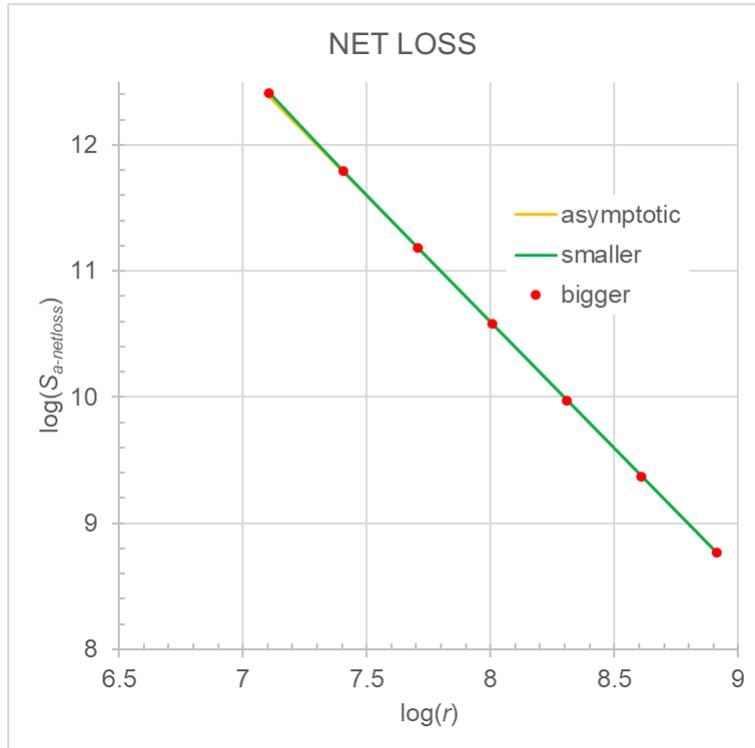


Figure 22: Net loss of two large (smaller, bigger) and dense spheres with asymptotic loss versus distance (CASE_3).

	CASE_2	CASE_3
r	S_F/S_{F0}	S_F/S_{F0}
6371002.0	0.9999999942	
6377373.0	1.0000000084	
6434712.0	1.0000000068	
7008102.2	1.0000000035	
8919402.8	1.0000000030	
9556500.0	1.0000000029	
12742000.0	0.9999999747	0.9999999726 0.99999999999990
31855000.0	1.0000000023	
63710000.0	1.0000000021	1.0000000021
318550000.0	1.0000000020	1.0000000020
637100000.0	1.0000000020	1.0000000020

Table 15: Computed S_F over theoretical S_{F0} force factor ratio versus distance for CASE_2 and CASE_3; second value with decreased tolerance at $r = 12742000.0$ m

We recall that the gravitational force in Eq. 85 was derived by a simple “reverse engineering approach” from the associated single (or lone) effective masses of each sphere, to which we attached their characteristic absorptivity. We then confirmed the same outcome from the integration of equations constituted from first PG principles. We have now further confirmed the same outcome with an alternative approach involving three integrations (the “surface” method), which allowed much improved precision with much shorter computation times including the cases of high absorption with A_R values close to unity. Thus, if we normalize the computed force factor S_F over the theoretical (reverse engineering) force factor denoted by S_{F0} , we invariably find unity as a function of distance, as can be seen by some representative results now shown in Table 15. The deviation from unity beyond 9 decimal places is caused by the set relative integration tolerance of errors, namely, of $eps_{rel} = E - 06$, $E - 05$ and $E - 04$ for the first, second and third integral. By decreasing the tolerance by four orders of magnitude in each integral, we obtain $S_F/S_{F0} = 0.99999999999990$ (at $r = 12742000.0$, see table).

Now, the variation of effective mass with distance, i.e. a decrease of mass with decrease of distance, means that Newton’s gravitational law, if applied using the effective masses, yields a smaller value of force than if the masses were independent of distance as is assumed by Newton. PG finds exactly the latter, i.e. the force behaves as if the masses were invariant without actually being so! This is a novel understanding, unexpected by Newton. It is a consequence of the underlying mathematical (better physical) relationship among the PG parameters involved via the gravion absorption.

The above difference between PG and Newton gravitational laws is fundamental, although they can be both written by a similar equation. They differ in the concept of mass, which is not trivial. Newton does not distinguish the “lone” mass from the mass acted upon by another mass. Thus, we can now distinguish between the two laws as follows:

If we use the actual masses (i.e. acting or effective) then Newton’s force should be:

$$F_{Newton} = G \frac{M_{e1r} M_{e2r}}{r^2} = G \frac{(M_{e1r-single} + M_{e1r-joint})(M_{e2r-single} + M_{e2r-joint})}{r^2} \quad (213)$$

whereas PG law using the far away lone masses (see also 85) is:

$$F_{PG} = G \frac{M_{e1} M_{e2}}{r^2} \quad (214)$$

The ratio of the above forces can be written also as a ratio involving the S_- parameters via the proportionality Eq. 202:

$$\frac{F_{Newton}}{F_{PG}} = \frac{S_{a1r} S_{a2r}}{S_{a1} S_{a2}} = \frac{(S_{a1r-single} + S_{a1r-joint})(S_{a2r-single} + S_{a2r-joint})}{S_{a1} S_{a2}} \quad (215)$$

We illustrate the difference between Newton and PG in Fig. 23 by plotting the above ratio (normalizing) as a function of distance for the same three cases. The importance of this difference can now be better appreciated by considering the involvement of the mathematical derivations dictated by PG and presented

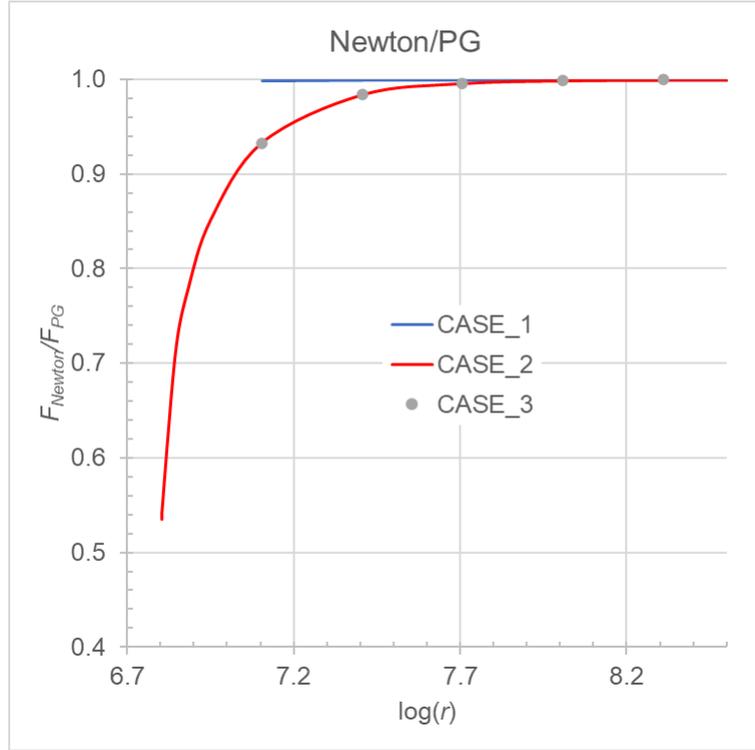


Figure 23: Ratio of Newton/PG forces against distance for three cases of density and size of spherical masses

in the Appendix. The coincidence in mathematical form is uncanny, except that Newton’s gravitational law is both empirical and approximate, whereas PG gravitational law is derivable and precise.

We see that case_1 (Earth-Moon) is very close to Newtonian behavior, but not so when the absorptivity A_R increases. Has conventional physics found how very dense bodies interact at close range, or only now PG reveals exactly that behavior? The present findings can afford us a deeper understanding of the notions of mass and force. We note that the initial integrand during the derivation of force and net loss have an identical factor apart from the trigonometric factors used to calculate the directional (projection of) flow of gravions. This applies with both methods, bulk or surface (see Appendix). There is clearly a common denominator under the idea of force and mass. The common denominator is the push gravions. Whereas the effective mass is made from gravions being absorbed from all possible directions, force is made from the net directional absorption of gravions between the centers of spheres, or presumably between the “centers of gravity” of any two bodies in a general case (to be shown in later PG development). In other words, effective mass and force have a common cause, namely, the rate of gravion absorption. Mass is the omnidirectional gravion absorption resulting in a scalar quantity, whilst force is the component of gravion absorption in a particular direction defined by two bodies at a distance apart, resulting to a vector quantity.

If PG can be experimentally confirmed and if Fig. 23 correctly reveals the deviation of Newtonian mechanics from actual physical processes, then EP, the equivalence principle emanating from Newtonian experience and carried over to GR, ceases to apply. That means that EP is not only redundant as previously suggested, but it may be an invalid one. We may have to reassess our own stance vis-a-vis EP (see Section 15.2), i.e. we may have to go from a defensive argument to a critical one against it. Here, it is important to clarify that the critics of PG citing the equivalence principle as reason for rejection of PG have presumably done so for different reasons, because they did not have our development of PG in front of them. In the light of the present findings, we have to say that PG seems to contradict the EP, but this is no reason to oppose PG. Conversely, it may be a reason to reassess the theories based on EP. It may be the reason why current theories have unsolved fundamental problems. Then, use of EP as basis for rejection of PG would be arbitrary. It may eventually be that EP is the pivot point between GR and PG, i.e. which direction to take. This can only be arbitrated by a proper experimental assessment of PG, whilst all hitherto tests of EP may have been simply inadequate.

Should we then realign the conventional meaning of “intrinsic” matter? Should this be the effective mass of a body away from other material bodies, because the effective mass varies with the distance from other

bodies, albeit imperceptibly in Newtonian mechanics? This is clearly explained under the platform of PG. We distinguish the three components of effective mass, namely, “joint”, “single” and “loss” mass. The sum of these three components is a constant (at the prevailing J_0 , or g_0 or Λ) equal to the isolated lone body’s mass, which we may now call “intrinsic” mass, or use another term to describe the new understanding (sooner or later, we will have to deal with the question of terminology in a more consistent and comprehensive way, but we can await for more results, especially a confirmation of PG). The finding that the force is derived from the product of such intrinsic masses, whilst they are actually smaller at a given distance, could be the key for explaining why the equivalence principle has imposed itself for such a long time. It provides a fortuitous condition that yields correct answers for things that don’t really exist. It assumes masses that don’t exist, but their assumed value yields correct outputs such as the gravitational law. It may be that PG for the first time deciphers a trick that nature has played on science.

It should be appreciated that throughout this report we have loosely used the notions of mass, force and energy, which is an unavoidable situation, if we have to evolve from our standard education in physics; not only pedagogical but fundamental errors may creep in science for these and other concepts. Like GR redefines these concepts in terms of space-time, etc., PG now also seems to afford us an alternative understanding of the same concepts, as they evolve from an assumed universal graviton absorption. Only by persistence in developing and evolving PG theory backed by purposeful experiments, may we ultimately understand and re-define the concepts of energy, mass and force. We have started with a set of principles, which themselves will evolve dialectically (back-and-forth) as we move forward and the pig picture of cosmos unravels more clearly.

The new understanding of mass and force can further lead to an ultimate understanding of energy in the form of work ($force \times distance$), potential and kinetic energy and the $E = mc^2$ relationship. An initial check does not equate the potential energy with the mass-energy equivalence conversion, but this should be of no concern. We should bear in mind that a system of bodies acted upon by their associated gravitational fields does not constitute a closed system. The bodies are inside an “aether” of gravitons coupled with them. They constitute an open thermodynamic system that exchanges gravitons with the surrounding universal reservoir of gravitons. The bodies must always be seen together with the surrounding universe acting from both short and long range. This universe is intimately connected with any system of bodies, small or big, at any moment of time and space, “here and now” (whatever this may mean!). The interconnection is such that the body resists any change of its steady-state status. This resistance is the conventional “inertia”. Therefore, when the gravitational “pulling” force is doing “work” by displacing a body between two points, the interchange between work and gravitational energy must be computed as a balance of graviton exchange with the surrounding universe. An analogous system in physics is an electronics signal (current or power) amplifier, whereby a given signal input is amplified in conjunction with an external supply circuit of electrons (energy).

Actually, the new notion of mass variation inside the gravitational field of interacting bodies is not limited to dense and massive bodies in astrophysics, but it must apply also to very small but dense particles. This seems to be consistent with particle physics phenomena. Let’s recall that within a given radius, there is an upper limit of effective mass that we can pack. This upper limit is determined by the surrounding maximum J_0 (or g_0 , see also Eqs. 199 and 202). If we can think of a particle in an accelerator that it sweeps more push particles at more speed, it is like creating an artificially increased g_0 enabling the particle to pack more effective mass over and above the maximum it might have prior to its entry in the accelerator. When the particle “stops”, it cannot retain the extra mass above the limit it acquired during its acceleration. Then, it has to shed this “unnatural” load of mass in the form of new particles inside the accelerator. Those new particles themselves usually have very short lives, if they still have to readjust and obey the PG law of an upper limit of effective mass. As it was also theorized in Section 14.7, the approach to an upper speed limit is not because the mass becomes mathematically infinite, but because the real mass becomes saturated with graviton absorption as A_R approaches the upper value of unity.

If the above understanding could prove correct, it would have enormous consequences. While more cases have been studied with PG formulations in the Appendix, they and many more can be the subject of continuing work by the scientific community under the new physics of PG. The case of three or more interacting spheres can be investigated by expanding the work presented in the Appendix; then, we can study the eclipses of Moon and Sun over and above the simplified study outlined in Section 12.4. All this requires more resources, but also makes it ever more imperative that PG should be verified by the relevant organizations.

17 Momentum or push gravity as the universal and unifying cause of all types of acceleration (force)

If gravity is finally proven to be caused by gravions under the working of PG, then it could be a logical conclusion that all forces may be attributable to a similar cause, albeit by different kind of push particles. What would then be needed is that each kind of particles have a mean free path much longer than the dimensions of the masses (particles) acted upon. This requirement is already fulfilled for planets and stars by gravions, to which we may also refer as the first type-I push particles. The force is then generated by the law of conservation of momentum and energy. This momentum force is well established in physics as it is also a tangible phenomenon, i.e. understood by common experience. We may then extend the proposal to apply to all kinds of force fields regardless of the size of the field generating body. This may be a sensible proposition, because size should not be an obstacle at least for all experimentally known particles. Given that the size of an atom is of the order of $\approx 10^{-9}$ m, we still have 16 orders of magnitude to reach the Planck particle, the length ℓ_P of which is defined as:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad (216)$$

where \hbar is the reduced Planck constant. The Planck length is about 10^{-20} times the diameter of a proton.

Thus, nucleons may be maintained by their own surrounding push (momentum) particles, the nucleus may be maintained by yet another kind of push particles, and so on for atoms, each group maintained by their own associated type of momentum particles. The universal situation may be that the space is filled by particles with a wide range of sizes (energies/wavelengths) corresponding to an equally wide range of mean free paths, acting as push particles to their matching (corresponding) relatively much bigger particles (bodies). The entire universe may then be thought as an agglomeration of varying concentrations of matter automatically sorting out themselves by the surrounding push particles. This proposition may then constitute a likely basis for a unification of all force fields in the cosmos from the smallest to the largest phenomena. Le Sage made a similar attempt to account for forces of different chemical strengths, by the existence of different species of "ultramundane" corpuscles of different sizes, whilst all this should be reconsidered and re-appraised in the light of modern particle physics, quantum mechanics, relativity and astrophysics.

Ultimately and inexorably, however, the above model only shifts the problem to what keeps the "ultimate" mysterious particle as a unity (re gravion), if not for an attractive force, according to Kant's philosophical reasoning. However, the lack of understanding of the nature of an ultimate particle is not yet reason good enough to reject a possible unifying model that allows us to concentrate our attention more to a smaller "area" of the cosmos that underlies as a common denominator to all other processes.

From the above broad model, we may narrow down the cosmological questions to assuming the existence of types of particles corresponding at least to the known force fields. Thus, gravions are type-I push particles that mediate the gravitational force, type-II push particles are those mediating the electrical field forces, type-III those mediating the nuclear forces, etc. Already, in quantum theory the electric field is thought to be due to a continuous stream of exchange of photons (say, here type-II particles). Dibrov (2011) believes that the core of electrons and positrons remains stable by pressure of the bombardment of "fations". The electron, in his proposed model, "*as against the static Abraham-Lorentz electron, is the dynamic object transforming the gravitational field energy into the energy of the electric field, and periodically exploding up.*" However, he probably means something very different to our proposed model in this report, because he talks about charge already being present in the electron, and he only tries to justify the re-emission of the "fation" energy in the form of electric field sub-particles. Considering various parameters quantitatively and his main conclusions, it is clear that his theory is not consistent with our findings. For example, "the active mass is not equal to the passive mass", he discovers a "violation of equivalence principle for the electron" and that the "gravitation constant G is not equal to the actual one", to mention a few aspects of his push gravity theory that are in clear variance to the ideas proposed herewith. Nevertheless, an "exploding electron" seems consistent with re-emission of absorbed gravion energy.

We may go on to elaborate on our general proposal (model). For example, the type-II particles, in particular, may be subdivided either in two sub-classes responsible for the positive and negative electrical force (such as opposite spin or an as yet unknown attribute), or may emanate from two complementary types of matter organization at the electron and positron level. The emanated energy (type-II) carriers exit as a result of the absorbed type-I push particles by protons and electrons, and so on and so forth.

In summary, for a field unification theory, it is logical and consistent to envisage and assume that all force fields are created by particles including gravity. This general idea of the underlying particles for all fields is then greatly facilitated by a push/momentum mechanism in a PG framework advanced in this report. A

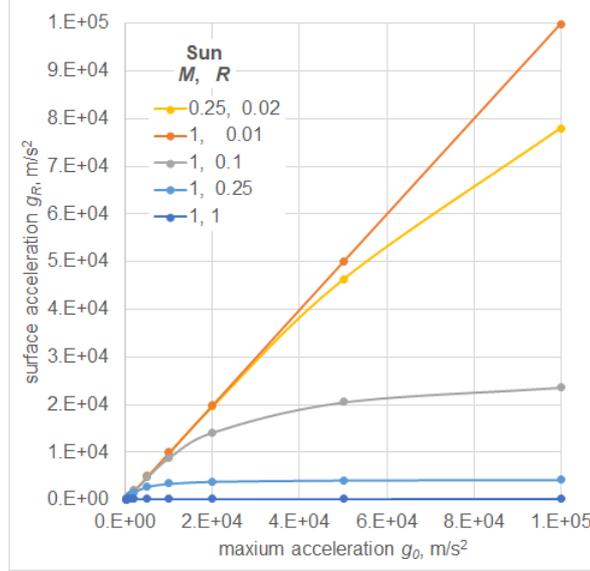


Figure 24: *Surface g_R against maximum g_0 for stars with mass and radius (M, R) in units of Sun mass and Sun radius.*

general push particle field principle may then be seen as one kind of self-similarity as used in fractal theory. Could self-similarity be used to recreate a new standard model in physics?

18 White dwarfs, neutron stars and black holes

It is reported that the gravitational field on a white dwarf is of the order of 10^6 m/s², whilst that on a neutron star is of the order of 10^{12} m/s², and much greater on black holes. If these extreme accelerations are caused by gravions (in that case being the universal cause of all gravitational fields), then it might be unlikely that we can practically detect them directly by the methods proposed here, because of the need for extremely sensitive gravimeters. However, if it were found that the maximum g_0 is, say, around 30000 m/s² by some careful measurement, then we would be faced to explain the super high values of acceleration on white dwarfs, neutron stars and black holes. Then, one possible explanation would be to assume that those extreme accelerations may be caused by different, more energetic types of push particles. Before we further speculate on these other hypothetical types of push particle, let us apply a little further the already found PG relationships below.

We continue our investigation from where we left off in Section 9.1. By increasing the maximum prevailing acceleration g_0 in the neighboring universe, we inversely decrease the corresponding k (see Eq. 60) by

$$k = \frac{\pi G \rho}{g_0} = \frac{3GM}{4g_0 R^3} \quad (217)$$

so that by keeping the mass and radius of a star constant, the PG equation is reduced to the value provided by Newton, namely, to $\frac{4}{3}\pi G \rho R = GM/R^2$, which is the saturation (asymptotic) value(s) observed in Fig. 10, when k becomes sufficiently small.

Let us now see the values of surface acceleration g_R against the prevailing maximum acceleration g_0 possible in a particular space of the universe for stars having various combinations of masses M with radii R . This is shown in Fig. 24, where both mass and radius are expressed in units of Sun (\odot) mass and radius. The masses used are those of the apparent Sun mass but taken to be the real mass of a hypothetical star as a first approximation to get a feel of the situation. Then as expected, the pair $(M, R)=(1, 1)$ reproduces the curve in Fig. 10 very close to the abscissa (visibly touching it) with an asymptotic value approaching the Newtonian value of surface acceleration of our Sun. The additional curves now show the outcomes of different values of the pair (M, R) , which can be understood by the above Eq. 217: For any fixed, g_0 and M , the value of k increases very fast with a decrease of radius, which forces the surface acceleration to be well below the saturation values is reached, as noted by the curves on the figure. When R reduces below a sufficiently low value, k becomes so large that the factor A_R in PG becomes unity and $g = g_0$, which is the straight line at unity slope in the figure.

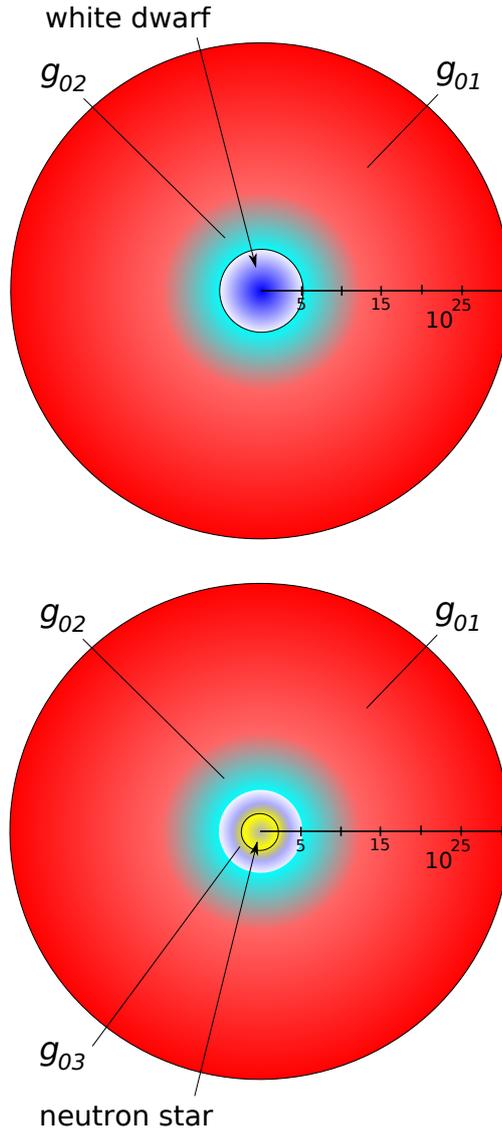


Figure 25: *Diagrammatic perception of a white dwarf (above) with its surrounding PG type-II (g_{02}) field inside the universal type-I (g_{01}) field, and a neutron star (below) with its surrounding PG type-III field (g_{03}) inside a dwarf type-II (g_{02}) field inside the universal type-I (g_{01}) field; the scales in m are logarithmic and approximate.*

In the event that we can safely measure g_0 and find that this is not as high as on the surface of a dwarf, however, it may be sufficient to trigger gravitational collapse in the presence of a critical mass. After the collapse, a white dwarf is formed that may be sustained also by push particles of a different kind (type-II). Likewise, upon formation of a neutron star, the forces holding it together may further be provided by push particles of a third kind (type-III) as they evolve upon the onset of a further collapse. This proposal forms initially a qualitative model, which is depicted with some hypothetical quantitative dimensions in logarithmic scales (powers of 10) in Fig. 25. The validity of such a hypothesis should by all means be cross-examined against existing data and theories in astrophysics to be further refined or even rejected, if not appropriate.

For the general reader and to better describe the proposed model here, it is helpful to summarize the current understanding of these dense bodies by conveniently referring to a brief description provided in relevant articles by Wikipedia contributors (2019f). The summary descriptions below are needed to precede a new idea here attempting to connect the neutron star field with the atomic nuclear field, both unified under the proposed PG field model.

A white dwarf is a very dense stellar core remnant composed mainly of electron-degenerate matter. It has a mass like the Sun with a volume like the Earth. Because it no longer undergoes fusion reactions, it has no source of energy, so that it cannot support itself by fusion heat against gravitational collapse. It is supported by electron degeneracy pressure and is extremely dense. Accretion takes place by accumulating particles into

a massive object, typically gaseous matter. Galaxies, stars, and planets, are formed by accretion processes. Neutrinos are radiated by white dwarfs through the Urca process (Wikipedia contributors, 2019e), which is a neutrino-emitting process playing a central role in the cooling of neutron stars.... White dwarfs have masses from about 0.07 to 10 M_{\odot} .

An astronomical body can collapse by its own gravity drawing matter inward toward its center. Gravitational collapse is a fundamental mechanism for structure formation in the universe. It can all start from relatively smooth distribution of matter gradually collapsing to form pockets of higher density, stars and planets, stellar groups and clusters of galaxies.

A giant star with a total of between 10 and 29 solar masses collapses to a neutron star (Wikipedia contributors, 2019b). Other than black holes, neutron stars are the smallest and densest stars with a radius on the order of 10 km and a mass less than 2.16 solar masses. They are produced from the supernova explosion of a massive star, and together with gravitational collapse achieve the density of atomic nuclei.

Binary systems of neutron stars can undergo accretion making the system bright in X-rays and a source of short-duration gamma-ray bursts, as well as produce gravitational disturbance. At soaring temperatures, electrons and protons combine to form neutrons via electron capture, releasing a flood of neutrinos. It is important for our model proposed here to quote verbatim from Wikipedia the following: *“When densities reach nuclear density of 4×10^{17} kg/m³, neutron degeneracy pressure halts the contraction. The in-falling outer envelope of the star is halted and flung outwards by a flux of neutrinos produced in the creation of the neutrons, becoming a supernova. The remnant left is a neutron star. If the remnant has a mass greater than about 3 M_{\odot} , it collapses further to become a black hole”*.

The temperature inside a newly formed neutron star is from around 10^{11} to 10^{12} Kelvin. However, the huge number of neutrinos it emits carry away so much energy that the temperature of an isolated neutron star falls within a few years to around 10^6 kelvin. At this lower temperature, most of the light generated by a neutron star is in X-rays.

A neutron star has some of the properties of an atomic nucleus, including density (within an order of magnitude) and being composed of nucleons. In popular scientific writing, neutron stars are therefore sometimes described as ‘giant nuclei’. However, in other respects, neutron stars and atomic nuclei are quite different. A nucleus is held together by the strong interaction, whereas a neutron star is held together by gravity. The density of a nucleus is uniform, while neutron stars are predicted to consist of multiple layers with varying compositions and densities.”

It is the above last statement that we can seize upon to support the PG model here, namely, we say here **that a nucleus and a neutron star are both held by the same force: That force is the pressure exerted by the presumed type-III push particles.** We propose that the strong nuclear interaction is no different from the neutron star gravity, namely, both being created by push particles of the same type. In consequence of this model, the space around any atomic nucleus inside the electron orbitals is occupied by push particles holding the nucleus together. The current understanding is that this space seems relatively more "empty" than the interplanetary space, so there is nothing weird about our hypothesis that push particles small enough occupy this space fulfilling the requirements of PG with regard to mean free path and absorption coefficient of the nucleus. There is plenty of "room" for such superfine particles on the scale all way down to the Plank length. It may turn out that these push particles are x-rays and gamma-rays of sufficiently short wavelength, which would be consistent with the strong x-ray emission by neutron stars. That may also provide the existence/mechanism of x-ray emission by the orbital electrons in atoms adjusting to different energy levels, as well as somehow explain the original mystery of stable electron orbitals of accelerating orbital charges. Thus, the atom is a micro-neutron star created from (after) breaking down a neutron star. We might want to call the corresponding type-III push particles *neutrions* (neutron + $\iota\acute{o}\nu$) in analogy to gravions. However, in proposing this model, it may not be clear how to differentiate between a nucleus and a neutron particle, so that we may have to refine the various distinctions of push particles mediating strong and weak interactions and all other sub-nuclear forces. The proposed model is only a general approach towards a unification of fields, which requires the cooperation of particle physics and astrophysics.

The above proposed scheme for neutron stars and atoms may not be acting alone, as it requires the simultaneous cooperation of a type-II push particles holding electrons and nuclei together in the atom. In a similar fashion, white dwarfs are the plasma state by free electrons and nuclei having released their mediating binding (type-II) particles around the white dwarf. These mediating particles responsible for the appearance of electric field might be called *electrions* (from electricity + $\iota\acute{o}\nu$). “Neutrions” and “electrions” are finally redistributed after explosion to form atoms.

At any rate, the above general model could be described in more specific terms of particle physics such as: Gluons participate in the strong interaction in addition to mediating it. This is unlike the photons, which mediate the electromagnetic interaction but lacks an electric charge. Gluons also share this property

of being confined within hadrons. One consequence is that gluons are not directly involved in the nuclear forces between hadrons. The force mediators for these are other hadrons called mesons. Although in the normal phase of QCD single gluons may not travel freely, it is predicted that there exist hadrons that are formed entirely of gluons — called glueballs. There are also conjectures about other exotic hadrons, in which real gluons (as opposed to virtual ones found in ordinary hadrons) would be primary constituents.

The above intermittent extracts from established theories and observations from astrophysics and particle physics serve only to stimulate further discussion, one way or another, that could involve the push theory principle consistent with the findings of this report.

In continuation to the proposed “aether of gravions” in Section 14.1 and generalized push particles in Section 17, it is envisaged that other types of push particles finally “leak out” into space together with gravions achieving some steady state concentration in various regions of space. Whilst their concentration is highest around their associated specific fields, which they mediate, like around white dwarfs, neutron stars and black holes, and various atomic and nuclear fields, the “aether” is a “soup” of various types of extremely fine particles. The “electrions” after mediating the electric field in matter, they leak out of bodies and fill the space. Thus, they might mediate also the propagation of photons in space, which is not an absolute vacuum, whilst they are also entrained by bodies. Electrions are also “energetic” particles, like gravions activating (or mediating) corresponding physical processes in electricity; they permeate not only space but also matter. Electrions seem to be responsible for charge and electricity, in a similar fashion to gravions being responsible for creating effective (active) mass, i.e. the conventional mass we are familiar with. Nikola Tesla, forgotten genius of electricity and the man who invented the twentieth century (Lomas, 1999), may have already envisaged the medium for the electric field and the propagation of lightning, when he was suggesting that we swim inside an inexhaustible source of energy.

18.1 More on mass, energy and black holes

We now continue from where we left off in Section 14.6 canvassing the possibility that black holes [consist mainly of passive mass per Eq. 139](#).

From the preceding discussion on massive bodies in general, we have proposed the possibility that these bodies are surrounded by different layers of different fields generated by the corresponding different types of push particles. For the dynamics of such bodies (kinematics and kinetics) in push gravity, it is important to consider the distance between any two given bodies. For long enough distance, they will be governed by our familiar long range gravity field due to gravions, but at closer range other types of field will take over or prevail, like the fields around white dwarfs, neutron stars and black holes. It is not yet known to the present author whether the proposed hypothesis agrees with data from astronomy and astrophysics. It is not known how the “gravitational” fields and masses have been calculated, whether these data are amenable to review, and overall if PG and the data can be inter-interpreted, re-interpreted, and/or mutually adapted, or the PG theory must be modified or be aborted. These questions need to be addressed by many more workers with appropriate expertise. Nevertheless, some tentative ideas are put forward next by way of trialing conjectures.

In a recent report, an active galactic nucleus (AGN) believed to be the explosion from a supermassive black hole has been announced (Giacintucci *et al.*, 2020). It looks like an outburst in slow motion that *“would take at least 240 Myr to rise to the current radius moving with the sound speed, which is an upper limit on the velocity, so the actual age would be greater than that”*. In our proposed model of a black hole, a small disturbance could create an instability to the spherical geometry exposing some internal mass near the surface to the action of push particles, in turn, setting off a chain reaction to a full blown explosion. The explosion would be necessary because, otherwise, the stable sphere can only hold a maximum limit of effective mass. As more passive mass is accessed by push particles, more effective mass is created by a process continuing for a long time. The internal passive mass of a black hole may resemble black powder waiting to explode. The end result is the creation of active mass (effective mass) in a huge cloud that will later start yet another cycle of galaxy and star formation according to prevailing theories.

The internal stuff, from which black holes are made of, is completely shielded from the action of push particles of various types, like gravions, electrions, neutrions and black-hole-field push particles. All these types of particles can actually exist in all of our everyday materials according to the preceding model. Even black hole stuff may be present in the smallest possible quantities corresponding to what we initially have called “real mass”. By way of self-similarity, the universe is deconstructed and reconstructed recursively.

We may have a tangible illustration of particle-matter transformation to energy in the $E = mc^2$ equation and conversely.

The above may not be implausible, as there is some similarity with current theories about the Higgs boson (field) giving mass to subatomic particles. Without the Higgs field, we wouldn’t exist, i.e. mass would “evaporate”. Similarly, we may say that push particles maintain the integrity of material bodies at all levels of organization, whilst in the absence of said push particles (fields), the effective mass would cease to exist too.

We may now have a reason, why the Higgs field exists. Matter cannot exist in absolute vacuum, and so PG theory is on the same page with some prevailing theories. There remains to work out an understanding on the relationship between gravions and black hole passive matter, as well as the interaction between gravions and all matter manifested in the universal constant Λ , i.e. the number of scattering events per unit mass thickness (area density).

Mathematics provide relationships about things, but not what the things are, whilst theorists often are not concerned as to what it means. Thus, we hope that PG can fulfill such a gap haunting several areas in physics. The physics/maths ratio needs a substantial overhaul, where physics lags behind mathematical formulations; mathematics is supposed to serve physics, whereas physics has become subservient to mathematics. For example, the mathematicians(?) invented the Higgs field, but they don't tell us what it is, not to mention a long standing difficulty in conceptualizing relativity outside a narrow circle of high expertise scientists.

We have now canvassed various possibilities for a fully fledged PG theory poised for verification by a number of feasible experiments **and potential data available** within the reach of many laboratories and organizations around the world. Only experiment will determine, which of the canvassed ideas above is the best suitable for a correct PG.

Therefore, PG must be incorporated in the body of official and mainstream science and must remain on the table as an active candidate for possible explanations of existing data and theories for sufficient time before it may ever be abandoned again.

19 Discussion

CAVEAT: Throughout this report, no claim is made or implied that PG theory will ultimately prevail, other than the assertive wish to be put to the test by objective means and not by means of another incomplete or erroneous theory. It never states that the theory of relativity is invalid other than it may be expanded and improved. The report is supplied as an open source publication with no financial or employment strings attached prior, during or after publication. It is motivated purely by a scientific urge of the author to overcome his ignorance on outstanding questions in physics during his free time outside life's mundane tasks. By obtaining a new set of derivations for ostracized push gravity, it was felt compelling to share the novel findings publicly. It would be a great personal satisfaction, if the scientific community could engage in some way towards verification (i.e. testing) and further elaboration of PG. In particular, should the veracity of PG become proven, it would be to the greatest benefit of science, whilst, otherwise, the author will be content to feel that it was at least "*a good try with some novel work*", but without the need for exorcisms.

Subject to the above caveat, it is now decided to release some discussion in relation to important topics such as the gravitational law, expanding universe, galaxies, perpetual motion universe and philosophy, which were omitted from the previous versions. They could constitute topics for further advances in PG later, but are provided only briefly on a tentative basis at present. This may at least help avoid unwarranted criticism based on some issues not yet addressed, even briefly, with the understanding that the following discussion does not reflect on the validity of a working PG per se. Even so, it is with some trepidation that the ideas are presented below. Some specific technical proposals may prove to be totally incorrect, whilst existing or new controversies can be dealt with by the relevant experts.

An interesting exposition of push gravity is presented by Thomas (2014). This provides a good philosophical basis of the concept of push/shadow gravity and a motive for further investigation. However, the positive aspects of PG should not be diminished by possible failure of certain specific interpretations of important issues. For example, the referenced gravitons are thought to be **a kind of strings as proposed therein**, which may or may not be proven correct, so that PG should not be bound by such specific technical claims. The Allais effect is attributed to some sort of "lensing" mechanism of the gravitons around the Moon, but we have showed that the effect can be readily interpreted and even measured by the PG derivations in Part 1. These and other speculative technical interpretations, if found incorrect, should by no means reflect on PG in general. We have now tried to create an alternative paradigm of PG by building the mathematics on a set of postulates in order to arrive at the established laws of physics and beyond. One fundamental difference from all prior PG theories is that the gravion absorption need not be weak and linear, but must be exponential in accordance with established laws of absorption theory.

The present author's main expertise lies outside the fields directly pertaining to this report. As a result, Part 2, in particular, may not be as authoritative as it should be, whilst Part 1 could be seen as an attempt to produce and report new data and evidence in support of a long standing hypothesis to explain gravity. It is hoped that others may use and apply the latter findings in a better way, or as they see fit. In this context, the primary aim would have been achieved, namely, to place PG within the mainstream of physics. For the latter, it would be an even greater achievement, if work is undertaken to test the veracity of the

present findings within the programs of various institutions and organizations. Should an affirmative finding be achieved, then PG could immediately find its rightful place in science. At any rate, the present author should be excused for possible “collateral” errors, whilst attention to the novel disclosures may not diminish. It is in this context that we discuss some ideas necessitated for expanding the general PG theory.

19.1 Expanding universe

We already canvassed the various possibilities of mean free path (m.f.p) of gravions in Part 1. The case of an infinite m.f.p would result in a universal “attractive” force regardless of distance, pretty much the same as would be expected by Newton’s gravitational law. However, such a system would beg to explain how gravions interact with matter but never between themselves, i.e. how they have an affinity with matter but not with each other. As a result, it is more plausible to accept and consider a finite m.f.p., which also implies that gravions would behave like a gas in the vast universe.

The idea of push gravity occurred to this author during work on gas flows in an environmental scanning electron microscope (ESEM) (Danilatos, 1997) using the direct simulation Monte Carlo (DSMC) method (Bird, 1995). The latter method made possible the study of gaseous flows under specific conditions (e.g. novel differential pumping stages by Danilatos (2012)). It was tested and confirmed the idea of two spherical bodies being “attracted” to each other, when the mean free path of gaseous molecules was much greater than the diameter of the spheres, while absorption to any degree was also present. It was then thought that the same might happen, if planets and stars are immersed in a medium with particles having mean free paths much greater than the size of the celestial bodies. That would then be the cause of gravitational acceleration and force to start with. For distances much greater than the mean free path, no “attraction” force is generated, while the celestial bodies will float around.

The implication of this would go much further, if we can consider the analogy between a real (familiar) gas and the universe gas (gravion gas): Dust particles in a real gas under the above condition of mean free path would appear to attract each other (at close range), while also, if the overall gas is set free to expand, the dust particles would also expand in unison with the surrounding gas. Likewise, the stars, planets and celestial bodies are like dust particles attracting each other at relatively short distances but move apart from each other as a whole, when the distances become far greater than the mean free path of the gravions behaving like an expanding gas. We may then say that PG is consistent with an expanding universe and consistent with the conclusions of the theory of a big bounce or bang, but not with the theory itself (based on different premises). The visual picture of a starry night through a high resolution telescope conveys the impression of a dusty space, which might be more than a coincidence to treat it like an expanding dusty gas. In proposing such a model, the observed accelerated expansion rate of the universe might correspond to the initial accelerated rate of expansion of a gas, which, however, eventually reaches supersonic and hypersonic speeds, after which it expands at a constant rate. Therefore, if this model is correct, our universe exists still at an initial expanding rate state. Finally, if this model were to be accepted, dark energy (repulsion) of current theories might have a tangible explanation too via PG. Terminology is again important to avoid confusion. If the expanding gravion gas is responsible for pushing stars and galaxies apart (not “attracting” them), then we might as well call this type of field “negative push gravity” (NPG), consistent with “dark energy”.

Prior to all this, we take for granted that redshift can only be explained by an expanding universe and that we understand the nature of light and its propagation: We take for granted also that the Michelson-Morley experiment can only be interpreted by the exclusion of aether, while we are unclear about the wave–particle duality principle and the double slit paradox. It seems that we may have to revisit all these experiments and phenomena anew under PG.

None of the above should be less plausible than various other hypotheses already on record to explain the observed expanding rate of the universe, like: A dark energy inherent in the fabric of space itself, or the quintessence field that expands space at changing rates, or a phantom energy and so and so forth. In any event, the said applications of PG in this area are only on a tentative basis, which should be adjusted as new data and information are compiled on any of these controversial topics.

19.2 Galaxies

Let us next consider the case where the mean free path is, say, several times the size of our heliosphere. The gravitational law derived in Part One will gradually degenerate and cease to apply at longer distances. The generated field will continue to generate a push for a significant transition region, after which stars will float around as previously described. Thus, there is a transition region corresponding to the transition region from free molecule flow to continuum flow in gas dynamics.

The galactic spiral shapes resemble closely to the spirals of weather storm clouds on Earth seen from space. This might be more than a coincidence, as it might provide a classically intuitive explanation for galaxies. The spiral storm formations are caused by pressure gradients in the Earth's atmosphere in conjunction with Coriolis forces. They belong to an atmospheric barometric low (bad weather), but a similar weather pattern is formed with a barometric high (good weather) circling in the opposite direction. We should then examine the possibility of galactic formations created by the gravion pressure gradients (barometric high) in conjunction with an as yet unknown cause for circular motion. The gravion gas beyond the galaxy in the greater universe may have its own "weather" patterns. The stars in galaxies may correspond to the condensed water droplets in clouds. They are stormy regions of a more general cosmic weather system. Galaxies have high concentrations of gravions at their center creating gravion pressure gradients toward the periphery. Macroscopic gravion pressure may play a major part in galaxy formation. This suggestion could be consistent with a "mock-gravity"-like of the creation not only of the chemical elements but also of the condensation of matter into galaxies (Gamow, 1949; Hogan, 1989; Wang & Field, 1989; Field, 1971). Gamow proposed that such a "mock gravity" could have played a role in galaxy formation after the Big Bang. Although there has been much controversy over such theories, it is envisaged now that the new PG could help re-appraise all these theories by incorporating them under a bigger framework for a new understanding of cosmic motion beyond the "local" gravitational fields.

PG then might provide a good explanation why galaxies rotate faster than the existing laws of physics predict, and the motion of vast clusters of galaxies in the universe. We are presented with an opportunity to consider ideas that are still possible and rule out others completely. Thus, one more anomaly may be readily accounted for by PG.

Furthermore, PG very nicely removes the singularities (infinities) of current theories, as the maximum force that can be generated is limited by the upper boundary of push gravion flux density. There is an asymptotic approach to this limit by an increase in mass or density of mass. The forces transmit at the speed of gravions, which can be the speed of light.

Galaxies are generally considered to date to be gravitationally bound systems of stars and not weather-like systems, as we propose now, so that the above ideas are totally out of established beliefs. Dark matter has been used to explain the galaxy rotational anomaly. However, if dark matter and dark energy have been invented to fill the shortcomings of other theories, PG may also be entitled for expansion (development) at long distances as well. For it might be ultimately easier to comprehend and apply weather-like systems in the cosmos than imaginary forces acting at vast distances.

These and other anomalies reported, like by extra massive hydrogen clouds and extra energetic photons, should be also re-examined in the light of general PG theory for possible explanations.

19.3 Perpetual motion of universe

The biggest challenge of PG is to understand and explain the recycling of the gravions in the universe overall. Our proposed model suggests that they are transformed successively to various types of push particles with a correspondingly smaller mean free path until they diffuse out back into space without an obvious direct trace to us yet, but somehow finding their way by accretion back into exploding massive stars, dwarfs, neutron stars and black holes. By such means, the universe must be regenerated and overall frictionless in contrast to a forecast thermodynamic thermal death of the cosmos. The idea of a static universe recycling itself has been advocated before among others by Edwards (2007), who adopts yet another approach to PG. His model seems much more complex in an attempt to base it on (or use) existing theories, whereas our model of PG is being built from simple principles in the simplest possible terms. Then we try to see if it is consistent with existing data and theories. However, his central thesis might apply at least in some aspect. The central thesis of his model is an inter-conversion of photon and graviton energy, whereby the gravitons cumulatively establish a quantum lattice connecting all masses. "*Photons incident on the filaments of this lattice impart energy to the gravitons, while at the same time losing a portion of their original energy. This loss of energy corresponds in the model to the cosmological redshift in a static universe*". Whilst the perpetual motion machine is readily rejected by thermodynamics, we may not say the same for the universe overall or parts thereof. Otherwise, the universe would come to a grinding halt, from which we would still require an exit without resorting to god; the Big Bang is only shifting the perpetual motion to a more distant past. Nothing should prevent the existence of frictionless regions in the universe, albeit extremely small and "invisible" to our instruments as yet.

If the entire universe existed in the form of a gravion gas only, we might say that the second law of thermodynamics has had its sway (has prevailed), i.e. to which everything has succumbed. However, the fluctuation theorem is also universally applicable and operates by way of another **law** for undoing the second law in no uncertain terms. This fact is often, or mostly overlooked in science. Fluctuation results in order

and creation, whilst the second law results in disorder and destruction. It may be that quantitatively the amount of gravions constitutes an overwhelming majority of mass over and above the visible mass of the universe. Nevertheless, both laws coexist intent on undoing each other's work. This is exactly a manifestation of another overriding universal law, namely, that of the coexistence of opposites. A one-sided view of things can lead to error and impasse, whereupon we should be spurred on to find the missing (overlooked) side of things.

The above is consistent with the thinking of the universe as continuously and continually recycling itself and appearing in different forms of matter and energy, all of which is spun from a common entity that pervades all that we can experience.

19.4 Philosophy of physics and a theory of everything

To say that the human mind cannot conceive the most intricate workings of nature (when a particular theory becomes complex and unintelligible), may also be a cowardice preventing us from moving forward. Skepticism inevitably leads to religion and to the end of science. This author subscribes to the school of thought that humans can and will ultimately comprehend nature and in the simplest of terms.

There is now an opportunity and a need to disassemble certain ideas about rest mass, gravitational mass, inertial mass and relativistic mass under existing theories and re-assemble them under the platform of PG and the newly found concepts of effective and real mass.

Could the limiting speed of light for all material bodies be explained and not postulated? Could it be that, by matter organization from gravion level up, nothing can be pushed faster than gravions?

If there is a unifying theory of everything, would there be a unifying common denominator? What is it that would unify them? A common particle? Some common entity and what is its nature? If mass and energy are equivalent, and if energy can appear in different forms but always conserved, we should be able to reduce all those forms to a common denominator. That common denominator may be the gravions at different levels of organization.

Overall, nothing is invariable in the cosmos, but universal constants appear under "local" provisos and conditions, which are only recurring over the entire cosmos.

The universe consists of particles distributed over a wide range of mean free paths that allow auto-sorting of particles and bodies with the end result of a self-assembling universe like a DNA.

In attempting to conceptualize the deeper meaning and application of the second law of thermodynamics together with the re-emission of gravion energy, we may have come to a better understanding of quantum mechanics too. In quantum mechanics, anything that is possible to happen can happen governed by the probabilities of that situation. The latter provides a probabilistic relational description of the states of particles but not the origin or an explanation why quantum particles move about incessantly in certain patterns (e.g. electron orbitals). By analogy, general relativity provides an accurate relational explanation of various parameters but it does not provide a hint about how and why gravity exists, or why the spacetime around a mass is bent and warped. PG via an incessant gravion flow may provide the basis for understanding both quantum mechanics and relativity at the same time. The ever flowing gravions pass through various levels of material organizations via quantitative accumulations leading to qualitative transformations from level to level. The universal relationship between quantity and quality can be seen at all levels of organization of matter, starting from the smallest quantum mechanical states, to chemical and mechanical systems of ordinary sizes, and all the way up to white dwarfs, neutron stars and black holes. Actually, the smallest of entities may not even be subject to quantum mechanical rules, if quantum mechanics has so far described an "intermediate" level of the universe. Quantum mechanics may be simply a macroscopic description of other underlying processes, like pressure and temperature are macroscopic statistical properties of a gas. Likewise, those other underlying of quantum mechanics processes may be ultimately the simplest ones waiting for us to discover.

A perpetual motion of matter/energy can rightfully belong to the universe as a whole, a principle which has been attributed to Heraclitus (Wikipedia contributors, 2019a). All these ideas eventually lead to the need to understand the nature of gravions and its interactions with matter and with themselves.

20 Conclusion

An attempt has been made to modify and advance the old principle of push gravity theory to a stage where gravity may be seen from a totally novel perspective. It constitutes a daring step, because it challenges and potentially provokes a re-consideration of long standing ideas and principles. This has already required a daunting determination especially as it comes from a non-established expert in the field of gravity.

The basic new element is the use of a gravity particle absorption coefficient that is not limited only to very low values as in prior PG theories. The consequences of that can be dramatic.

The theory of PG has now been brought to a stage ready for verification with several proposed tests and methods. Should these tests yield a positive outcome, they could provide explanation to many outstanding issues in science. Otherwise, the test may prove insufficient pending further instrument refinements. Alternatively, if one produces sufficient evidence to reject PG once and for all, that would compel science to concentrate on other pathways, as it does, even more. At any rate, it should be appreciated that the proposed tests are inexpensive at least in relative terms for many organization to engage.

In summary, new work provides sufficient evidence for a genuine re-appraisal of push gravity. A novel quantitative theory has been advanced on the basis of a set of primary principles (postulates), from which the derivation of classical acceleration and force by stationary massive bodies in the steady state is possible. In contrast to prior conceptions, it is shown that the absorption of gravity particles by matter need not be extremely weak and linear, in order to derive and explain the observed classical laws of gravity. Any value of the absorption coefficient by a uniform spherical mass produces a gravitational field obeying the inverse square of distance law. The gravitational constant (big G), is itself a function of the ratio of the absorption coefficient over the density of matter. The latter ratio now becomes the new universal constant of the cosmos, whilst G can vary in different locations of the universe. The measured mass of planets and stars is only an effective or apparent mass actually smaller than the real mass due to a self-shadowing or shielding effect of the absorption of gravitational particles. Any given mass appears quantitatively different depending on its spatial distribution including orientation. We now find that Newton's gravitational law uses only the apparent (or effective) masses with a potentially variable G, but the inverse square distance relationship is preserved in the cosmos. The radiant flux of energetic particles being uniform over a region of space creates a maximum acceleration of gravity for all material bodies in that region, so that any further mass accretion over a certain upper limit does not create additional acceleration; this limit is reached when practically all gravitational particles are absorbed (saturation state) by the massive body above a saturation mass. The latter limit should be measurable, for which some tentative situations and experiments are proposed for prospective experiments and tests. The internal field of a spherical mass and the external field of a two layered sphere have been derived. The superposition principle of gravity fields has been reformulated and the Allais effect explained. The equivalence principle can now be properly understood and explained in a way that the principle per se becomes redundant. We can now better understand the meaning of matter, inertia and mass. For moving bodies, the established relationships from special and general relativity may continue to operate within the gravitational fields created by push particles, but may need to be adapted and re-aligned within the greater framework of push gravity principles operating at any distance.

An attempt is made to overcome the main remaining objection of presumed catastrophic thermal accretion of absorbed particles. A further attempt is made also for the push-gravity principles to explain the vastly higher intensity gravitational fields of white dwarfs, neutron stars and black holes. It is proposed that the field of white dwarf stars is created also by push particles but of a different kind, namely, by those responsible for mediating the electric field. In the same way, the field of neutron stars is created by yet a third kind of push particles, namely, those responsible for mediating the nuclear field. In general, push particles may exist with different energy (or mass) having different mean free paths as they traverse different concentrations of masses (black holes, neutron stars, dwarfs, stars, planets, ordinary masses, atoms, nuclei, protons and all the known or unknown sub-nuclear particles). The invariable principle of momentum transfer (push) by particles directly relating to their absorption rate by the various concentrations (density) of masses could be the basis and the starting principle for a prospective unification theory of everything. The first part of this report, if verified, should create the basis for new physics across many fields of physical science. Pending such a verification, we may also work towards the development of a general PG including a theory of the dynamics of the observed motion of celestial bodies.

If there is a "theory of everything", then gravions could provide an underlying mechanism not only in gravity but also in quantum mechanics. Gravions may be responsible for both the gravitational fields and the associated masses being nothing else than effective masses. It may be that we can make one step closer to a better philosophical understanding of the cosmos, if we can grasp the nature of the gravion, perhaps, as being the embodiment of the coexistence of opposites in a perpetual flow of the universe.

ACKNOWLEDGMENT

Zenodo should be gratefully acknowledged for providing the best platform to disseminate this special science report encompassing many different disciplines in physics. Whilst review and moderation definitely make better scientific standards for specialist journals, it could also be a hindrance, if working on the present

paper depended on a few only specialist opinions, or a single endorser. The possibility of successive versions is helpful, whilst it can provide a platform for a joint project with collaborating participants from diverse fields.

APPENDIX

A Gravitoids

Let us be reminded that a prolate spheroid is a surface of revolution obtained by rotating an ellipse about its major axis, whereas an oblate spheroid is obtained by rotating about its minor axis. It is well known that spheroids acting as gravitational bodies would produce precession or regression of the elliptical orbit of a planet around it. This arises by the gravitational force being slightly greater or lesser than the inverse of the square of the distance. In other words, it is the distance (not the mass) responsible for these phenomena (here, we are not referring to the relativistic cause of the extra Mercurial precession). It is interesting to examine and clarify what happens with PG theory in this connection via the following observations.

Now, by virtue of Eq. 38, the self-shadowing effect produces a gravitational force (acceleration) less than the value expected from simple Newtonian attraction by a sphere. By increasing k with all else constant, the force increases in proportion to it (or the density) by Newtonian mechanics, but to a lesser degree by PG theory, on account of the exponential decrease along a chord (straight line) of the sphere in Fig. 2. That is, the Newtonian length that would produce an attractive force effectively contracts (shrinks) to produce the correct force. Each elemental component is then equivalent to having a lesser length at a given density, whilst in reality there is additional mass for the remainder of the length of the chord. This becomes, in effect, a virtual mass distributed inside a spheroid-like shape. It may initially look like an oblate spheroid, but it has a peculiar shape dependent on the distance OP and k . For a sufficiently long distance OP (i.e. r), the lines u are nearly parallel (very small angles φ), whilst as we approach the surface at point C, the same lines radiate at large angles φ , and the shape becomes more like a compressed egg along its axis while being inflated at right angles. We can see these and other effects by plotting the corresponding lengths and shapes quantitatively for two positions of the point O, namely, at the surface of the sphere and at a relatively long distance of $n_R = 100$.

On the surface of the sphere, we show pairs of the chord lengths and body shapes between Newtonian and virtual PG cases for three different values of k , in Fig. 26. To clarify, because point O lies on the surface of the sphere, any distance from the fixed point O to any other point on the surface defines the chord length, via which we also plot the sphere. Thus, these graphs show simultaneously both the chords and volumes of revolution corresponding to the real sphere and virtual shape yielded by PG. As expected, for $k < 0.01$, PG shapes become gradually indistinguishable from Newton. Otherwise, the difference increases significantly.

Next, we plot the virtual chord lengths for a sphere with unity radius from a distance $r = 100$ units. Planet Mercury approximately has this distance from the Sun at its aphelion. We consider again three values for $k = 0.01$, $k = 0.1$ and $k = 1$ in Fig. 27 together with the same **real** chord lengths of the same sphere (in black). We have used the same Eqs. 12 and 47.

Finally, we can visualize the corresponding virtual shapes of the sphere (here, like the Sun) from the same distance of 100 sphere radii (as from Mercury) with the same corresponding values of k in Fig. 28. This is obtained by adding the PG chord length by 47 to the corresponding u_1 provided by Eq. 16, i.e. we use the virtual end points of u_{e2} in PG given by Eq. 48.

The above spheroid-like shapes are bounded by the red lines together with overlapping black lines on the left. We note that a shallow dimple appears on the far side, the depth of which increases as we further increase k , effectively producing a *dimpled spheroid-like* shape.

As previously noted, the real shapes (and sizes) of a sphere effectively act as some peculiar virtual shapes, fictitious and invisible, for which we may collectively use (coin) the new term *gravitoids*. Their mass may be used with linear absorption as in Newton's law to yield the force as predicted by PG.

Below, we also present the analytical expressions already used to plot these gravitoids in Fig. 6 and discussed in Section 6.4. We follow the steps in finding the volume of a sphere to illustrate the point of deviation (departure) between the two approaches:

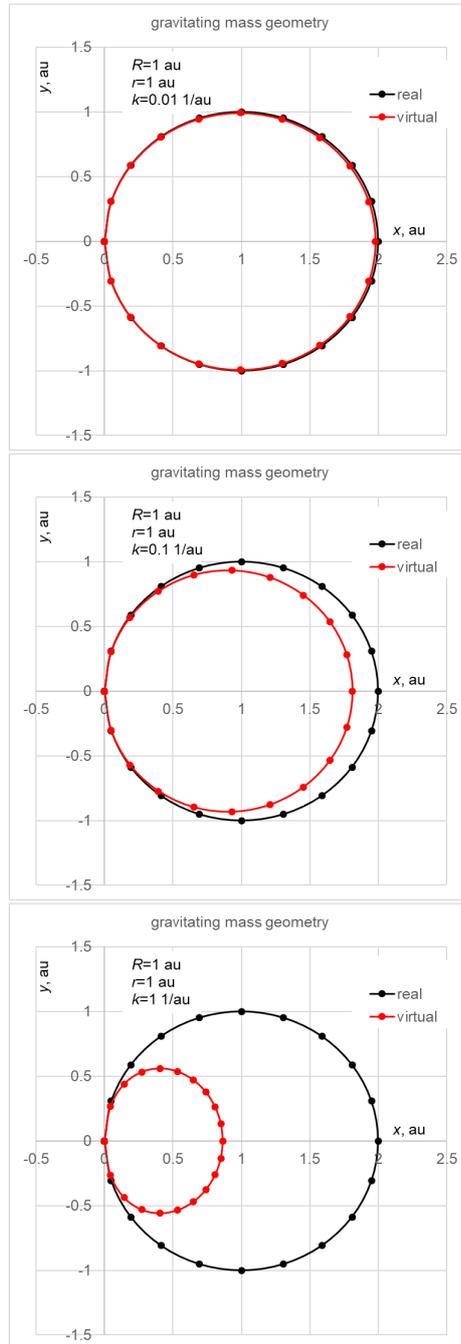


Figure 26: Real sphere in black, gravitoid (virtual) shape in red for three values of k and $r = R = 1$.

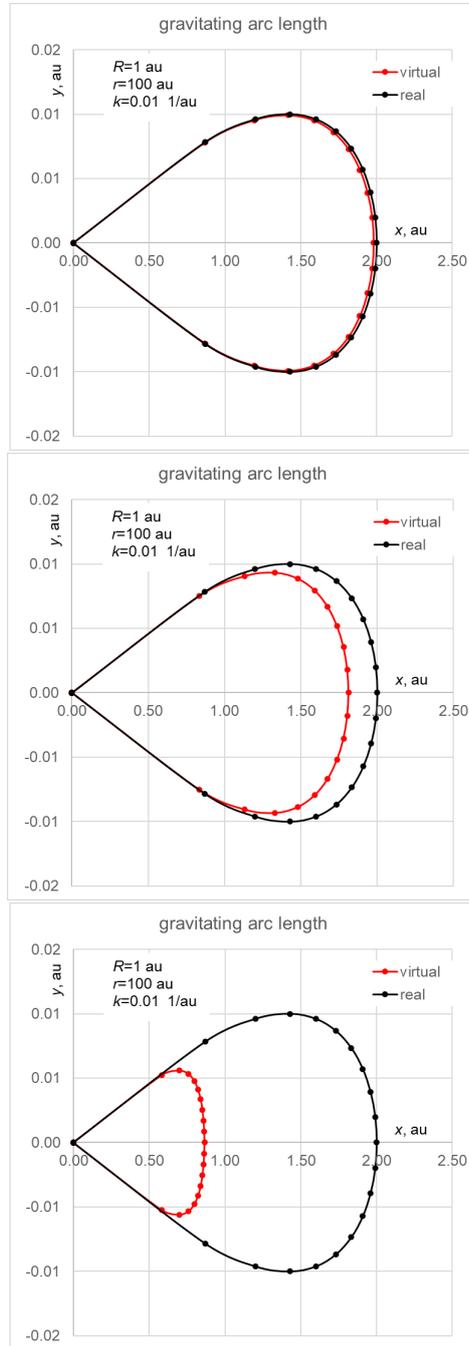


Figure 27: Arc length for real sphere in black and gravitoid (virtual) in red for three values of k at $n_R = 100$.

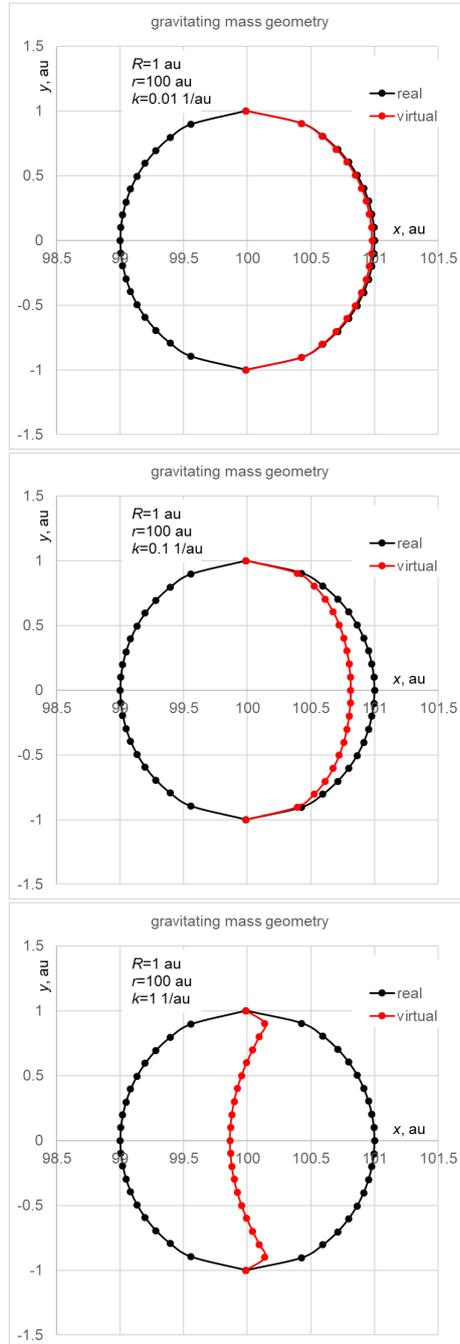


Figure 28: Real sphere in black, gravitoids (virtual) shape in red (together with black left of red line) for three values of k and $n_R = 100$.

$$\begin{aligned}
V_{sphere} &= \int_0^{\varphi_0} \int_{u_1}^{u_2} 2\pi \sin \varphi d\varphi \cdot u^2 du = \frac{2\pi}{3} \int_0^{\varphi_0} [\sin \varphi d\varphi \cdot u^3]_{u_1}^{u_2} \\
&= \frac{2\pi r^3}{3} \int_0^{\varphi_0} \sin \varphi d\varphi \left[\left(\cos \varphi + \sqrt{a^2 - \sin^2 \varphi} \right)^3 - \left(\cos \varphi - \sqrt{a^2 - \sin^2 \varphi} \right)^3 \right] \quad (218)
\end{aligned}$$

and by using the limits in Eqs. 16 and 17, it finally yields the expected result:

$$V_{sphere} = \frac{2\pi r^3}{3} \left[-2 \cos \varphi (\cos^2 \varphi + a^2 - 1)^{3/2} \right]_0^{\varphi_0} = \frac{4\pi R^3}{3} \quad (219)$$

Similarly, starting from the same elementary volume equation

$$V_{gravitoid} = \int_0^{\varphi_0} \int_{u_1}^{u_2} 2\pi \sin \varphi d\varphi \cdot u^2 du = \frac{2\pi}{3} \int_0^{\varphi_0} [\sin \varphi d\varphi \cdot u^3]_{u_1}^{u_2} \quad (220)$$

but using the limits in Eqs. 16 and 48 we obtain:

$$\begin{aligned}
V_{gravitoid} &= \frac{2\pi}{3} \int_0^{\varphi_0} \sin \varphi d\varphi \cdot \\
&\left[\left(r \left(\cos \varphi - \sqrt{a^2 - \sin^2 \varphi} \right) + \frac{1}{k} - \frac{1}{k} \exp \left(-2kr \sqrt{a^2 - \sin^2 \varphi} \right) \right)^3 - \left(r \left(\cos \varphi - \sqrt{a^2 - \sin^2 \varphi} \right) \right)^3 \right] \quad (221)
\end{aligned}$$

by which we finish up with a different curve shape for the volume of the gravitoid. This shape filled with the actual (real) density may be used with Newton's law to reproduce the same force yielded by PG. The above examples simply illustrate that the initial common integration for a volume diverges on account of the different integration limits in the corresponding theories of Newton and PG. They illustrate the formal relationships between the two theories. The above integration has been performed numerically and plotted against k in Fig. 6 after it is normalized by dividing by the sphere in Eq. 219, as was done for the effective spherical volume defined by Eq. 52.

We note that the effective volume generally lies above the gravitoid, as it should, because it is further away from the gravitoid relative to the reference point O. If they both contain the same real density matter, then both yield the correct value of acceleration by applying Newton's equation. We further note that the gravitoid volume (effective mass) increases, as we move away from the gravitating mass (e.g. compare the obvious corresponding sizes provided by Figs. 26 and 28). However, this does not affect the inverse of r^2 dependency, because this effective mass increase is compensated by the integration to a lower upper limit of angle (i.e. over a smaller angle range). For a possible precession to be generated, we need to consider the time effects also in PG as in the corresponding GR theory.

B Field formulations around a single sphere

We have initially derived the absorption fraction of gravions at a point outside a sphere based on its axial symmetry around the line joining the point with the center of the sphere. However, we can also generally derive the same fraction by considering a Cartesian reference frame of x, y, z axes for later use in non-axially symmetric systems. We integrate the gravion absorption by revolving an elementary solid angle around each of the x, y, z axes to yield three components of absorption corresponding to the classical vector of acceleration. For simplicity, here we consider only a sphere intersecting one coordinate plane of symmetry along its diameter, but the derivation can be expanded for any location of the test point located outside the sphere; points inside the sphere are considered during the two sphere "bulk" formulation.

With reference to Fig. 29, the plane of symmetry intersecting the sphere is the yOz and we define and use the following notations of constants and parameters:

$$(AB)_z = 2\sqrt{(y_0 \sin \varphi_z \cos \theta_z + z_0 \cos \varphi_z)^2 - (r^2 - R^2)} \quad (226)$$

$$(AB)_z = 2\sqrt{OM(\varphi_z, \theta_z)^2 - (r^2 - R^2)} \quad (227)$$

$$(AB)_z = 2r\sqrt{a^2 - \sin^2 \psi} \quad (228)$$

Now, the operand under the square root must be positive, which sets the limits of azimuth angle as a function of zenith angle by solving the equation:

$$(y_0 \sin \varphi_z \cos \theta_z + z_0 \cos \varphi_z)^2 - (r^2 - R^2) = 0 \quad (229)$$

$$y_0 \sin \varphi_z \cos \theta_z + z_0 \cos \varphi_z = \pm\sqrt{(r^2 - R^2)} \quad (230)$$

$$\theta_z = \text{acos} \left(\frac{-z_0 \cos \varphi_z + \sqrt{(r^2 - R^2)}}{y_0 \sin \varphi_z} \right) \quad (231)$$

where we use the positive root sign because $OM = y_0 \sin \varphi_z \cos \theta_z + z_0 \cos \varphi_z$ must be positive and, when $OM=ON$, it becomes tangent at the limits:

$$\theta_{z1} = -\text{acos} \left(\frac{-z_0 \cos \varphi_z + \sqrt{(r^2 - R^2)}}{y_0 \sin \varphi_z} \right)$$

$$\theta_{z2} = \text{acos} \left(\frac{-z_0 \cos \varphi_z + \sqrt{(r^2 - R^2)}}{y_0 \sin \varphi_z} \right)$$

$$\varphi_{z1} = \varphi - \text{asin}(a)$$

$$\varphi_{z2} = \varphi + \text{asin}(a)$$

With the chord length AB found, the absorption fraction for the z-axis is given by the double integral:

$$f_{gz} = \int_{\varphi_{z1}}^{\varphi_{z2}} \int_{\theta_{z1}}^{\theta_{z2}} (1 - \exp(-k(AB)_z)) \sin \varphi_z \cos \varphi_z d\varphi_z \quad (232)$$

Similarly, we follow the same steps for the y axis by interchanging the corresponding parameters and notations and adding $\pi/2$ to φ_z as follows:

$$(AB)_y = 2\sqrt{(z_0 \sin \varphi_y \cos \theta_y + y_0 \cos \varphi_y)^2 - (r^2 - R^2)} \quad (233)$$

with limits:

$$\theta_{y1} = -\text{acos} \left(\frac{-y_0 \cos \varphi_y + \sqrt{(r^2 - R^2)}}{z_0 \sin \varphi_y} \right)$$

$$\theta_{y2} = \text{acos} \left(\frac{-y_0 \cos \varphi_y + \sqrt{(r^2 - R^2)}}{z_0 \sin \varphi_y} \right)$$

$$\varphi_{y1} = \varphi - \text{asin}(a)$$

$$\varphi_{y2} = \varphi + \text{asin}(a)$$

with which we obtain the integration around y axis for the absorption component f_{gy} , so that the total absorption fraction is $f_g = \sqrt{f_{gz}^2 + f_{gy}^2}$. Noted $(AB)_z = (AB)_y = AB$. The above equations are valid while the sphere does not cross any axis. In the case when it crosses one axis (let's say the z axis), there are two consecutive sub-ranges of the angles with limits:

First:

$$\begin{aligned}\theta_{z11} &= -\pi \\ \theta_{z21} &= \pi \\ \varphi_{z11} &= 0 \\ \varphi_{z21} &= \varphi - \text{asin}(a)\end{aligned}$$

Second:

$$\begin{aligned}\theta_{z12} &= -\text{acos}\left(\frac{-y_0 \cos \varphi_y + \sqrt{(r^2 - R^2)}}{z_0 \sin \varphi_y}\right) \\ \theta_{z22} &= \text{acos}\left(\frac{-y_0 \cos \varphi_y + \sqrt{(r^2 - R^2)}}{z_0 \sin \varphi_y}\right) \\ \varphi_{z22} &= \varphi - \text{asin}(a) \\ \varphi_{z22} &= \varphi + \text{asin}(a)\end{aligned}$$

with which we obtain two sub-components for this axis to be added as $f_{gz} = f_{gz1} + f_{gz2}$

We repeat the same when the sphere crosses the other axis. Likewise, when the sphere crosses both axes. For negative values of $-\varphi$, we replace with positive φ , whilst for $\varphi > \pi/2$ we replace φ with $\pi - \varphi$.

B.1 Alternative formulation for the normal component

Beyond the general formulations above, the normal component of acceleration can be deduced with an alternative simpler way: The component of absorption around the normal axis y can be found concurrently during rotation around z axis inside the same solid angle used for axis z . As we rotate around the fixed z axis, we can project and find the component of the chord AB on the fixed y -axis by multiplying with azimuth $\cos \theta_z$ times zenith $\sin \varphi_z$, so the normal component is:

$$f_{gy} = \int_{\varphi_{z1}}^{\varphi_{z2}} \int_{\theta_{z1}}^{\theta_{z2}} (1 - \exp(-kAB)) \cos \theta_z \sin^2 \varphi_z d\varphi_z \quad (234)$$

The two components f_{gz} and f_{gy} must subsequently themselves be projected on the line OP to obtain the required total absorption fraction, namely:

$$f_g = f_{gz} \cos \varphi + f_{gy} \sin \varphi$$

We can re-write all above in a combined expression as:

$$f_g = \int_{\varphi_{z1}}^{\varphi_{z2}} \int_{\theta_{z1}}^{\theta_{z2}} (1 - \exp(-kAB)) (\sin \varphi_z \cos \varphi_z \cos \varphi + \cos \theta_z \sin^2 \varphi_z \sin \varphi) d\varphi_z \quad (235)$$

Although we have already described the field around the axis of symmetry at the outset of PG theory with the simplest equations, the above formulations are more than a theoretical exercise, because they are needed in more complex mass distributions like the two-sphere problem examined later.

Further theoretical processing and analysis of the above derivations can be done separately, but the above can be used immediately as "raw" material for numerical integration to obtain some early results without further ado.

C Force between two spherical masses - bulk method

For the formulation of the problem of force between two material spheres, we have used two different methods. One method involves the points (elements) inside the bulk of one sphere followed by integration over the entire bulk of the sphere. The other method involves the points (elements) on the surface of one sphere and integration over the entire surface of the sphere. The outcomes are equivalent (equal) since traces of gravions passing through any point inside the bulk of a sphere must also cross the surface of the sphere and vice-versa. The bulk method involves four integrals and takes far longer integration times with numerical methods. The second method has its own complexity, but it involves three integrals requiring much shorter integration times.

With reference to Fig. 30, we define and use the following notations of constants and parameters: We have sphere_1 (*sphere*₁) and sphere_2 (*sphere*₂) with corresponding radii $R_1 = P_1N$ and $R_2 = P_2M$, and with uniform material densities and hence uniform absorption coefficients k_1 and k_2 ; the distance between the centers of the spheres is $P_1P_2 = r$. We choose a random point O inside sphere_2 forming an angle $\angle OP_1P_2 = \varphi_2$ with corresponding differential semi-angle $d\varphi_2$ and maximum subtended angle $\angle P_2P_1S = \varphi_{20}$. From point O, we define the direction u along OP_1 , around which we draw a random solid angle Ω_1 with semi-angle $\angle P_1OB_1 = \varphi_1$ with corresponding differential angle $d\varphi_1$ and maximum subtended angle $\angle P_1ON = \varphi_{10}$. The solid angle Ω_1 is enveloped by line u' , which crosses sphere_1 at points A₁ and B₁, and sphere_2 at points A₂ and B₂; we will also need the points A and B, at which the axis u crosses the sphere_2. Normal to the axis u is the axis ν with a third axis y (not shown) normal to the plane of $u\nu$, thus defining the coordinate system (νuy) . Another coordinate system (xzy') is rotated around y' by the angle φ_2 with the axis z aligned with the two centers P₁ and P₂ and y' normal to the plane xz , i.e. parallel to y .

Point O is at a distance u from the center P₁, i.e. $OP_1 = u$, and the usual parameters are:

$$a_1(u) = R_1/u = \sin \varphi_{10} \quad (236)$$

$$a_2 = R_2/r = \sin \varphi_{20} \quad (237)$$

$$P_1B = u_1(\varphi_2) = r \cos \varphi_2 - r \sqrt{(a^2 - \sin^2 \varphi_2)} \quad (238)$$

$$P_1A = u_2(\varphi_2) = r \cos \varphi_2 + r \sqrt{(a^2 - \sin^2 \varphi_2)} \quad (239)$$

The normal from P₂ to u crosses AB at mid point M, so that we obtain for the segments:

$$OM(u, \varphi_2) = u - r \cos \varphi_2 \quad (240)$$

$$MP_2(\varphi_2) = r \sin \varphi_2 \quad (241)$$

The length A_1B_1 is derived per Eq. 12 again as:

$$A_1B_1(\theta, \varphi_1, u) = 2u \sqrt{a_1^2 - \sin^2 \varphi_1} \quad (242)$$

which is independent of (being constant with) the azimuth angle of rotation around axis u ; we introduce the azimuth because we further require to know the chord lengths OB_2 and OA_2 , which vary by rotating the line u' around the axis u at constant angle φ_1 ; the azimuth angle of rotation θ is not shown for simplicity of drawing. We derive the latter chords by solving the equations of sphere_2 and line u' per analytical geometry as follows:

$$y^2 + (\nu - MP_2)^2 + (u - OM)^2 = R_2^2 \quad (243)$$

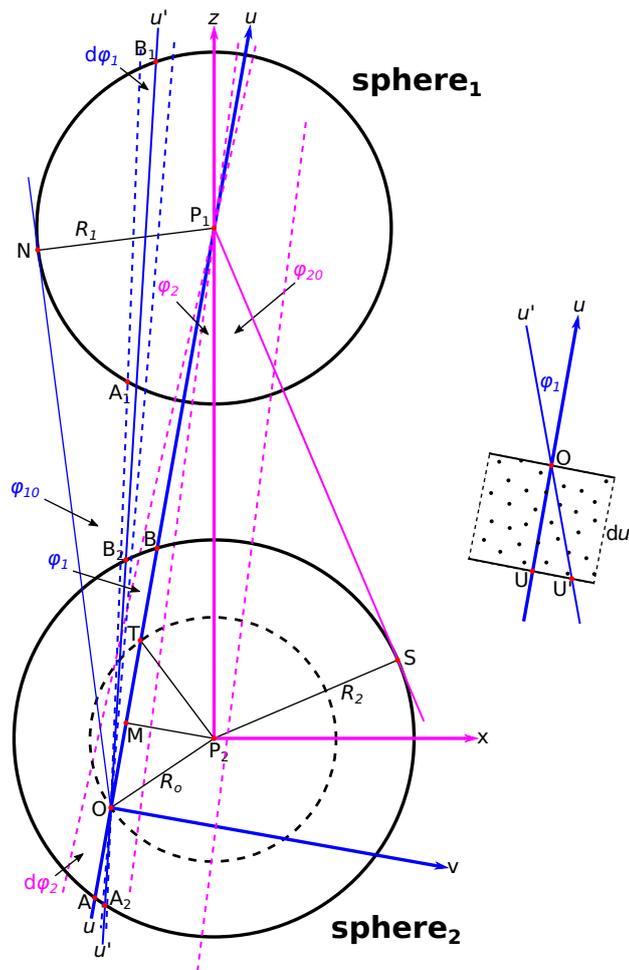


Figure 30: Diagram for the derivation of the force exerted between two spherical masses with different diameters and different but uniform densities

$$v = (OB_2) \sin \varphi_1 \cos \theta \quad y = (OB_2) \sin \varphi_1 \sin \theta \quad u = (OB_2) \cos \varphi_1 \quad (244)$$

The simultaneous solution of above equations gives the required lengths

$$OA_2(\theta, \varphi_1, u, \varphi_2) = MP_2 \sin \varphi_1 \cos \theta + OM \cos \varphi_1 - \sqrt{(MP_2 \sin \varphi_1 \cos \theta + OM \cos \varphi_1)^2 + R_2^2 - OM^2 - MP_2^2} \quad (245)$$

$$OB_2(\theta, \varphi_1, u, \varphi_2) = MP_2 \sin \varphi_1 \cos \theta + OM \cos \varphi_1 + \sqrt{(MP_2 \sin \varphi_1 \cos \theta + OM \cos \varphi_1)^2 + R_2^2 - OM^2 - MP_2^2} \quad (246)$$

where it is important to take the absolute value of the above lengths (when they are negative) in the exponent of the exponential factor used to derive the graviton absorption below.

The general strategy is briefly as follows: We consider all graviton flows in all possible directions at every given point and vectorially sum the flows (forces) over all points inside the sphere. The components of flow in the direction P_1P_2 are responsible for the force, whilst those perpendicular (normal) to that direction contribute no force between the spheres. We first group all the components of flow in the direction of axis u and all the components in the normal direction of axis v , and then project the two outcomes in the direction of z defined by the centers of the two spheres, whilst all components in the normal direction x vanish. In the latter stage, the useful absorption fraction $f_g(O)$ at point O is the absorption $f_g(O)_z$ in direction (projection) z derived from the absorption $f_g(O)_u$ in direction u plus the absorption $f_g(O)_v$ in direction v as follows:

$$f_g(O) \equiv f_g(O)_z = f_g(O)_u \cos \varphi_2 + f_g(O)_v \sin \varphi_2 \quad (247)$$

This will be used in a final integration to find the force F by:

$$F = 2\pi \frac{J}{c} \int_0^{\varphi_{20}} \left(\int_B^A f_g(O) u^2 k_2 du \right) \sin \varphi_2 \cos \varphi_2 d\varphi_2 \quad (248)$$

The above provides only a general idea of what we better explain and clarify with detailed steps next.

C.1 u -axis

There are two components of graviton absorption by integration around this axis, namely, those crossing both spheres by joint traces and those crossing only sphere_2.

C.1.1 Joint crossing (*variable_{uj}*)

Gravitons passing through point O and crossing both spheres (“jointly”) along the line $B_1A_1B_2OA_2$ undergo exponential absorption before they arrive at point O , the difference of which multiplied by the usual product $\sin \varphi_1 \cos \varphi_1$ yields the absorption component along the u axis:

$$f_{g-uj}(\theta, \varphi_1, u, \varphi_2) = [\exp(-|k_2OA_2|) - \exp(-k_1A_1B_1 - k_2OB_2)] \sin \varphi_1 \cos \varphi_1 \quad (249)$$

which we integrate around the axis over the full azimuth angle θ and over the zenith angle φ_1 within the subtended solid angle by sphere_1 at the given angle φ_2 of the axis with P_1P_2 and the given distance u of O from P_1 :

$$F_{uj}(u, \varphi_2) = \int_0^{\text{asin}(a_1(u))} \int_{-\pi}^{\pi} f_{g-uj}(\theta, \varphi_1, u, \varphi_2) d\theta d\varphi_1 \quad (250)$$

The subscripts $_{uj}$ stand for u axis (direction) and “joint” traces. The above integrated graviton component exerts a pressure J_0/c on an elementary thin material segment disposed normal to the axis of rotation (u), so that the product of the pressure with the area of the element times the absorption coefficient k_2 of material

sphere_2 over the elementary thickness du produces an elementary force at point O. We may omit the constant factor J_0/c from the interim formulations until we obtain the end result for the absorption factor. Now, we need to multiply by the area $u^2 d\Omega_2$, where $d\Omega_2$ is the elementary solid angle subtended by the surface element at the center P_1 times $k_2 du$ to allow for the absorption along du generating an elementary force:

$$d^2 S_{uj}(u, \varphi_2) = F_{uj}(u, \varphi_2) u^2 k_2 du d\Omega_2 \quad (251)$$

It is noteworthy at this point that the above factor has acquired the dimensions of an area being an elementary surface (S) after initially being a pure number. From the above, we find the total absorption along the length of chord $A_2 B_2$

$$dS_{uj}(\varphi_2) = \left[\int_{u_1(\varphi_2)}^{u_2(\varphi_2)} F_{uj}(u, \varphi_2) u^2 k_2 du \right] d\Omega_2 \quad (252)$$

Here and in following derivations, when we use an elementary solid angle $d\Omega$, we replace it either with $d\Omega = \sin \varphi d\varphi$ in an asymmetrical rotation by involving the azimuth angle, or with the elementary annular solid angle $d\Omega = 2\pi \sin \varphi$ being the integral around a rotational symmetry without use of the azimuth angle. We do this without explicitly stating it. Thus, and by projecting the above force on $P_1 P_2$ by $\cos \varphi_2$, we finally integrate over the entire bulk of sphere 2 by

$$S_{uj} = \int_0^{\text{asin}(a_2)} dS_{uj}(\varphi_2) \cdot 2\pi \sin \varphi_2 \cos \varphi_2 d\varphi_2 \quad (253)$$

C.1.2 Single crossing (*variable_{us}*)

For traces of gravions crossing only sphere_2, to which we refer with the term “single” or “lone” (with subscript _s), the corresponding integrand of the innermost integral is a little simpler by:

$$f_{g-us}(\theta, \varphi_1, u, \varphi_2) = [\exp(-|k_2 OA_2|) - \exp(-k_2 OB_2)] \sin \varphi_1 \cos \varphi_1 \quad (254)$$

We repeat the same steps except that we integrate with respect to zenith angle from φ_{10} (i.e. $\text{asin}(a_1(u))$) to $\pi/2$, so that by changing the notation of “j” to “s”, we get:

$$F_{us}(u, \varphi_2) = \int_{\text{asin}(a_1(u))}^{\pi/2} \int_{-\pi}^{\pi} f_{g-us}(\theta, \varphi_1, u, \varphi_2) d\theta d\varphi_1 \quad (255)$$

and finally

$$S_{us} = \int_0^{\text{asin}(a_2)} dS_{us}(\varphi_2) \cdot 2\pi \sin \varphi_2 \cos \varphi_2 d\varphi_2 \quad (256)$$

We find that the terms for “single” absorption are not needed for the final force derivation, because they cancel out exerting a null force. However, we consider the steps involved not only for completeness, but also because we need to follow the corresponding steps during derivation of mass or energy in subsequent sections.

C.2 ν -axis

Again, there are two kinds of components of gravion absorption by integration around this axis, namely, those crossing both spheres by joint traces and those crossing only sphere_2. The latter “single” crossings contain two subgroups, namely, those in the complementary angle to the joint zenith angle (i.e. outside the joint zenith angle up to $\pi/2$) and those in the supplementary joint azimuth angle within the joint zenith angle (i.e. outside the joint azimuth angle $2\pi - \text{joint_azimuth}$ angle). We explain this in the following three steps.

C.2.1 joint crossing (*variable* $_{\nu j}$)

The notation of angles (θ, φ_1) could have been designated as (θ_u, φ_{1u}) for the u -axis and as $(\theta_\nu, \varphi_{1\nu})$ for the ν -axis, because the angles with reference to the normal axis ν are different, but there is no ambiguity to retain the same notation with both axes noting that the range of integration angles are different, for which special care is required to avoid possible errors; by use of the correct integration limits, we do not need to change notation of azimuth and zenith angles.

Now, A_1B_1 is found by a different expression:

If *operand* = $(u \sin \varphi_1 \cos \theta)^2 + R_1^2 - u^2 > 0$ then

$$A_1B_1 \equiv A_1B_{1\nu j}(\theta, \varphi_1, u) = 2\sqrt{\text{operand}} \quad (257)$$

From the condition of a positive operand above, we obtain the range of angles:

$$\theta_1 = -\text{acos} \left(\sqrt{u^2 - R_1^2} / (u \sin \varphi_1) \right)$$

$$\theta_2 = +\text{acos} \left(\sqrt{u^2 - R_1^2} / (u \sin \varphi_1) \right)$$

$$\varphi_{11} = \pi/2 - \text{asin}(a_1(u))$$

$$\varphi_{12} = \pi/2 + \text{asin}(a_1(u))$$

The above limits ensure that the integration contains only joint traces. Note that φ_1 must cover the range on either side from $\pi/2$. The lengths OA_2 and OB_2 are given by the same Eqs. 245 and 246 provided we apply the above (correct) range of angles. Then, we follow the same steps changing the notation of u with ν accordingly:

$$f_{\nu j}(\theta, \varphi_1, u, \varphi_2) = [\exp(-|k_2OA_2|) - \exp(-k_1A_1B_1 - k_2OB_2)] \sin \varphi_1 \cos \varphi_1 \quad (258)$$

finally obtaining:

$$S_{\nu j} = \int_0^{\text{asin}(a_2)} dS_{\nu j}(\varphi_2) \cdot 2\pi \sin \varphi_2 \cos \varphi_2 d\varphi_2 \quad (259)$$

C.2.2 Single (or lone) crossing

There are two terms for “single” (lone) crossing traces to derive in ν -direction:

(i) **ν -axis single complementary** - (*variable* $_{\nu s-c}$) This term arises in the zenith angle range $0 \rightarrow \pi/2 - \text{asin}(a_1(u))$ (being the complement of the joint zenith angle), with corresponding full azimuth angle range $-\pi \rightarrow +\pi$. We follow the same steps leading to the final integral:

$$S_{\nu s-c} = \int_0^{\text{asin}(a_2)} dS_{\nu s-c}(\varphi_2) \cdot 2\pi \sin \varphi_2 \cos \varphi_2 d\varphi_2 \quad (260)$$

(ii) **ν -axis single supplement** (*variable* $_{\nu s-s}$) This term arises in half of the joint zenith angle range $\varphi_{11} \rightarrow \varphi_{12}$ (below) with corresponding partial (i.e. supplementary) azimuth angle range $\theta_1 \rightarrow \theta_2$ (i.e. outside the “joint” crossings), where

$$\theta_1 = \text{acos} \left(\sqrt{u^2 - R_1^2} / (u \sin \varphi_1) \right)$$

$$\theta_2 = 2\pi - \text{acos} \left(\sqrt{u^2 - R_1^2} / (u \sin \varphi_1) \right)$$

$$\varphi_{11} = \pi/2 - \text{asin}(a_1(u))$$

$$\varphi_{12} = \pi/2$$

It should be noted that, while the zenith range is actually between $\pi/2 - \text{asin}(a_1(u)) \rightarrow \pi/2 + \text{asin}(a_1(u))$, we use only half of this range to avoid a second pass of the same single crossing in the second semi-range $\pi/2 + \text{asin}(a_1(u))$ of the zenith angle.

The above limits ensure that the integration contains only “single” chords in the function of absorption along the given line u' . We follow again the same steps yielding the other additional component now indexed with “ $_{\nu s-s}$ ”

$$S_{\nu s-s} = \int_0^{\text{asin}(a_2)} dS_{\nu s-s}(\varphi_2) \cdot 2\pi \sin \varphi_2 \cos \varphi_2 d\varphi_2 \quad (261)$$

C.3 Summation of terms

In the general case above, we have formulated five terms initially to be summed for the total force between the two spheres. These terms are all components projected along the line joining the spheres, so that we must take their algebraic sum. Each derivation provides a positive number for each component. However, the configuration of Fig. 30 is such that all terms of the ν -axis are pointing in the negative direction and hence they must enter with a negative sign in the sum for the total S_F :

$$S_F = S_{uj} + S_{us} - S_{\nu j} - S_{\nu s-c} - S_{\nu s-s} \quad (262)$$

As already mentioned, the sum of all “single” terms vanishes, because $S_{us} = S_{\nu s-c} + S_{\nu s-s}$, leaving only the joint components:

$$S_F = S_{uj} - S_{\nu j} \quad (263)$$

The latter is to be finally multiplied by J_o/c to yield the force.

C.4 Alternative joint ν -axis component along with joint u -axis (combination) (*variable_{uvj}*)

Similar to the alternative derivation by Eq. B.1, it is possible also to account for the joint component arising from the ν -axis. This is facilitated first because all “single” components of force contribute a null effect. Therefore, if only the “joint” components are important for the force derivation, then we need to find the ν -axis joint component in the same range of limits of integration along (concurrently) with u -axis joint component. With this approach, the equations for the lengths A_1B_1 , OA_2 and OB_2 are all the same in both cases. To account for the ν -component while revolving around the u -axis, we use the factors $\cos \theta \sin^2 \varphi_1$ (corresponding to $\cos \theta_z \sin^2 \varphi_z$ in Eq. 234) in the first double integral followed by the same steps, namely:

$$f_{uvj}(\theta, \varphi_1, u, \varphi_2) = [\exp(-|k_2 OA_2|) - \exp(-k_1 A_1 B_1 - k_2 OB_2)] \cos \theta \sin^2 \varphi_1 \quad (264)$$

$$F_{uvj}(u, \varphi_2) = \int_0^{\text{asin}(a_1(u))} \int_{-\pi}^{\pi} f_{uvj}(\theta, \varphi_1, u, \varphi_2) d\theta d\varphi_1 \quad (265)$$

then we form the third integral in the usual way by

$$d^2 S_{uvj}(u, \varphi_2) = F_{uvj}(u, \varphi_2) u^2 k_2 du d\Omega_2 \quad (266)$$

$$dS_{uvj}(\varphi_2) = \left[\int_{u_1(\varphi_2)}^{u_2(\varphi_2)} F_{uvj}(u, \varphi_2) u^2 k_2 du \right] d\Omega_2 \quad (267)$$

Since $d\Omega_2 = 2\pi \sin \varphi_2$ and projecting on P_1P_2 , however, now by $\sin \varphi_2$, we finally integrate over the entire bulk of sphere 2 by

$$S_{uvj} = \int_0^{\text{asin}(a_2)} dS_{uvj}(\varphi_2) \cdot 2\pi \sin^2 \varphi_2 d\varphi_2 \quad (268)$$

Thus, we can write as an alternative formulation of the total force factor:

$$S_F = S_{uj} - S_{uvj} \quad (269)$$

which again multiplied by the pressure J_o/c yields the total force between the two spheres.

C.5 Quadruple integral

We have conducted numerical integration of all of the above formulations and have confirmed the expected equivalent results with all cases within the set integration tolerance. The starting formulations may be thought of as the “raw” constituting equations of PG theory. They can be further worked out. They are amenable to further theoretical analysis and processing, which can be done separately. For now, we can summarize with a general quadruple integration of all the above by taking advantage of the common integration limits when using the alternative ν -axis joint component as follows:

$$S_F = 2\pi k_2 \int_0^{\varphi_{20}} \left\{ \sin \varphi_2 \cos \varphi_2 \int_{u_1}^{u_2} \left[\left(\int_0^{\varphi_{10}} \int_{-\pi}^{\pi} f_u() d\theta d\varphi_1 \right) u^2 \right] du - \sin^2 \varphi_2 \int_{u_1}^{u_2} \left[\left(\int_0^{\varphi_{10}} \int_{-\pi}^{\pi} f_v() dx dy \right) u^2 \right] du \right\} d\varphi_2 \quad (270)$$

where

$$f_u() = [\exp(-|k_2 O A_2|) - \exp(-k_1 A_1 B_1 - k_2 O B_2)] \sin \varphi_1 \cos \varphi_1 \quad (271)$$

$$f_v() = [\exp(-|k_2 O A_2|) - \exp(-k_1 A_1 B_1 - k_2 O B_2)] \cos \theta \sin^2 \varphi_1 \quad (272)$$

By re-writing we get

$$S_F = 2\pi k_2 \int_0^{\varphi_{20}} \int_{u_1}^{u_2} \int_0^{\varphi_{10}} \int_{-\pi}^{\pi} [f_u() \sin \varphi_2 \cos \varphi_2 - f_v() \sin^2 \varphi_2] u^2 \cdot d\theta d\varphi_1 du d\varphi_2 \quad (273)$$

or

$$S_F = 2\pi k_2 \int_0^{\varphi_{20}} \int_{u_1}^{u_2} \int_0^{\varphi_{10}} \int_{-\pi}^{\pi} f() [\sin \varphi_1 \cos \varphi_1 \sin \varphi_2 \cos \varphi_2 - \cos \theta \sin^2 \varphi_1 \sin^2 \varphi_2] u^2 \cdot d\theta d\varphi_1 du d\varphi_2 \quad (274)$$

where

$$f() = \exp(-|k_2 O A_2|) - \exp(-k_1 A_1 B_1 - k_2 O B_2) \quad (275)$$

with integration limits and chord lengths or segments as provided during the preceding detailed derivations.

We have also performed a numerical integration of the above and found consistency with all other other part term computations. With every computation, we have normalized by the factor $(\pi^2 A_1 A_2)/r^2$ where

$$A_1 = \left[R_1^2 - \frac{1}{2k_1^2} + \frac{\exp(-2k_1 R_1)(2k_1 R_1 + 1)}{2k_1^2} \right] \quad (276)$$

$$A_2 = \left[R_2^2 - \frac{1}{2k_2^2} + \frac{\exp(-2k_2 R_2)(2k_2 R_2 + 1)}{2k_2^2} \right] \quad (277)$$

as used with the original “reverse engineering” derivation by Eq. 85. The normalization has invariably resulted in unity within the prescribed tolerance of the integrals. Having said that, the original (“raw”) derivations of the various terms of absorption (like “single” and “joint”) are also needed to study the physics and underlying mechanisms of force and mass or energy not otherwise directly obvious from the end Eq. 85. This is done in the theory of the main body of this report in Section 16.

D Effective Mass or Energy for one and two spherical masses - bulk method

In this section, we formulate the problem of finding the graviton absorption rate by one (single) or two interacting spheres. Since we have also established that we can deduce the corresponding effective mass or energy from the graviton absorption rate per Sections 15.7 and 16, we can compare the expressions of mass or energy with the expressions of force.

D.1 General case for two spheres

The mass or energy, being a scalar, can be derived from summing (integrating) the absorption rate of gravitons by the elementary volume around points O inside the bulk of the sphere. This is done first by summing, instead of subtracting, the two terms of Eq. 249 corresponding to the absorption lengths along the direction u' on either side of point O, so that the absorption factor f_{a-} is:

$$f_{a-uj}(\theta, \varphi_1, u, \varphi_2) = [\exp(-|k_2OA_2|) + \exp(-k_1A_1B_1 - k_2OB_2)] \sin \varphi_1 \quad (278)$$

without the factor $\cos \varphi_1$, which was necessary to obtain the projection of graviton flow along the direction u . We conduct the double integration by setting:

$$M_{uj}(u, \varphi_2) = \int_0^{\text{asin}(a_1(u))} \int_{-\pi}^{\pi} f_{a-uj}(\theta, \varphi_1, u, \varphi_2) d\theta d\varphi_1 \quad (279)$$

Then, we come to the third integral and account for the gravitons absorbed by the elementary material slice facing in the direction u . The gravitons absorbed by the elementary thickness $du = OU$ (see inset in Fig. 30) is actually proportional to the elementary length $OU' = du/\cos \varphi_1$, which must also be multiplied by the same $\cos \varphi_1$ to account for the cosine law reduction (oblique incidence) of the arriving gravitons; the net result is that the absorption is proportional to the elementary thickness du times k_2 times the elementary area $u^2 d\Omega_2$, as with the force:

$$d^2 S_{a-uj}(u, \varphi_2) = M_{uj}(u, \varphi_2) u^2 k_2 du d\Omega_2 \quad (280)$$

from which we correspondingly obtain the absorption along the length of chord A_2B_2

$$dS_{a-uj}(\varphi_2) = \left[\int_{u_1(\varphi_2)}^{u_2(\varphi_2)} M_{uj}(u, \varphi_2) u^2 k_2 du \right] d\Omega_2 \quad (281)$$

Now, we again have $d\Omega_2 = 2\pi \sin \varphi_2$, but there is no reason to project the scalar quantity of absorbed gravitons on P_1P_2 to obtain the last integration over the entire bulk of sphere_2, so that we finally obtain

$$S_{a-uj} = \int_0^{\text{asin}(a_2)} dS_{a-uj}(\varphi_2) \cdot 2\pi \sin \varphi_2 d\varphi_2 \quad (282)$$

We must further add the absorption by the “single” sphere term in the remaining zenith angle from $\text{asin}(a_1(u))$ to $\pi/2$. Here, the absorption factor f_{a-} is a simpler expression:

$$f_{a-us}(x, y, u, \varphi_2) = [\exp(-|k_2OA_2|) + \exp(-k_2OB_2)] \sin \varphi_1 \quad (283)$$

as the gravitons trace only the chord A_2B_2 of sphere_2, and

$$M_{us}(u, \varphi_2) = \int_{\text{asin}(a_1(u))}^{\pi/2} \int_{-\pi}^{\pi} f_{a-us}(\theta, \varphi_1, u, \varphi_2) d\theta d\varphi_1 \quad (284)$$

We do the third and fourth integral in the same way by replacing “j” with “s”, so that finally we have

$$S_{a-us} = \int_0^{\text{asin}(a_2)} dS_{a-us}(\varphi_2) \cdot 2\pi \sin \varphi_2 d\varphi_2 \quad (285)$$

The total graviton absorption rate is then the sum of the above two terms:

$$S_a = S_{a-uj} + S_{a-us} \quad (286)$$

The latter is again a characteristic area, the product of which with J_0 , i.e. $S_a J_0$, yields the total graviton rate of absorption by sphere_2.

D.2 Single sphere

We have used and confirmed the following two alternative bulk method formulations for graviton absorption rate by a single sphere over and above the method already provided in Section 15.7.

D.2.1 Single sphere general

For a single sphere, say sphere_2, we can apply the preceding Eq. 285 by setting $\text{asin}(a_1(u)) = 0$, i.e. by initially integrating over the entire zenith angle ($0 < \varphi_1 < \pi/2$), i.e. by vanishing sphere_1 with $R_1 = 0$ at any arbitrary distance r :

$$M_{us-full}(u, \varphi_2) = \int_0^{\pi/2} \int_{-\pi}^{\pi} f_{a-us}(\theta, \varphi_1, u, \varphi_2) d\theta d\varphi_1 \quad (287)$$

Then, we follow with the same third and fourth integrals. The final integral is similar to Eq. 285 but for the full zenith angle (in the above integral):

$$S_{a-us-full} = \int_0^{\text{asin}(a_2)} dS_{a-us}(\varphi_2) \cdot 2\pi \sin \varphi_2 d\varphi_2 \quad (288)$$

D.2.2 Single sphere bulk alternative

With reference to Fig. 7 used to find the internal acceleration, we can derive a simple formulation for the graviton absorption rate as follows:

We now add the scalar terms of absorption to obtain the absorption factor at point O:

$$f_{aO} = 2\pi \sin \varphi d\varphi \cdot \left[\exp \left(-k\sqrt{R^2 - (R_O \sin \varphi)^2} + kR_O \cos \varphi \right) + \exp \left(-k\sqrt{R^2 - (R_O \sin \varphi)^2} - kR_O \cos \varphi \right) \right] \quad (289)$$

We integrate to find the absorption from all the traces radiating out from this point:

$$M_{aO} = \int_0^{\pi/2} 2\pi \sin \varphi d\varphi \cdot \left[\exp \left(-k\sqrt{R^2 - (R_O \sin \varphi)^2} + kR_O \cos \varphi \right) + \exp \left(-k\sqrt{R^2 - (R_O \sin \varphi)^2} - kR_O \cos \varphi \right) \right] \quad (290)$$

This is the same for all points on the internal spherical surface with radius R_O , so that the spherical shell with thickness dR_O absorbs $M_{aO} \cdot 4\pi R_O^2 \cdot kdR_O$, from which we sum (integrate) for the total bulk of the sphere by:

$$S_a = \int_0^R M_{aO} \cdot 4\pi k R_O^2 dR_O \quad (291)$$

D.3 Net loss of absorption rate between two spheres - bulk method

There is an important quantity arising by the difference of the absorption rate of one sphere in the presence of another from the absorption rate it has, when it is alone (without the absence of other bodies in the neighborhood). In this difference, there is a common term $f_{a-us}(x, y, u, \varphi_2)$ in the interval from $\text{asin}(a_1(u))$ to $\pi/2$ of the zenith angle, leaving only the difference between the “single” and “joint” terms in the common zenith interval from 0 to $\text{asin}(a_1(u))$:

$$f_{a-netloss}() = f_{a-us}() - f_{a-uj}() = [\exp(-|k_2OA_2|) - \exp(-k_1A_1B_1 - k_2OB_2)] \sin \varphi_1 \quad (292)$$

which is identical to the force term in Eq. 249 except for the $\cos \varphi_1$ that was necessitated to project the graviton flow on the axis joining the centers of the two spheres. We use the term “net” loss to distinguish it from the generally present (current) steady-state absorption in any or both spheres. The net loss is not “current”, i.e. it represents a loss of gravitons that is not present absorption in any of the spheres, but which was present prior to the interaction between the two spheres and now has gone “missing”. We will return to this net loss and the common mathematical factor again, when we consider the “surface” derivations of graviton absorption.

E Effective Mass or Energy for one and two spherical masses - surface method

The “bulk” method for the force and effective mass appeared to be the logical or standard way to start with. It provides good insight on the various fractions of graviton absorption and their possible inter-relationships, whilst it forms a basis for further elaboration. However, it presents a practical disadvantage by the long computation times, if we opt to follow this method. Fortunately, there is also an alternative method by expanding the approach used in Section 15.7 for a single sphere. We have thus developed the “surface” method with one less integration and much faster computation time, but subject to considerable complexity in defining the integration ranges as we move around the surface of one sphere relative to the other. We explain this method in detail below.

E.1 General case for two spheres

With reference to Fig. 31, we have sphere_1 and sphere_2, for which we designate certain variables and relationships needed for the intended formulations. The three tangents from the sphere centers, namely, P_2D and P_2W and P_1L together with the two mutual tangents IS and KT cross the surface of sphere_2 at points G , S , L , T , E and F signaling a transition of the angle $\omega = \angle qP_2z$ from $0 \rightarrow \omega_G \rightarrow \omega_S \rightarrow \omega_L \rightarrow \omega_T \rightarrow \omega_E \rightarrow \pi$. Concomitant with these lines and points, we need the following lengths and relationships:

$$\begin{aligned} r &= P_1P_2, & r_1 &= P_1N, & r_2 &= P_2N, & r_3 &= P_2M \\ R_1 &= P_1K, & R_2 &= P_2T, & \frac{R_2}{r_2} &= \frac{R_1}{r_1} \\ r_1 + r_2 &= r, & r_1 &= \frac{R_1}{R_1 + R_2}r, & r_2 &= \frac{R_2}{R_1 + R_2}r, & r_3 &= \frac{R_2}{R_2 - R_1}r \\ \omega_G &= \angle P_1P_2D = \text{asin} \varphi_{01} \\ \omega_S &= \text{acos} \left(\frac{R_2}{r_2} \right) = \text{acos} \left(\frac{R_1 + R_2}{r} \right) \\ \omega_L &= \text{acos} \left(\frac{R_2}{r} \right) \\ \omega_T &= \text{acos} \left(\frac{R_2}{r_3} \right) = \text{acos} \left(\frac{R_2 - R_1}{r} \right) \end{aligned}$$

We consider all points O on the surface of this sphere and all the traces of gravitons through this point crossing this sphere alone or both spheres. The line from the center of sphere_2 to point O on the surface constitutes the axis z of revolution, around which we calculate the first two integrals with respect to azimuth angle θ and zenith angle $\varphi_z = \angle zOB$. For this moving coordinate system with origin O , we further require the distance d of center P_1 and angle φ of OP_1 with axis z , which are easily found to be:

$$\begin{aligned} d &= OP_1 = \sqrt{r^2 + R_2^2 - 2rR_2 \cos \omega}. \\ \varphi &= \angle P_1Oz = \omega + \text{asin} \left(\frac{R_2}{d} \sin \omega \right). \end{aligned}$$

We further require the usual limiting angles subtended by each sphere as:

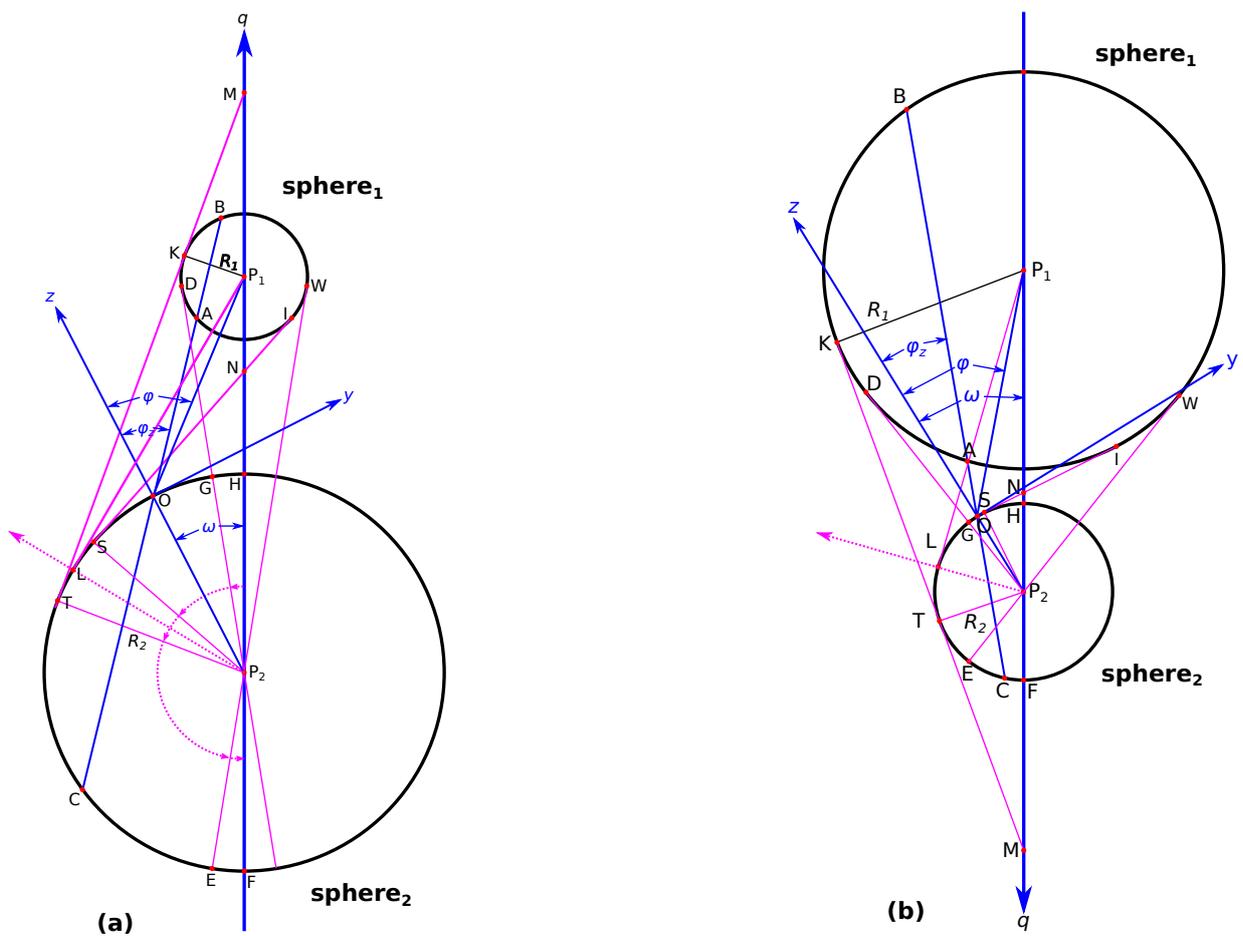


Figure 31: Alternative energy absorption rate formulation

$$a = \frac{R_1}{d}, \quad a_{01} = \frac{R_1}{r} = \sin \varphi_{01}, \quad a_{02} = \frac{R_2}{r} = \sin \varphi_{02}$$

Now, with such a configuration, there is always (at all zenith angles) a chord OC of sphere_2 with every trace through this point (even with one vanishing length), whilst only within the subtended solid angle by sphere_1 there are traces crossing sphere_1 along the chords AB. The length of these corresponding chords are:

$OC = 2R_2 |\cos \varphi_z|$, which always enters as a positive number in the exponential factor of absorption and

$$(AB)_z = 2\sqrt{(y_0 \sin \varphi_z \cos \theta_z + z_0 \cos \varphi_z)^2 - (d^2 - R_1^2)},$$

where $z_0 = d \cos \varphi$ and $y_0 = d \sin \varphi$, as we have found in Appendix B.

Next, we proceed to find the two fractions of absorption (“single” or “joint”) by this sphere by first establishing the integrand for absorption around axis P_2z . Since there is an axis of symmetry defined by the centers of the two spheres (P_1P_2), the situation must be the same for all points on the surface annulus around this axis at angle ω . Thus, we can find the absorption rate for the elementary surface annulus and then integrate with respect to ω in the interval $0 < \omega < \pi$. For the surface element at point O, we note that sphere_2 is always located entirely on one side of the plane of the element. However, sphere_1 changes location relative to the said surface element plane in ways that it can be (i) entirely on one side of the plane, or the other, i.e. on the same side as, or the alternate side from sphere_2, or (ii) the plane crosses sphere_1. In the latter case, sphere_1 can be crossed either above or below its “equator”, i.e. sphere_1 lies more than half above the horizon or more than half below the horizon of sphere_2, or (iii) the axis of rotation P_2z crosses sphere_1. The latter is also important to bear in mind, when we obtain the double integral of graviton absorption with respect to zenith and azimuth angles. During integration, we must either avoid doubling up the same absorption traces, or we must consistently do so and then divide by 2 (necessary to avoid dubious errors). Depending on how we decide to formulate the problem, we can finish up with 4, 5 or 6 absorption terms, the situation also depending on the relative size, configuration and distance of the spheres. For consistency and to avoid confusion, let us consider sphere_2 the one, for which we wish to find its absorption as affected by the other sphere_1.

It is helpful to clarify again that ω_G is the semi-angle subtended by sphere_1 with $\omega_G = \text{asin}(a_{01})$, when the z -axis becomes tangent to sphere_1; ω_S is at the point where the normal axis y (i.e. the horizon plane of sphere 2) first becomes tangent to sphere_1, ω_L is at the point where the horizon plane crosses the center of sphere_1, ω_T is at the point where the horizon plane becomes again tangent with sphere_1, ω_E is where the z -axis becomes again tangent with sphere_1 and $\omega_F = \pi$. This subdivision is necessitated in order to apply the rules and limits of integration for a single sphere (sphere_1) as described in Appendix B regarding the various positions of sphere_1 relative to the frame of reference with origin at O. However, sphere_2 always being on one side of (“below” or “above”) the tangent plane, the rule is straightforward with absorption chords OC always radiating from point O. The need to distinguish the six intervals of angle ω arises from the changing integration limits of the first double integral. Thus, we integrate separately in each of these intervals and then sum all the results. In summing all the latter integrals, we should note that each chord is traced twice (once from each of its ends), so that we must divide the final result by a factor 2. Alternatively, we can take care to include only those terms (four or five terms), which exhaust all chords once, namely, by avoiding to integrate over the zenith angle below the horizon of sphere 2; then we should not divide by the factor 2, whilst it saves us dealing with unnecessary terms and speeds up the computation work. In the latter case, again care should be taken for the “single” traces not to cross the other sphere either above or below the horizon.

Further, it is important to note that, while the integration ranges of ω are for the most of distance r as depicted by Fig. 31(a), the situation is different at close enough distance depending on the given set of radii R_1 and R_2 and their relative location with regard to which sphere is under examination. For example, in the first diagram 31(a), point G lies between H and S, whilst in the second diagram Fig. 31(b), point S lies between points G and H. This cross-over of points S and G occurs as we decrease the distance so that

$$\omega_G = \omega_S = \varphi_{01}. \text{ Then } \text{acos} \left(\frac{R_1 + R_2}{r} \right) = \text{asin} \left(\frac{R_1}{r} \right), \frac{R_1 + R_2}{r} = \sqrt{1 - \left(\frac{R_1}{r} \right)^2} \text{ and:}$$

$$r_S = \sqrt{(R_1 + R_2)^2 + R_1^2} \quad (293)$$

It is also possible sometimes to have a cross-over of the corresponding points T and E, when $\omega_T = \omega_E =$

$$\pi - \varphi_{01}. \text{ Then } \text{acos} \left(\frac{|R_1 - R_2|}{r} \right) = \text{asin} \frac{R_1}{r}, \frac{R_2 - R_1}{r} < \sqrt{1 - \left(\frac{R_1}{r} \right)^2} \text{ and:}$$

$$r_T = \sqrt{(R_1 - R_2)^2 + R_1^2} \quad (294)$$

No diagram is shown for the latter condition here, but it can be easily envisaged. The latter characteristic situation can arise only when one of the spheres is sufficiently smaller than the other, whilst the two spheres do not merge (overlap). In summary, care should be taken to establish when and if the distance r is in the ranges:

$$r \leq r_T \leq r \leq r_S \leq r \quad (295)$$

in order to correctly define the corresponding ranges of ω with correct integration limits of the double integral.

As with the bulk method, we consider the two fractions of absorbed gravions, i.e. (a) along lines jointly crossing both spheres and (b) along lines singly crossing the sphere under investigation (let's say sphere_2).

For the joint absorption, in all traces of gravions through point O in the figure, we have $1 - \exp(-k_2 OC)$ absorbed from the side (direction) of sphere_2 and $\exp(-k_1 AB) - \exp(-k_1 AB)\exp(-k_2 OC)$ from the side (direction) of sphere_1. Their sum yields the integrand as a function of the azimuth and zenith angle by

$$f_{a-zj}(\theta_z, \varphi_z, \omega) = (1 - \exp(-k_2 OC)) (1 + \exp(-k_1 AB)) \sin \varphi_z \cos \varphi_z \quad (296)$$

where the factor $\cos \varphi_z$ is again introduced to allow for the oblique incidence of gravions relative to the sphere surface element. However, it is not canceled out now as it was for the gravions absorbed by the elementary material slice $du = OU$ (see inset in Fig. 30), because we use the integrated absorption through the entire chord length OC already containing the same factor in its exponential (not like the elementary length $OU' = du/\cos \varphi_1$). The distinction is subtle, but underlies an important mechanism: Both involve an elementary surface attached to an elementary mass slice in the bulk method but to the entire massive chord in the surface method. This saves us from the integration along this chord, which is a significant advantage of the "surface" method. Since the above integrand is not rotationally symmetric around the variable axis z , we must integrate with respect to the interdependent azimuth and the zenith angle to obtain the intermediate factor

$$M_{a-zj}(\omega) = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} f_{a-zj}(\theta_z, \varphi_z, \omega) d\theta_z d\varphi_z \quad (297)$$

It is important to obtain the limits of integration such that all traces fall inside the subtended solid angle by sphere_1 (see Appendix B). The above result is rotationally symmetric about the line joining the centers of the spheres and hence we multiply times the elementary annular surface $2\pi \sin \omega R_2^2 d\omega$ and integrate within each of the above defined ω intervals between points H, G, S, L, T and E, the limits denoted by ω_1 and ω_2 in the final integration for each term:

$$S_{a-j} = \int_{\omega_1}^{\omega_2} M_{a-zj}(\omega) \cdot 2\pi \sin \omega R_2^2 d\omega \quad (298)$$

It should be noted that the absorption factor given by Eq. 296 remains the same in all intervals of ω , because the absorption along chord OC is the same from whichever end we look at (as can be easily verified).

For single absorption, we follow the same steps: The absorption in each direction of the trace through point O is $(1 - \exp(-k_2 OC))$, and the starting integrand is the simplest by

$$f_{a-zs}(\theta_z, \varphi_z, \omega) = 2(1 - \exp(-k_2 OC)) \sin \varphi_z \cos \varphi_z \quad (299)$$

so that the first double integral gives:

$$M_{a-zs}(\omega) = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} f_{a-zs}(\theta_z, \varphi_z, \omega) d\theta_z d\varphi_z \quad (300)$$

Here, the limits of integration are the supplementary angle of the limits used for the azimuth (i.e. $2\pi - \theta_{joint}$) in joint absorption and the complementary angles used for the zenith ($0 \rightarrow \varphi_1$) and/or ($\varphi \rightarrow \pi/2$) of the joint absorption (in one or two parts). Extra care should be taken that the gravion trace does not cross sphere_1 in either direction as we rotate the trace. The final integral is similar by:

$$S_{a-s} = \int_{\omega_1}^{\omega_2} M_{a-zs}(\omega) \cdot 2\pi \sin \omega R_2^2 d\omega \quad (301)$$

While the distance r between the two spheres is fixed and not explicitly seen in the integrands above, it is clear that the two absorption fractions are a function of distance, and we could also use the subscript r as we do in Section 16. The characteristic total surface factor S_a is the sum of the above:

$$S_a = S_{a-j} + S_{a-s} \quad (302)$$

The gravion absorption rate is the product $S_a J_0$

E.2 Net loss of absorption rate between two spheres - surface method

Like in Appendix D.3, we can derive the same net loss of absorption rate $f_{a-netloss}$ using the surface method. Again, we find the difference of the absorption rate of one sphere in the presence of another from the absorption rate it has when it is alone (in the absence of other bodies in the neighborhood). This is obtained from the difference of Eqs. 299 and 296 :

$$f_{a-netloss} = f_{a-zs}(\theta_z, \varphi_z, \omega) - f_{a-zj}(\theta_z, \varphi_z, \omega) = (1 - \exp(-k_2 OC)) (1 - \exp(-k_1 AB)) \sin \varphi_z \cos \varphi_z \quad (303)$$

in the common integration solid angle subtended by sphere_1, outside of which the terms of single absorption cancel out. Thus, we form the double integral:

$$M_{a-netloss}(\omega) = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} f_{a-netloss} d\theta_z d\varphi_z \quad (304)$$

The final integral yields a characteristic net-loss-surface-quantity $S_{a-netloss}$

$$S_{a-netloss} = \int_{\omega_1}^{\omega_2} M_{a-netloss}(\omega) \cdot 2\pi \sin \omega R_2^2 d\omega \quad (305)$$

We repeat the clarification as previously: We use the term “net” loss to distinguish it from the generally current (present) absorption in any or both spheres. This loss is not a “current” absorption loss, i.e. it represents a loss of gravions that is not present in any of the spheres at the given distance, but which was present prior to the interaction (at very long distance) between the two spheres but now gone “missing”.

F Force between two spherical masses - surface method

We can derive the total force exerted on sphere_2 by summing (integrating) all the forces exerted along all the chords traced by gravions in all possible directions through the point O on the surface. The “lone” or “single” crossings cancel out leaving only the “joint” ones. For any given chord OC, there is a force component from the direction that is free from (out of the way of) sphere_1 and caused by the absorption fraction $1 - \exp(-k_2 OC)$. The other component in the opposite direction arises from the arriving diminished intensity of absorption by a factor $\exp(-k_1 AB)$ after passing through sphere_1, while it traverses chord OC; The gravion absorption by this second beam of gravions is then $\exp(-k_1 AB) - \exp(-k_1 AB)\exp(-k_2 OC)$. The net force is the difference of the second from the first with a net force factor $(1 - \exp(-k_2 OC))(1 - \exp(-k_1 AB))$. To prepare the integrand for integration around axis z , we should multiply by the usual product of $\sin \varphi_z$ (to account for the elementary solid angle around the chord) and by $\cos \varphi_z$ to account for the per unit area oblique incidence with regard to absorption per se at that point. However, this is incomplete until we multiply again by $\cos \varphi_z$ to project that absorption flow on the axis z . Thus, the integrand for this component of force around this axis is given by:

$$f_{g-z}(\theta_z, \varphi_z, \omega) = (1 - \exp(-k_2 OC)) (1 - \exp(-k_1 AB)) \sin \varphi_z \cos^2 \varphi_z \quad (306)$$

with first double integration around this axis taken inside the solid angle subtended by sphere_1:

$$F_z(\omega) = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} f_{g-z}(\theta_z, \varphi_z, \omega) d\theta_z d\varphi_z \quad (307)$$

yielding the intermediate force factor per unit area on the surface at this point O. We multiply times the elementary annular surface at angle ω by $2\pi \sin \omega R_2^2$ as preciously, but we must also multiply by $\cos \omega$ to project this component of force on the line joining the centers of the two spheres. The above yields a characteristic surface area, which finally integrated is S_F :

$$S_{Fz} = \int_{\omega_1}^{\omega_2} F_z(\omega) \cdot 2\pi \sin \omega \cos \omega R_2^2 d\omega \quad (308)$$

Last, we must repeat the same steps for the normal component of force around y axis. It is easier (less complicated) to use the alternative method as per Appendix C.4. That is, we deduce the y component concurrently with the revolution used for integrating around z axis by using the same (common) limits of integration (see also Eq. 234). We again need to multiply Eq. 303 by the azimuth and zenith product $\cos \theta_z \sin \varphi_z$ as in:

$$f_{g-y}(\theta_z, \varphi_z, \omega) = (1 - \exp(-k_2 OC)) (1 - \exp(-k_1 AB)) \cos \theta_z \sin^2 \varphi_z \cos \varphi_z \quad (309)$$

and follow by the next two steps:

$$F_y(\omega) = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} f_{g-y}(\theta_z, \varphi_z, \omega) d\theta_z d\varphi_z \quad (310)$$

$$S_{Fy} = \int_{\omega_1}^{\omega_2} F_y(\omega) \cdot 2\pi \sin^2 \omega R_2^2 d\omega \quad (311)$$

noting that we used $\sin \omega$ to project this component on the line joining the centers, and the usual $2\pi \sin \omega$ for the elementary annular angle.

The algebraic sum $S_{Fz} + S_{Fy}$ yields a component of the total force for each interval $\omega_1 \rightarrow \omega_2$ prescribed for the angle ω , so that the final grand total S_F is the sum from all these intervals. Again, the total force acted upon sphere_2 by sphere_1 is given by the product $\frac{J_0}{c} S_F$. In the steady state considered by this report, this force is the same for both spheres (see also the symmetry of the force factor above).

Attention is drawn again (like with the bulk method) to the common factor presenting itself in the corresponding Eqs. 303, 306 and 309 between net loss and and net force, which are discussed in the main body of the report in Section 16.

Whilst further theoretical processing and analysis of all of the above derivations can be done separately, we have used them to obtain some immediate results with numerical integration with simple Python codes and a good laptop computer. Computation time can be a practical problem with the “bulk” method, but this depends on each case. The “surface” method is the fastest that has allowed us to accumulate a significant amount of results. Computation time has been greatly reduced also by parallel running of codes in a multi-core CPU computer, i.e. separate codes are executed concurrently for the various terms involved in the mass and force derivations. Key cases have been run with both “bulk” and “surface” formulations yielding identical results within the integration tolerances set, which has provided further reassurance that no errors are involved with the derivation of equations or the computer codes. It is intended to make the “personal” codes available in public separately, as soon as they can be edited with sufficient commentary and consistent symbolism matching the presented theory; this is necessary to make it readily understood and applicable by the general user. In the meantime, this task may also be better undertaken by expert computer programmers, who can to develop a dedicated integrator for the fundamental needs of PG. As a reference example, the computer specifications used at present are: x64based PC, MS Windows 7 Home Premium, Processor: Intel(R) Core(TM) 3612QM CPU @2.10 GHz up to 3.0 GHz with turbo boost, 4-Core, 8-Logical Processors, 8 GB RAM.

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