Improved Circular Model on Forecasting Arrivals from Western European Countries to Sri Lanka

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Received 15 August 2020;
Accepted 30 September 2020;
Available online 20 October
2020

Abstract: The Circular Model (CM) is a newly joined member to the family of Univariate Statistical Models. Development of the CM was based on; Newton's Law of Circular Motion, Fourier Transformation and Least Square Regression. The CM could be applied for either stationary or non-stationary series and is capable of capturing both seasonal and cyclical variations. However, the applicability of CM is restricted to trend free series, hence it was intended to overcome the limitation. The Improved Circular Model; named as, Sama Circular Model (SCM), tests on tourist arrival data from Western European countries to Sri Lanka. Forecasting ability of the SCM was compared with the Decomposition techniques and Seasonal Auto Regressive Integrated Moving Average. It is concluded that the SCM is capable in forecasting arrivals from Western European countries and superior to the other tested models.

Keywords: Circular model; circular motion; Fourier transformation

1. Introduction

Modeling wave patterns were an immense interest over the centuries, as wave like patterns is common in; Natural Sciences, Medical Sciences, Economic Sciences and many more. In general, waves are viewed in time domain analysis or frequency domain [1]. Time domain analyses waves with respect to time, whilst the frequency domain analyses a signal with respect to frequency. The time domain analysis is known as Time Series Analysis; the frequency domain analysis is known as the Spectral Analysis or Fourier Analysis. The Decomposition Technique; Auto Regressive Integrated Moving Average (ARIMA); Seasonal Auto Regressive Integrated Moving Average (SARIMA) are well known time series techniques used for analyzing wave patterns. The Circular Model (CM) analyzes a wave with respect to its spectrum. Most important properties of the CM is that, the CM could be applied for either stationary or non-stationary series. Further, the model is capable of capturing both seasonal and cyclical patterns of a time series.

1.1 Research Problem

Sri Lanka has been an attractive tourism destination for centuries. The tourism industry in Sri Lanka shows a rapid growth from year 2009; drawn the attention of researchers. Konarasinghe [2] has emphasized the importance of forecasting arrivals to Sri Lanka from various regions of the

world, and attempted to forecast arrivals from Western European region. Konarasinghe [3] has tested; Decomposition techniques and SARIMA, and found them successful for the purpose. Yet, the author was unable to differentiate the Seasonal Variations and Cyclical Variations of the series.

The Cyclical variations are long term wave like patterns, while the seasonal variations are short term wave like patterns. Seasonal patterns are observed within a year, but the cyclical patterns are observed in longer period; at least more than a year. In general, decomposition techniques are used to capture the cyclical patterns. In decomposition models; a time series is described as a function of four components; Trend (T), Cyclical influence (C), Seasonal influence (S) and the random error (e). In order to capture the cyclical pattern, Decomposition technique follows several steps; firstly, fit the trend model and then obtain the de-trend series; secondly, find the seasonal indices for de-trended data and de-seasonalize them; finally model the de-seasonalized series by trigonometric functions. However, this method is time consuming and cumbersome. In contrast, the CM is easy to use and less time consuming. Yet, the applicability of the CM is limited to trend free series [4].

1.2 Objectives of the Study

The Objectives of the study were twofold; primary and secondary.

Primary Objective: Improve the Circular Model, in order to forecast the series with trend.

Secondary objective: Testing the applicability of Improved Circular Model on forecasting arrivals from the Western European Region.

2. Materials and Methods

The study is based on the CM of Konarasinghe [5]. Development of the CM was based on the theory of Uniform Circular motion, Fourier Transformation and Least Square Regression. The Fourier transformation (FT) can be used to transform a real valued function $f(x)$ into a series of trigonometric functions [6]. The FT has two versions; discrete transformation and continuous transformation. The discrete version of Fourier transformation is:

$$f(x) = \sum_{-\infty}^{\infty} a_n e^{-kn\theta} \quad (1)$$

According to De Moivre's theorem,

$$e^{-k\theta} = \cos k\theta + i \sin k\theta \quad (2)$$

where, i is a complex number. Therefore, $f(x)$ can be written as:

$$f_x = \sum_{k=1}^n a_k \cos k\theta + b_k \sin k\theta \quad (3)$$

where a_k and b_k are amplitudes, k is the harmonic of oscillation.

A particle P , which is moving in a horizontal circle of centre O and radius a is given in Fig. 1, ω is the angular speed of the particle.

Angular speed is defined as the rate of change of the angle with respect to time. Then,

$$\omega = \frac{d\theta}{dt} \quad (4)$$

$$\int_0^{\theta} d\theta = \int_0^t \omega dt \quad (5)$$

Hence,

$$\theta = \omega t \quad (6)$$

Substitute Equation (6) in Equation (3),

Fig. 1 - Motion of a particle in a horizontal circle

$$f_x = \sum_{k=1}^n a_k \cos k\omega t + b_k \sin k\omega t \quad (7)$$

At one complete circle $\theta=2\pi$ radians. Therefore, the time taken for one complete circle (T) is given by:

$$T = 2\pi / \omega \quad (8)$$

In circular motion, the time taken for one complete circle is known as the period of oscillation. In other words, the period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with f peaks in N observations, its period of oscillation can be given as:

$$T = \frac{\text{total number of periods}}{\text{total number of peaks}} = \frac{N}{f} \quad (9)$$

Equating Equation (8) and Equation (9), we obtain

$$\frac{2\pi}{\omega} = \frac{N}{f} \quad (10)$$

However, Equation (3) is a deterministic model, does not capture the randomness in real life. Therefore, Equation (3) is modified for a random variable Y_t as follows:

$$Y_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (11)$$

The model in Equation (11) was named as the Circular Model.

Model Assumptions are:

Y_t is a continuous random variable

$t \geq 0$,

$k \in R^+$

Series, $\sin k\omega t$ and $\cos k\omega t$ are independent

ε is Normally distributed with 0 mean and constant variance

2.1 Improved Circular Model

The CM cannot be applied, if Y_t has a wave like pattern with the trend. This study suggests the method of differencing to mitigate the limitation of the CM. In usual notation, differencing series of Y_t are as follows:

First differenced series:

$$Y'_t = Y_t - Y_{t-1} = (1-B)Y_t \quad (12)$$

Second differenced series:

$$\begin{aligned} Y''_t &= Y'_t - Y'_{t-1} = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2}) \\ &= Y_t - 2Y_{t-1} + Y_{t-2} = (1-B)^2 Y_t \end{aligned} \quad (13)$$

Similarly, d^{th} order difference is:

$$Y_t^d = Y_t - Y_{t-d} = (1-B)^d Y_t \quad (14)$$

where, B is the Back Shift operator; $BY_t = Y_{t-1}$.

Assume Y_t^d is trend free. Let, $Y_t^d = X_t$. Then X_t could be modeled as:

$$X_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (15)$$

Hence,

$$Y_t - Y_{t-d} = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \quad (16)$$

$$Y_t = Y_{t-d} + \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t$$

The model in Equation (16); improved Circular Model, is named as the Sama Circular Model (SCM). The term "Sama" is taken from the first part of the name of the author; Samanthi.

2.2 Population and Sample of the Study

It is a known fact that the Western European countries contribute highly into the Sri Lankan tourism market. As such; monthly arrival data for three leading counties; UK, Germany and France were collected for the period of April 2008 to December 2016 from Sri Lanka Tourism Development Authority. Firstly, outliers of the series were tested with the help of Box Plots and adjusted. Time Series plots and Auto Correlation Functions were used for pattern recognition. Then the Sama Circular Model (SCM) was tested by using the statistical software, MATLAB. The Goodness of fit tests and measurements of errors was used in the model validation. The Auto Correlation Functions (ACF) of residuals and Ljung-Box Q statistics (LBQ) were used to test the independence of residuals. The Anderson Darling test was used to test the normality of residuals. Forecasting ability of the models was assessed by Mean Square Error (MSE) and Mean Absolute Deviation (MAD). Finally, the forecasting ability of SCM was compared with the forecasting ability of Decomposition Models and SARIMA models.

3. Results and Findings

At first, box plots of monthly arrival series were obtained and check whether outliers exist. If so, they were adjusted by taking the moving average of the previous three months. Time series plots were used for pattern recognition of the series. Fig. 2 shows the wave like patterns with increasing trend for all

Fig. 2 - Time series plot of arrivals

Fig. 3 - Time series plot of arrivals from the UK (Y_t)

Fig. 4 – The first difference of the same series (X_t) in Fig.3

countries. Log transformed arrivals were used in the analysis; the differencing technique was used to obtain the trend free series.

For example, Fig. 3 shows the arrivals from the UK (Y_t), while Fig. 4 shows the first difference of the same series (X_t).

Fig. 3 and Fig. 4 are irregular waves, but Fig. 3 shows an increasing trend, while the first difference series is trend free. Therefore, differenced series were tested on the CM, and found that the C was well fitted to the third difference series of arrivals from the UK. The fitted model is:

$$X_t = 0.0039967 + 0.12063 \sin \omega t \quad (17)$$

where $\omega = 1.08^\circ$. The Anderson Darling test confirmed the normality of residuals; the LBQ test confirmed the independence of residuals. Measurement of errors is satisfactorily small, the MSE in model fitting is 0.029, while 0.031 in model verification; the MAD in model fitting is 0.137, while 0.146 in model verification.

$$X_t = Y_t''' = (1-B)^3 Y_t$$

$$= (1 - 3B + 3B^2 - B^3) Y_t \quad (18)$$

$$= Y_t - 3Y_{t-1} + 3Y_{t-2} - Y_{t-3}$$

Hence, estimates of arrival from the UK can be obtained by the model:

$$Y_t = 3Y_{t-1} + 3Y_{t-2} - Y_{t-3} + 0.0039967 + 0.12063 \sin \omega t \quad (19)$$

Table 1 - Summary of fitting SCM

Country	Best Fitting Model
UK	$Y_t = 3Y_{t-1} + 3Y_{t-2} - Y_{t-3} + 0.12 \sin \omega t$
Germany	$Y_t = 2Y_{t-1} + Y_{t-2} - 0.06 \sin \omega t + 0.15 \cos \omega t$
France	$Y_t = 2Y_{t-1} + Y_{t-2} + 0.01 - 0.07 \sin \omega t + 0.17 \cos \omega t - 0.07 \cos 5 \omega t$

The same procedure was repeated with arrival series of; Germany, France and Netherlands; summary of the analysis is given in Table 1.

The model in Equation (19) comprises only one periodic function, $\sin \omega t$. The period of oscillation of $\sin \omega t$ is 6 months, hence the arrivals from the UK follows only seasonal pattern. The fitted model for German comprises two periodic functions; $\sin \omega t$ and $\cos \omega t$, with the period of oscillation 5 months and 6 months respectively. Therefore, arrivals from the Germany follow only seasonal patterns. The fitted model for France comprises three periodic functions, but the period of oscillation of each function is less than 12 months. Therefore, arrivals from the France follow seasonal patterns, but not cyclical patterns

4. Conclusion

The CM is a univariate time series technique, which can be used to model the wave like patterns. The literature reveals that the CM can be applied only for trend free series. Hence the present study was focused on improving the CM and testing the applicability of improved model in real life data sets. The CM was improved by adopting the differencing technique; the improved model is named as the Sama Circular Model (SCM). The SCM was tested on tourism arrivals to Sri Lanka from the Western European countries, and found that the SCM fits into tested data sets. Then the forecasting ability of the SCM was compared with SARIMA and Decomposition models and found that the SCM is superior to the other two methods in forecasting arrivals from Western European countries to Sri Lanka.

The study concludes that the SCM mitigates the restriction of the CM; can be applied for a series with trend. It is recommended to test the SCM for more real life data sets in different fields of research.

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